Chapter

Nash Equilibrium Study for Distributed Mode Selection and Power Control in D2D Communications

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Abstract

One of the main challenges of LTE-advanced (LTE-A) is to recover the local-area services and improve spectrum effciency. In order to reach those goals technical capabilities are required. D2D is a promising techniques for the 5G wireless communications system using several applications, as: network traffic offoading, public safety, social services and applications such as gaming and military applications. In this chapter, we investigate both mode selection and distributed power control in D2D system. Indeed, the mode selection is provided while respecting a predetermined SINR threshold relative to cellular and D2D users. The amount of minimum and maximum power are then derived to fulffill the predetermined requirements, by limiting the interference created by underlaid D2D users. In order to realize our proposed power control step, a new distributed control approach is proposed using game theory tools for several cellular and D2D users. This distributed approach is based on the mode selection strategy already proposed in the previous step. Finally, simulations were established in order to compare the proposed distributed algorithm in terms of coverage probability which is based on game theory, with other conventional centralized algorithms.

Keywords: mode selection, power control, distributed, Nash equilibrium

1. Introduction

The Internet of Things (IoT) is a developing and promising innovation, which were able to revolutionize the world [1]. IoT manages low-powered gadgets, using the internet by interacting with one another. IoT interconnect "Things" and also helps in machine-to-machine (M2M) communication, which is a way of data communication between varied gadgets without human intercession [2].

IoT applications can be classified into six main categories, such as [1]: smart cities, smart business, smart homes, healthcare, security and surveillance. Regarding these different applications, several requirements should be maintained, like [2]: (1) high scalability, (2) security and privacy, (3) high capacity, (4) security and privacy, (5) energy saving, (6) reduced latency, (7) quality of service (QoS), (8) built-in redundancy, (9) heterogeneity and (10) efficient network and spectrum.

The 5G enabled IoT contains a number of key communication techniques for IoT applications, in order to make the network with faster speeds and greater accessibility. A network that reaches all over the world and is accessible to all. Since 5G technology offers greater connectivity, more and more applications for this technology are likely to come to the field. Four main categories can be cited in the this context: (1) Wireless Network Function Virtualization, (2) Architecture of 5–IoT, (3) Heterogeneous Network (HetNet) and (4) Device to Device (D2D) Communication.

In this chapter, we mainly focus on the last type especially on D2D Communication [3–8], which allows the exchange of data between user equipment without the use of the base station. The short distance communication between two devices (D2D) becomes a challenging way to transmit data, since it benefits the 5–IoT with low power consumption, load balancing and better QoS for edge users. Indeed, in IoT over 60% of applications require low power, a long battery and also wide connectivity coverage. Hence, for these reasons more light should needs to be shed on low-power wireless networks and their prospects in meeting these requirements. Integrating D2D in cellular networks poses challenges and design problems, in order to offer adequate Radio Resource Management (RRM) schemes [4–6, 9–11] and this taking into account all the constraints imposed by the different users. As has already been mentioned in the literature, RRM techniques can be classified into four groups as: (1) Mode Selection (MS): where the Mobile Station determines whether D2D candidates in the proximity of each other should communicate in direct mode using the D2D link or in cellular mode [3–5], (2) Power Control (PC): is an efficient solution to mitigate the interference for D2D underlaid cellular network, in order to improve the overall of the system [6, 9]. (3) Pairing: is a concept which exists only when D2D links are reusing cellular resources and consists on assigning one cellular uplink user (CUE) and one or more D2D uplink user (DUE) links for each resource block [18] and (4) Resource Allocation: is a process of selecting radio resources for each cellular and D2D link, this can be done jointly with MS and pairing [3, 4].

Several approaches have already been proposed in the literature in order to achieve MS and PC management, these approaches can be: (1) Centralized management: where the base station (BS) allocates directly to the designated DUE and require the knowledge of D2D links' Channel State Information (CSI) at the BS level and (2) Distributed (or decentralized) management, in which the BS informs D2D users which Resource Blocks (RBs) they can use. In this chapter, we focus on the RRM algorithms in underlay D2D communication, for both centralized and distributed MS and PC, in order to improve the overall of the system using game theory tools. In [3–7, 9, 11, 13–17, 19–22], the authors have proposed centralized and distributed PC approaches with perfect channel state information (CSI) using powerful mathematical tools, such as game theory [7–9], stochastic approximation [20], mechanism design [21] etc.

In fact, Game theory (GT) is a branch of applied mathematics that provides models and tools for analyzing situations where multiple rational users interact to achieve their goals [23, 24]. Several examples based on Wireless Communications are investigated in the literature, as in PC, congestion control, load balancing, etc. In [6], a centralized and distributed PC algorithms are developed and evaluated for a D2D underlaid cellular system using stochastic geometry. The authors in [9] have focused on maximizing the total sum-rate in an heterogeneous network (HetNet) via game theoretic approaches. The authors in [11] have proposed a distributed PC, based on an appropriate interference management scheme in D2D underlaid Cellular Network by using GT approach. An iterative distributed power allocation algorithm for the two kinds of game: pure and mixed has investigated. Distributed vs. centralized MS and PC approaches have been suggested in [25] using GT tools.

This chapter investigates both MS and PC in D2D communications using centralized and distributed approaches based on GT tools. In the proposed MS approach, the CUE and DUE list, the minimum and maximum quantities of power are derived according to CUE and DUE signal-to-interference plus-noise-ratio (SINR) thresholds. The expression of the minimum amount of power known in the literature as the Pareto power [6, 9, 11], has always been used until now without mathematical proof. We propose in this chapter to show mathematically this Pareto power, which is considered as a key of the PC process. Then, a pure strategy non-cooperative game between the two kind of users is modeled as a distributed PC approach, based on the derived minimum and maximum power, where two utility functions are investigated for both type of users. This chapter reviews the work previously published by the authors in [12]. For this, all the proofs and demonstrations of the different results stated in this chapter will not be provided since they are already explained in [12].

