Chapter

Verification of Diffusion Hydrodynamic Model

Theodore V. Hromadka II and Chung-Cheng Yen

Abstract

The efficacy of the one- and two-dimensional diffusion hydrodynamic model (DHM) for predicting flow characteristics resulting from a dam-break scenario is tested. The model results, for different inflow scenarios, are compared with the standard United States Geological Survey (USGS) K-634 model. The sensitivity of the model results to grid spacing and the chosen time step are presented. The model results are in close agreement.

Keywords: floodplain, hydrograph, unsteady flows, initial flow condition, spatial grid size, transient simulation

1. Introduction

An unsteady flow hydraulic problem of considerable interest is the analysis of dam breaks and their downstream hydrograph. In this section, the main objective is to evaluate the diffusion form of the flow equations for the estimation of flood depths (and the floodplain), resulting from a specified dam-break hydrograph. The dam-break failure mode is not considered in this section. Rather, the dam-break failure mode may be included as part of the model solution (such as for a sudden breach) or specified as a reservoir outflow hydrograph.

The use of numerical methods to approximately solve the flow equations for the propagation of a flood wave due to an earthen dam failure has been the subject of several studies reported in the literature. Generally, the flow is modeled using the one-dimensional equation wherever there is no significant lateral variation in the flow. Land [1, 2] examined four such dam-break models in his prediction of flooding levels and flood wave travel time and compares the results against observed dam failure information. In the dam-break analysis, an assumed dam-break failure mode (which may be part of the solution) is used to develop an inflow hydrograph to the downstream floodplain. Consequently, it is noted that a considerable sensitivity in modeling results is attributed to the dam-break failure rate assumptions. Ponce and Tsivoglou [3] examined the gradual failure of an earthen embankment (caused by an overtopping flooding event) and present detailed analysis for each part of the total system: sediment transport, unsteady channel hydraulics, and earth embankment failure.

In another study, Rajar [4] studied a one-dimensional flood wave propagation from an earthen dam failure. His model solved the St. Venant equations using either a first-order diffusive or a second-order Lax-Wendroff numerical scheme. A review of the literature indicates that the most frequently used numerical scheme was the method of characteristics (to solve the governing flow equations) as described in Sakkas and Strelkoff [5], Chen [6], and Chen and Armbruster [7].

Although many dam-break studies involve flood flow regimes which are truly two-dimensional (in the horizontal plane), the two-dimensional case has not received much attention in the literature. Katopodes and Strelkoff [8] used the method of bicharacteristics to solve the governing equations of continuity and momentum. The model utilizes a moving grid algorithm to follow the flood wave propagation and also employs several interpolation schemes to approximate the nonlinearity effects. In a much simpler approach, Xanthopoulos and Koutitas [9] used a diffusion model (i.e., the inertial terms are assumed negligible in comparison to the pressure, friction, and gravity components) to approximate a two-dimensional flow field. The model assumed that the flow regime in the floodplain is such that the inertial terms (local and convective acceleration) are negligible. In a one-dimensional model, Akan and Yen [10] also used the diffusion approach to model hydrograph confluences at channel junctions. In the latter study, comparisons of modeling results were made between the diffusion model, a complete dynamic wave model solving the total equation system, and the basic kinematic-wave equation model (i.e., the inertia and pressure terms are assumed negligible in comparison to the friction and gravity terms). The differences between the diffusion model and the dynamic wave model were small, showing only minor discrepancies.

The kinematic-wave flow model has been used in the computation of dam-break flood waves [11]. Hunt concluded in his study that the kinematic-wave solution is asymptotically valid. Since the diffusion model has a wider range of applicability for varied bed slopes and wave periods than the kinematic model [12], the diffusion model approach should provide an extension to the referenced kinematic model.

