Chapter

Estimation of the Efficiency Indices for Operating the Vertical Transportation Systems

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Abstract

Various lifts' systems with different control rules are considered. It is suggested to use the efficiency indexes: customer's average waiting in lift cabin time and average total time, including the time of delivering the customer to the desired floor. Various control rules are introduced: *Odd-Even*, where one lift serves only customers in *Odd* floors and other lift only does that in *Even* floors *Up-Down* control rule where one lift serves only customers who are going from the first floor to the destination floor $2, 3, \ldots, k;$ another lift serves customers from the first floor to the upper floor $k + 1, k + 2, \ldots, n$. The results of simulation, allowing to compare various control rules relatively to the efficiency indexes, are given. It is introduced an optimal number of lifts, which minimizes number of lifts, minimizing a customer's average waiting time. For some systems, the method of finding the optimal number of lifts, is suggested. Necessary figures demonstrating the operation of the lifts' systems and the results of the simulation allow to estimate the efficiency indexes.

Keywords: simulation of various lifts' systems, *odd-even*, *up-down*, situation of control rules, customer's average waiting time and total service time

1. Introduction

The world economy suffered a lot of losses after the coronavirus pandemic and it will take a long time for its rehabilitation. An important role in the development of the world economy will have the transportation and communication systems because it is necessary to renovate the economic communications among countries. For the investigation of the transportation and communication systems, mathematical models of queuing systems with moving servers are widely used. Typical examples of queues with moving servers are the lifts' systems. Lifts and communication systems, the traffic, the airport and the shipping facilities have a lot of similarities. All of them are united by the same principle – servers are moving in these systems. Hence, mathematical models of lifts' systems can be applied for the investigation of other systems with moving servers. As investigation of such systems by analytical approaches faces troubles, the use of the modern computers can allow to simulate their behavior. The simulation of such systems, various systems of programming (*Wolfram Mathematica* and others) allow to get close to reality, the numerical data of the desired parameters and give some advice for applications. The simulation also can also be a hint for the continuation of the analytical research.

Today it is difficult to imagine the development of modern cities, such as New York, Moscow, Shanghai, Istanbul and others, without skyscrapers. The process of designing skyscrapers needs an effective planning and operating of the lifts' systems, which allows to improve various characteristics (customer's waiting and service times) and to reduce energy expenses. An important problem is also to introduce various control rules for the lifts' systems. At a first view, it seems that the lifts' systems have simple structures. In fact, from a scientific point of view, these systems have complicated structures and the construction of mathematical models needs some non-standard approaches. Moreover, although these problems are formulated in the frame of standard queues models with moving servers, for their investigations it is necessary to develop new methods and approaches, using different fields of mathematics. The lifts' systems, the traffic problems, the airport and the shipping facilities have a lot of similarities. The investigation of these systems can allow to estimate the main operating parameters (customers' waiting and service times, energy expenses and others) and to make the necessary recommendations for constructors and engineers. There are many publications in this field e.g. [1–3] and even special scientific journals are published *(International J. Transportation Science and Technology, Research in Transportation Business & Management* and many others*)*. Unfortunately, the complicated lifts' systems, with various control rules, are not yet investigated widely. In [4–7], various mathematical models of lifts' systems with, different control rules, were introduced. The construction of mathematical models of lifts' systems and their research by analytical approaches, face some difficulties, because as it was mentioned above, these models have complicated structures. Hence, one of the effective methods are the simulation and the collection of simulated data, which can be used for estimating the various parameters of such lift systems, by comparing different control rules, finding optimal regimes for their operation. As the customers' arrival process into the lifts' systems has a stochastic structure, hence it leads to constructing and investigating the new stochastic models, approaches and programs for their simulation.

In this paper, the authors consider various lift systems with different parameters and different control rules. This paper can be regarded as a continuation of the authors' investigations presented in [4, 5]. Hence, we follow the notations introduced in these papers.

2. Control policies for the lifts' systems

There are many various control rules for the lifts' systems. We will consider only some of them, for instance, the *Odd-Even* system, where some lifts serve customers at the odd floors and other lifts, at the even floors. Another control rule, we call it following to [4], the *Up-Down* system, where some lifts serve customers going from the first floor to the Down floors $1, 2, \dots, k,$ others serve customers going from the first floor to the Upper floors$1, k+1, k+2, \dots, n$. This control was introduced in [5]. For some systems by simulation, the numerical values of optimal $k_{opt.}$, which minimize the value of *CWT,* was found. All these control rules can improve the service, i.e. to reduce the customer's waiting and service times and also diminish energy expenses. Some methods of investigation of queues with a finite service capacity can be used for the research of the lifts' systems [8].

An interesting unofficial control policy was created in the seventy years of the XXth century, by the students in the dormitory of the Moscow Lomonosov State University. There are 18 floors in the student dormitory and two lifts' halls with four lifts in each. The first lift hall operates from the *1st to the 12th, 14* th, *16th* *and 18th* floors. In order that the lifts work more rapidly, it was skipped the odd numbered floors, after the 12th. There is also a second lift hall for serving on the 1st-10th floors. If in the first hall, a lift came to the first floor and the first student yelled the word *"Higher",* then, the lift would be filled by students who are going up only to the higher floors *(16th and 18th)* and the next lift will be filled by students who are going to the *12th, 14th, 16th* floors and upper. If the first call had been *"LOWER",* then the lift would have operated between the lower floors (*12th, 14th* and afterward, to the other upper floors). The students called it a *Higher-Lower* system.

In [7], it was introduced the so called "situation control rule" for systems with two lifts. If both lifts are going from up to down, then all arrived customers (at the different floors) will be distributed between lifts. This control rule allows to exclude stopping both lifts almost at the same time, at the same floors. Such systems work effectively for high intensity of customers' flows. For instance, if both lifts are going from up to down, then each lift system defines the floors where the lift must stop and serve the customers. In the case of a customer's arrival at the new floor system, it must be recalculated the number of the floors where the lift must stop. Such a control rule allows using lifts capabilities in a uniform way. Although the "situation control rule" needs some additional software and technical equipment, nevertheless it improves the service (reducing customer's waiting and service times), it saves energy expenses and increase the lifetime of the lifts.