The structure of this chapter is given as: In Section 2, the system model of a D2D communication and CUE and DUE SINR and coverage probability expressions are defined. In Section 3, the closed form expression of both minimum and maximum amounts of power with a mathematical demonstration are provided, based on the predetermined CUE and DUE SINR thresholds. In Section 4, our proposed centralized MS and PC approaches are investigated, which consists of generalizing a classical centralized MS and PC approaches. The Section 5, outlines an iterative NE distributed power approach which is proposed for both CUE and DUE and is based on the minimum and maximum amounts of power, derived from Section 3 and on the GT tools. This proposed distributed approach aims to achieve a better compromise between different users in terms of allocated powers is presented in Section 6. Several simulations are provided in order to assess the performance of the allocation approaches of the MS and PC thus proposed in Section 7. Section 8 is followed with conclusion and future scope.

2. System model

In this section, a D2D uplink underlaid cellular network is considered illustrated in this section, which is shown in **Figure 1**. A system which is composed by a single-cell cellular network, where K_1 CUE and K_2 DUE communicate with as it is illustrated in **Figure 1**, where the BS is in the center of circular coverage area and each D2D user refers to a source-destination pair. The total number of users K is defined as $K = K_1 + K_2$. We assume that all users (CUE and DUE) are drawn in a circular disk C with radius R and are randomly distributed in the whole $\mathbb{R}^2$ and modeled as an independently homogeneous Poisson Point Process (PPP) Φ with density λ. For each kind of user (CUE or DUE) k, we denote y_k as the received signal defined as follows.

- CUE mode: if $1 \le k \le K_1$, y_k is the received signal at the CUE k from the BS.
- DUE mode: if $K_1 + 1 \le k \le K$, y_k is the received signal at the k^{th} DUE receiver from the k^{th} DUE transmitter.

Let $g_{k,i}$ denotes the instantaneous channel gain from the k^{th} transmitter to the i^{th} receiver, where $k, i \in \mathbf{K} = \{1, ..K\}$. Further, we denote $\mathbf{K_1} = \{1, ..K_1\}$ and $\mathbf{K_2} = \{K_1 + 1, ..K\}$.

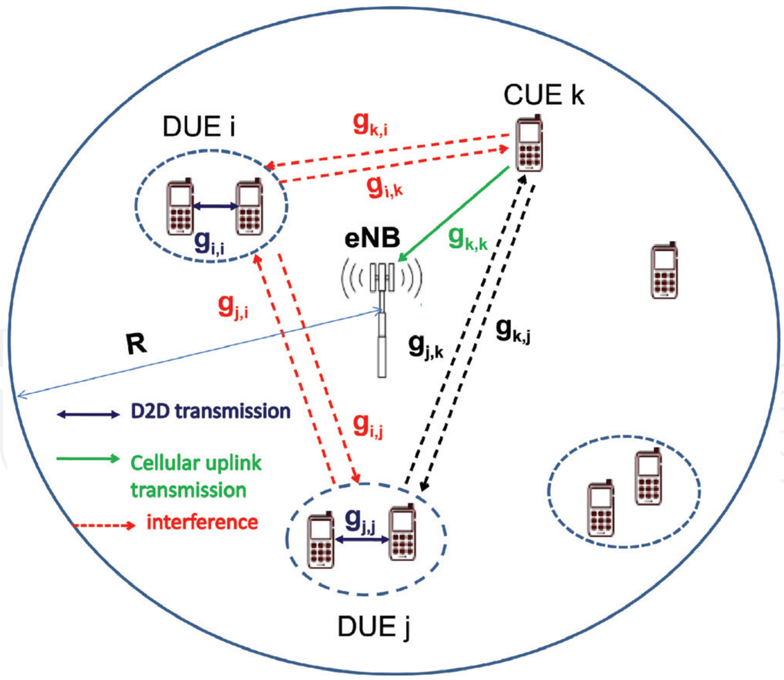

Figure 1.
System model of D2D communication.

2.1 CUE and DUE SINR expressions

In order to ensure a QoS in terms of γ_c^{th} and γ_d^{th}, as SINR thresholds of both CUE and DUE (respectively), we assume the following statement for each user k, as performance yardsticks

$$\begin{cases} \gamma_k(\mathbf{P}) = \dfrac{g_{k,k}p_k}{\sum_{i=1,i\neq k}^{K} g_{k,i}p_i + \sigma^2} \geq \gamma_c^{th}, \forall k \in \mathbf{K_1} : \text{cellular mode}, & (1.a) \\ \gamma_k(\mathbf{P}) = \dfrac{g_{k,k}p_k}{\sum_{i=1,i\neq k}^{K} g_{k,i}p_i + \sigma^2} \geq \gamma_d^{th}, \forall k \in \mathbf{K_2} : \text{D2D mode}, & (11.b) \end{cases} \tag{1}$$

where p_k is the amount of the transmit powers for the k^{th} user (CUE or DUE), σ^2 is the receiver noise power, $\mathbf{P} = \left(p_1, ..p_K\right)$ is the vector of transmit powers and $g_{k,i}$ is defined as follows

$$g_{k,i} = |h_{k,i}|^2 d_{k,i}^{-\alpha}, \tag{2}$$

where, $h_{k,i}$ and $d_{k,i}$ are respectively the distance-independent fading and the distance from the transmitter k to receiver i and α is the path loss.

Let us define for each user k (CUE and DUE), the SINR threshold γ_k^{th}, as

$$\gamma_k^{th} = \begin{pmatrix} \gamma_c^{th} \text{ if } k \in \mathbf{K_1} : \text{Cellular mode} \\ \gamma_d^{th} \text{ if } k \in \mathbf{K_2} : \text{D2D mode}. \end{pmatrix} \tag{3}$$

