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Nature of Temporal ($t > 0$) Quantum Theory: Part I

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Abstract

It is our science governs the mathematics and it is “not” our mathematics governs our science. One of the very important aspects is that every science has to comply with the boundary condition of our universe; dimensionality and temporal ($t > 0$) or causality. In which I have shown that time is real and it is not an illusion, since every aspect within our universe is coexisted with time. Since our universe is a temporal ($t > 0$) subspace, everything within our universe is temporal. Science is mathematics but mathematics is not science, we have shown that any analytic solution has to be temporal ($t > 0$); otherwise, it cannot be implemented within our universe. Which includes all the laws, principles, and theories have to be temporal? Uncertainty principle is one of the most fascinated principles in quantum mechanics, yet Heisenberg principle was based on diffraction limited observation, it is not due to the nature of time. We have shown it is the temporal ($t > 0$) uncertainty that changes with time. We have introduced a certainty principle as in contrast with uncertainty principle. Of which certainty subspace can be created within our universe; which can be exploited for application. Overall of this chapter is to show that; it is not how rigorous the mathematics is, it is the physical realizable paradigm that we embrace.

Keywords: temporal universe, timeless space, physical realizable, uncertainty principle, certainty principle, quantum mechanics

1. Introduction

Strictly speaking every scientific solution has to be proven whether it is physical realizable before considering for experimentation, since analytical solution is mathematics. For example, if an elementary particle has proven not a temporal ($t > 0$) or a timeless ($t = 0$) particle, it has no reason to spend that big a budget for experimentally searching a timeless ($t = 0$) particle since timeless particle does not exist within our universe. Similarly, a mathematician discovers a 10-dimensional subspace, would not you want to prove that his 10-dimensional subspace is a temporal ($t > 0$) subspace, before experimentally search for it since mathematical solution is virtual.

Nevertheless at the dawn of science, scientists have been using a piece or pieces of papers; drawn models and paradigms in it and using mathematics as a tool analyzing for possible solution. But never occurs to them the back ground of that piece of paper represented a mathematical subspace that is “not” existed within our universe, for which practically all the laws, principles, and theories were developed

from a piece or pieces of papers, which are timeless ($t = 0$) and strictly speaking are virtual.

Since science is mathematics but mathematics is “not” equaled to science, it is vitally important for us to understand what science really is. In order to understand science, firstly we have to understand what supported the science? For which the supporter must be the subspace within our universe. In other words, any scientific solution has to be proven existed within our universe; otherwise, it may be fictitious and virtual as mathematics is, since science is mathematics. In which we see that, our universe is a physical subspace that supports every physical realizable aspect within her space, “if and only if” the scientific postulation complies within the existent condition of our universe; dimensionality and causality or temporal ($t > 0$).

The essence of our temporal ($t > 0$) universe is that; if a mathematical solution is “not” complied within the temporal ($t > 0$) condition of our universe, it cannot exist within our universe. Since quantum mechanics is one of the pillars in modern science, I will start with one of the most intriguing principles in quantum mechanics; uncertainty principle. I will carry on the principle onto a newly found “certainty” principle. In which I will show Heisenberg’s principle was based on diffraction limited observation, instead upon on “nature” of time, developing his principle. I will also show the mystery of coherence theory can be understood with principle of certainty. In which I will show that; certainty subspace can be created within our temporal ($t > 0$) universe. Samples as applied to synthetic aperture imaging and wave front reconstruction will be included.

2. Science and mathematics

There is a profound relationship between science and mathematics, in which we have seen that without mathematics there would be no science. In other words, science needs mathematics but mathematics does not need science. Although science is mathematics but mathematics is not science. For example, any mathematical solution if it cannot be proven it exists within our universe, then her solution is “not” a “physical realizable” solution that can be “directly” implemented within our temporal ($t > 0$) universe.

But this is by no means to say that; the solutions are not temporal ($t > 0$) or timeless ($t = 0$) solutions there are not science. In fact practically all the fundamental laws, principles, and theories are timeless ($t = 0$) or time-independent. And these timeless ($t = 0$) laws, principles, and theories were and “still” are the corner stone and foundation of our science, as I will call them timeless ($t = 0$) or time-independent science; a topic I will elaborate in a different occasion. For simplicity, let me take one of the simplest examples; Einstein’s energy Eq. (1) as given by;

$$E = mc^2 \quad (1)$$

where E is the energy, m is the mass and c is the velocity of light. This equation is one of the most famous equations in science, yet it is timeless ($t = 0$). Although this equation has been repeatedly used and applied in practice, but strictly speaking; it cannot be directly implemented within our temporal ($t > 0$) universe, since it is not a time variable function. Let us transform Einstein’s equation into a time variable equation as given by [1].

$$\frac{\partial E(t)}{\partial t} = -c^2 \frac{\partial m(t)}{\partial t}, t > 0 \quad (2)$$

where $\partial E(t)/\partial t$ is the rate of increasing energy conversion, $-\partial m/\partial t$ is the corresponding rate of mass reduction, c is the speed of light, and $t > 0$ denotes a forward time-variable equation. In which we see Eq. (2) is a time-dependent equation exists at time $t > 0$, which represents a forwarded time variable function that only occurs after time excitation at $t = 0$. Incidentally, this is the well-known “causality” constraint (i.e., $t > 0$) [2] as imposed by our temporal ($t > 0$) universe.

Nevertheless in mathematical, a postulation is first needed to proof that there is solution existed before we search for the solution, although it is not guarantee that we can find it. But it seems to me it does not have a criterion to proof that a hypothetical science is existed within our universe, before we search for the science. For example, an analytically solution indicates that it exists an “angle particle” from a complicated mathematical analyses, will not you want to find out first is the solution existed within our temporal ($t > 0$) universe before experimentally to search for it. And this is precisely that we shall know first before experimentation is taken place, since it is a very costly in time and in revenue to find a physical particle.

Although science needs mathematics, but without simplicity mathematically approximation, science would be very difficult to learn and to facilitate. And this is precisely the reason practically all the fundamental laws are point-singularity approximated. In which we see precisely, science is a “law of approximation” and mathematics is “an axiom of certainty”. Again we take Einstein’s energy equation of Eq. (1) as an example, no dimension and size and it is a typical point-singularity approximated equation. It is discernible; if we include all the negligibly terms, “physical significances” of this equation would be over whelmed by the terms of mathematics. For which we see that an ounce of good approximation worth more than tons of mathematical calculation!

Let me stress that the essence of simplicity in science is that without the symbolic substitution and approximation, it will be extremely difficult or even impossible to develop science since science itself is already very complicated. Yet simplicity representation of science has also been misinterpreted as referred them as “classical and deterministic (i.e., classical physics).” The implication of deterministic or classical is a totally misled by our part, since our predecessors who developed those fundamental laws and principles were “precisely” understood the deficiency of approximation. Yet without the approximated presentation, how can we develop science? Instead of ignoring our predecessors’ wisdom, turns around we had treated them “deterministic” or classical, which were “never” been our predecessors intention. Again without the point-singularity approximated science, please tell me how we can develop those simple and elegant laws, principles, and theories. Although those laws, principles, and theories were timeless ($t = 0$), most of them were and “still” are the foundation and corner stone of our science. Nevertheless, mathematics is a “symbolic” langue of science, but mathematics is not science.

Since all laws, principles, and theories were made to be broken or revised or even to replace, as science advances into sub-subatomic scale regime and moving closer to near real time processing, those timeless ($t = 0$) laws, principles, and theories could produce incomprehensible consequences; particularly as applied them directly confronting the temporal ($t > 0$) constraint of our universe. For example, as applying superposition principle to quantum computing and communication, since superposition is a timeless ($t = 0$) principle [3].

3. Temporal ($t > 0$) subspace

In this section, I will show several subspaces that have been used by the scientists, in the past as depicted in **Figure 1**. It is reasonable to stress that why subspace

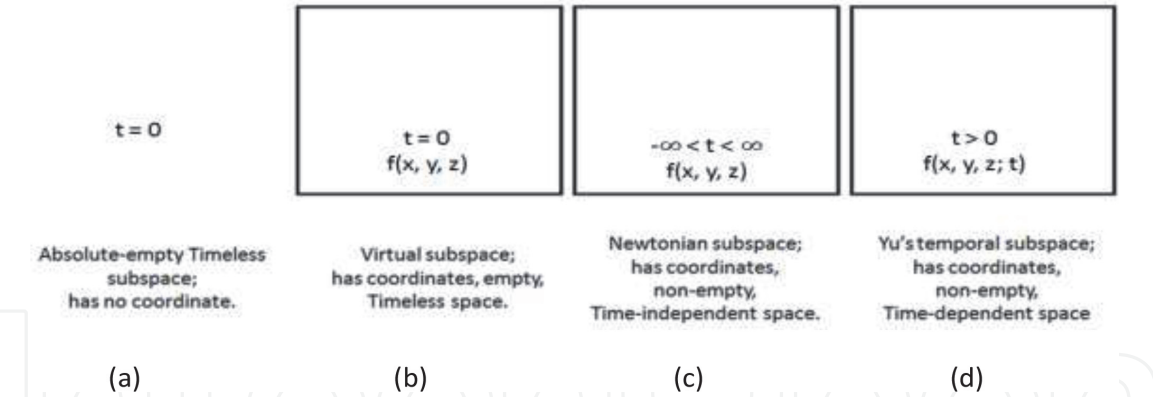


Figure 1.
(a) Shows an absolute-empty space, (b) a virtual mathematical space, (c) a Newtonian space, and (d) a temporal ($t > 0$) space, respectively.

of a scientific model embedded is crucially important is that any analytical solution produced follows the “limitation” of the subspace, because it is the subspace dictates the science but “not” the mathematics changes the subspace.