Because the diffusion modeling approach leads to an economic two-dimensional dam-break flow model (with numerical solutions based on the usual integrated finite difference or finite element techniques), the need to include the extra components in the momentum equation must be ascertained. For example, evaluating the convective acceleration terms in a two-dimensional flow model requires approximately an additional 50 percent of the computational effort required in solving the entire two-dimensional model with the inertial terms omitted. Consequently, including the local and convective acceleration terms increases the computer execution costs significantly. Such increases in computational effort may not be significant for one-dimensional case studies; however, two-dimensional case studies necessarily involve considerably more computational effort, and any justifiable simplifications of the governing flow equations is reflected by a significant decrease in computer software requirements, costs, and computer execution time.

Ponce [13] examined the mathematical expressions of the flow equations, which lead to wave attenuation in prismatic channels. It is concluded that the wave attenuation process is caused by the interaction of the local acceleration term with the sum of the terms of friction slope and channel slope. When local acceleration is considered negligible, wave attenuation is caused by the interaction of the friction slope and channel slope terms with the pressure gradient or convective acceleration terms (or a combination of both terms). Other discussions of flow conditions and the sensitivity to the various terms of the flow equations are given in Miller and Cunge [14], Morris and Woolhiser [15], and Henderson [16].

It is stressed that the ultimate objective of this effort is to develop a two-dimensional diffusion model for use in estimating floodplain evolution, such as those that occur due to drainage system deficiencies. Prior to finalizing such a model, the requirement of including the inertial terms in the unsteady flow equations needs to be ascertained. The strategy used to check on this requirement is to evaluate the accuracy in predicted flood depths produced from a one-dimensional diffusion model with respect to the one-dimensional United States Geological Survey (USGS) K-634 dam-break model which includes all of the inertial term components.

2. One-dimensional analysis

2.1 Study approach

To evaluate the accuracy of the one-dimensional diffusion model [Chapter 1, Eq. 22) in the prediction of flood depths, the USGS fully dynamic flow model K-634 [1, 2] is used to determine channel flood depths for comparison purposes. The K-634 model solves the coupled flow equations of continuity and momentum by an implicit finite difference approach and is considered to be a highly accurate model for many unsteady flow problems. The study approach is to compare predicted (1) flood depths and (2) discharge hydrographs from both the K-634 and the diffusion hydrodynamic model (DHM) for various channel slopes and inflow hydrographs.

It should be noted that different initial conditions are used for these two models. The USGS K-634 model requires a base flow to start the simulation; therefore, the initial depth of water cannot be zero. Next, the normal depth assumption is used to generate an initial water depth before the simulation starts. These two steps are not required by the DHM.

In this case study, two hydrographs are assumed; namely, peak flows to 120,000 and 600,000 cfs. A base flow of 5000 and 40,000 cfs was used for hydrographs with peaks of 120,000 and 600,000 cfs, respectively, for all K-634 simulations. Both hydrographs are assumed to increase linearly from zero (or the base flow) to the peak flow rate at 1 h and then decrease linearly to zero (or the base flow) at 6 h (see **Figure 1** inset). The study channel is assumed to be a 1000-feet-width rectangular section of Manning's n equal to 0.040 and various slopes S_0 in the range of $0.001 \le S_0 \le 0.01$. **Figure 1** shows the comparison of modeling results. From the figure, various flood depths are plotted along the channel length of up to 10 miles. Two reaches of channel lengths of up to 30 miles are also plotted in **Figure 1** which correspond to a slope $S_0 = 0.0020$. In all tests, grid spacing was set at 1000-feet intervals. Time steps were 0.01 h for K-634 and 7.2 s for DHM.

From **Figure 1**, it is seen that the diffusion model provides estimates of flood depths that compare very well to the flood depths predicted from the K-634 model. For downstream distances at up to 30 miles, differences in predicted flood depths are less than 3% for the various channel slopes and peak flow rates considered.

In **Figures 2–5**, good comparisons between the diffusion hydrodynamic and the K-634 models are observed for water depths and outflow hydrographs at 5 and 10 miles downstream from the dam-break site. It should be noted that the test conditions are purposefully severe in order to bring out potential inaccuracies in the diffusion hydrodynamic model results. Less severe test conditions should lead to more favorable comparisons between the two model results. Although offsets do occur in timing, volume continuity is preserved when allowances are made for differences in base flow volumes.

