Chapter

Using Multi-Criteria Optimization in Decision Support under Risk

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Abstract

The chapter presents an extension of a previous method for decision support under risk. The decision-making process is modeled by a multi-criteria optimization problem, in which the individual evaluation functions represent the results of decisions in several possible scenarios with associated risks. The decision support method is an interactive decision-making process. The choice is made by solving the problem depending on the control parameters that define the aspirations of the decision maker as well as on an evaluation of the obtained solutions. The decision maker selects a set of parameters representing various risks' impacts that influences a solution, and then he/she evaluates the obtained solution by accepting or rejecting it. In another case, the decision maker selects a new value and the problem is solved again for the new parameter. In this chapter, an example of supporting decision-making under risk is presented.

Keywords: decision under risk, multi-criteria optimization, symmetrically efficient decision, scalarizing function, method of decision selection

1. Introduction

In real decision-making problems, the evaluation of a decision is usually nondeterministic, because each problem concerns future activities and is evaluated in terms of future results. A significant portion of the parameters determining the decision conditions and assessment of the results may change, for example, raw material prices, product prices, currency exchange rates, and the sales potential of a given product.

The paper [1] presents a method of modeling decisions under risk in the form of a multi-criteria optimization problem. In this chapter, this approach is developed to apply multi-criteria optimization to supporting decision-making under risk.

As pointed out in [1], when making a decision, the decision maker must take into account both the choice of decisions and the risk's conditions that may occur in his/her environment. Depending on the degree of knowledge of the decision-making situation (features of the problems being solved and the nature of the environment), decisions can be made in a situation of certainty, uncertainty, or risk. This chapter extends the previous work to represent three types of decision-making under risk. The first type of decision-making: decisions are made under conditions of certainty when the decision maker has accurate and reliable information on which to base his/her actions. The effects of the actions can be predicted with high accuracy. The second type of decision-making: a decision maker has a situation of uncertainty when he can determine what factors will affect the decision-making situation, but he cannot determine the probabilities of their occurrence and therefore also the risks' impacts of the decisions taken. The other type of decision-making is a decision in risk conditions that applies to situations where the decision maker can determine what factors will affect the decision situation and determine the probabilities of their occurrence. The decision maker, using his experience and information from the environment, can determine with known or estimated probability the effects of decisions as well as the circumstances surrounding them.

Decision-making under risk is a process in which the results of actions taken by the decision maker are uncertain due to the potential of unforeseen circumstances, factors interfering with these circumstances, or disruptive factors, for example, ambient conditions, called scenarios. These, in turn, are caused by factors independent of the decision maker and have a significant impact on the results of the decision. Examples of scenarios can be: good or bad weather in the future; decline, stabilization, or rising stock values on the stock exchange in the future; and different price values and order volumes for a company operating in the future. Each such variant is a scenario. At the same time, each scenario clearly defines the implementation of results for individual decisions. Only the past is known from experience; we observe the present and try to predict the future. Such predictions are related to the construction of probable scenarios based on statistical analysis of the past data in order to find indications about the future and to anticipate it as accurately as possible. The decision maker is not able to determine with certainty which actions will lead to a result, but he can calculate the probability that a given result will occur. Specific scenarios correspond to the appropriate implementation of the assessment function. For each scenario, we are interested in the best evaluation value [1, 2].

As pointed out in [1], the theory of decision-making under risk refers to utility function and two-criterion techniques (Markowitz-type models). The utility function of the decision maker ensures complete order. If it is known, then the optimal decision is one that maximizes the expected utility [3–5].

This chapter shows an extension of [1] on how the decision problem under risk can be modeled with the multi-criteria optimization, that is, simultaneous minimization of a vector evaluation function whose particular coordinates represent the result of the decision when the given scenario is under risk occurred. The traditional approach [1] to solving a multi-criteria optimization problem requires the introduction of a single scalar objective function valuating individual y vectors and hence the decision vectors x. The solution of the decision problem is then reduced to determining the solution of the optimal single-criteria optimization problem. This approach implies the assumption that the preference relationship can be described using the utility function, u. The major difficulty in solving multi-criteria decision problems is due to the inability to determine a single aggregate quality indicator a priori, while the utility function is just such an indicator. Multi-criteria optimization techniques allow you to solve such a problem without using utility function models. This provides to the interactive multi-criteria techniques for decision support under risk. There are tools of the interactive analysis to define decision support process. They depend on additional preference information gained interactively from the decision maker, allowing simultaneously the decision maker to learn the problem during the process with possible evolvement of the preferences. The effective decision support is using the reference point method. Using the multi-criteria optimization approach, there is no need to identify the utility function of the decision maker. This approach is good for any decision maker who makes decisions under risk conditions (in a cost problem where less is better) for which less is better. This is consistent with first-order stochastic dominance.