2.1.1 CUE and DUE coverage probabilities

In order to simplify the notations used in the chapter, we will consider vector rather than analytical expressions. According to each kind of user, we define the coverage probabilities expressions denoted as $P_{c,cov}(\mathbf{P}, \Gamma_c^{th})$ and $P_{d,cov}(\mathbf{P}, \Gamma_d^{th})$ for both CUE and DUE (respectively) as

$$P_{c,cov}(\mathbf{P}, \Gamma_c^{th}) \triangleq Prob(\Gamma_c(\mathbf{P}) \geq \Gamma_c^{th}), \tag{4}$$

$$P_{d,cov}(\mathbf{P}, \Gamma_d^{th}) \triangleq Prob(\Gamma_d(\mathbf{P}) \geq \Gamma_d^{th}), \tag{5}$$

where,

$$\Gamma_c(\mathbf{P}) = (\gamma_1(\mathbf{P}), ..\gamma_{K_1}(\mathbf{P})), \quad \Gamma_c^{th} = (\gamma_c^{th}, ..\gamma_c^{th}). \tag{6}$$

$$\Gamma_d(\mathbf{P}) = (\gamma_{K_1+1}(\mathbf{P}), ..\gamma_K(\mathbf{P})), \quad \Gamma_d^{th} = (\gamma_d^{th}, ..\gamma_d^{th}). \tag{7}$$

3. The minimum and maximum amount of power: $(\mathbf{P_{min}}, \mathbf{P_{max}})$

This section investigates the study of the existence of the two minimum and maximum powers $\mathbf{P_{min}}$ and $\mathbf{P_{max}}$, necessary to verify the constraints imposed by the previous system (1). First, based on this system (1), the minimum power $\mathbf{P_{min}}$ is derived, already known in the literature under the name of Pareto Power. Second, by limiting the quantity of power by a quantity, which we denote $\mathbf{P_{max}}$ from $\mathbf{P_{min}}$, we make sure more that the system (1) remains satisfied as long as we are in the power range $[\mathbf{P_{min}}, \mathbf{P_{max}}]$.

To do this, we start by providing another vector form of the (1) system to build this Pareto power.

3.1 Vector form of system

Let us take into consideration the following definition and proposition (1),

Definition 1. If $\mathbf{A} = (a_{i,j})_{1 \leq i,j \leq K}$ and $\mathbf{B} = (b_{i,j})_{1 \leq i,j \leq K}$ are two matrices, then we define

$$\mathbf{A} \geq \mathbf{B} \Leftrightarrow a_{i,j} \geq b_{i,j}, \quad 1 \leq i,j \leq K. \tag{8}$$

Proposition 1. The previous Eq. (1) can be written compactly in the following vector form as

$$(\mathbf{I}_K - \mathbf{F})\mathbf{P} \geq \mathbf{b}, \tag{9}$$

where $\mathbf{I}_K$ denotes the identity matrix of order K, $\mathbf{F} = \left(f_{k,i}\right)_{k,i \in \mathbf{K}}$ and $\mathbf{b} = (b_k)_{k \in \mathbf{K}}$, $\forall k, i \in \mathbf{K}$, are defined as below [11, 12].

$$f_{k,i} = \begin{cases} \dfrac{g_{k,i}}{g_{k,k}} \gamma_k^{th} & \text{if } k \neq i \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad b_k = \frac{\sigma^2 \gamma_k^{th}}{g_{k,k}}. \tag{10}$$

3.2 Power lower band $\mathbf{P_{min}}$

All previous works [6, 8, 9, 11] have dealt with the resolution of the problem presented in (9) by using a minimum power $\mathbf{P_{min}}$ without proof, which is defined as

$$\mathbf{P_{min}} = (\mathbf{I_K} - \mathbf{F})^{-1}\mathbf{b}, \tag{11}$$

The authors in [6, 8, 11] have shown that: if $\rho(\mathbf{F}) < 1$ then the matrix $(\mathbf{I_K} - \mathbf{F})$ is invertible. Next, we propose to suggest another sufficient condition on the matrix $\mathbf{F}$, which guarantees the existence of the inverse of the matrix $(\mathbf{I_K} - \mathbf{F})$, in order to derive the quantity of minimum power $\mathbf{P_{min}}$, already given in Eq. (11).

To do this, we propose theorems, definitions and propositions in order to outline all the necessary steps which allow to build this sufficient condition. Obviously, to make reading easier, all the demonstrations relating to these theorems and propositions are already detailed in [11, 12].

Theorem 1. *We assume that* $\rho(\mathbf{F}) < 1$, *the following statement is true*

$$(\mathbf{I_K} - \mathbf{F})\mathbf{P} \geq \mathbf{b} \Rightarrow \mathbf{P} \geq \mathbf{P_{min}} = (\mathbf{I_K} - \mathbf{F})^{-1}\mathbf{b}. \tag{12}$$

Hence, if $\rho(\mathbf{F}) < 1$ *then the minimum power* $\mathbf{P}_{min}$ *defined in Eq. (11) exists and we can consider the following notation*

$$\mathbf{P_{min}} = \left(\underbrace{P_{min}(1), ..P_{min}(K_1)}_{\mathbf{P_{c,\,min}}}, \underbrace{P_{min}(K_1+1), \;...P_{min}(K)}_{\mathbf{P_{d,\,min}}} \right). \tag{13}$$

We can note in another way

$$\mathbf{P_{min}} = (\mathbf{P_{c,\,min}}, \mathbf{P_{d,\,min}}). \tag{14}$$

Proposition 2. Let consider the following iterative power process, relative to each iteration i, as [26].

$$\mathbf{P}(i+1) = \mathbf{F}\mathbf{P}(i) + b. \tag{15}$$

Hence,

$$\lim_{i \to +\infty} \mathbf{F}^i = 0 \Rightarrow \sum_{i=0}^{+\infty} \mathbf{F}^i \mathbf{b} = (\mathbf{I_K} - \mathbf{F})^{-1}\mathbf{b}. \tag{16}$$