2.2 Grid spacing selection

The choice of the time step and grid size for an explicit time advancement is a relative matter and is theoretically based on the well-known Courant condition [17]. The choice of grid size usually depends on available topographic data for nodal elevation determination and the size of the problem. The effect of the grid size

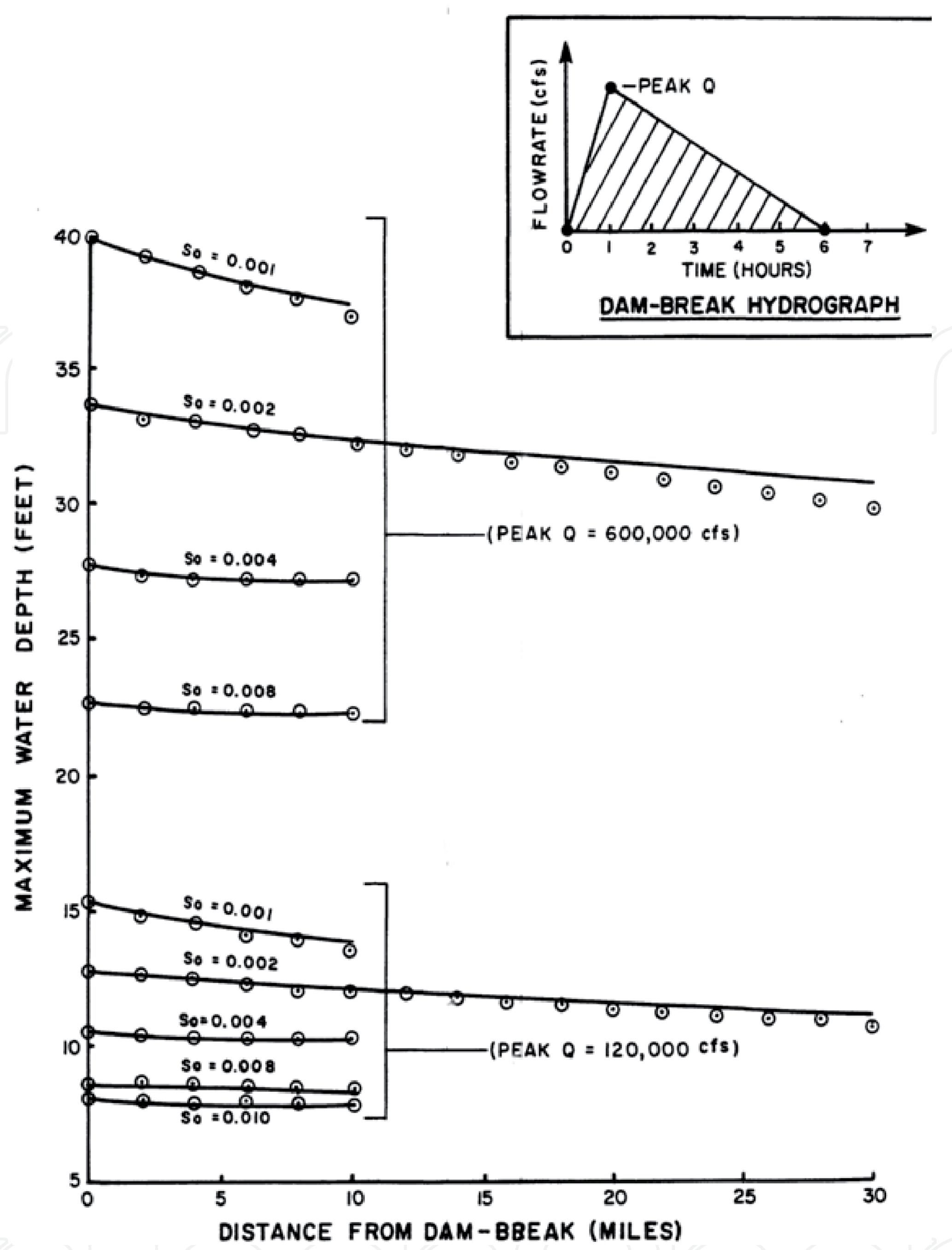

Figure 1.
Diffusion hydrodynamic model (⊙) and K-634 model results (solid line) for 1000-feet-width channel, Manning's n = 0.040, and various channel slopes, S_0.