The chapter is organized as follows:

- Section 2 presents a modeling approach of decision under risk.
- Section 3 defines a symmetrically effective decision that resolves the decision problem under risk conditions.
- Section 4 discusses the technique of generating symmetrically effective decisions and the method of supporting the decision maker.
- Section 5 gives an example of the application of the proposed decision support approach to a discrete problem.
- Section 6 provides a conclusion of this chapter.

2. Modeling of decision under risk

This section discusses how multi-criteria optimization methods can be used to model decisions under risk. The problem of multi-criteria optimization in the decision space and in the assessment of decision space is formulated.

Decision-making under risk is modeled by introducing scenarios, which represent possible states of the environment. Scenarios are factors that influence the outcome of a decision but are beyond the influence of the decision maker. For example, the risk factors can be raw material prices, product prices, currency exchange rates, deposit rates, and demand (e.g., sales opportunities for a given product) which may change. There may also be a catastrophic event changing the situation, for example, closing of the sales or supply market (e.g., due to embargo), customer insolvency, loss of license, etc.

The scenarios representing the risk factors are presented according to their probability distribution. If we assume that the probability of each scenario is a rational number, then by repeating relevant scenarios, it is possible to approach a situation where the probability of each scenario is the same, for example, selection between random variables Y' and Y'':

$$P(Y' = x) = \begin{cases} 1/2 & x = 2 \\ 1/2 & x = 4 \end{cases} P(Y'' = x) = \begin{cases} 1/4 & x = 1 \\ 3/4 & x = 5 \end{cases}$$

is equivalent to the problem of choosing between two lotteries $y' = (2,\ 2,\ 4,\ 4)$ and $y'' = (1,\ 1(?),\ 5,\ 5)$ with equally probable outcomes, where the order of outcomes is not important.

The number of occurrences of a specific scenario corresponds to the probability assigned to it. The specific set of scenarios $S_i, i = 1, \dots, m$ corresponds to the appropriate realization environment conditions associated with the evaluation function $f_i(x), i = 1, \dots, m$, where $x \in X_0$, a decision set that belongs to the set of admissible decision. There is an assessment function associated with each scenario. At the same time, each scenario clearly defines the implementation of results for the individual evaluation function. For each scenario, a lesser value of the evaluation function is preferred [4, 6, 7].

They are given as:

- the feasible decision set $X_0 \subset R^n$;
- the set of scenarios $S_1, S_2, \dots, S_{\mathrm{m}}$ and the set of probabilities $p_1, p_2, \dots, p_{\mathrm{m}}$ of occurrence of each scenario, and these probabilities are assumed to be known to the decision maker; and

- the decision assessment function, x, at the scenario $f_i(x), i = 1, \ldots, m$, where there is one evaluation function, $f_i(x)$, associated with each scenario.

The problem of decision under risk is modeled in the form of some kind of multi-criteria optimization problem:

$$\min_x \{ \left(f_1(x), \ldots, f_m(x) \right) : \quad x \in X_0 \} \tag{1}$$

This is a special problem of multi-criteria optimization in the sense that all assessment functions are expressed in the same units. This differs from the standard multi-criteria optimization problem, where evaluation functions can be expressed in different units. In the case of modeling decisions in risk conditions, individual assessments, although generated by different functions, are all expressed on the same scale, which allows comparison of their values.

There are as many assessment functions in a multi-criteria problem as there are scenarios. Each scenario has a different assessment function. You want to have the best score for all scenarios.

In the problem of multi-criteria optimization, all values for all scenarios are taken into account and by not looking at the values in each scenario (and not looking at individual coordinates). The result of the decision is the grade vector. You want to have the best score for all scenarios. One grading vector that gives the best score for all scenarios is sought.

The function, f, assigns to each decision variable vector, $x \in X_0$, an evaluation vector, $\mathrm{y} = \mathrm{f}(x)$, which measures the quality of decisions, x, from the point of view of the determined system of evaluation functions, $f_1, \ldots, f_\mathrm{m}$. For each individual assessment at a given scenario, the lower rating means a better evaluation. The formulation of the multi-criteria optimization problem is expressed in the decision space. This is a way for representating the decision problem, where the goal is to choose the right decision, given a set of criteria associated with decision's risk.