3. The mathematical models of the lift systems

For constructing the mathematical models of the lifts' systems, we use conceptions and parameters introduced in [4, 5]. The followings notations are introduced:

n – is the number of the floors in the building;

k –is the number of the lifts in the building;

$L_kF_nC_{xx}$ – is the systems with k lifts, n floors and control policy xx;

i - is an ordered in time identifying number of a customer during simulation;

$f_a\ (i)$ - is the floor of appearance of the *i-th* customer;

$f_d(i)$- is the floor of destination of the *i-th* customer.

It is necessary to note that for some different i the $f_a\ (i)$ and $f_d(i)$ can take the same value.

$t_a\ (i)$ - is the instant of appearance of the *i-th* customer;

$t_b\ (i)$ - is the instant of the beginning service of the *i-th* customer in lift cabin;

$t_e(i)$ - is the instant of end service of the *i-th* customer;

$t_{c(j)}$- is the instant when lift on *j-th* cycle is returning to the *1-st* floor;

n – number of the floors in the building;

r – roominess, restriction of maximum possible number of customers, who can be in the lift cabin;

h_f - time necessary for the lift to move up or down, between two neighboring floors;

h_d– time which is spent for opening and closing the floor's door;

Usually, in practice, approximately $h_d = 2h_f$. If we consider the stationary input flow, then, the following parameters are used:

λ_{f1f2} –is the intensity of customers' flow, which appears at the f_1*-th* floor and want to go to f_2*-th* floor;

$\lambda_1 = \sum_{k=2}^{n} \lambda_{1k}$ - is the intensity of customers' flow, which appears at the first floor and are going to upper floors;

$\lambda_2 = \sum_{k=2}^{n} \lambda_{k1}$ – is the intensity of customers' flow, which appears on the upper $\{2, 3, \dots, n_f\}$ floors, who want to go down to the first floor;

CWT(S) – a customer's average Waiting Time in the system *S*, i.e. the mean time from the instant when a customer arrives at the system and waits until the instant when he gets the lift;

CST(S) – a customer's average Service Time in the system *S*, i.e. the mean time from the instant when the customer gets in the lift, until the instant when he gets off the lift;

CTT(S) = CWT(S) + CST(S) – a customer's average Total Time in the system *S,* which is measured as a mean time from the instant when the customer arrives into the system until he gets off the lift (arrival to ordered floor).

For instance, $CTT(L_kF_nC_{xx})$ is a customer's average total time, for a system in a building with *k* lifts, *n* floors and control policy *xx.*

IL–independent lifts' system. It means that all the lifts are operating independently from each other, i.e. if at the preceding instant of a new customer's arrival, several lifts are free (empty), then, all of them will go to this customer's call. Such systems are often used in the buildings with two lifts.

DL – dependent lifts' system (for a customer' call, the nearest lifts going to him);

UD(k)- where one lift serves only customers who are going from the first floor to *2, 3, … , k;* and another one serves customers who are going from the first floor to upper *k + 1, k + 2, … , n;* In such systems, when an *Up* lift is going from j_1*-th* floor $(j_1 > k)$ to down, it can take customers from j_2*-th* floor $k < j_2 < j_1$, if there is an empty space in the cabin. Otherwise, the lift is directly going to the first floor. Similarly, when the *Do(wn)* lift is going from j_3*-th* floor $(j_3 < k)$ to the first floor, it can collect customers from j_4 *-th* floor $j_4 < j_3$, if there is empty space in the cabin.

$T^U\ (L_2F_nC_{UD(k)})$ - cycle time of the *Up* lift in the system $L_2F_nC_{UD(k)}$;

$T^D(L_2F_nC_{UD(k)})$- cycle time of the *Do* lift in the system $L_2F_nC_{UD(k)}$;

SC – situation control - there is some (robot) software, which depends on new customers' arrivals, gives commands to the lifts where to stop and which floors to pass by. The appearance of a customer at the new floors can change the system of commands;

LRC –Average *Lift Return Cycle* time, i.e. the average time interval between two comings of the lift to at the first floor.

We also introduce the new parameters for the lifts' systems, which describe the *lift energy expenses* and the *single race time*:

LEE_j *(S)* – Average value of the *j-th Lift Energy Expenses* in the system *S,* measured in *Kw* (kilowatt);

Note that Energy Expenses in *Kw* depend not only on the volume and weight of the cabin, but also on its speed, acceleration and deceleration. Empirically, electric Energy Expenses can be shown each day, on the electric counter of each lift.

SRT(t) – Average *Single Rate Time*, i.e. average time when the lift is moving without customers, during time *t*;

SEE(S) – average value of *System Energy Expenses*, i.e. average value of energy expenses of all the lifts in the system *(S).*

$SEE(S) = LEE_1(S) + LEE_2(S) + \ \dots\ + LEE_n(S);$

k_d – coefficient defining the lifts' energy expenses, during a unit time, for opening and closing the doors;

k_f – coefficient defining the lifts' energy expenses, during a unit time, for covering the distance between two neighboring floors.

There are different regimes of operating the lifts' systems.

Loading regimes, where customers from the first floor are going to upper floors. Such regimes are observed in the office buildings, in the morning (08.00–09.30) when customers are going to their offices. Similar regimes are observed in the residence buildings, in the evening (17.30–19.00), when people come back home from their work.

Unloading regimes, in the office buildings, in the evening (17.00–18.00), customers stop working and go back by lifts, from their offices to the first floor.

There also exist **mixed regimes**, when customers from the first floor are going to the upper floors and vice versa. Moreover, there are customers who are going from j_1-*th* floor to the j_2-*th* ($j_1, j_2 = 2,3,..,n$). In this paper, only loading and unloading regimes will be considered. Some investigations of the mixed regimes can be found in [5].