For example, when you are designing a submarine, the subspace that the submarine is supposed to be situated within is vitally important; otherwise, your submarine will very “likely” not to survive thousands of feet underwater pressure. Therefore, it is necessary to know the subspace that a postulated science to be implementing into it; otherwise, the postulated science is very likely “cannot” be existed within the subspace.

In view of **Figure 1**, we see that; there is an absolute-empty space, a mathematical virtual space, a Newtonian’s space [4], and a temporal ($t > 0$) space. An absolute-empty space or just empty space has no substance and has no time. A mathematical virtual space is an empty space which has no substance in it, but mathematicians and theoretical scientists can implant coordinate system in it, since mathematics is virtual and theoretical scientists are also mathematicians.

We note that mathematical virtual space has been used over centuries by scientists at the dawn of science, but this is a virtual space that does “not” exist within our temporal (i.e., $t > 0$) universe. The next subspace is known as Newtonian space [4]; it has substance and coordinates in it, but treated time as an “independent” variable, for which Newtonian and mathematical spaces are virtual the “same.” Since Newtonian space is time independent, it “cannot” be exist within our temporal ($t > 0$) space since time and substance has to be “mutually coexisted” within our temporal ($t > 0$) universe. Yet scientists have been using Newtonian space for their analyses over centuries and not knowingly it is a virtual space.

The last subspace is known as temporal ($t > 0$) space [5], where time and substance are interdependently “coexisted” and time is a forward “dependent variable” runs at a “constant speed”. We stress that this temporal ($t > 0$) subspace is currently “only” physical realizable space, where the space was created by Einstein energy Eq. (2).

Physical reality is that any scientific hypothesis that deviates “away” the boundary condition that imposed by our temporal ($t > 0$) universe is “not” a physically realizable solution. But this is by no means that the virtual mathematical empty space and Newtonian space are useless. The fact is that all the physical sciences were developed within timeless ($t = 0$) or Newtonian subspaces “inadvertently,” at the dawn of science. Practically all the fundamental laws, principles, and theories were derived from a timeless ($t = 0$) subspace, which was from the background subspace of a piece of paper although not intentionally [6]. In which we see that practically all the laws, principle, and theories are timeless ($t = 0$).

Nevertheless what temporal ($t > 0$) space means is that any subspace is coexisted with time, where time is a forward dependent variable with respect to its subspace and its speed has been well settled when our universe was created. This means that before the creation of our temporal universe, there is a “larger” temporal space that our universe is embedded in; otherwise, our universe will “not” be existed. Nevertheless every subspace within our universe is a time varying stochastic [7] subspace, in which every substance or subspace changes with time. Strictly speaking our universe is a “temporal ($t > 0$) stochastic expanding subspace.” For which we see that; any postulated law, principle, and theory has to comply with the temporal ($t > 0$) condition within our universe; otherwise, it is virtual as mathematics.

4. Timeless ($t = 0$) space

Let me show what mathematicians can do within a virtual subspace as depicted in **Figure 2**. Since quantum mechanists are also mathematicians, they can implant coordinate system within an empty space as they wishes, regardless whether the model is physically realizable or not.

The basic difference between **Figure 2(a)** and **(b)** is that there is a virtual coordinate system that has been added in **Figure 2(b)** by quantum mechanists. Once the coordinate system is implanted, dimensionality of the sub-atomic particles cannot be ignored. The reason is that for the atomic model to be existed within the subspace, the atomic model has to “comply” with the existence conditions within the subspace, since it is the subspace affects the solution and not the solution changes the subspace. In which we see that neither **Figure 2(a)** nor **Figure 2(b)** are “not” physical realizable paradigms. For which solutions obtained from these empty subspace models will be timeless ($t = 0$).

Aside the non-physical realizable paradigms of **Figure 2**, I will show what a timeless ($t = 0$) subspace can do for substances within the subspace. Let me assume we have three particles situated within an empty space, as normally do on a “piece of paper”, shown in **Figure 3**.

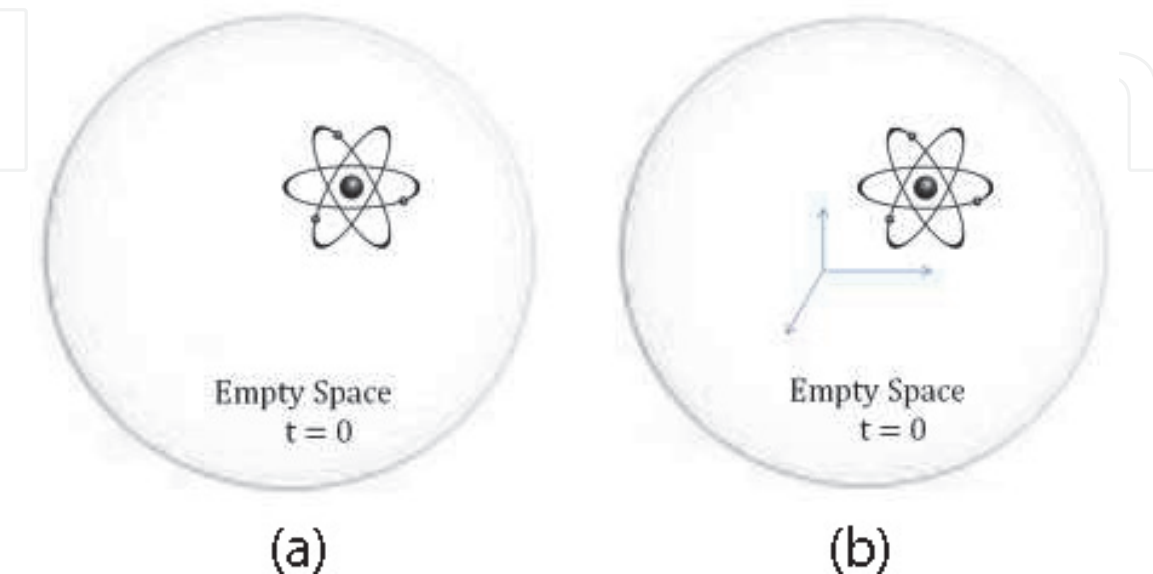


Figure 2.
A set of atomic models embedded within virtual empty subspaces. (a) shows a singularity approximated atomic model is situated within an empty space, which has no coordinate system. (b) shows an atomic model is embedded within empty space that has a coordinate system drawn into it.

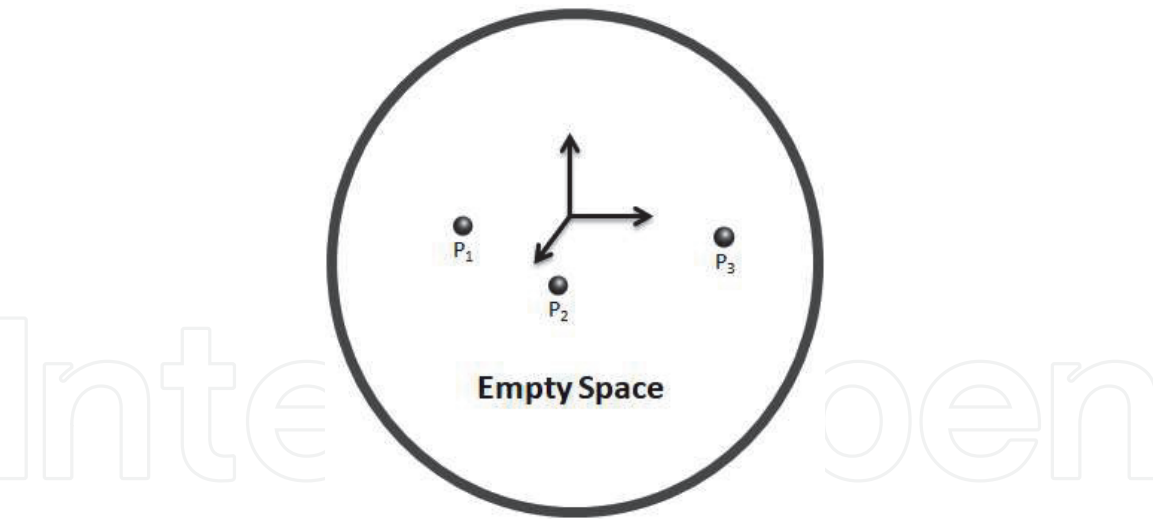


Figure 3.
A hypothetical scenario shows three particles are embedded within an empty subspace.

Since empty subspace has “no time,” all particles within the subspace collapse or “superimposing” instantly all together at $t = 0$, because time is distance and distance is time. This is precisely the “simultaneous and instantaneous” superposition principle does in quantum mechanics [3]. The reason particles collapsed at $t = 0$, it is because the subspace has “no time.” And the other reason that particles superimposed together, since within a timeless ($t = 0$) space, it has “no distance” or no space.

By virtue of energy conservation, we see that superimposed particles has a mass equals to the sum of entire superimposed particles, but it has “no size.” In view of timelessness space, we see that the superimposed particles can be found everywhere within the entire timeless ($t = 0$) subspace, since timeless ($t = 0$) subspace has “no” distance, as depicted hypothetically in **Figure 4**. In which we see that Schrödinger’s fundamental principle of superposition is existed within a virtual timeless ($t = 0$) subspace, and it cannot be existed within our temporal ($t > 0$) universe, since timeless and temporal are “mutually exclusive.”