There is a transformation $f = \left(f_1, f_2, \ldots, f_m \right)$ of a set of feasible decisions, X_0, into a set of achievable assessment vectors, Y_0. The problem of choosing the best decision arises naturally. The choice of decision only considers grade vectors and decisions with identical grade vectors that are equally good. Thus, the problem of determining the best decision can be limited to the issue of choosing the best grade vector in the set of achievable grades (achievable grade vectors):

$$Y_0 = \{y : \ y = f(x), \ x \in X_0\}$$

This leads to a multi-criteria model in the assessment space:

$$\min_x \left\{ y = \left(y_1, \ldots, \mathrm{y_m} \right) : \ y_i = f_i(x) \ \forall i, \ x \in X_0 \right\} \tag{2}$$

where grades are directly specified as individual variables.

Each vector x in the set X_0 corresponds to the vector y for the set Y_0. The vector from the set Y_0 is selected and one sees the decision from the set X_0.

3. Symmetrically efficient solutions

This chapter extended the way of defining a symmetrically effective decision compared to the work [1]. This chapter provides the basic definition of a symmetrically effective decision. It is a decision that is a solution to a specific multi-criteria optimization problem, a problem used to support decisions under risk. Decision assessments must meet an additional condition—the condition of anonymity of preference relationships.

The model of the decision problem under risk in the form of a multi-criteria optimization problem imposes additional properties of preference relations and, consequently, limits the choice of decisions to an appropriate subset of the entire set of effective decisions. In the problem of making decisions under risk, minimizing all assessments is equally important.

Decision problems under risk are, when the decision is based on minimization of a vector outcome with various realizations under several scenarios. The preference model leads to the Pareto efficiency with respect to the realizations under scenarios understood as multiple criteria. The case of equally probable scenarios leads to the concept of symmetric optimization (efficiency) of multi-criteria corresponding to realizations under scenarios. The solution should have the feature of anonymity: no distinction is made between results that differ in their orientation coordinates. This solution of the problem, called a symmetrically efficient decision, is an efficient decision that possesses an additional property, that is, that of preference relation anonymity.

Nondominated solutions (*optimum Pareto*) are defined with the use of preference relations which answer the question of which one of a given pair of evaluation vectors $y^1, y^2 \in R^m$ is better. This is the following relation:

$$y^1 \succ y^2 \Leftrightarrow y_i^1 \le_i^2 \forall i = 1, \dots, m \wedge \exists \mathrm{j} \; y_j^1 < y_j^2 \tag{3}$$

The vector of evaluation $\hat{y} \in Y_0$ is called the *nondominated vector*, provided there is no vector $y \in Y_0$ such that $\hat{y}$ is dominated by y. The domination structure is shown in **Figure 1**.

The set of *nondominated solutions* is defined as follows:

$$\hat{Y}_0 = \left\{ \hat{y} \in Y_0 : \left(\hat{y} + \tilde{D} \right) \cap Y_0 = \varnothing \right) \} \tag{4}$$

where $\tilde{D}$ is a positive cone without the top. The positive cone can be $\tilde{D} = R_+^m$.

The set $\hat{Y}_0$ is shown in **Figure 2**.

A decision $\hat{x} \in X_0$ is called *an efficient decision* (*Pareto optimal*) if there is no $\overline{x}$ such that $y_i = f_i(\overline{x}) \le \hat{y}_i = f_i(\hat{x})$ for $i = 1, \dots, m$ with strict inequality for at least one i [5, 8].

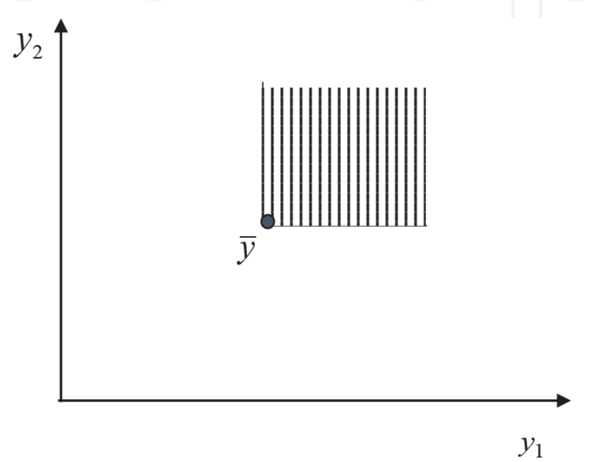

Figure 1.
Dominance structure in R^2.