In the unloading regimes, when lifts are going from the upper j_1-*th* floor to the first floor, the lifts can take customers from j_2-*th* floor ($j_2 < j_1$), if there is a free space in the cabin. If, at some floor, the number of customers in the cabin became r (roominess), then the lift would go directly to the first floor, without stopping. This policy is observed in all the regimes.

Remind that $L_2F_nC_{n1,n2}$ is the system with 2 lifts, n floors and after completing the customer's service, one lift (with empty cabin) must go to n_1-*th* floor, if there is no lift, otherwise, it should go to n_2-*th* floor. Below, in the **Figures 1** and **2**, axes x means current time;

- means that the lift is occupied;

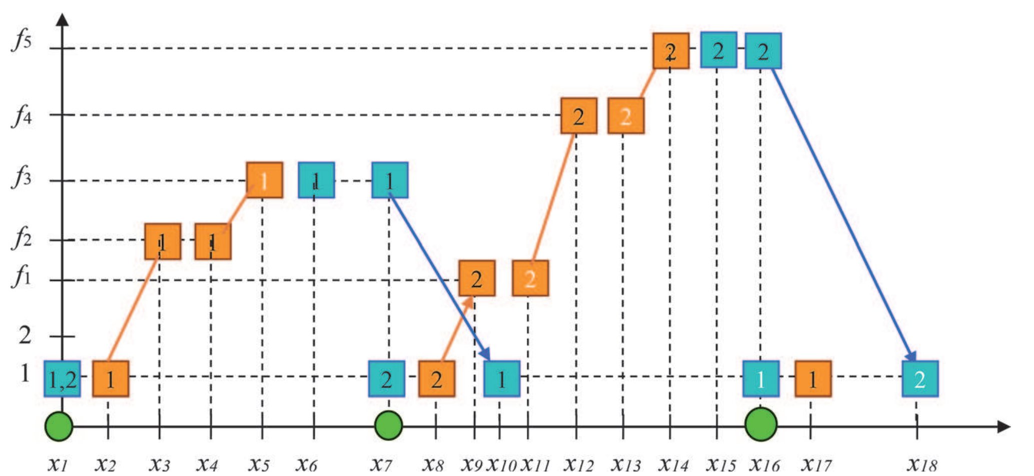

Figure 1.
Example of loading regime for $L_2F_nC_{IL}$.

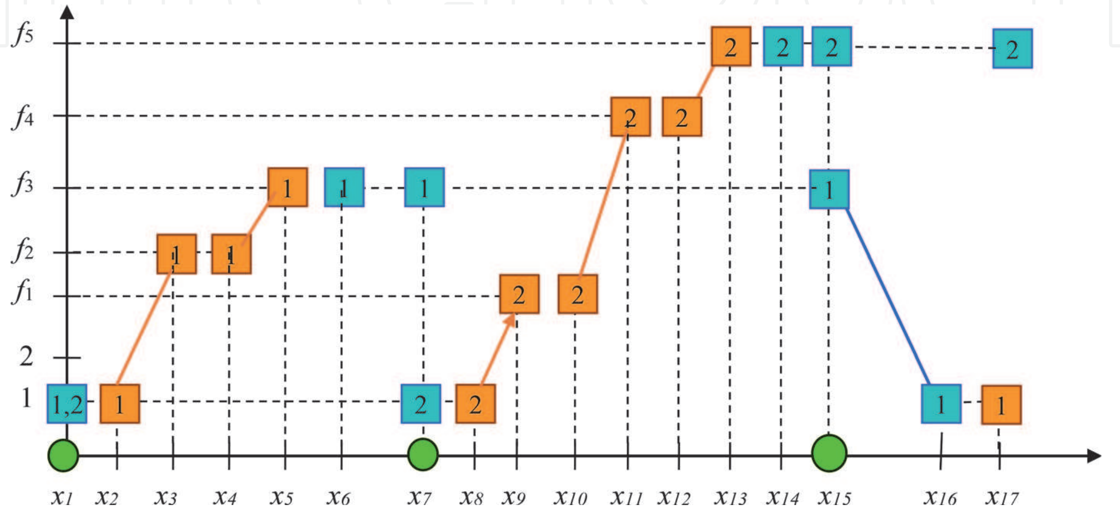

Figure 2.
Example of loading regime for $L_2F_nC_{DL}$.

- means that the lift is empty (free);
- means the instant of the customers' arrival instant.

Definition. The flow of customers is called *rare* for the lift system $L_kF_nC_{xx}$, if at the preceding instant of the customer's arrival, among the k lifts there is at least one (non-occupied) lift, which goes to the customer's call.

4. The systems $L_2F_nC_{IL}$ and $L_2F_nC_{DL}$ in the loading regimes

We will compare both systems $L_2F_nC_{IL}$ and $L_2F_nC_{DL}$ with rare flow of customers, in the loading regime, relatively to a customer's waiting time (CWT). In the **Figures 1** and **2**, axes x means current time and axes y, an ordinal number of the floor, where the lift delivers the customers.

Below, in the **Figure 1**, the lifts' positions at the preceding instants of the customer's arrival are presented (rare flow) (see, **Figure 1**). If the input flow is rare, then, for the system $L_2F_nC_{IL}$ in loading regime, at the preceding instant of a customer's arrival one lift is located at the first floor and another is located at j-*th* floor, where $j = 2,3,\dots, n.$ (see, **Figure 1**).

$x_1 = t_a(1) = t_a(2), x_2 = t_b(1) = t_b(2) = x_1 + h_d, x_3 = x_2 + (f_2–1)h_f, x_4 = t_e(1) = x_3 + h_d,$
$x_5 = x_4 + (f_3\text{-}f_2)h_f, x_6 = t_e(2) = x_5 + h_d, x_7 = t_a(3) = t_a(4) = t_a(5),$
$x8 = t_b(3) = t_b(4) = t_b(5) = x7 + h_d, x_9 = x_8 + (f_1–1)h_d, x_{10} = x_7 + (f_3–1)h_f,$
$x_{11} = t_e(3) = x_9 + h_d, x_{12} = x_{11} + (f_4\text{-}f_1)h_f, x_{13} = t_e(4) = x_{12} + h_d, x_{14} = x_{13} + (f_5\text{-}f_4)h_f,$
$x_{15} = t_e(5) = x_{14} + h_d, x_{16} = t_a(6), x_{17} = t_b(6) = x_{16} + h_d, x_{18} = x_{16} + (f_5–1)h_f.$

Consider the system $L_2F_nC_{DL}$ with rare input flow in loading regime. Then, at the preceding instants of a customer's arrival, both lifts occupy the floors $2,3,..,n.$ (see, **Figure 2**).