3.3 Power upper band $\mathbf{P_{max}}$

To let $\mathbf{P_{max}}$ greater than $\mathbf{P_{min}}$ for all users, it is proposed in this paragraph to build a quantity of power $\mathbf{P_{max}}$ from $\mathbf{P_{min}}$, in order to guarantee the conditions required by the users already depicted in Eq. (11). The two maximum power quantities dedicated to the different CUE and DUE users, denoted as $\mathbf{P}_{c,max}$ and $\mathbf{P}_{d,max}$ (respectively), are defined as follows

$$\begin{cases} \mathbf{P}_{c,max} = \mathbf{P}_{c,min} + \Delta\mathbf{P}_c \\ \mathbf{P}_{d,max} = \mathbf{P}_{d,min} + \Delta\mathbf{P}_d, \end{cases} \tag{17}$$

where, $\Delta\mathbf{P}_c$ and $\Delta\mathbf{P}_d$ are the power margins allocated to different users CUE and DUE (respectively), to ensure that both $\mathbf{P}_{c,max}$ and $\mathbf{P}_{d,max}$ remain greater than $\mathbf{P}_{c,min}$ and $\mathbf{P}_{d,min}$ (respectively). Almost all previous work [6, 8, 9, 11] carried out in this context proposes an amount of powers not to be exceeded for both the CUEs and the DUEs. Since these maximum powers quantities may not verify the criteria already mentioned in the system (1), it is proposed in this chapter to guarantee this condition, by assuming that the power $\mathbf{P_{max}}$ remains always greater than $\mathbf{P_{min}}$. The difference between the two power quantities $\mathbf{P_{max}}$ and $\mathbf{P_{min}}$ is denoted by $\Delta\mathbf{P}_c$ for the CUEs and by $\Delta\mathbf{P}_d$ for the DUEs.

Likewise, we consider the following notation

$$\mathbf{P_{max}} = \left(\underbrace{P_{max}(1), ..P_{max}(K_1)}_{\mathbf{P_{c,\,max}}}, \underbrace{P_{max}(K_1+1), \; ... P_{max}(K)}_{\mathbf{P_{d,\,max}}} \right). \tag{18}$$

Hence,

$$\mathbf{P}_{max} = (\mathbf{P}_{c,max}, \mathbf{P}_{d,max}). \tag{19}$$

4. On the proposition of a centralized MS and PC approaches

This section investigates centralized MS and PC approaches, which aims to select CUE and DUE from a predetermined list and to minimize the consumed amount of power, in order to satisfy the QoS depicted in Eq. (1). A centralized approach is proposed in this section, which is a generalized version of the algorithm CPCA (denoted GCPCA) to more than one CUE.

The condition assumed during the MS process is $\rho(\mathbf{F}) < 1$. Thus only the users who check this last condition are retained in the final list. Then, the minimum power $\mathbf{P_{min}}$ is allocated to the different types of users (CUE and DUE) based on this selection criterion, in order to optimize the amount of power.

4.1 Proposed generalized centralized power control algorithm (GCPCA)

Unlike the CPCA algorithm which is based on a MS relating to a system containing only one CUE, the GCPCA (see algorithm 1) generalizes this latter for several CUEs, based on the same condition $\mathbf{K_1} \geq 1$. In fact, this assumption is more realistic and illustrates a more real case.

As shown in step 1 from algorithm 1, we first test if the matrix $\mathbf{F}^l$ (relative to the iteration l), verifies the condition $\rho(\mathbf{F^l}) < 1$. If this is true, the Pareto power $\mathbf{P_{min}}$ already defined in the Eq. (11) is assigned to admitted users, as the steps 5 and 6 indicate. Otherwise, we select the $\hat{k}$-th user transmitter (CUE or DUE) who can increase the maximum of interference power compared to other receivers, as shown in step 2. Mathematically, this results in finding user $\hat{k}$, such as: $\hat{k} = argmax_k \|f_k^l\|_2$.

Thus, this user $\hat{k}$ will now be eliminated from the list of users admitted into the system, as is confirmed at step 3.

his most annoying user elimination strategy is repeated until the constraint is verified. Finally, the matrix obtained satisfies the sufficient condition $\rho(\mathbf{F^l}) < 1$, for the existence of the pareto power $\mathbf{P_{min}}$ (see step 5).

All these steps are more detailed in algorithm 1.

Algorithm 1: Proposed GCPCA

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initialisation: F^l for $l = 0$, for all active CUE and DUE. [Step 1]
Step 1: if $\rho(\mathbf{F}^l) < 1$, go to step 5 and 6. Otherwise, go to Step 2.
Step 2.$\hat{k} = argmax_k \|f_k^l\|_2$
Step 3. remove the $\hat{k}$-th column and row vectors of the matrix $\mathbf{F}^l$.
Step 4. update: $\mathbf{F}^{l+1} = \mathbf{F}^l$, $l = l + 1$. Go to Step 1.
Step 5. evaluate the power $\mathbf{P_{min}}$ using the equation (11).
Step 6. $\mathbf{P} = \mathbf{P_{min}}$

This GCPCA algorithm converges after an iteration number, since the condition $\rho(\mathbf{F}^l) < 1$ must each time be checked by the selected users during each iteration l. In fact, the proposition 2 developed in the previous section provides a convergence certificate of this algorithm.

The maximum power $\mathbf{P}_{max}$ deduced from the previous Eq. (17), will be useful in the next section in order to limit the powers allocated for each type of user.

5. On the proposition of a distributed PC approach based on GT

The power control problem proposed in this paper is considered as a distributed strategies non-cooperative game, where the utility functions as well as the strategies adopted by each user are defined and justified.

5.1 Proposed utility functions

Several utility functions have suggested in [3, 9, 27, 28], using a pricing coefficient to enhance both efficiency and fairness among users. The proposed CUE and DUE utility functions considered in this section are defined as follows [6, 11],

1. **CUE utility function:** The utility function $u_k(\mathbf{P})$ relative to a CUE k is defined as

$$u_k(\mathbf{P}) = -\left(\gamma_k(\mathbf{P}) - \gamma_c^{th}\right)^2, \quad \forall k \in \mathbf{K}_1. \tag{20}$$

2. **DUE utility function:** The utility function $u_k(\mathbf{P})$ relative to a DUE k is defined as

$$u_k(\mathbf{P}) := Rew_k(\mathbf{P}) - Pen_k(\mathbf{P}), \quad \forall k \in \mathbf{K}_2, \tag{21}$$

where

- The reward function $Rew_k(\mathbf{P})$, relative to the k^{th} DUE user, evaluates the payoff of the k^{th} DUE based on both γ_d^{th} and on a nonnegative weighting factor pricing coefficient a_k, as follows

$$Rew_k(\mathbf{P}) = 1 - e^{-a_k\left(\gamma_k(\mathbf{P}) - \gamma_d^{th}\right)}, \quad \forall k \in \mathbf{K}_2, \tag{22}$$