(for constant time step for 7.2 s) on the diffusion model accuracy can be shown by example where nodal spacings of 1000, 2000, and 5000 feet are considered. The predicted flood depths varied only slightly from choosing the grid size between 1000 and 2000 feet. However, an increased variation in results occurs when a grid size of 5000 feet is selected. For the example of peak flow rate test hydrograph of 600,000 cfs, the differences of simulated flow depths between 1000 and 5000-feet grid are 0.03, 0.06, and 0.17 feet at 1, 5, and 10 miles, respectively, downstream from the dam-break site for the maximum flow depth with the magnitude of 30 feet.

Because the algorithm presented is based upon an explicit time stepping technique, the modeling results may become inaccurate, should the time step size versus grid size ratio become large. A simple procedure to eliminate this instability is to half the time step size until convergence in computed results is achieved. Generally,

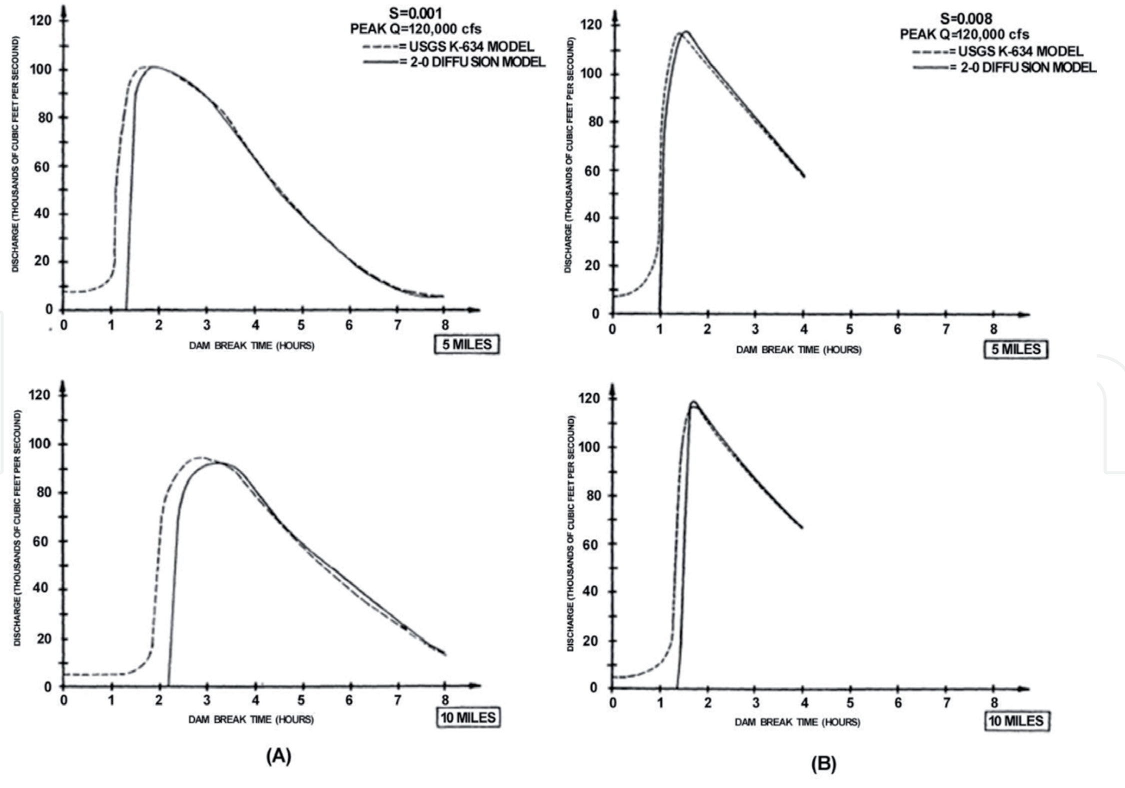

Figure 2.
Comparisons of outflow hydrographs at 5 and 10 miles downstream from the dam – break site (peak Q = 120,000 cfs) (A) S = 0.001 (B) S = 0.008.