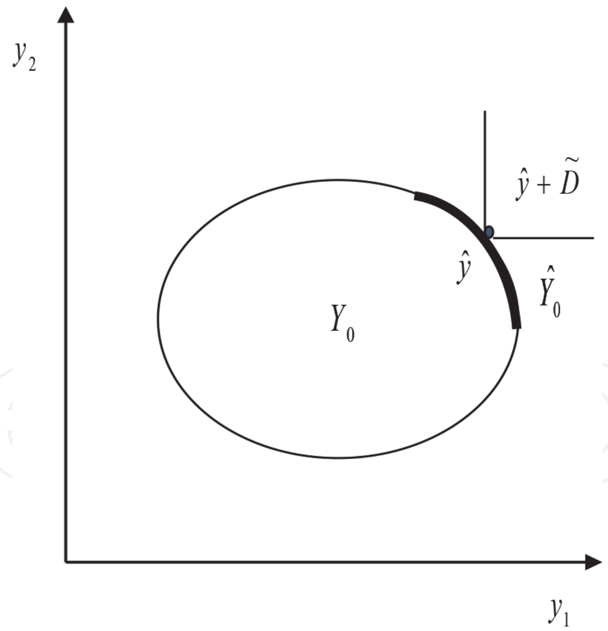

Figure 2.
Nondominated solutions.

In the problem with homogeneous and equally important assessments, the relation of the decision maker preferences should be impartial due to individual assessment functions. That is, for a given set of evaluation functions, only the distribution of the values achieved by these functions for a given decision is important, and it is not important which function it took. This requirement is formulated mathematically as a property of the anonymity of preference relationships. The risk assessment vector should meet the property of anonymity.

The relation is called an *anonymous* (*symmetric*) relation if, for every vector $y = (y_1, y_2, \dots, y_m) \in R^m$ and for any permutation, P, of the set $\{1, \dots, m\}$, the following is true:

$$\left(y_{P(1)}, y_{P(2)}, \dots, y_{P(m)}\right) \approx (y_1, y_2, \dots, y_m) \tag{5}$$

We look at the whole with the help of an anonymous preference relationship, and all scenarios are considered rather than on individual results for given scenarios. Anonymous preference relationship is a superstructure over a preference relationship—an additional condition of anonymity is added.

A nondominated vector satisfying the anonymity property is called *a symmetrically nondominated vecto*r. The set of symmetrically nondominated solutions is marked as follows: $\hat{Y}_{os}$. In the decision space, symmetrically efficient decisions are specified. The decision $\hat{x} \in X_0$ is called *a symmetrically efficient decision*, if the corresponding evaluation vector $\hat{y} = f(\hat{x})$ is a symmetrically nondominated vector. The set of symmetrically efficient decisions is marked as follows: $\hat{X}_{os}$ [9, 10].

The domination structure of symmetric dominance depends on the location of an evaluation vector, y, relative to the line $y_1 = y_2 = \dots = y_m$. The domination structure is shown in **Figure 3**.

The relation of symmetric domination can be expressed as the domination of evaluation vectors with coordinates ordered in no decreasing order. This can be formalized with the map $T : R^m \to R^m$ such that $T(y) = (T_1(y), T_2(y), \dots, T_m(y))$, where $T_1(y), \geq T_2(y) \geq, \dots, \geq T_m(y)$ and a permutation, P, of the set $\{1, \dots, m\}$ exists such that $T_i(y) = y_{P(i)}$ for $i = 1, .., m$.

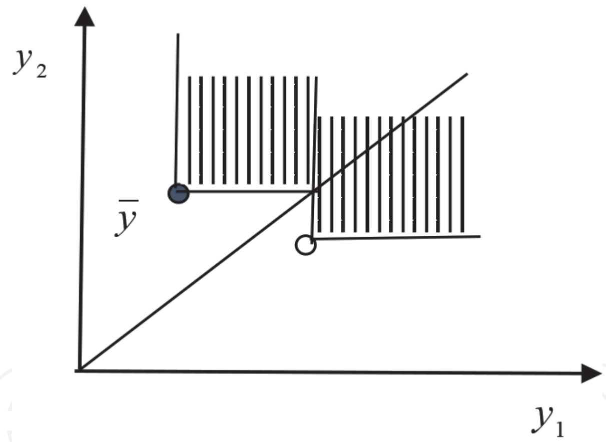

Figure 3.
Symmetric dominance structure in R^2.