By the way, this is precisely the superposition principle that Einstein was objecting to, which he called it spooky. As I quote from a 1935 The New York Times’ article (i.e., **Figure 5**), “Einstein and two scientists found quantum theory is

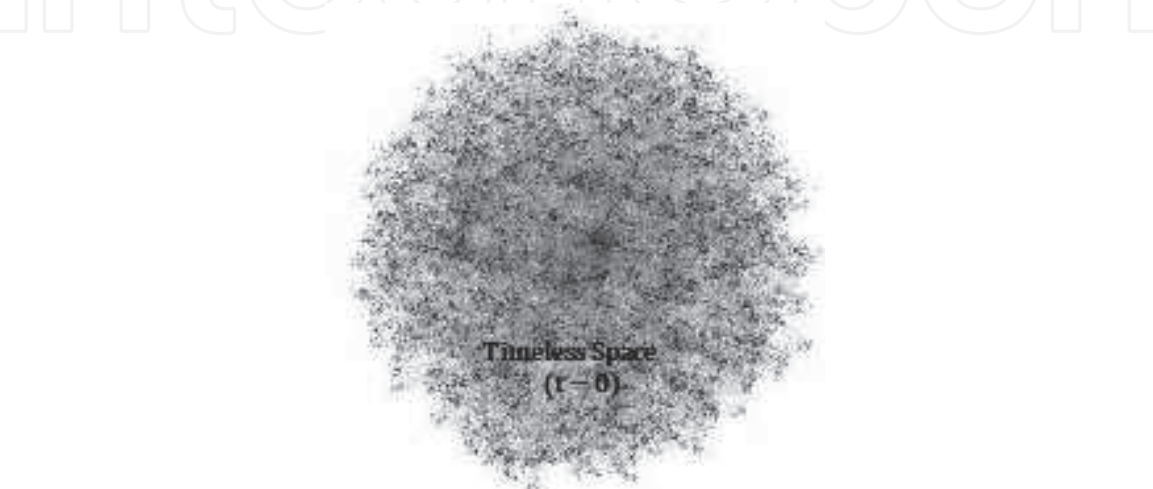


Figure 4.
Superimposed particle existed “simultaneously and instantaneously” all over the entire timeless ($t = 0$) subspace.

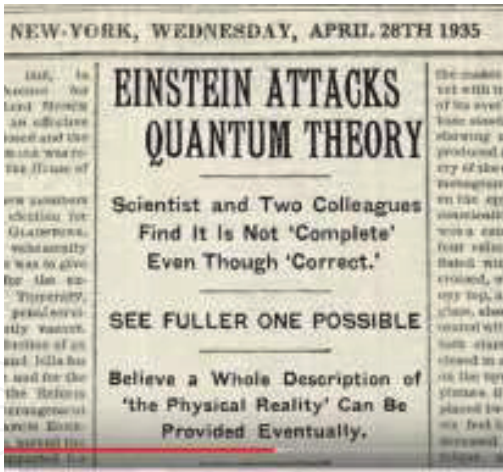


Figure 5.
A 1935 New York times' article.

incomplete even though correct” [8]. In view of preceding illustration, we see that Schrödinger’s superposition principle is “correct” but only within a timeless ($t = 0$) subspace and it is “incorrect” within our temporal ($t > 0$) space,” since timeless space cannot exist within our temporal universe.

5. Time is not an illusion but real

As we accepted subspace and time are coexisted within our temporal ($t > 0$) universe, time has to be real and it cannot be virtual, since we are physically real. And every physical existence within our universe is real. The reason some scientists believed time is virtual or illusion is that; it has no mass, no weight, no coordinate, no origin, and it cannot be detected or even be seen. Yet time is an everlasting existed real variable within our known universe. Without time there would be no physical matter, no physical space, and no life. The fact is that every physical matter is associated with time which including our universe. Therefore, when one is dealing with science, time is one of the most enigmatic variables that ever presence and cannot be simply ignored. Strictly speaking, all the laws of science as well every physical substance cannot be existed without the existence of time. For which we see that time “cannot” be a dimension or an illusion. In other words, if time is an illusion, then time will be “independent” from physical reality or from our universe. And this is precisely that many scientists have treated time as an “independent” variable such as Murkowski’s space [9], for which the space can be “curved” or time-space can be changed by gravity [10]. If time-space can be curved, then we can change the “speed” of time. In other words, is our universe exists with time, or time exists with universe? The answer is our universe exists with time, although space and time are interdependent but is not time exists with our universe.

As time is coexisted with subspace, we see that any subspace within our temporal ($t > 0$) universe cannot be empty and speed of time is the same everywhere within our universe. This means that the speed of time within a subspace is “relatively” with respect to the different subspaces, as based on Einstein’s special theory of relativity [9]. For example, subspaces closer to the edge of our universe, their time runs faster “relative” to ours, but the speed of time within the subspaces near the edge as well within our subspace are the “same,” which has been determined by the speed of light as our universe was created by a big bang theory using Einstein equation as given by [5];

$$\frac{\partial E(t)}{\partial t} = -c^2 \frac{\partial m(t)}{\partial t}, t > 0 \quad (3)$$

where $\partial E/\partial t$ is the rate of increasing energy conversion, $-\partial m/\partial t$ is the corresponding rate of mass reduction, c is the speed of light and $t > 0$ represents a forward time-variable. In which we see that it a “time-dependent” equation exists at time $t > 0$; a well-known causality constraint (i.e., $t > 0$) [2] as imposed by our universe. Similarly preceding equation can be written as:

$$\frac{\partial E}{\partial t} = -c^2 \frac{\partial m}{\partial t} = [\nabla \cdot S(v)] = -\frac{\partial}{\partial t} \left[\frac{1}{2} \epsilon E^2(v) + \frac{1}{2} \mu H^2(v) \right], t > 0 \quad (4)$$

where ϵ and μ are the permittivity and the permeability of the deep space, respectively, v is the radian frequency variable, $E^2(v)$ and $H^2(v)$ are the respective electric and magnetic field intensities, the negative sign represents the “out-flow” energy per unit time from an unit volume, $(\nabla \cdot)$ is the divergent operator, and S is known as the Poynting Vector or “Energy Vector” of an electro-magnetic radiator as can be shown by $S(v) = E(v) \times H(v)$ [11].

In view of this equation, we see how our universe was created as depicted by a composited diagram in **Figure 6**, in which we see that radian energy (i.e., radiation) diverges from the mass, as mass reduces with time. In which we see that our universe enlarges and her boundary expands at speed of speed of light.

Figure 7 shows a schematic diagram of our temporal ($t > 0$) universe, which depicts approximately the behavior of subspace changes as her boundary expands with speed of light. In which we see that, subspace enlarges faster closer toward the boundary, but solid substance $m(t)$ changes little within the subspace. We also see that the out-ward speed of particle (or subspace) increases “linearly” as boundary increases with light speed. For example; out-ward speed of particle 2 is somewhat faster than particle 1 (i.e., $v_2 > v_1$). For which we see that our universe is a dynamic temporal ($t > 0$) “stochastic” universe that simple geometrical equation or mathematical abstract space can describe. One of the important aspects of our universe is that every subspace, no matter how small it is, “cannot” be empty and it has time.

For instance, in order for us to be existed within our planet, we must be temporal ($t > 0$): that is we have time and must change with time; otherwise, we

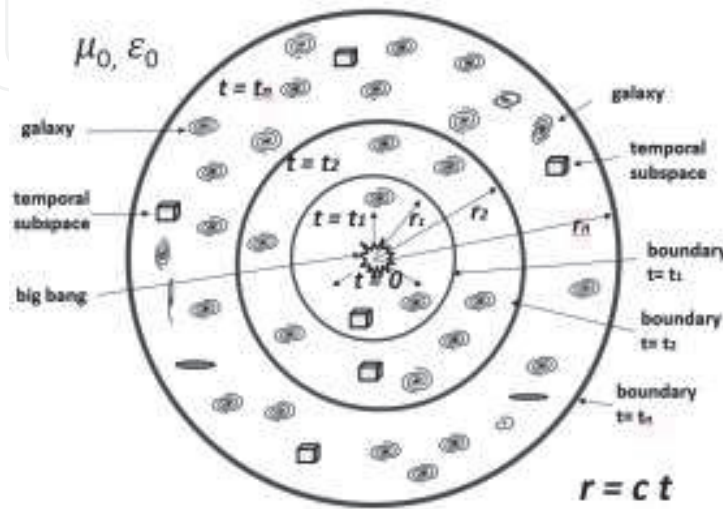


Figure 6.

Composite temporal ($t > 0$) universe diagrams. $r = ct$, r is the radius of our universe, t is time, c is the velocity of light, and ϵ_0 and μ_0 are the permittivity and permeability of the space.