$x_1 = t_a(1) = t_a(2), x_2 = t_b(1) = t_b(2) = x_1 + h_d, x_3 = x_2 + (f_2–1)h_f, x_4 = t_e(1) = x_3 + h_d,$
$x_5 = x_4 + (f_3\text{-}f_2)h_f, x_6 = t_e(2) = x_5 + h_d, x_7 = t_a(3) = t_a(4) = t_a(5),$
$x_8 = t_b(3) = t_b(4) = t_b(5) = x_7 + h_d, x_9 = x_8 + (f_1–1)h_d, x_{10} = t_e(3) = x_9 + h_d,$
$x_{11} = x_{10} + (f_4\text{-}f_1)h_f,$
$x_{12} = t_e(4) = x_{11} + h_d, x_{13} = x_{12} + (f_5\text{-}f_4)h_f, x_{14} = t_e(5) = x_{13} + h_d, x_{15} = t_a(6),$
$x_{16} = x_{15} + (f_3–1)h_f,$
$x_{17} = t_b(6) = x_{16} + h_d.$

Thus, we have $CWT(L_2F_nC_{IL}) = h_d$ and $CWT(L_2F_nC_{DL}) = nh_f/6+ h_d(1),$
$CWT(L_2F_nC_{IL}) < CWT(L_2F_nC_{DL})$. If an intensity of input flow is increasing, then the difference $(CWT(L_2F_nC_{IL})\text{-} CWT(L_2F_nC_{DL}))$ is decreasing and goes to zero.

After some critical value of intensity λ^* this difference $(CWT(L_2F_nC_{IL})$ - $CWT(L_2F_nC_{DL}))$ is increasing until to some other value of intensity λ^{**}.

Afterward, it is again decreasing and goes to zero, for a high value of intensity. It is clear, that for a high intensity of the input flow, an operating of the systems $L_2F_nC_{IL}$ and $L_2F_nC_{DL}$ is becoming close to each other (see, **Figure 3**). In the **Figures 3–5**, axes x means intensity of the input flow and axes y means the value of the *CTT*. If roominess of the lift is bounded, then for a high intensity of the input flow it is not necessary to introduce any control, because both lifts stop at each floor and the system is operating like deterministic (at each floor there is always at least one customer).

Remark. For small values of intensity of the input flow, the system $L_2F_nC_{IL}$ is preferable than the system $L_2F_nC_{DL}$, i.e. $CTT(L_2F_nC_{IL}) < CTT(L_2F_nC_{DL})$. There exists some interval $(\lambda^*, \lambda^{**})$ of intensity which can be calculated by simulation),

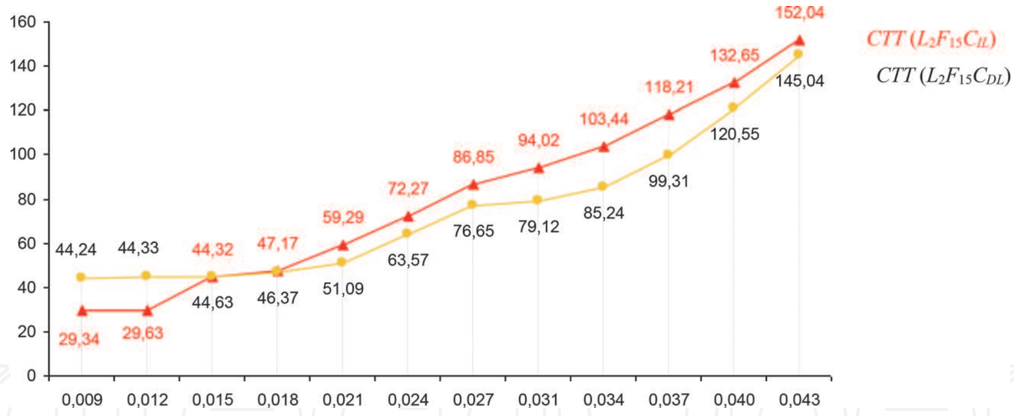

Figure 3.
Graphs of the CTT($L_2F_{15}C_{IL}$) *and* CTT($L_2F_{15}C_{DL}$).

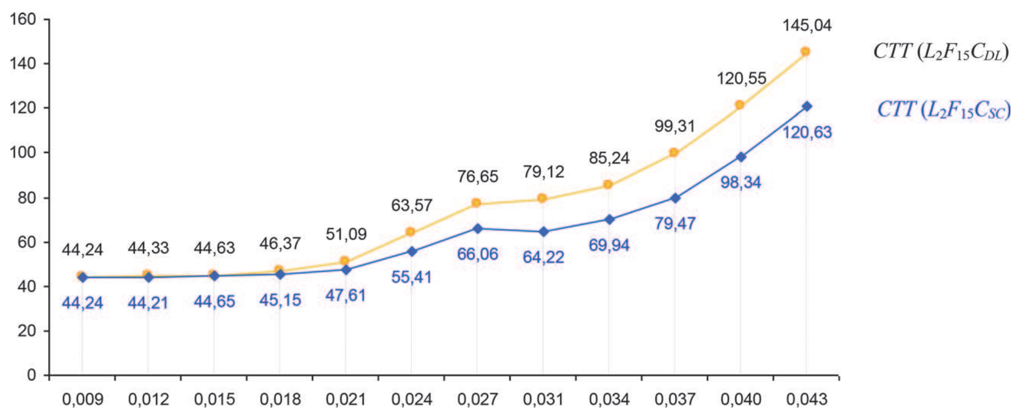

Figure 4.
The graphs of the CTT($L_2F_{15}C_{DL}$) *and* CTT($L_2F_{15}C_{SC}$).