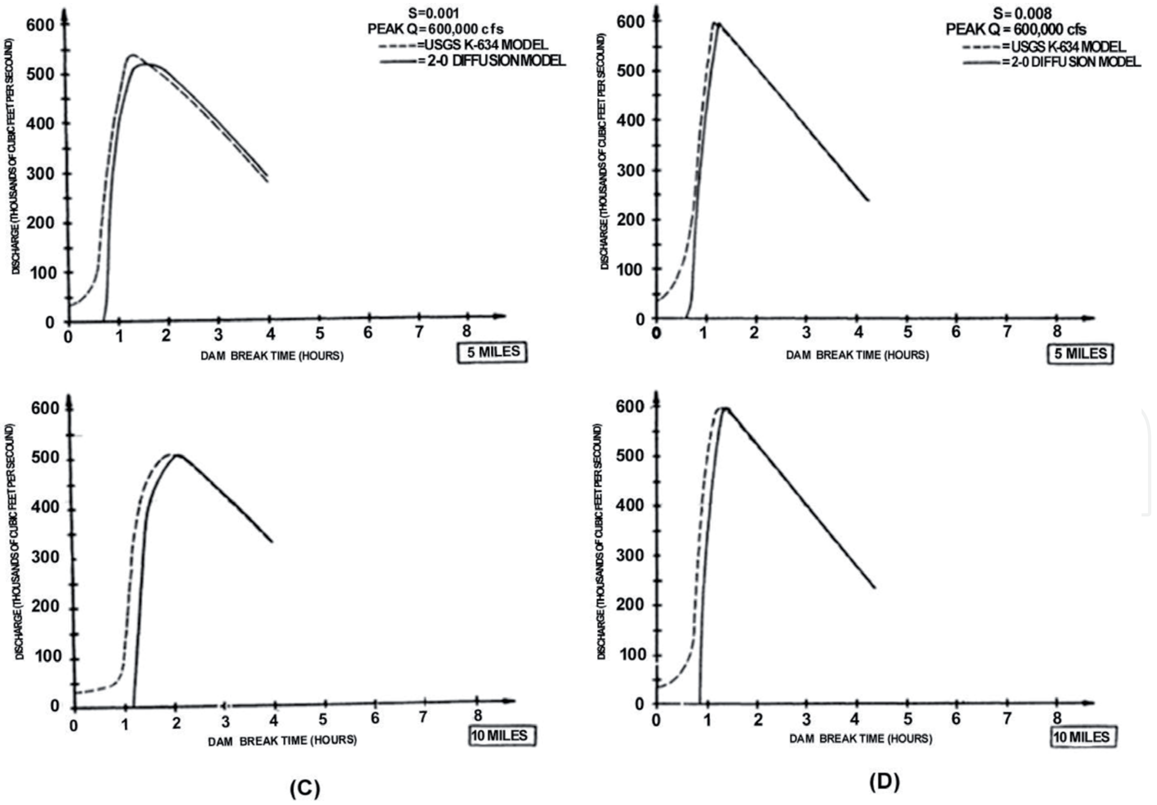

Figure 3.
Comparisons of outflow hydrographs at 5 and 10 miles downstream from the dam – break site (peak Q = 600,000 cfs) (C) S = 0.001 (D) S = 0.008.

such a time step adjustment may be directly included in the computer program for the dam-break model. For the cases considered in this section, the time step size of 7.2 s was found to be adequate when using the 1000–5000-feet grid sizes.