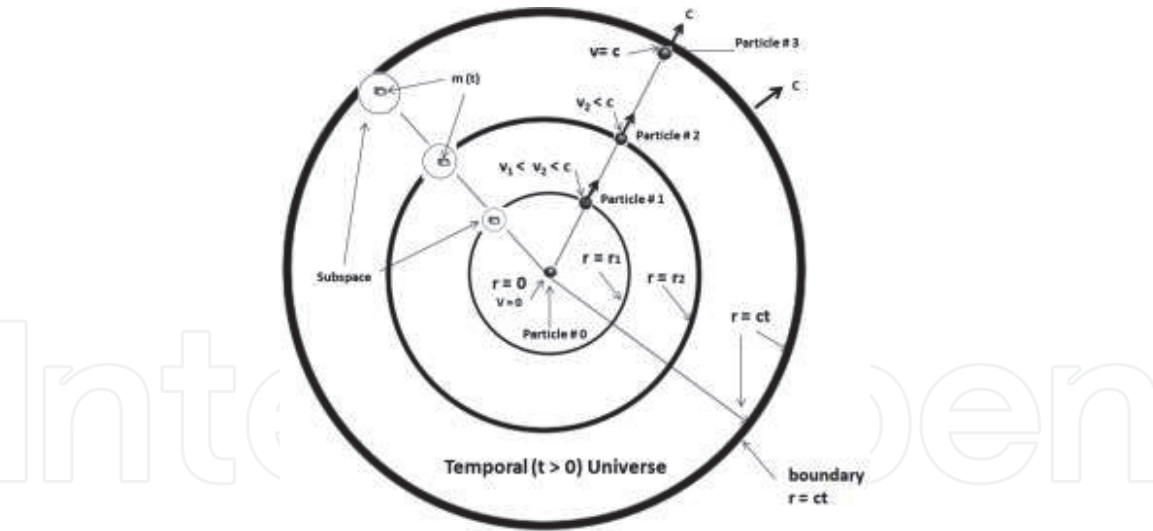


Figure 7.
A schematic diagram of our temporal ($t > 0$) universe. c is the speed of light, $m(t)$ is the temporal mass, and v is the radial velocity.

cannot exist within our universe. In other words, our time is the same as our planet and the universe but the velocity of our planet is different from other subspaces. For example, subspaces near the edge of our universe are moving faster than us, for which it has “relative” speed of time between us and a subspace closer to the edge of our universe. On the other hand, if we assume that we are timeless ($t = 0$), we could “not” have existed within our universe, since time and timelessness are mutually exclusive.

I further note that any subspace within our universe cannot empty, since subspace is coexisted with time. Although subspace is coexisted with time, but time is neither equaled to subspace. Yet, space is time and time is space since time and space are mutually inclusive. For example, substance has dimension (or space), but time has no dimension and no mass. In which we see that time is “not” a dimension but it is “dependently” existed with respect to subspace. In which we stress that it is our universe governs the science and it is not the science changes our universe.

Once again, we have shown that time is “not” an illusion or virtual, time is physically real because everything existed within our living space is physical real; otherwise, it will not be existed within our temporal universe. In other words, everything within our universe is temporal ($t > 0$), of which I have discovered that practically all the laws, principles, theories, and paradoxes of science were developed from a timeless ($t = 0$) platform (i.e., a pieces or pieces of papers) for centuries, at the dawn of science “inadvertently” [6].

Nevertheless, one of the important aspects within our universe is that every subspace has a price, an amount of energy ΔE , and a section of time Δt to create (i.e., ΔE and Δt), and it is “not free.” For example, a simple facial tissue takes a huge amount of energy ΔE and a section of time Δt to create. It is, however, a “necessary” but not sufficient condition, because it also needs an amount of information ΔI to make it happen (i.e., ΔE , Δt , and ΔI) [12].

In short, I would stress that if there is a beginning then there is an end. Since time and space are coexisted, then time and space have no beginning and no end. In which we see that time-space [or temporal ($t > 0$) space] is ever existed, since existence and non-existence are mutually exclusive. In other words, emptiness and non-emptiness are mutually excluded, then time “always” exists with space. Thus, time is real because the space is real, for which time-space has no beginning and has no end. And this must be the art of temporal ($t > 0$) universe.

6. Law of uncertainty

One of the most intriguing principles in quantum mechanics [13] must be the Heisenberg’s Uncertainty Principle [14], as shown by the following equation:

$$\Delta p \Delta x \geq h \tag{5}$$

where Δp and Δx are the momentum and position errors, respectively, and h is the Planck’s constant. As reference to “wave-particle dynamics,” the momentum p of a “photonic particle” is presented by a “quanta” of energy $h\nu$ as given by:

$$p = h/\lambda = h\nu/c \tag{6}$$

where h is the Planck’s constant, λ is the wavelength, ν is the frequency, and c is the velocity of light.

In which we see that Heisenberg’s principle was based on “wave-particle duality” existed within an “empty space.” The essence of the Heisenberg’s uncertainty principle is that one cannot precisely determine the position x and the momentum p of a particle “simultaneously under observation”, as illustrated in **Figure 8**. In which we see that; it is “independent” of time, since Heisenberg’s principle was based on “observation” stand point which has nothing to do with changing naturally with time. Yet we know that if there is “no” time there is “no” uncertainty.

In view of **Figure 8**, Heisenberg principle was derived on an empty timeless ($t = 0$) subspace and it has “nothing to do or independent” with the “underneath subspace” that the particle is situated. Strictly speaking, it is “not” a physical realizable paradigm should be used in the first place, since particle and empty subspace are “mutual exclusive.” Secondly, the position error Δx of Heisenberg was based on a “diffraction limited” microscopic observation, where the “spatial” ambiguity of Δx is given by [15]:

$$\Delta x = 0.6 \lambda / \sin \alpha \tag{7}$$

where λ is the observation wavelength, $2(\sin \alpha)$ is the “numerical aperture” of the microscope and α is subtended half-angle of observation aperture. In which we

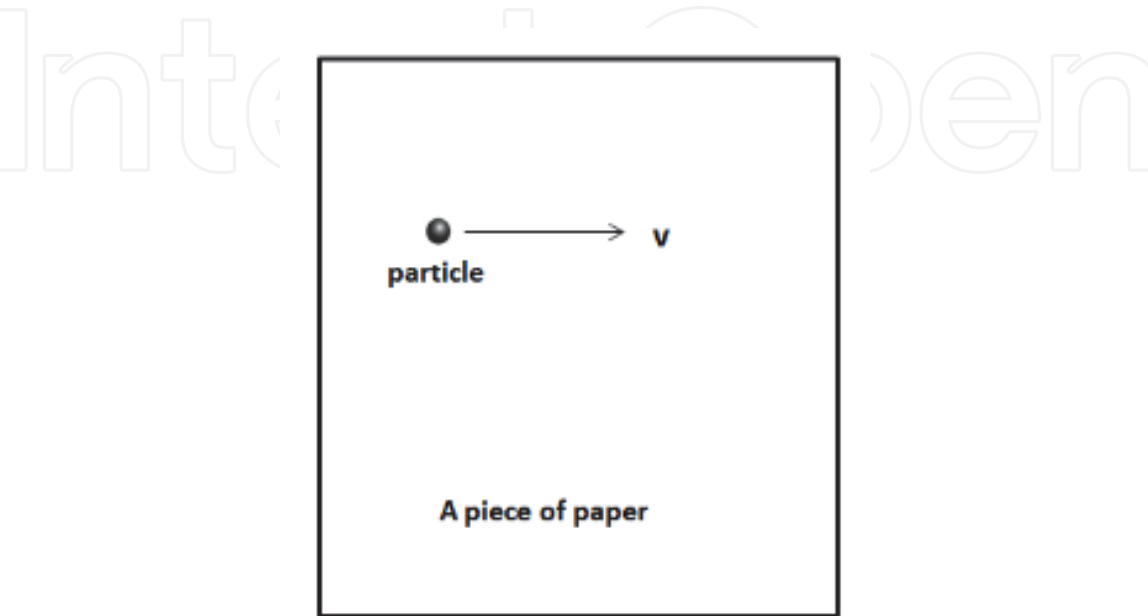


Figure 8.
A particle in motion within an “empty” subspace. v is the velocity. Note that background paper has been treated as an “empty” subspace for centuries.

see that the position error Δx is “not” due to particle in motion, but based on the diffraction limited aperture. This is precisely why Heisenberg’s position error Δx has been interpreted as an “observation error” which is independent with time. But uncertainty changes naturally with time, since without time it has no uncertainty.

Secondly, the momentum error Δp as I quote [15]: after collision the particle being observed, the photon’s path is only to lie within a cone having semi-vertical angle α in which momentum of the particle is uncertain by the amount as given by:

$$\Delta p = h(\sin \alpha)/\lambda \quad (8)$$

where λ is the wavelength of the quantum leap of $h\nu$. In which we see that; momentum error Δp is “not” due to band width $\Delta \nu$ of quantum leap since any physical radiator has to be band limited. In other words, the momentum error Δp of preceding Eq. (8) is a singularity approximated λ , which is “not” a band limited $\Delta \lambda$ of physical reality.

As we look back at the subspace that Heisenberg’s principle developed from, it was an “inadvertently” timeless ($t = 0$) subspace as shown in **Figure 8**. Aside the timeless ($t = 0$) subspace, it is the uncertainty mainly due to diffraction limited observation, which is a “secondary cause” by human intervention, but not due to naturally change with time. This is similar to entropy theory of Boltzmann [16]: entropy increases naturally with time within an enclosed subspace. In which we see that uncertainty should be increasing with time, without human intervention. As I have noted, without time, there would be no entropy and no uncertainty.

Nevertheless, momentum error Δp and position error Δx are mutually “coexisted.” In principle they can be traded. But the trading cannot without constraint, since time is a dependent forward variable. But Heisenberg uncertainty; Δp and Δx are “not” mutually dependent, since his position error Δx is due to diffraction limited observation, which is nothing to do with time. For which it poses a physical “inconsistency” within our universe, although Heisenberg principle has been widely used without any abnormality. But it is from the “physical consistency” standpoint, Heisenberg’s position error Δx was based on diffraction limited observation has “nothing” to do with time. And also added and his momentum error Δp was based on singularity wavelength λ which is “not” a band limited reality.