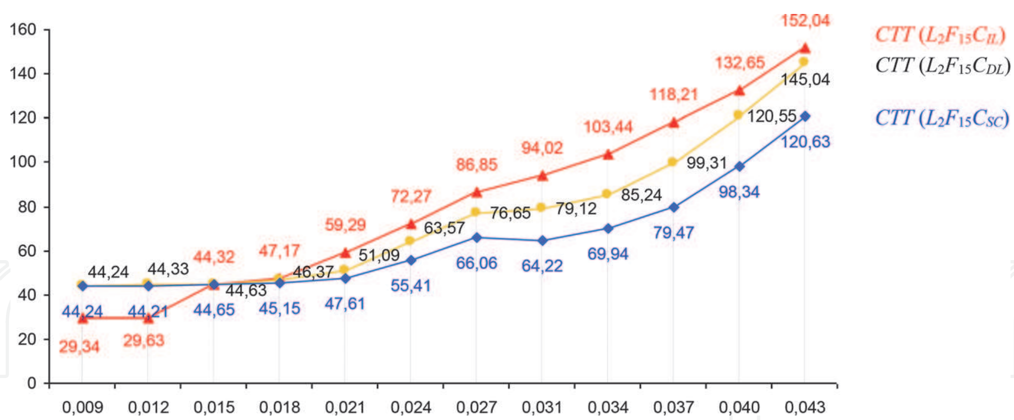

Figure 5.
The graphs of the CTT($L_2F_{15}C_{IL}$), CTT($L_2F_{15}C_{DL}$) *and* CTT($L_2F_{15}C_{SC}$).

where the system $L_2F_nC_{DL}$ is preferable than the system $L_2F_nC_{IL}$, because $CWT(L_2F_nC_{DL}) < CWT(L_2F_nC_{IL})$. Consider the system $L_2F_nC_{UD(k)}$, where one of the two lifts, let us call it *Do-lift,* serves customers who are going from the first floor to *2,3,..,k* floors. Another lift, let us call it *Up-lift,* serves customers from the first floor to the upper floors $k + 1, k + 2,..,n$. Remind that $T^U(L_2F_nC_{UD(k)})$ is the cycle time for the *Up*-lift, and $T^D(L_2F_nC_{UD(k)})$ is the cycle time for the *Do-lift.* The cycle time of a lift is defined as the time interval between two sequential comings of the lift to the first floor. For this system we also introduce the floor number f_{opt}. (optimal border cut), which can be found from the equation, when the cycle time of the *Up*-lift closes to the cycle time of the *Do*-lift. In other words, $f_{opt.}$ is found from the following ratio.

$f_{opt.} = \{k: min/T^U(L_2F_nC_{UD(k)}) - T^D(L_2F_nC_{UD(k)})/\}k$

where **/./**means the absolute value of (.). Below, as the result of the simulation, various systems are given. In the **Table 1**, for different number of the floors *(n)*, the value of $f_{opt.}$ is given. For simulation, there were used the following lifts' parameters $h_f = 4$, $h_d = 7(Sec.)$.

Simulation shows (see, **Table 1**) that typically.

$2n/3 \le f_{opt.} \le 3n/4$

Below, in the **Table 2**, the results of simulation for comparison of the systems $L_2F_nC_{IL}$ and $L_2F_nC_{OE}$ relatively to the *CTT*, are given. In the **Figure 3**, the graphical behavior of the *CTT* for both systems $L_2F_nC_{IL}$ and $L_2F_nC_{OE}$ is given. The results of simulation show that relatively to the *CTT,* the system $L_2F_nC_{OE}$ is preferable, than the system $L_2F_nC_{IL}$.

$CTT(L_2F_nC_{OE}) \le CTT(L_2F_nC_{IL}))$ (see, **Table 2** and **Figure 3**).

Consider the systems $L_2F_{15}C_{IL}$and $L_2F_{15}C_{DL}$. It is necessary to note denote that by introducing the control rules, can be reduced not only the *CWT* (customer's waiting time) but also the *CST* (customer's service time) and hence, the *CTT* (customer's total time). Below, we will consider the *CTT* for all the systems.

n	$f_{opt.}$	h_d	h_f
12	8	7	4
12	8	7	4
15	11	7	4
15	10	7	4
22	15	7	4

Table 1.
The values of $f_{opt.}$ *For buildings with various floors*.

λ	$CTT(L_2F_{15}C_{IL})$	$CTT(L_2F_{15}C_{DL})$
0,009	29,34	44,24
0,012	29,63	44,33
0,015	44,32	44,63
0,018	47,17	46,37
0,021	59,29	51,09
0,024	72,27	63,57
0,027	86,85	76,65
0,031	94,02	79,12
0,034	103,44	85,24
0,037	118,21	99,31
0,040	132,65	120,55
0,043	152,04	145,04

Table 2.
The values of the CTT ($L_2F_{15}C_{IL}$) *and CTT* ($L_2F_{15}C_{DL}$).

n		$CTT(L_2F_nC_{IL})$	$CTT(L_2F_nC_{OE})$	***Gain (%)***
1	*0,075*	*57,5*	*57,5*	*0,0*
2	*0,3*	*63,63*	*61,2*	*3,8*
3	*0,45*	*70,32*	*64,6*	*8,1*
4	*0,6*	*77,7*	*69,5*	*10,6*
5	*0,75*	*81,2*	*71,6*	*11,8*
6	*0,9*	*84,2*	*74,9*	*11,0*
7	*1,05*	*91,85*	*78,1*	*15,0*
8	*1,2*	*96,2*	*80,7*	*16,1*
9	*1,5*	*118,2*	*91,4*	*22,7*
10	*1,8*	*152,4*	*118,6*	*22,2*

Table 3.
The values of the $CTT(L_2F_{15}C_{IL})$ *and* $CTT(L_2F_{15}C_{DL})$.