- The penalty function, $Pen_k(\mathbf{P})$, relative to the k^{th} DUE user, is defined as

$$Pen_k(\mathbf{P}) = b_k \frac{C(p_k, \mathbf{P}_{-k})}{I_k(\mathbf{P}_{-k})}, \quad \forall k \in \mathbf{K}_2 \tag{23}$$

where,

$$C(p_k, \mathbf{P}_{-k}) = \sum_{j=1, j\neq k}^{K} p_k g_{k,j} \text{ and } I_k(\mathbf{P}_{-k}) = \sum_{j=1, j\neq k}^{K} p_j g_{k,j} + \sigma^2, \tag{24}$$

and b_k is a constant and nonnegative weighting factor, which reflects the relative impact of the k^{th} DUE user in terms of power. We denote $\mathbf{P}_{-k}$ as the vector of transmit powers of all users other than k, defined as follows

$$\mathbf{P}_{-k} = \left(p_1, \ldots p_{k-1}, p_{k+1}, \ldots p_K\right), \quad \forall k \in \mathbf{K}. \tag{25}$$

From which it follows

$$\mathbf{P} = \left(p_k, \mathbf{P}_{-k}\right). \tag{26}$$

Afterwards, we denote the utility function vector as: $\mathbf{u}(\mathbf{P}) = (u_1(\mathbf{P}), u_2(\mathbf{P}), ..u_K(\mathbf{P}))$, where $u_k(\mathbf{P})$ can be evaluated from (20) or (21), depending on whether the user k is CUE or DUE (respectively).

5.2 Pure strategies game

We denote our game $\mathbf{G} = \{\mathbf{K}, \mathbf{P}, \mathbf{u}(\mathbf{P})\}$ of complete information between $\mathbf{K}$ players. The strategies of such game are considered to be the vector power $\mathbf{P} \in \Omega$, where Ω is given by

$$\Omega = \left\{\mathbf{P} = \left(p_1, p_2, ..p_K\right), p_k \in \Delta_k, \forall k \in \mathbf{K}\right\}, \tag{27}$$

where,

$$\Delta_k = \left[p_{min}(k), p_{max}(k)\right]. \tag{28}$$

The two powers $p_{min}(k)$ and $p_{max}(k)$ are derived from Eqs. (13) and (18) (respectively).

The NE is a strategy profile in which the strategy used by each user is at least as good a reply as any other strategy available to him to the strategies played by the other users. In this sense, to derive the NE of our proposed game, we propose in the following paragraph to study the best response relative to each user k, by improving the utility function of each user (CUE and DUE).

5.3 Nash equilibrium

Definition 2. Best-response: The best-response function $BR_k(\mathbf{P}_{-k})$ of a user k (CUE or DUE) to the profile of strategies $\mathbf{P}_{-k}$, is a set of strategies p_k^* for that user k^* should satisfy the following condition [24]

$$BR_k(\mathbf{P}_{-k}) = \left\{p_k^* \in \Delta_k^*, u_k\left(p_k^*, \mathbf{P}_{-k}\right) \geq u_k\left(p_k, \mathbf{P}_{-k}\right), \forall p_k \in \Delta_k\right\}. \tag{29}$$

Hence, each element of the best-response function $BR_k(\mathbf{P}_{-k})$ is a best-response of the user k, relative to other strategies $\mathbf{P}_{-k}$.

Definition 3. Nash Equilibrium (NE) A pure strategies NE (PSNE) $\mathbf{G} = \{\mathbf{K}, \mathbf{P}, \mathbf{u}(P)\}$, is a set of strategies $\mathbf{P}^* = \left(p_1^*, p_2^*, ...p_K^*\right)$, such that no player can unilaterally enhance its own utility [24], i.e.,

$$u_k\left(p_k^*, \mathbf{P}_{-k}^*\right) \geq u_k\left(p_k, \mathbf{P}_{-k}^*\right), \quad \forall p_k \in \Delta_k. \tag{30}$$

Hence, a PSNE $\mathbf{P}^*$ is a stable outcome of a game $\mathbf{G}$, if no user has any incentive to change its strategy.

5.4 Example: 2-users game

5.4.1 Best-responses expressions of CUE and DUE

We assume that $(K_1 = K_2 = 1)$ and $(a_1 = a_2 = a$ and $b_1 = b_2 = b)$, the expressions of the two Best-responses relatives to CUE and DUE are studied and evaluated in this section. As already explained in the preceding sections, the amount of power for each type of user k should be greater than a minimum power ($p_{c,min} = p_{min}(1)$ for CUE and $p_{d,min} = p_{min}(2)$ for DUE) and also less than a maximum power ($p_{c,max} = p_{max}(1)$ for CUE and $p_{c,max} = p_{max}(2)$ for DUE). Hence, if we denote $\mathbf{P} = (p_1, p_2)$ as the allocated power vector, where p_1 is the power relative to the CUE which should belong to Δ_1 and p_2 is the power relative to the DUE which should belong to Δ_2. We remind that Δ_1 and Δ_2 are already defined in Eq. (28).

In this case, the feasible region of the power is defined as a region where the amount of power $\mathbf{P} = (p_1, p_2) \in \Omega$ should verify the following condition

$$p_{c,\min} \leq p_1 \leq p_{c,\max} \Leftrightarrow p_1 \in \Delta_1 \tag{31}$$

$$p_{d,\min} \leq p_2 \leq p_{d,\max} \Leftrightarrow p_2 \in \Delta_2 \tag{32}$$

Proposition 3. The Best-response relative to the first user (CUE), denoted as $BR_1(p_2)$, is given by

$$BR_1(p_2) = \left\{ p_1 \in \Delta_1, p_1 = \gamma_{th}^c \left(\frac{g_{1,2}}{g_{1,1}} p_2 + \frac{\sigma^2}{g_{1,1}} \right), \quad p_2 \in \Delta_2 \right\}. \tag{33}$$