Yet, uncertainty principle can be made temporal ($t > 0$), similar to entropy theory of Boltzmann. For which we have a “law of uncertainty” as stated: uncertainty of an isolated particle increases naturally with time. Or more specific: uncertainty of an isolated particle within an isolated subspace, increases with time and eventually reaches to a maximum amount within the isolated subspace. For which we see that there it exists a profound connection between uncertainty and entropy.

7. Temporal ($t > 0$) uncertainty

Since it is our universe governs the science and it is not the science governs our universe. Therefore, every principle within our universe has to comply with the temporal ($t > 0$) condition within our universe; otherwise, the principle cannot be existed within our universe. Which includes all the laws, principles and theories; such as Maxwell’s Electro-Magnetic theory, Boltzmann’s entropy theory, Einstein’s relativity theory, Bohr’s atomic model, Schrödinger’s superposition principle, and others. Of which uncertainty principle cannot be the exception?

Let us now assumed a temporal ($t > 0$) particle $m(t)$ is situated within a temporal ($t > 0$) subspace as depicted in **Figure 9**. Strictly speaking any particle existed within a temporal subspace must be a temporal ($t > 0$) particle; otherwise, the particle cannot be existed within our temporal ($t > 0$) universe.

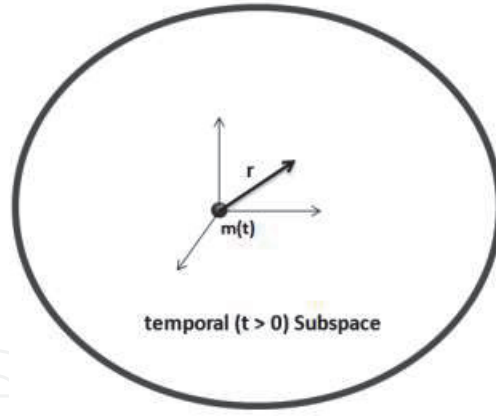


Figure 9.

A temporal ($t > 0$) particle $m(t)$ within a temporal ($t > 0$) subspace. r is the radial direction. Note: it is a “physical realizable” paradigm, since a temporal particle $m(t)$ is embedded within a temporal subspace.

For simplicity, we further assume $m(t)$ has no time or “pseudo-timeless,” after all science is a law of approximation. The same as Heisenberg’s assumption, the particle is a photonic particle (i.e., a photon), as from wave particle-duality standpoint [17] momentum of a photon is given by:

$$p = h/\lambda = h \nu/c \quad (9)$$

where h is the Planck’s constant, λ is the wavelength and ν is the frequency of the photonic particle. As I have mentioned earlier, within our universe any radiator has to be band limited. Thus the momentum error is naturally due changes of band-width $\Delta\nu$, as given by;

$$\Delta p = h \Delta\nu/c \quad (10)$$

Instead of using a cone of light as Heisenberg had postulated. By virtue of time-bandwidth product $\Delta\nu \Delta t = 1$, $\Delta\nu$ “decreases” with time. For which position error can be written as:

$$\Delta r = c \Delta t \quad (11)$$

where r is the radial distance, we have the following uncertainty relationship;

$$\Delta p \Delta r = [h \Delta\nu/c] [c \Delta t] = h \Delta\nu \Delta t \quad (12)$$

In which we see that; $\Delta\nu \cdot \Delta t$ is the “time-bandwidth” product. As we imposed the optimum energy transfer criterion on time-bandwidth product [12], as given by:

$$\Delta\nu \Delta t \geq 1 \quad (13)$$

Since lower bound for a photonic particle is limited by Planck’s constant, we have the following equivalent form as given by:

$$\Delta E \Delta t \geq h \quad (14)$$

Nevertheless, in view of Eq. (13), momentum uncertainty principle can be shown as:

$$\Delta p \Delta r \geq h, t > 0 \quad (15)$$

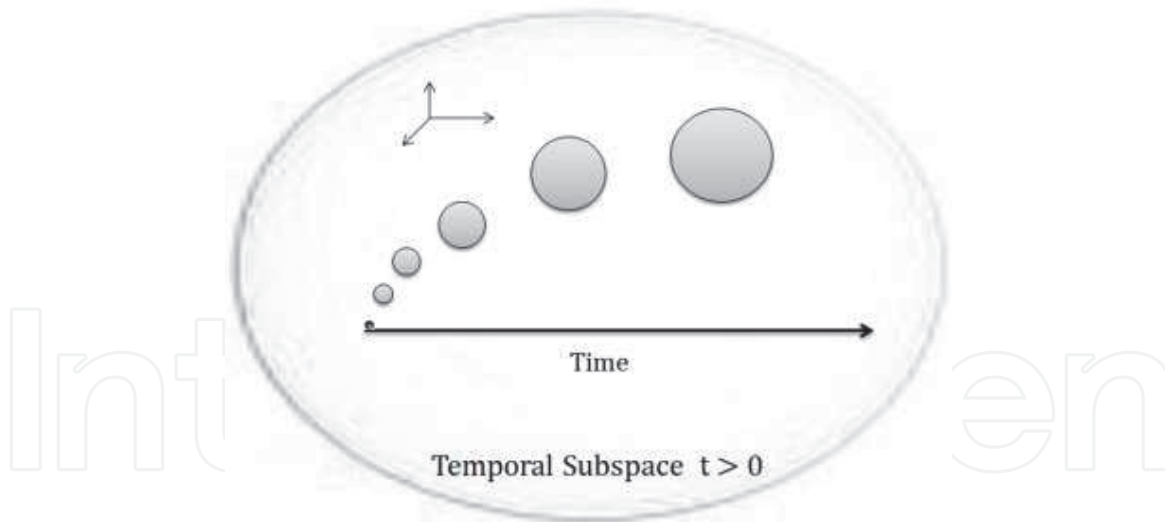


Figure 10.
 Position error Δr (i.e., sphere of Δr) enlarges naturally with time within a temporal ($t > 0$) subspace: Δr represents a position error of the particle, at various locations as time moves constantly.

where ($t > 0$) denotes that uncertainty principle is complied with the temporal ($t > 0$) condition within our universe. In view of either conservation of momentum or energy conservation, we see that position error Δr increases naturally with time. Which shows that momentum error Δp “decreases” naturally with bandwidth Δv , as in contrast with Heisenberg’s assumption; momentum error Δp has “nothing” to do with the changes of Δv . This is precisely the “law of uncertainty” as I have described earlier, uncertainty of an isolated particle increases naturally with time.

Since the increase in position error Δr is due to time, it must be due to the dynamic expansion of our universe [5]. For example, as the boundary of our universe constantly expanding at the speed of light, by virtue of energy conservation, it changes every dynamic aspect within our universe. As time moves on naturally, the larger the position error Δr increases with respect to that starting point, as illustrated in **Figure 10**.

Therefore we see that uncertainty is “not” a static process it is a temporal ($t > 0$) dynamic principle, as in contrast with Heisenberg’s position error Δr is “independent” with time and his momentum error Δp is “independent” with Δv . In which we see that if there is no time, there is no uncertainty and no probability. Nevertheless, each of the uncertainty unit or cell, such as $(\Delta p, \Delta x)$, $(\Delta E, \Delta t)$ and $(\Delta v, \Delta t)$ is self-contained. In other words, ΔE and Δt are coexisted which they can be bilateral traded, but under the constraint of time as a forward moving dependent variable. In other words, if a section of Δt has been used, we cannot get the “same” section back, but can exchange for a different section of Δt . In which we see that we can trade for a narrower Δt with a wider ΔE or wider Δt with a narrower ΔE . But we “cannot” trade Δt for ΔE , since Δt is a real dependent variable has “no” substance to manipulate.

8. Certainty principle

One of the important aspects of “temporal uncertainty” is that subspace within our universe is a temporal ($t > 0$) uncertain “subspace.” In other words, any subspace is a temporal ($t > 0$) stochastic subspace, such that the dynamic behavior of the subspace changes “dependently” with time. In which any change within our universe has a profound connection with the constant expanding universe. In which we have shown that uncertainty increases naturally with time, even though without

any other perturbation or human intervention. Similar to the myth of Boltzmann's entropy theory [16], entropy increases naturally with time within an enclosed subspace, which has been shown is related to the expanding universe [5].

Similarly, there is a profound "connection" between coherence theory [18] and "certainty" principle as I shall address. Nevertheless, it is always a myth of coherence, as refer to **Figure 11**, where coherence theory can be easily understood by Young's experiment. In which degree of coherence can be determined by the "visibility" equation as given by:

$$\nu = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \quad (16)$$

where I_{\max} and I_{\min} are the maximum and minimum intensities of the fringes. But the theory does not tell us where the physics comes from. For which, it can be understood from "certainty principle," as I shall address.

It is trivial that if there is an uncertainty principle, it is inevitable not to have a certainty principle. This means that, as photonic particle we are looking for is "likely" to be found within a "certainty" subspace. Since "perfect certainty" (or absolute uncertainty) occurs at $t = 0$, which is a timeless ($t = 0$) virtual subspace not exist within our universe. Nevertheless, "certainty principle" can be written in the following equivalent forms;

$$\Delta p \Delta r < h, (t > 0) \quad (17)$$

$$\Delta E \Delta t < h, (t > 0) \quad (18)$$

$$\Delta \nu \Delta t < 1, (t > 0) \quad (19)$$

where ($t > 0$) denotes that equation is subjected to temporal ($t > 0$) constrain. In view of the position error Δr in Eq. (17), it means that it is "likely" the photonic particle can be found within the certainty subspace. Since the size of the subspace is limited by Planck constant h , it is normally used as limited boundary "not" to be violated. Yet within this limited boundary, certainty subspace had been exploited by Dennis Gabor for his discovery of wave front reconstruction in 1948 [19] and as well it was applied to synthetic aperture radar imaging in 1950s [20].