The evaluation vector y' symmetrically dominates the vector y'' if the following condition is satisfied:

$$y^1 \succ_a y^2 \Leftrightarrow T(y^1) \le T(y^2) \tag{6}$$

The relation of symmetrical domination $\succ_a$ is a simple vector domination for evaluation vectors with no decreasing coordinates of evaluation vector [9, 10].

For the problem of decisions under risk expressed in the form of a multi-criteria optimization problem, the solution is a set of symmetrically effective decisions.

4. Technique of generating symmetrically efficient decisions

This chapter extended the way of defining a symmetrically effective decision compared to the work [1]. This chapter discusses how to support decisions under risk. It is an interactive IT system that processes relevant data for a given decision situation and assists the decision maker in recognizing the decision problem in the sense of understanding his own preferences. The decision maker's role is paramount. The system is not a substitute for the decision maker at any stage of decision-making. Such a system is to support, not replace in the final selection of the decision maker.

In multi-criteria decision problems, the relation of preferences is not known a priori, and therefore the final choice of solution can be made only by the decision maker. Given the numerous set of solutions, this selection is made using the appropriate interactive information system—the decision support system. Such a system processes important data for a given decision situation but also supports the decision maker in recognizing the decision problem in the sense of understanding his own preferences. The decision maker's role is paramount. The system is not a substitute for the decision maker at any stage of decision-making. Decision support system is to support, not replace in the final selection of the decision maker. In the problem of multi-criteria optimization, you cannot impose an optimal solution on the decision maker, you should support it—give the decision maker the opportunity to review such solutions that give the best results—symmetrically nondominated solutions. The decision maker chooses the decision by looking at the symmetrically nondominated set. This system enables a controlled review of the set of symmetrically efficient solutions. On the basis of the values of certain control parameters given by the decision maker, the system presents various solutions that are symmetrically efficient for analysis.

Solutions of a symmetrically efficient multi-criteria problem can be determined by solving the optimization of a multi-criteria problem:

$$\min\left\{\left(y_1, \ldots, y_m\right) : x \in X_0\right\} \tag{7}$$

with the scalarizing function $y : R^m \to R$ defining the preference relation:

$$y^1 \succ_a y^2 \Leftrightarrow s\left(y^1\right) > s\left(y^2\right) \tag{8}$$

If the relation fulfills the condition of anonymity, the efficient solution generated by this scalarization is also a symmetrically efficient solution to the multi-criteria problem (1).

Symmetrically efficient decisions for a multiple criteria problem (1) are obtained by solving a special problem in multi-criteria optimization, that is, a problem with coordinates of the vector of evaluation arranged in a no decreasing order. This problem is as follows:

$$\min_y \{(T_1(y), T_2(y), \ldots, T_m(y)) : y \in Y_0\} \tag{9}$$

where $y = \left(y_1, y_2, \ldots, y_k\right)$ is an evaluation vector,

$T(y) = (T_1(y), T_2(y), \ldots, T_m(y))$, where $T_1(y), \geq T_2(y) \geq, \ldots, \geq T_m(y)$ is an ordered evaluation vector,

Y_0 is the set of evaluation vectors.

An efficient solution of multi-criteria optimization problem (9) is a symmetrically efficient solution of the multi-criteria problem (1).

The method of determining individual symmetrically efficient decisions involves the solution of a parametric scalarization of a multi-criterion problem. This is a problem of single objective optimization using a specially created scalarizing function of two variables: the evaluation vector, $y \in Y$, and control parameter, $\overline{y} \in \Omega \subset R^m$; thus, we have $s : Y_0 \times \Omega \to R^1$:

$$\min_x \{s\left(y_1, \ldots, y_m\right) : x \in X_0\} \tag{10}$$

The parameter $\overline{y} = \left(\overline{y}_1, \overline{y}_2, \ldots, \overline{y}_m\right)$ is available to the decision maker, enabling him or her to review the set of symmetrically efficient solutions.