λ	$CTT(L_2F_{15}C_{DL})$	$CTT(L_2F_{15}C_{SC})$
0,009	44,24	44,24
0,012	44,33	44,21
0,015	44,63	44,65
0,018	46,37	45,15
0,021	51,09	47,61
0,024	63,57	55,41
0,027	76,65	66,06
0,031	79,12	64,22
0,034	85,24	69,94
0,037	99,31	79,47
0,040	120,55	98,34
0,043	145,04	120,63

Table 4.
The values of the $CTT(L_2F_{15}C_{DL})$ *and* $CTT(L_2F_{15}C_{SC})$.

The results of simulation (see, **Table 2**) and the graphical behavior (see, **Figure 3**) of the *CTT* (customer's total time) are presented.

The **Table 2** and **Figure 3** show that for a small intensity of the input flow of customers, we have $CTT(L_2F_{15}C_{IL}) < CTT(L_2F_{15}C_{DL}))$. It follows from the fact that for a small intensity of the input flow at the preceding instant of a customer's arrival in the system $L_2F_{15}C_{IL}$, one lift occupies the first flow and another one, *j-th* floor (j = 2,3,..,n). In the system $L_2F_{15}C_{DL}$, for a small intensity at the preceding instant of a customer's arrival, both lifts occupy the first floor and hence, an average distance from lifts' position to customer call, far than in the system $L_2F_{15}C_{IL}$ (**Table 3**).

In the **Table 3** and **Figure 3**, the values of the *CTT*, depending on the intensity of the input flow for various systems, are shown. For a high intensity of the input flow, a difference between $CTT(L_2F_nC_{IL})$ and $CTT(L_2F_nC_{OE})$ is increasing, when the intensity of the input flow goes up (see, **Figure 3**), because in this case all the lifts stop at each floor and the system $L_2F_nC_{IL}$ operates like a system with one lift

λ	$CTT(L_2F_{15}C_{IL})$	$CTT(L_2F_{15}C_{DL})$	$CTT(L_2F_{15}C_{SC})$
0,009	29,34	44,24	44,24
0,012	29,63	44,33	44,21
0,015	44,32	44,63	44,65
0,018	47,17	46,37	45,15
0,021	59,29	51,09	47,61
0,024	72,27	63,57	55,41
0,027	86,85	76,65	66,06
0,031	94,02	79,12	64,22
0,034	103,44	85,24	69,94
0,037	118,21	99,31	79,47
0,040	132,65	120,55	98,34
0,043	152,04	145,04	120,63

Table 5.
The values of the $CTT(L_2F_{15}C_{IL})$, $CTT(L_2F_{15}C_{DL})$ *and* $CTT(L_2F_{15}C_{SC})$.

$(L_1F_nC_{IL})$ but with double roominess (see. **Figure 3**). Consider the systems with two lifts, with a situation control rule and denote it $L_2F_nC_{SC}$, where the *SC* means situation control. At each given time-unit, the software is checking a new customer's arrival into the system and depending on this, each lift gets command, at which floor to stop for the customer's service. Below, in the **Table 1**, the result of simulation of such a system, for a building with 15 floors in an unloading regime, is presented. It is assumed that the roominess of the lifts is quite large and the lifts can take all the customers waiting on the floor. In simulation, we take $h_f = 4$, $h_d = 7$. Below, the results of the simulation (see, **Table 4**) and the graphical behavior (see, **Figure 3**) of the *CTT* (customer's total time), are presented. It is necessary to note that introducing of the control rules, can be reduced not only the *CWT* (customer's waiting time), but also the *CST* (customer's service time) and hence, the *CTT* (customer's total time).

5. Situation control rule

Introducing the *SC* (situation control) allows to reduce the *CTT* for a high intensity of the input flow. Below, the results of the simulation (see, **Table 4**) and the graphical behavior of the $CTT(L_2F_{15}C_{DL})$ and $CTT(L_2F_{15}C_{SC})$ (see, **Figure 4**), are presented:

Data of the **Table 4** show that by increasing of the intensity of the input flow, the gain in the *CTT* is going up. In **Figure 4**, there are given the results of the simulation for the systems $L_2F_{15}C_{DL}$ and $L_2F_{15}C_{SC}$. It is clear that for small and high values of intensity, it is not necessary to introduce the situation control, because both systems are almost the same and moreover, for high values of intensity, they coincide and the efficiency indexes can be calculated. There exists some interval where a difference between efficiency indexes takes maximal value and afterward it goes to zero, because for high values of customers' intensity flows, the lifts must stop at each floor, hence both systems have the same behavior (see **Figure 4**).

Below, the results of simulation (see, **Table 5**) and graphical behavior (see, **Figure 5**) of the *CTT(customer's total time)* for all the three systems, are presented. It is necessary to note that by introducing the control rules, can be reduced not only the *CWT (customer's waiting time*) but also the *CST(customer's service time*) and hence, the *CTT(customer's total time).*

6. Energy expenses

Now we will show that introducing of the control rules, will be to reduced not only the *CWT* and the *CTT,* but also the *LEE* (*lift energy expenses*). Note, as it was mentioned above, for rare input flows it is not necessary to introduce the control rule *DL*, because.

$CTT(L_2F_nC_{IL}) < CTT(L_2F_nC_{DL})$

and moreover, from formula (1), it follows

$LEE(L_2F_nC_{IL}) = k_dh_d$ and $LEE(L_2F_nC_{DL}) = k_f\, nh_f/6 + k_dh_d$

i.e. $LEE(L_2F_nC_{IL}) < LEE(L_2F_nC_{DL})$.