Since the size of certainty subspace is exponentially enlarging as the position error Δr increases, for which the "radius" of the certainty sub-sphere is given by:

$$\Delta r = c \Delta t = c / (\Delta \nu) \quad (20)$$

where c is the speed of light, Δt is the time error, and $\Delta \nu$ is the bandwidth of a light source or a quantum leap $h\nu$. Thus we see that position error Δr is inversely proportional to bandwidth $\Delta \nu$, as plotted in **Figure 12**.

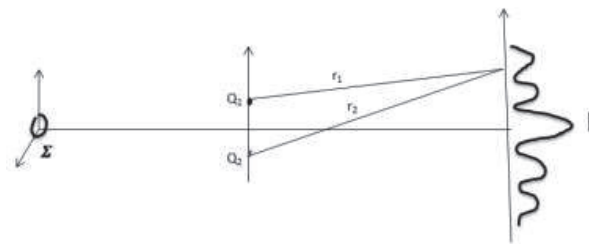


Figure 11.

Young's experiment. Σ represents an extended monochromatic source, Q_1 and Q_2 are the pinholes, and " I " represents the irradiance distribution.

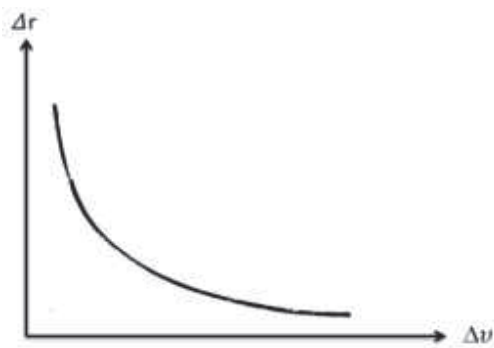


Figure 12.
A plot of position error Δr versus bandwidth $\Delta \nu$.

In view of this plot, we see that when bandwidth $\Delta \nu$ decreases, a larger certainty subspace enlarges “exponentially” since the volume of the subspace is given by:

$$\text{Certainty subspace} = (4\pi/3)(\Delta r)^3 \tag{21}$$

In which we see that, a very “large” certainty subspace can be realized within our universe which is within limited Planck’s constant h as depicted in **Figure 13**, where we see a steady state radiator A emits a continuous band limited $\Delta \nu$ electro-magnetic wave as illustrated. A “certainty subspace” with respect to an assumed “photonic particle” A for a give Δt can be defined as illustrated within $r = c \Delta t$, where $\Delta t = 1/\Delta \nu$. In other words, it has a high degree of certainty to relocate particle A within the certainty subspace. Nevertheless, from electro-magnetic disturbance standpoint; within the certainty subspace provides a high “degree of certainty” (i.e., degree of coherence) as with respect to point A .

As from coherence theory stand point, any other disturbances away from point A but within the certainty subspace (i.e., within $r < c \Delta t$) are mutual coherence (i.e., certainty) with respect A ; where $r = c \Delta t$ is the radius of the “certainty subspace” of A . In other words, any point-pair within $d < c \Delta t$, where $\Delta t = 1/\Delta \nu$, are “mutual coherence” within a radiation subspace. On the other hand, distance

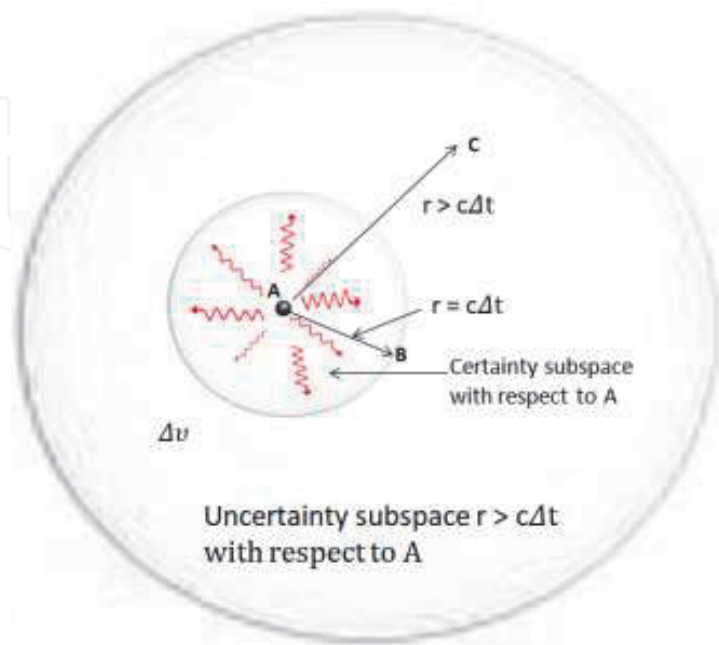


Figure 13.
A certainty subspace is embedded within uncertainty subspace. A is assumed a steady state photonic particle emits a band limited $\Delta \nu$ radiation, r is the radius with respect to the emitter A ; and B represents the boundary of certainty subspace of A .

greater than $r > c \Delta t$ from point A is a mutual “uncertainty” subspace with respect to A. In other words, any point-pair distance is larger than $d > c \Delta t$ within the radiation space are mutually “incoherent.” In which we see that; it is more “unlikely” to relocate a photonic particle, after it has been seen at point A, within a “certainty subspace.”

Since certainty subspace represents a “global” probabilistic distribution of a particle’s location as from particle physicists stand point, which means that it is “very likely” the particle can be found within the certainty subspace. In which we see that a postulated particle firstly is temporal ($t > 0$) or has time; otherwise, there is no reason to search for it. Then after it has been proven it is a temporal (i.e., $m(t)$) particle, it is more favorable to search the particle, within a certainty subspace.

The essence of “wave-particle duality” is a mathematical simplistic assumption to equivalence a package of wavelet energy as a particle in motion from statistical mechanics stand point, in which the momentum $p = h/\lambda$ is conserved. However one should “not” treated wave as particle or particle as wave. It is the package of wavelet energy “equivalent” to a particle dynamics (i.e., photon), but they are “not” equaled. Similar to Einstein’s energy equation, mass is equivalent to energy and energy is equivalent to mass, but they are not equaled. Therefore as from energy conservation, bandwidth Δv “decreases” with time is the physical reality instead of treating a package of wavelet as a particle (i.e., photon), which was due to the classical mechanics standpoint, treats quantum leap momentum $p = h/\lambda$. In which we see that photon is a “virtual” particle although many quantum scientists have been regarded photon as a physical particle?

We further note that any point-pair within the certainty subspace exhibits some degree of certainty or coherence, which has been known as “mutual coherence” [18]. And the mutual coherence can be easily understood as depicted in **Figure 14**, in which a steady state band limited Δv electro-magnetic wave is assumed existed within a temporal ($t > 0$) subspace. As we pick an arbitrary disturbance at point B, a certainty subspace of B can be determined within $r \leq c \Delta t$, as shown in the figure.

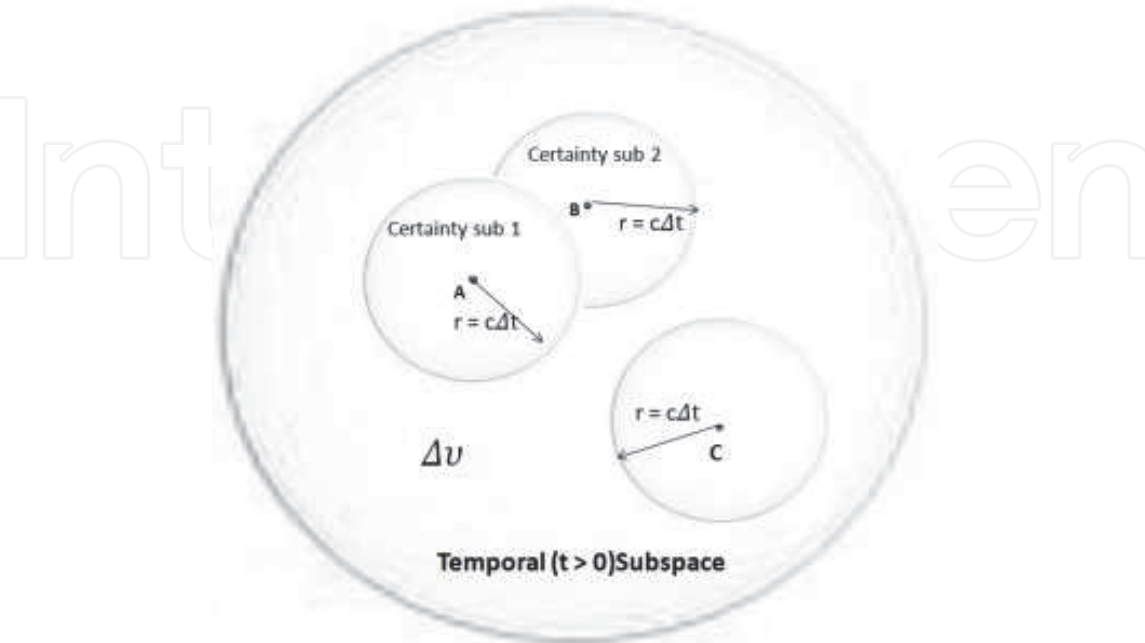


Figure 14. Various certainty subspace configurations, as with respect to various disturbances within a steady state band limited Δv electro-magnetic environment within a temporal ($t > 0$) subspace.