To ensure the anonymity of the relationship, it is necessary and sufficient that the scalarizing function is symmetrical, that is,

$$s\left(y_{P(1)}, y_{P(2)}, \ldots, y_{P(m)}\right) \approx s\left(y_1, y_2, \ldots, y_m\right) \tag{11}$$

for any permutation, P, of the set $\{1, \ldots, m\}$.

Complete and sufficient parameterization of the set of symmetrically efficient solutions can be achieved, using the method of the reference point for problem (9). In this method, aspiration levels are applied as control parameters. An aspiration level is a value of the evaluation function that satisfies the decision maker.

The scalarizing function defined in the method of the reference point is as follows:

$$s(y, \overline{y}) = \min_{1 \leq i \leq m}\left(T_i(y) - T_i(\overline{y})_i\right) + \varepsilon \cdot \sum_{i=1}^{m}\left(T_i(y) - T_i(\overline{y})_i\right) \tag{12}$$

where $y = \left(y_1, y_2, \ldots, y_m\right)$ is an evaluation vector,

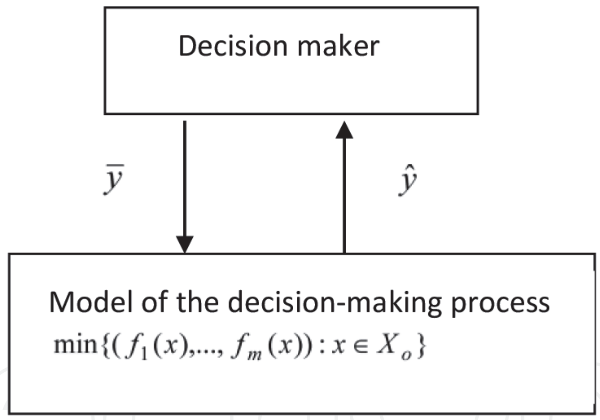

Figure 4.
The method of supporting decision selection.

$T(y) = (T_1(y), T_2(y), \ldots, T_m(y))$, where $T_1(y), \geq T_2(y) \geq, \ldots, \geq T_m(y)$ is a no decreasing ordered evaluation vector,

$\bar{y} = (\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_k)$ is a vector of aspiration levels,

$T(\bar{y}) = (T_1(\bar{y}), T_2(\bar{y}), \ldots, T_m(\bar{y}))$, where $T_1(\bar{y}) \geq T_2(\bar{y}) \geq, \ldots, \geq T_m(\bar{y})$ is a no decreasing order vector levels of aspiration,

ε is an arbitrary small, positive adjustment parameter.

This kind of scalarizing function is called *a function of achievement*. The aim is to find the solution closest to the specific requirements, that is, the aspiration levels. Maximizing this function determines the symmetrically efficient solution, $\hat{y}$, and the symmetrically efficient decision, $\hat{x}$. Note that the symmetrically efficient solution, $\hat{x}$, depends on the aspiration level, $\bar{y}$ [9, 11].

The solution to the multi-criteria optimization problem is a set of efficient solutions. The choice of solution should be made by the decision maker using an IT system. Such a system allows him to browse the entire set of solutions and make choices freely. The final choice of the solution among the set of efficient solutions can only take place based on the user's preferences. A tool for searching the set of solutions is the function (12). The maximum of this function depends on the parameter, $\bar{y}$, which is used by the decision maker to select a solution. The method of supporting decision selection is an iterative method consisting of the alternating performance of:

- calculations, that is, finding another symmetrically efficient solutions;
- interaction with the system, that is, dialog with the decision maker, which is a source of additional information about his or her preferences.

The method of supporting decision selection is shown in **Figure 4**.

This method of supporting decision-making, which does not impose a rigid scenario for the analysis of the decision-making problem upon the decision maker, enables modification of his or her preferences during the analysis of the problem. The decision maker plays a key role in the decision-making process.

5. Example: selecting a decision

The problem of selecting a decision is shown in order to illustrate the method of supporting a decision under risk [12]. The costs of 10 alternatives in three scenarios

Decision	S_1	S_2	S_3
Decision x_1	59	65	75
Decision x_2	50	58	71
Decision x_3	68	72	60
Decision x_4	69	72	62
Decision x_5	53	60	63
Decision x_6	51	59	65
Decision x_7	68	71	77
Decision x_8	56	57	75
Decision x_9	62	58	80
Decision x_{10}	62	55	70

Table 1.
Scenarios of 10 decisions.

are presented in **Table 1**. The probabilities of particular scenarios are as follows: P1 = 0.3, P2 = 0.6, and P3 = 0.1.