Energy expenses linearly depend on the *CTT* and also on the *SRT* (single rate time). As it follows from **Table 2**, the introduction of the SC (situation control) reduces the value of the *CTT,* by up to 25%. In [4] it was shown that for the $CTT(L_2F_nC_{IL})$ in an unloading regime and rare flow of customers, the following ratio is true:

$CWT(L_2F_nC_{IL}) = h_f\,(n\text{-}1)/2 + h_d$, $CST(L_2F_nC_{IL}) = h_f(n\text{-}1)/2 + h_d$ and

$CTT(L_2F_nC_{IL}) = h_f\,(n\text{-}1) + 2h_d$, $LEE(L_2F_nC_{IL}) = k_f(n\text{-}1)h_f + 2k_d\,h_d$,

$SRT = k_f\,h_f\,[3(n\text{-}1)/4] + m\,k_dh_d$. Then, for Poisson flow of customers with intensity λ during the time interval $[0,t)$, we have $SRT(t) = \lambda(k_f\,h_f\,3(n\text{-}1)/4 + mk_d\,h_d)t$. As λT is an average number of arrivals during the time *T*, then $\lambda Tk_f\,h_f\,(n\text{-}1)$ is an average energy which lift spends for serving the customers (motion of lift), during time *T.* As at each arrival instant, there is an average number of customers equal to *m,* then λTm is the average number of customers who arrived during the time *T,* into the system. For each customer's arrival, the lift spends the time h_d for opening and closing the door. If we assume that each customer spends the time h_c coming in and getting off a lift, then, the mh_c is the time, which was spent for the *m* customers (coming in and getting off). Hence, a customer average energy spent for opening and closing the door, for customers coming in the lift and getting off equals $2\lambda_1Tk_dh_d + 2\lambda_1Tmh_c = 2\lambda_1T(k_dh_d + mh_c)$. Thus, we have $LEE(L_1F_nC_{IL}) = \lambda Tk_f\,h_f\,(n\text{-}1) + 2\lambda T(k_dh_d + mh_c)$. Below, for simplicity, we assume $h_c = 0$, which means that during the time h_d, all the customers, who want to come in and get off a lift, can do it.

7. Analysis of the two lifts system in planning office buildings

Suppose it is a plan to construct a 15 floors office building with two similar lifts, which will carry in the morning, the customers to their offices and back, to the 1st floor, at the end of their work. It is necessary to introduce parameters of the lifts, e.g. roominess, velocity of lifts going up and down between floors and times for opening the doors on floors. Here, we consider unloading regime, where all the customers leave offices at the end of work hours. The offices will be placed on the floors *{2, 3, … ,15}.* The number of customers working on these floors will be *{12, 12, 15, 16, 12, 10, 17, 12, 14, 14, 16, 11, 18, 14}* i.e. all together in the building will be $n_c = 193$ customers. They should leave their offices during an interval of *1800 sec* (half an hour) in the evenings. The probability density $p[s] = 2(1800\text{-}s)/1800^2$, $0 < s < 1800$ of customers to leave their offices is given in **Figure 6**:

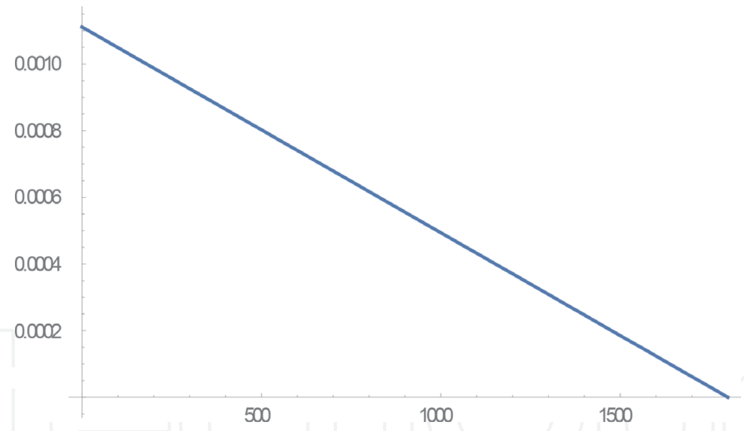

Figure 6.
The decreasing probability density p[s] *during an interval of* 1800 Sec.

The main efficient parameters of the lifts' systems are *the customers' average waiting times (CWT)* and *customers' average total times (CTT).* Remind that the *CWT* is defined as an average time from the instant when the customer presses the button at the unloading, to get the lift cabin. *The CTT* is defined as the sum of the *CWT* and *CST,* i.e. the average time from the instant when a customer arrives into the system until the instant when it he has left the cabin, at the desired floor. Simulated data can be used to obtain estimates of the *CWT, CST* and *CTT.* The results of the lifts which are operating, can be described by initial histories of customers $\{i, f_a, t_a, f_d\}$, $i = 1, 2, \ldots, n_c$, where n_c is the total number of customers in the building, i is the ordinal number of customer, f_d - destination floor. For simulating an unloading regime, it is assumed that $f_d = 1$ and the lift spends $h_f = 2.5$ sec. to cross the distance between two neighboring floors. If the lift stops at some floor, then the time for opening and closing the door is $h_d = 5$ *sec.* Let's assume that only one lift with roominess $r = 10$or$r = 20$, is operating. If the lift is located at the first floor ($f_d = 1$) and its cabin is empty, then it immediately goes up to the highest *15th* floor. It means that we consider an unloading regime, where the lift is going from up to down and collects customers at the lower floors, if roominess allows it. Using our simulated program for unloading regimes, we have obtained six sets with initial customer histories, $\{i, t_a, f_a\}$: three the sets with roominess $r = 10$ and three the sets with roominess $r = 20$. Using programming *Wolfram Mathematica*, we created the program, which transforms initial customer histories $\{i, t_a, f_d\}$ of n_c customers, into full histories $\{i, f_a, t_a, t_b, t_e\}$. Here t_b is the instant when the *i-th* customer goes in the lift cabin and t_e is the instant when the customer leaves the cabin at the *1st* floor. For simulating the 3-dimensional vector $\{i, t_a, f_a\}$, the similar program has been created for loading regime. This program was used by comparing the full histories for a lift with $r = 10$ and $r = 20$.