This means that any point disturbance within in the certainty subspace has a strong certainty (or coherence) with respect to point B disturbance. Similarly if we pick an arbitrary point A, then a certainty subspace of A can be defined as illustrated in the figure, of which we see that a portion is overlapped with certainty subspace of B. Any other disturbances outside the corresponding subspaces of certainty A, B, and C are the uncertainty subspace. It is trivial to see that a number of configurations of certainty subspaces can be designed for application. In which we see that multi wavelengths, such as $\Delta\nu_1$, $\Delta\nu_2$, and $\Delta\nu_3$, can also be simultaneously implemented to create various certainty subspace configurations, such as for multi spectral imaging or information processing application.

One of the commonly used for producing certainty subspaces for complex wave front reconstruction is depicted in **Figure 15** [21]. In which we see that a band limited $\Delta\nu$ laser is employed, where a beam of light is split-up by a splitter BS. One beam B_2 is directly impinging on a photographic plate at plane P and other beam B_1 diverted by a mirror and then is combined with beam B_2 at the same spot on the photographic plate P. It is trivial to know that if the difference in distances between these two beams is within the certainty subspace, then B_1 and B_2 are “mutually” coherence (or certainty); otherwise, they are mutually incoherence (or uncertain). In which we see that the distance between B_1 and B_2 is required as given by:

$$|d_1 - d_2| < c \Delta t = c / \Delta \nu \tag{22}$$

where d_1 and d_2 are the distances of beam B_1 and B_2 , respectively, from the splitter BS. In which we see that radius of certainty subspace of BS is written by;

$$\Delta r = |d_1 - d_2| < c \Delta t = c / \Delta \nu \tag{23}$$

where $|d_1 - d_2| = c / \Delta \nu$ is the “coherent length” of the laser. In which we see that by simply reducing the bandwidth $\Delta\nu$, a larger certainty subspace can be created within a temporal ($t > 0$) subspace.

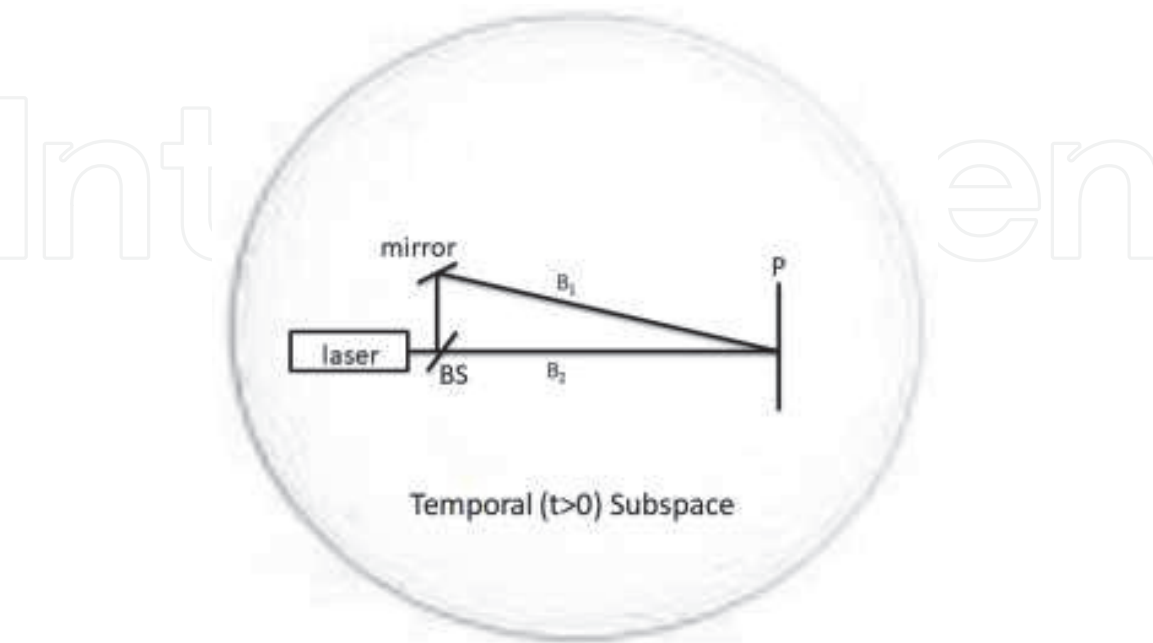


Figure 15.
An example of exploiting certainty subspace for wave front reconstruction. BS, beam splitter; P, photographic plate.

9. Essence of certainty principle

Since every substance or subspace within our universe was created by an amount of energy ΔE and a section of time Δt [i.e., $(\Delta E, \Delta t)$], any changes of ΔE changes the size of certainty subspace Δr . This is a topic that astrophysicists may be interested. Similarly to particle physicists, subatomic particle has to be temporal ($t > 0$); otherwise, the particle must be a virtual particle cannot exist within our universe. Secondly, it is more “likely” a temporal ($t > 0$) particle to be found within its certainty subspace; otherwise, it will be searching a timeless ($t = 0$) particle “forever” within our temporal ($t > 0$) universe. In view of the certainty unit: ΔE and Δt are mutually coexisted in which time is a forward dependent variable. Any changes of ΔE can “only” happen with an expenditure of a section time Δt , but it “cannot” change the speed of time. Since the energy is “conserved,” Δt is a section of time required to have the amount of ΔE within a certainty unit of $(\Delta E, \Delta t)$. In other words, ΔE and Δt can be traded; for example, a wider variance of ΔE is traded for a narrower Δt .

Nevertheless, time has been treated as an “independent” variable for decades, as normally assumed by scientists. But whenever a section of time Δt has been used, it is not possible to bring back the “original” moment of Δt , even though it is possible to reproduce the same section of Δt . This similar as we reconstructed a damaged car, but we cannot bring back the “original” car that has been crashed. And this is precisely the “price of time” to pay for everything within our universe. Then my question is that if time is a forward dependent variable with respect to its subspace, how can we “curve” the space with time? Similarly, we are coexisted with time, how can we get back the moment of time that has passed by?

Since certainty subspace changes with bandwidth $\Delta \nu$ as illustrated in **Figure 16**, in which we see that as bandwidth $\Delta \nu$ decreases a very large certainty subspace can be created within our universe as depicted in **Figure 16(a)–(c)**.

High resolution observation requires shorter wavelength but shorter wavelength inherently has broader bandwidth $\Delta \nu$ that creates a smaller certainty subspace, which can be used for high resolution wave front reconstruction [21]. On the other hand, for a larger certainty subspace, it required a narrower bandwidth of $\Delta \nu$ which has a larger certainty subspace for exploitation, such as applied to side looking radar imaging [20]. In which we see that the size of the certainty subspace can be manipulated by the bandwidth $\Delta \nu$ as will be shown in the following:

Since narrower bandwidth $\Delta \nu$ offers a huge certainty subspace that can be exploited for long distance communication, in which I have found that the certainty subspace is “in fact” the coherence subspace as I have discussed in the preceding. In other words, within a certainty subspace it exhibits a “point-pair certainty” or coherent property among them as illustrated in **Figure 17**. In other words, it has a

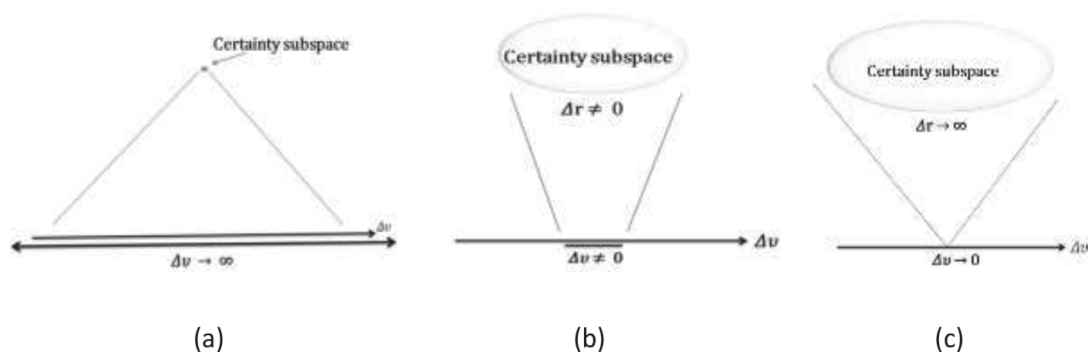


Figure 16.

Size of certainty subspace enlarges rapidly as band width $\Delta \nu$ narrows. (a) shows a very small size of certainty subspace as the result of $\Delta \nu$ approaching to very wide. (b) shows the size of certainty subspace reduces as $\Delta \nu$ continues to reduce. And (c) shows a huge size certainty subspace can be created as band width $\Delta \nu$ narrows.

high degree of certainty within a certainty subspace between points. This means that, if a photonic particle as it has been started at point u_1 , then it has a high degree of certainty that the particle to be found at the next instantly Δt at u_2 , since distance is time within a temporal ($t > 0$) subspace.