The decision maker's problem is to select one of 10 decisions with three possible future scenarios. Since the configuration of conditions that will apply during the decision is unknown, this problem is a selection decision under risk [13].

The problem of decision-making under risk is modeled as a multi-criteria optimization problem:

$$\min_x x\left\{y^1, y^2, y^3, y^4, y^5, y^6, y^7, y^8, y^9, y^{10} : \ x \in \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}\right\}, \tag{13}$$

where the results of particular decisions are the following vectors:

$$\begin{aligned}
y^1 &= (59, 65, 75) \text{ for decision } x_1,\\
y^2 &= (50, 58, 71) \text{ for decision } x_2,\\
y^3 &= (68, 72, 60) \text{ for decision } x_3,\\
y^4 &= (69, 72, 62) \text{ for decision } x_4,\\
y^5 &= (53, 60, 63) \text{ for decision } x_5,\\
y^6 &= (51, 59, 65) \text{ for decision } x_6,\\
y^7 &= (68, 71, 77) \text{ for decision } x_7,\\
y^8 &= (56, 57, 75) \text{ for decision } x_8,\\
y^9 &= (62, 58, 80) \text{ for decision } x_9,\\
y^{10} &= (62, 55, 70) \text{ for decision } x_{10},
\end{aligned}$$

in which particular coordinates of evaluation vectors occur with probabilities: $P_1 = 0.3$, $P_2 = 0.6$, and $P_3 = 0.1$.

The problem consists in selecting a decision for which the evaluation vector has the minimum value in the sense of symmetrical dominance.

The repeating of relevant scenarios results in a situation in which the probability of each scenario is the same and, that is, $P = 1/10$. The result is a problem equivalent to the starting problem in which the results for each decision, namely: $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$ are the following evaluation vectors with equally probable coordinates:

$$
\begin{aligned}
y^1 &= (59, 59, 59, 65, 65, 65, 65, 65, 65, 75),\\
y^2 &= (50, 50, 50, 58, 58, 58, 58, 58, 58, 71),\\
y^3 &= (68, 69, 68, 72, 72, 72, 72, 72, 72, 60),\\
y^4 &= (69, 69, 69, 72, 72, 72, 72, 72, 72, 62),\\
y^5 &= (53, 53, 53, 60, 60, 60, 60, 60, 60, 63),\\
y^6 &= (51, 51, 51, 59, 59, 59, 59, 59, 59, 65),\\
y^7 &= (68, 68, 68, 71, 71, 71, 71, 71, 71, 77),\\
y^8 &= (56, 56, 56, 57, 57, 57, 57, 57, 57, 75),\\
y^9 &= (62, 62, 62, 58, 58, 58, 58, 58, 58, 80),\\
y^{10} &= (62, 62, 62, 55, 55, 55, 55, 55, 55, 70).
\end{aligned}
$$

In order to compare the vectors in the sense of symmetrical dominance, the coordinates of vectors are ordered in no decreasing order and the results are the following evaluation vectors for each decision:

$$
\begin{aligned}
T(y^1) &= (75,\ \ 65, 65, 65, 65, 65, 65, 59, 59, 59),\\
T(y^2) &= (71, 58, 58, 58, 58, 58, 58, 50, 50, 50),\\
T(y^3) &= (72, 72, 72, 72, 72, 72, 68, 69, 68, 60),\\
T(y^4) &= (72, 72, 72, 72, 72, 72, 69, 69, 69, 62),\\
T(y^5) &= (63, 60, 60, 60, 60, 60, 60, 53, 53, 53),\\
T(y^6) &= (65, 59, 59, 59, 59, 59, 59, , 51, 51, 51),\\
T(y^7) &= (77,\ \ 71, 71, 71, 71, 71, 71, , 68, 68, 68),\\
T(y^8) &= (75, 57, 57, 57, 57, 57, 57, , 56, 56, 56),\\
T(y^9) &= (80, 62, 62, 62, 58, 58, 58, 58, 58, 58),\\
T(y^{10}) &= (70, 62, 62, 62, 55, 55, 55, 55, 55, 55).
\end{aligned}
$$

The set of symmetrically nondominated vectors is as follows: $\hat{Y}_{os} = \{y^2, y^5, y^6, y^8, y^{10}\}$. Five decisions x_2, x_5, x_6, x_8, and x_{10} are symmetrically efficient decisions. When making a selection, one should choose from among them and the decisions x_1, x_3, x_4, x_7, and x_9 should be rejected regardless of individual preferences. These five decisions are incommensurate with respect to a symmetrical preference relation. The choice between them depends on the individual preferences of the decision maker.