The estimates of the efficiency of the *CWT* and *CTT* are given in **Table 6**. It follows from **Table 6**, that roominess is the very essential parameter and the *CWT*

Days	*1st*		*2nd*		*3rd*	
r	*CWT*	*CTT*	*CWT*	*CTT*	*CWT*	*CTT*
10	*388.88*	*415.87*	*410.82*	*438.23*	*359.97*	*387.41*
20	*90.04*	*126.91*	*78.28*	*117.12*	*68.05*	*108.96*

Table 6.
The values estimation of the CWT *and* CTT *obtained during the simulation unloading, three times (days), different lifts, with roominess* r = 10, 20.

Days	*1st*		*2nd*		*3rd*	
Lift	*CWT*	*CTT*	*CWT*	*CTT*	*CWT*	*CTT*
L1	*46.86*	*68.90*	*40.28*	*62.53*	*39.59*	*61.00*
L2	*45.73*	*81.55*	*46.58*	*81.65*	*40.78*	*76.05*

Table 7.
The values estimation of the CWT *and* CTT *obtained for lifts L1 and L2, for data three "days".*

Days	*1st*		*2nd*		*3rd*	
Lift	*CWT*	*CTT*	*CWT*	*CTT*	*CWT*	*CTT*
L1	*59.26*	*87.07*	*48.55*	*76.47*	*51.14*	*79.17*
L2	*57.81*	*85.26*	*60.91*	*87.26*	*50.51*	*78.06*

Table 8.
The estimated values of the CWT *and* CTT *for lifts* L1 *and* L2, *for data three "days".*

and *CTT* are better if $r = 20$. From the engineering point of view, it is more practical to use a lift system with two different lifts *L1* and *L2,* with roominess $r = 10$ each.

Below, in **Table 7**, we illustrate the following control rules: lift *L1* serves lower floors, from floor *2* up to floor *10* and lift *L2* serves all the floors, from floor *11* up to floor *15.*

Note that we obtained in **Table 7**, better parameters, *for three days and two lifts,* than in **Table 6**. The above-considered data, for the *CWT* and *CTT*, correspond to three days. Note that our programs can simulate lifts for many days' operating data.

In **Table 8**, two lifts can stop *L1,* on *{1, 3, 5, 7, 9, 11, 13, 15}*- odd floors, *L2,* on *{1,2, 4, 6, 8, 10, 12, 14}*- even floors, and both lifts have $r = 10$.

We introduce the lifts' systems dispatcher (computer with special control lifts programs), as controller of the traffic of the moving lifts. Then, we can consider essentially many types of control rules for the lifts. For example, we can consider a system with two dependent similar lifts. They can stop, if their cabin contains less than r customers, follow specific rules at the floors with *waiting* customers, and if the system's dispatcher allows it.

8. Conclusion

Several mathematical models of lifts' systems, which have different control rules, are introduced and investigated. By simulation, the data customer's waiting time *CWT* and total time *CTT* were estimated under different control rules and they have been compared relatively to the efficiency indices for the introduced control rules. The result of the calculation shows that relatively to the *CTT* usage of the situation control *SC,* in comparison with *DL* control rule, a gain of around 25% is achieved. If the roominess of the lift cabin is unbounded, then, for a high intensity of the input flow, it is not necessary to introduce any control. Then, it follows that if at each floor there is at least one waiting customer and both lifts stop at each floor, the system is operating like deterministic. The simulation also shows that, in the case of two lifts and a rare customers' flow, it is not necessary to introduce *DL* or *SC* control rules, because the system *IL* (with independent lifts) is preferable than the *DL* and *SC* control rules. It was shown that for a high value of an input flow of customers, the introduced control rule also reduces energy expenses, even by 25%, in some cases, which confirms the advisability of the introduced control rules.

These results allow to make practical recommendations for reducing the various characteristics of the lifts' systems, such as the *CWT, CTT, SRT* (single rate time) and *LEE* (lift energy expenses). We completed the paper by examples with the calculation of a customer's waiting time *CWT* and a customer's average total time *CTT* for customers after work, for non-stationary cases, when there is an intensity of the customers' flow. It can be used for planning of the construction of the new office buildings with two similar lifts. The program can be extended for the case of several (more than two) lifts. We would like to underline that for the simulation of non-stationary cases, it is necessary to prepare a special program, which has a more complicated structure. In **Tables 6–8**, the results of the simulation for non-stationary cases, are given. We used the programming system *Wolfram Mathematica,* to create the programs for the simulation data and for a possible operation of two lifts. The results show that using simulation can help to estimate the appropriate values of roominess and find the optimal control rules, which can optimize the choice of the lifts' parameters (customer's waiting time, energy expenses and others). It can help for planning high floors buildings and future lifts' systems.

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References

[1] S.Markon, H.Kita, H.Kise, T.Bartz-Beielstein, “Control of Traffic Systems in Buildings”, Springer, 2006.

[2] G.C.Barney, “Elevator Traffic Handbook”, Taylor & Francis, 2003.

[3] Allen, A.O., Probability, statistics and queueing theory with computer science applications. New York, Academic Press, 1978.

[4] BelyaevYu.K., HajiyevA.H. Mathematical models of lift systems with various control rules. Gnedenko Forum. RT&A, No. 2(57), Volume 15, June, 2020.

[5] BelyaevYu.K., HajiyevA.H. Mathematical models of the lift systems and their simulation. Proceed. XIII ICMSEM, Canada. Springer, Ser. Electrical Engineering, 2019. p.507–519.

[6] Al-Sharif L., Abu-Alqumsan A., Ghanem W. Modelling of Elevator Traffic Systems Using Queuing Theory. Symposium on Lift and Escalator Technologies. Northampton, United Kingdom. Volume: 4

[7] БеляевЮ.К., ГаджиевА.Г., ГромакЮ.И., ДугинаТ.Н. Сравнительныйанализпростейших системвертикальноготра нспорта. Изв. АНСССР. Т ехническаякибернетика, №3, 1977, с. 97-103. (in Russian). (English transl.: Soviet Journal Computer and Systems Sciences).

[8] Aalto S. Some methods of investigation of queues with finite service capacity can be used for research of lifts’ systems. Mathematical Methods of Operations Research volume 51, p.263–285. 2000