Proof. Based on the expression of the CUE utility function $u_1(p_1, p_2)$, which is defined in Eq. (20) and where $\gamma_1(\mathbf{P})$ can be derived from the Eq. (1.a), we can easily deduce the following result

$$\frac{\partial u_1(p_1, p_2)}{\partial p_1} = 0 \Rightarrow -2 \frac{g_{1,1}}{g_{1,2} p_2 + \sigma^2} \left(\frac{g_{1,1} p_1}{g_{1,2} p_2 + \sigma^2} - \gamma_{th}^c \right) = 0. \tag{34}$$

So, the expression of $BR_1(p_2)$ found in (33) is derived by deducing the expression of p_1 according to p_2 from the last equation. This completes the proof. □

Proposition 4. The Best-response relative to the second user (DUE), denoted as $BR_2(p_1)$, is given by

$$BR_2(p_1) = \left\{ p_2 \in \Delta_2, p_2 = \frac{g_{2,1} p_1 + \sigma^2}{g_{2,2}} \left[-\frac{1}{a} \log \left(\frac{b g_{1,2}}{a g_{2,2}} \right) + \gamma_{th}^d \right]^+, \quad p_1 \in \Delta_1 \right\}. \tag{35}$$

Proof. Based on the utility function expression $u_2(p_1, p_2)$ defined in Eq. (21) and on the expression of $\gamma_2(\mathbf{P})$ defined in Eq. (1), we can easily get the following expression

$$\frac{\partial u_2(p_1, p_2)}{\partial p_2} = 0 \Rightarrow \frac{a g_{2,2}}{g_{2,1} p_1 + \sigma^2} e^{-a \left(\frac{g_{2,2} p_2}{g_{2,1} p_1 + \sigma^2} - \gamma_{th}^d \right)} - \frac{b g_{1,2}}{g_{2,1} p_1 + \sigma^2} = 0. \tag{36}$$

After simplification, the $BR_2(p_1)$ depicted in Eq. (35) is readily derived by deducing the expression of p_2 according to p_1 from the last equation. □

5.4.2 Simulations and result interpretations

The two best-response $BR_1(p_2)$ and $BR_2(p_1)$ of both CUE and DUE (respectively), which are derived from (33) and (35) (respectively) and the NE are depicted in **Figure 2**. The simulation parameters used in this figure are presented in **Table 1**. As a note from this figure, we can notice that the NE exists and is unique,

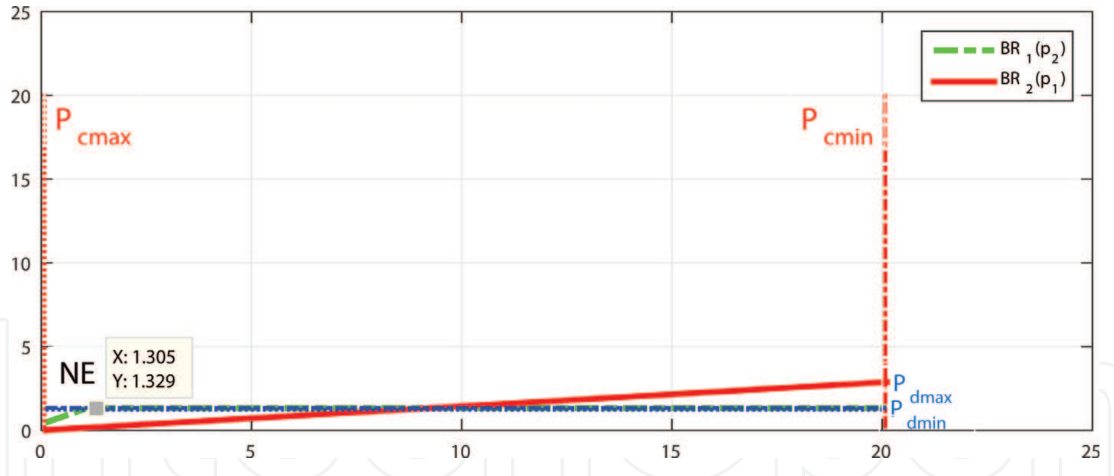

Figure 2.
Best-response $BR_1(p_2)$, $BR_2(p_1)$ and NE.

Parameters	Values
R	700 (m)
D2D link range $d_{k,k}$	50 (m)
D2D link density (λ)	5×10^{-5}
Path loss exponent (α)	4
γ_{th}^{c}	from −6 to 18 (dB)
γ_{th}^{d}	from −10 to 14 (dB)
Δp_c	$50mw$
Δp_d	$0.002mw$
σ^2 (for $1MHz$ bandwidth)	$-143.97(dBm)$
	25
b	1
λ	510^{-5}
ε	$1mw$
Iterations number	10^3

Table 1.
Simulation parameters.

because the two curves of $BR_1(p_2)$ and $BR_1(p_2)$ intersect at a single point in the feasible region Ω.

In the next section, we propose to extend this study for a system which contains K_1 CUE and K_2 DUE and furthermore to determine the NE SINR and power closed forms for both CUE and DUE, if it exists.

6. Proposed distributed power control algorithm based on GT

A NE Distributed PC Algorithm (NEDPCA) relative to both CUE and DUE is investigated in this section, in which our proposed game and CUE and DUE utility functions already defined in the previous section are considered. First, to do this, the SINR NE expressions for each user (CUE and DUE) are presented. Afterwards, the amount of power allocated to each user (CUE and DUE) relative to the derived NE are also studied. Thirdly, a power allocation algorithm will be suggested, based on the obtained results to derive the NE power quantities and the power limitation already discussed in Section 3.

6.1 SINR and power NE for CUE and DUE

The authors in [11] have shown that for a CUE and DUE k, the unique SINR NE $\gamma^*_{c,k}(\mathbf{P}^*)$ and $\gamma^*_{d,k}(\mathbf{P}^*)$ (respectively) have the following expressions:

$$\begin{cases} \gamma^*_{c,k}(\mathbf{P}^*) = \gamma_c^{th}, \quad \forall k \in \mathbf{K_1} \quad (37.a) \\ \gamma^*_{d,k}(\mathbf{P}^*) = \left[\frac{1}{a_k} \log\left(\frac{a_k g_{k,k}}{b_k \sum_{i \neq k} g_{j,k}}\right) + \gamma_d^{th}\right]^+ \quad \forall k \in \mathbf{K_2}, \quad (37.b) \end{cases} \tag{37}$$

where $[f]^+ = max(f, 0)$.