The method of the reference point for the problem with coordinates of the evaluation vector arranged in no decreasing order is used to determine the solution of the problem (13). The decision maker controls the selection of an investment project through the levels of aspiration by specifying the desired values of the aspiration vector for each scenario: $\overline{y} = \left(\overline{y}_1, \overline{y}_2, \overline{y}_3\right)$, where $\overline{y}_1$ is a level of the aspiration value for scenario 1, $\overline{y}_2$ is a level of the aspiration value for scenario 2, and $\overline{y}_3$ is a level of the aspiration value for scenario 3.

Iteration	
1. Aspiration level $\bar{y}$ Solution $\hat{x}$	$\bar{y} = (50, 55, 60)$ Decision x_6
2. Aspiration level $\bar{y}$ Solution $\hat{x}$	$\bar{y} = (55, 60, 60)$ Decision x_5
3. Aspiration level $\bar{y}$ Solution $\hat{x}$	$\bar{y} = (55, 56, 68)$ Decision x_2
4. Aspiration level $\bar{y}$ Solution $\hat{x}$	$\bar{y} = (55, 56, 74)$ Decision x_8
5. Aspiration level $\bar{y}$ Solution $\hat{x}$	$\bar{y} = (62, 56, 68)$ Decision x_{10}

Source: own calculations.

Table 2.
Interactive analysis of the search for a decision.

The multiple-criteria analysis is presented in **Table 2**.

At the beginning of the selection, the decision maker identifies the aspiration levels as the best values that can be achieved separately for each scenario, and in subsequent iterations, he or she changes the aspiration levels depending on his or her preferences.

In the first iteration, the decision maker determines the preferences as an aspiration level equal to the vector $\bar{y} = (50, 55, 60)$ and obtains the decision solution, x_6, as the solution. In the second iteration, the decision maker reduces the requirements for scenarios 1 and 2 without changing the requirement for scenario 3, states the vector $\bar{y} = (55, 60, 60)$ as the aspiration level, and obtains decision x_5 as the solution. In the third iteration, the decision maker does not change the requirements for scenario 1, increases them for scenario 2, and reduces them for scenario 3, and he or she states the vector $\bar{y} = (55, 56, 68)$ as the aspiration level and obtains decision x_2 as the solution. In the fourth iteration, the decision maker does not change the requirements for scenarios 1 and 2 and reduces the requirement for scenario 3. He or she states the vector $\bar{y} = (55, 56, 74)$ as the aspiration level and obtains decision x_8 as the solution. In the fifth iteration, the decision maker reduces the requirements for scenario 1 leaves the requirements for scenario 2 unchanged, and increases the requirements for scenario 3. He or she states the vector $\bar{y} = (62, 56, 68)$ as the aspiration level and obtains decision x_{10} as the solution.

The final selection of a specific solution depends on the decision maker's preferences. The example given here shows that the method enables the decision maker to discover his or her decision-making capabilities in the course of interactive analysis and obtain a satisfactory solution.

6. Conclusions

In the decision-making process, risk plays a significant role, influencing the final result of the decision. The decision maker should be able to analyze them when making decisions. Using his experience and information from the environment, he should make such decisions that will not bring unnecessary threat (risk) to the effects of the decision. Despite the use of objectified tools optimizing decision-making processes in the choice of solution, ultimately the decision maker takes responsibility for the decisions taken.

The chapter presents a method for the decision made under risky situations. The risk is introduced to the model with a set of scenarios with specified probabilities. The choice is made by solving the problem of multi-criteria optimization. This provides a systematic procedure to help a decision maker choose the most desirable and satisfactory decision under risk situations. Therefore, using this way, a decision can be made according to the decision maker's preference. This method is characterized by:

- The use of reference point method, that is, the concepts of aspiration levels and minimization of the achievement function to organize interaction with the decision maker.

- The assumption that the decision maker's preferences are not fully formed changes during the decision-making process, while the main problem of the decision support system is to support the decision maker's learning rather than the final act of choice.

- The method gives a whole set of solutions symmetrically effective decisions and allows the decision maker a free choice. This procedure does not replace the decision maker in making decisions. The whole decision-making process is controlled by the decision maker.

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