For example, given any two arbitrary complex disturbances $u_1(r_1; t)$ and $u_2(r_2; t)$, as long the separation between them is shorter than the radius Δr of the certainty subspace as given by:

$$d \leq c/(\Delta v) \tag{24}$$

the disturbances between $u_1(r_1; t)$ and $u_2(r_2; t)$ are “certainly” related (or mutually coherence). For which the “degree of certainty” (i.e., degree of coherence) between u_1 and u_2 can be determined by the following equation:

$$\gamma_{12}(\Delta t) = \frac{\Gamma_{12}(\Delta t)}{\Gamma_{11}(0)\Gamma_{22}(\Delta 0)} \tag{25}$$

where, “mutual certainty” (or mutual coherence) function between u_1 and u_2 can be written as:

$$\Gamma_{12}(\Delta t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T u_1(t; r_1) u_2^*(t - \Delta t; r_1) dt \tag{26}$$

Similarly, the respective “self certainty” (or self coherence) functions are, respectively, given by:

$$\Gamma_{11}(\Delta t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T u_1(t; r_1) u_1^*(t - \Delta t; r_1) dt \tag{27}$$

$$\Gamma_{22}(\Delta t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T u_2(t; r_2) u_2^*(t - \Delta t; r_2) dt \tag{28}$$

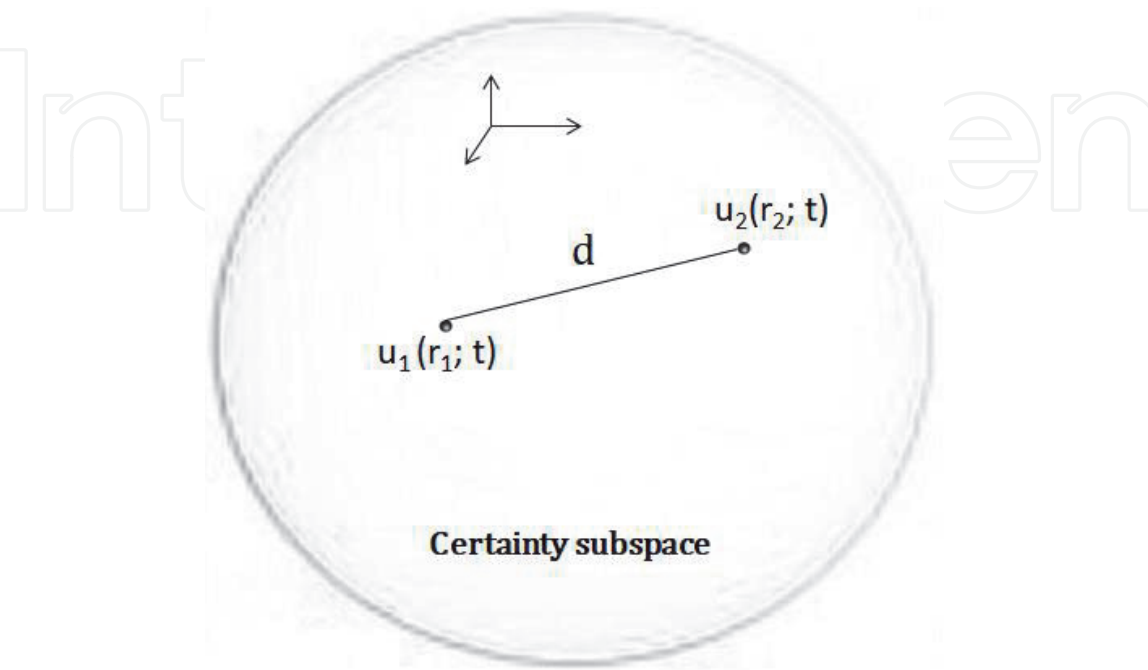
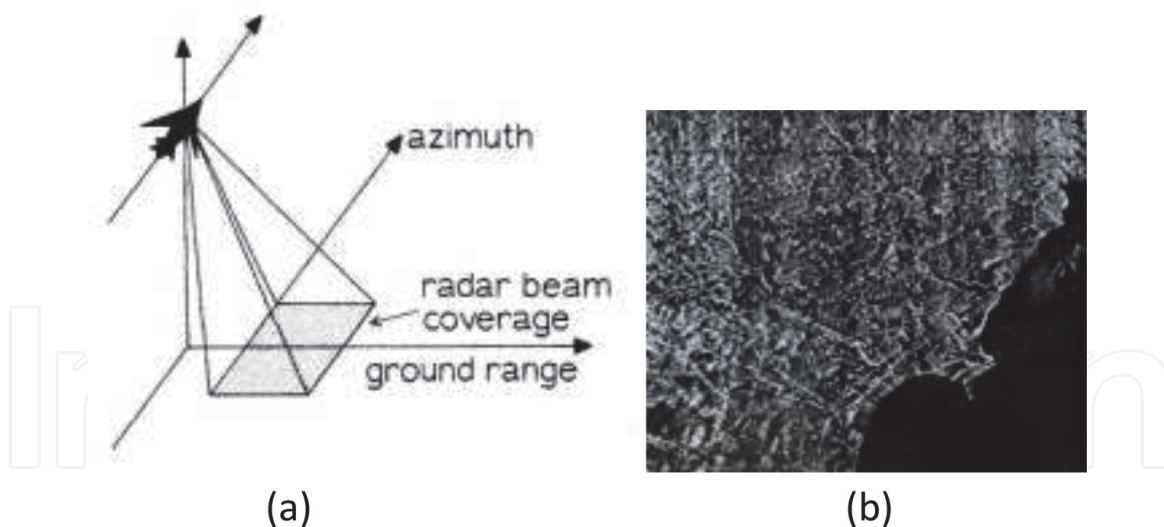


Figure 17.
Mutual certainty within a certainty subspace. $u_1(r_1; t)$ and $u_2(r_2; t)$ represent two arbitrary disturbances separated at distance d .

**Figure 18.**

Side-looking radar imaging within certainty subspace: (a) shows a side-looking radar scanning flight path; (b) shows an example of synthetic aperture radar imagery.

One of the interesting applications for certainty principle must be to synthetic aperture radar imaging as I have mentioned earlier is shown in **Figure 18**. In which we see an aircraft carried a side looking synthetic radar system shown in **Figure 18(a)**, emitting a sequence of radar pulses scanned across the flight path of the terrain. The returned pulses are combined with local radar pulses, which are “mutual coherence” (i.e., high degree of certainty), to construct a recording format that can be used for imaging the terrain, for which a synthetic imagery is shown in **Figure 18(b)**. In which we see a variety of scatters, including city streets, wooded areas, and farmlands and lake with some broken ice floes can also be identified on the right of this image. Since microwave antenna has a very narrow carrier bandwidth (i.e., $\Delta\nu$) and its certainty radius (i.e., $d = c \cdot \Delta t$) or the coherence length can be easily reached to hundreds of thousand feet. In other words, a very large certainty subspace for complex-amplitude imaging (or for communication) can be realized.

Finally I would address again within the certainty unite (Δp , Δr) [i.e., equivalently for (ΔE , Δt) and ($\Delta\nu$, t) unit] can be mutually traded. But it is the trading of Δp for Δr (or ΔE for Δt and $\Delta\nu$ for Δt) is physically visible, since time is not a physical substance but a forward constant dependent “variable” that we “cannot” manipulate. For which we see that the “section” of Δt that has been “used” cannot get it back. In other words, we can get back the same amount Δt , but “not” the same moment of Δt , that has been expensed. As I have shown earlier, everything within our universe has a price, an amount of energy ΔE , and a section of time Δt . Aside ΔE we can physically change, it is the moment of time Δt which has been expensed that is “preventing” us to get it back, because that moment of Δt is the “same moment” of time of our temporal ($t > 0$) universe that has been passed. And this is the “moment of time” Δt within our temporal ($t > 0$) universe, once the “moment” passes by and we can never able to get it back.

10. Conclusion

In conclusion, I would point out that quantum scientists used amazing mathematical analyses added with their fantastic computer simulations provide very convincing results. But mathematical analyses and computer animations are virtual

and fictitious, and many of their animations are “not” physically real; for example such as the “instantaneous and simultaneous” superimposing principle for quantum computing is “not” actually existed within our universe. One of the important aspects within our universe is that, one cannot get something from nothing there is always a price to pay, an amount of energy ΔE and a section of time Δt . The important is that they are not free!

Since any science existed within our universe has time or temporal ($t > 0$), in which we see that any scientific law, principle, theory, and paradox has to comply with temporal ($t > 0$) aspect within our universe; otherwise, it may not be science. As we know that science is mathematics but mathematics is not equaled to science. In which we have shown that any analytic solution has to be temporal ($t > 0$); otherwise, it cannot be implemented within our universe. Which includes all the laws, principles, and theories have to be temporal ($t > 0$)?

Since it is our universe governs our science and it is not our science changes our universe. In which we have shown every hypothetical science, law, principle, and theory has to be temporal ($t > 0$); otherwise, they are virtual and fictitious which cannot exist within our universe. Since time is a dependent variable coexisted with space, we have concluded that time is not an illusion but real, since we are real. As in contrast with most of the scientists, they believe that time is an independent variable and some of them even believe that time is an illusion?


Uncertainty principle is one of the most fascinating principles in quantum mechanics, yet Heisenberg principle was based on diffraction limited observation, it is not due to the nature of time or temporal ($t > 0$) nature of our universe. We have shown uncertainty increases with time, as in contrast with Heisenberg’s principle. We have also introduced a certainty principle, in which we have shown high degree of certainty within a certainty subspace can be exploited. For which we have shown that certainty subspace can be created within our temporal subspace for complex amplitude communication and imaging. Yet the important aspect of this chapter is that it is not how rigorous the mathematics is, but it is the physical realizability of science is, since mathematics is not science.

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