Assumption 1. Based on the Eq. (37.b), we assume the following statement for each DUE k

$$\frac{1}{a_k} \log\left(\frac{a_k g_{k,k}}{b_k \sum_{i \neq k} g_{j,k}}\right) \geq 0 \tag{38}$$

In fact, if: $\frac{1}{a_k} \log\left(\frac{a_k g_{k,k}}{b_k \sum_{i \neq k} g_{j,k}}\right) < 0$, we can have one of the two following cases

$$\begin{cases} \text{case } 1: \text{if } \dfrac{1}{a_k} \log\left(\dfrac{a_k g_{k,k}}{b_k \sum_{i \neq k} g_{j,k}}\right) + \gamma_d^{th} \geq 0 \Rightarrow \gamma^*_{d,k}(\mathbf{P}^*) < \gamma_d^{th}. \\ \text{case } 2: \text{if } \dfrac{1}{a_k} \log\left(\dfrac{a_k g_{k,k}}{b_k \sum_{i \neq k} g_{j,k}}\right) + \gamma_d^{th} < 0 \Rightarrow \gamma^*_{d,k}(\mathbf{P}^*) = 0. \end{cases} \tag{39}$$

The unique NE power $\mathbf{P}^*$ of both CUE and DUE can be derived as follows

$$\mathbf{P}^* = \left(p_1^*, ..p_{K_1}^*, ...p_K^*\right), \text{ where } p_k^* = \left[\frac{I_k(\mathbf{P}^*_{-k})\gamma_k^*}{g_{k,k}}\right]_{p_{min}(k)}^{p_{max}(k)}, \forall k \in \mathbf{K}, \tag{40}$$

where, $\gamma^*_{c,k}(\mathbf{P})$ and $\gamma^*_{d,k}(\mathbf{P})$ are defined in Eq. (37.a) if $k \in \mathbf{K_1}$ and (37.b) if $k \in \mathbf{K_2}$ (respectively).

We propose in the next step a distributed PC algorithm for the mentioned pure strategy game, which is based on the allocated power $\mathbf{P}^*$ previously defined in Eq. (40).

6.2 Proposed distributed power control algorithm based on GT

The algorithm depicted in 2 outlines the different steps of the proposed algorithm NEDPCA offered to each CUE and DUE, which is based on the previous pure strategy game. In fact, this algorithm NEDPCA offers a NE power for the two kinds of users, based on both the previous constraints (1) and on the power expression depicted in Eq. (40). First, the MS process is derived from the GCPCA, in order to guarantee the constraints imposed by the CUEs and the DUEs in terms of SINR thresholds (see Eq. (1)). This is shown in step 1 of the Algorithm 2. Second, the SINR NE $\gamma^*_{c,k}(\mathbf{P})$ and $\gamma^*_{d,k}(\mathbf{P})$ relative each user k (CUE and DUE) are obtained, which are evaluated in Eqs. (37.a) and (37.b) (respectively) (as it is shown from steps 2 and 3). Third, the PC process is investigated based on the iterative approach which is executed by using Eq. (40), as shown in steps 4 and 5. Step 6 allows to finalize the power distribution step, with an error of nearly δ for each user k. If $|p_k^{*(t+1} - p_k^{*t}| < \delta \ \forall$ user k, then the amount of power allocated to each user k, is given as: $\mathbf{P}_k^{*t} = \mathbf{P}_k^*$, otherwise we repeat this process and go to step 2.

Algorithm 2: Proposed NEDPCA

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initialisation: $t = 0, \mathbf{F}, \mathbf{b}, \delta > 0$ and $p_k = p_k^0, \forall k$ user.
Step 1: Evaluate from the Algorithm 1 GCPCA:
 1) the CUE and DUE set of users: $\mathbf{K_1}$, $\mathbf{K_2}$
 2) $\mathbf{P}_{min}$ and $\mathbf{P}_{max}$ using equations (17) and (19).
 2. for each CUE k, evaluate $\gamma^*_{c,k}(P)$ using equation (37).a.
Step 3. for each DUE k, evaluate $\gamma^*_{d,k}(\mathbf{P})$ using equation (37).b.
Step 4. Derive for each CUE and DUE k, the amount of power p_k^{*t} using equation (40), where $p_{min}(k)$ and $p_{max}(k)$ are derived from the 1st step. Evaluate $\mathbf{P}^{*t} = \left(p_1^{*t}, ..p_K^{*t}\right)$
Step 5: update $\mathbf{P}^{*(t+1)} = \mathbf{F}\mathbf{P}^{*t} + \mathbf{b}$.
Step 6. if $|p_k^{*\left(t+1-p_k^{*t}| < \delta, \forall k\right.}$, derive the solution $\mathbf{P}_k^{*t} = \mathbf{P}_k^*$, otherwise $t = t + 1$ and go to the 2st step.

All the NEDPCA steps relative to the distributed MS and PC for both CUE and DUE are detailed in Algorithm 2. Indeed, the first step of NECPCA makes it possible to realize the MS approach and all the other steps allow to deduce the PC approach.

Like the GCPCA algorithm, the NEDPCA algorithm converges after an iteration number, since it is based on the same condition $\rho\left(\mathbf{F^l}\right) < 1$ which must be checked during each iteration l by all the selected users. In fact, by applying the step 1 of the algorithm GCPCA (see Algorithm 2), the last condition should be guaranteed. It is also due to the proposition 2, that the convergence of NEDPCA is proved.

7. Analysis of simulations

In order to evaluate the performance of the algorithms already mentioned and proposed in the following sections, we consider in this section to study the simulations of these algorithms: GCPCA and NEDPCA. A Monte Carlo simulation is applied according to the Table 1, already given in the previous section.

The CUE and DUE Total powers are evaluated in **Figures 3** and **4** versus γ_c^{th} and γ_d^{th} (respectively).

We remain that the CPCA algorithm considers only one CUE and possibly several DUE. The GCPCA allocates to the different users the minimum power derived from Eq. (11), while respecting the condition $\rho(\mathbf{F}) < 1$. Thereby, any CUE and/or DUE that does not verify this condition will be eliminated from the user list.

First, we can notice from **Figure 3** that all the curves relative to GCPCA and NEDPCA algorithms are decreasing when γ_c^{th} increases. This is because when the threshold γ_c^{th} increases, it becomes difficult to find CUEs that check the constraints depicted in (1). On the other side from **Figure 4**, only the DUE performances are improved when GCPCA is considered and γ_d^{th} is improved. This is due to the fact that, the NE allows to maximize the utility function relative to each user k.

As it is shown from these two figures, the DUE coverage probability is significantly improved compared to that of DUE, because the DUEs benefit much more from TG compared to CUE, by using the NEPCA.

Second, the proposed centralized approach GCPCA which is a generalization of the CPCA approach offers less total power compared to the NEPCA, for both types of users CUE and DUE. This is due to the fact that when we want to reach a NE, by increasing the utility functions of each type of user, we should increase the total power consumed. Moreover, the difference between the two types of curves represents the power gain that must be added in order to reach this NE. From **Figures 3** and **4**, it is clear that for DUEs, this difference in terms of power is smaller

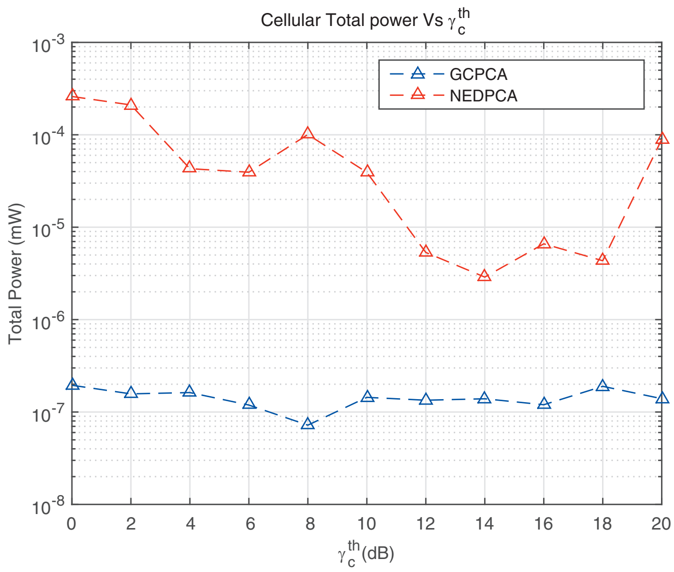

Figure 3.
CUE Total power vs. γ_c^{th}*.*

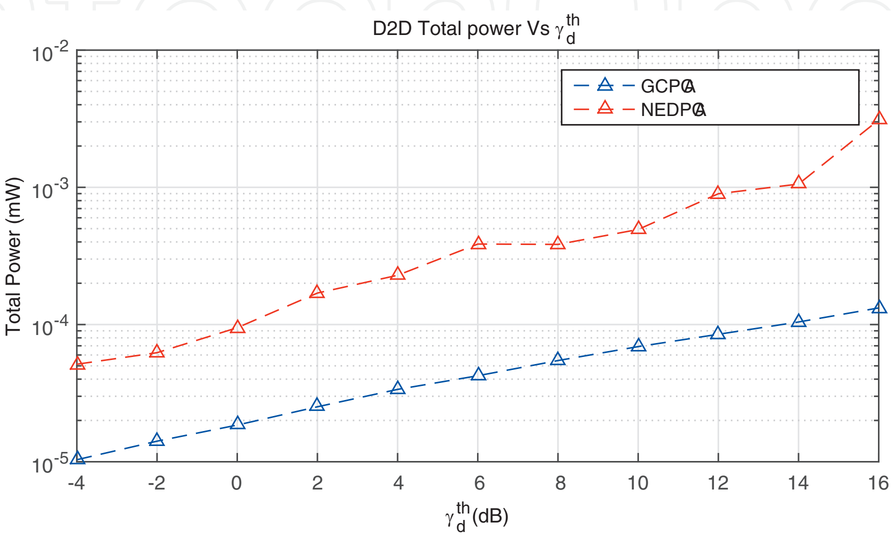

Figure 4.
DUE Total power vs. γ_d^{th}*.*

compared to that of the CUEs. This is explained by the fact that the DUEs require less power to transmit. Third, by limiting the power consumed in $[\mathbf{P}_{min}, \mathbf{P}_{max}]$ when GCPCA and NEDPCA are used, more flexibility and possibility to integrate the two types of users are offered. The greater the amount of maximum power $\mathbf{P}_{max}$, the higher the probability of coverage using the proposed GCPCA algorithm compared to CPCA one. It is for these reasons that it would be judicious to choose adequate margins of power $\Delta\mathbf{P}_c$ and $\Delta\mathbf{P}_d$, relative to the types of users (CUE and DUE).

8. Conclusions

This chapter allows to invoke the problem of selection mode and power control for a D2D underlaid cellular networks in 5G. The basic idea of this chapter is to generalize the classic allocation algorithms by applying Game Theory, for many CUEs and DUEs in system.

First, we assume that the amount of power allocated to each kind of user should be between two amounts of power: a minimum power defined as a Pareto solution and a maximum power. Thus, a mathematical demonstration was provided in this chapter, in order to prove the expressions of these two powers, based on constraints imposed by the users in terms of SINR thresholds to be respected.

Second, our proposed system is modeled as a non-cooperative pure game between the different types of users, where the utility functions should be maximized. From the built-in utility functions, NE SINR and PC solution existence and uniqueness are investigated and studied.

Third, simulations have been established in this context, in order to assess the performance of the algorithms thus proposed in terms of total powers relative to both users CUE and DUE. Through these simulations which compare these results without and with GT, we noticed that by applying the TG, the total power consumed increases in order to reach the NE for the two types of users: CUE and DUE. This is due to the fact that to increase the utility functions relating to the two types of users, a power margin must be added. However, this difference in terms of power becomes less important for the DUEs, since they require less power relative to the DUEs.

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