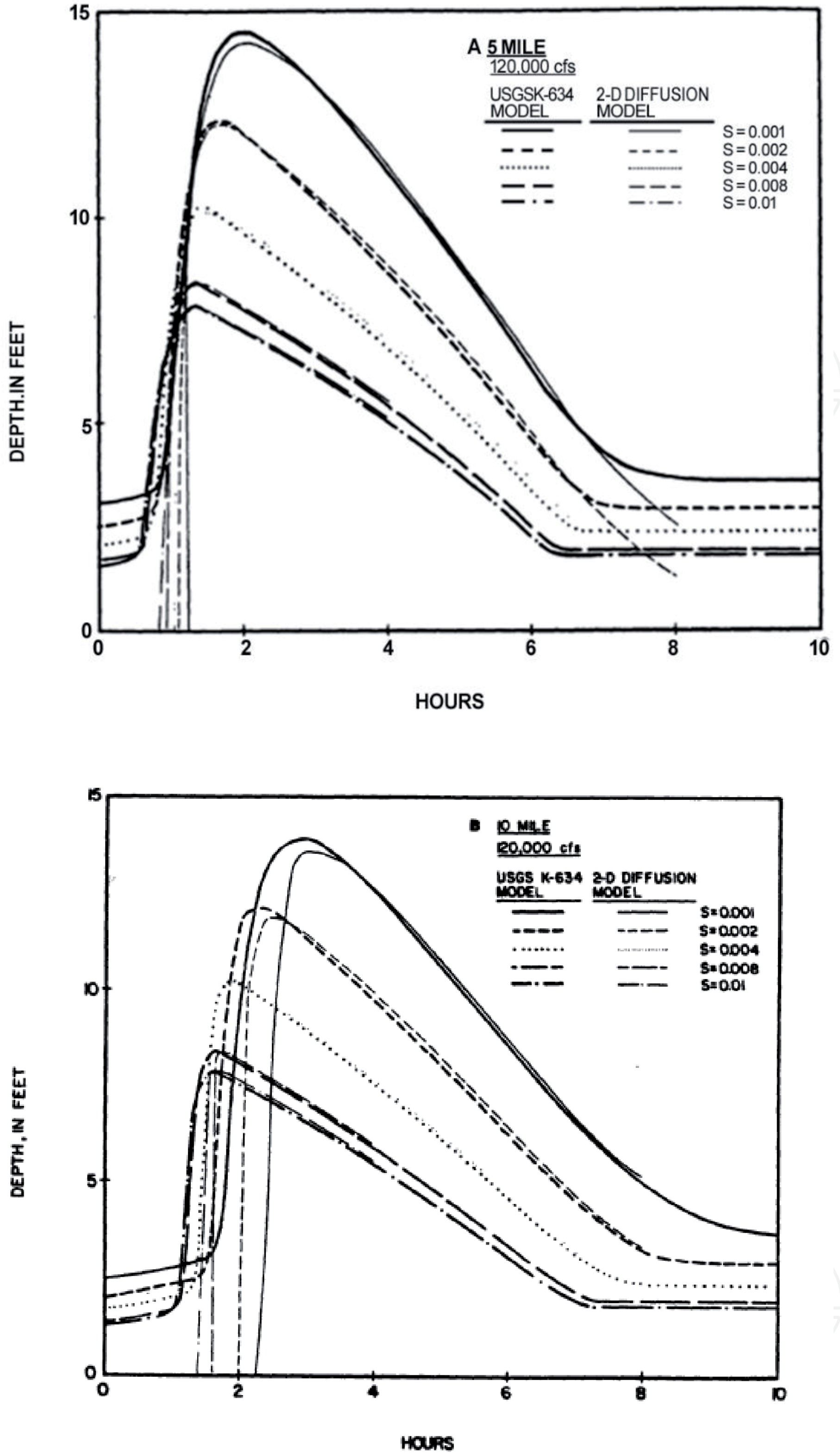

Figure 4.
Comparisons of depths of water at (A) 5 miles and (B) 10 miles downstream from the dam-break site (peak Q = 120,000 cfs).

2.3 Results

For the dam-break hydrographs considered and the range of channel slopes modeled, the simple diffusion dam-break model of Eq. (22) in Chapter 1 provides estimates of flood depths and outflow hydrographs which compare favorably to the results determined by the well-known K-634 one-dimensional dam-break model. Generally speaking, the difference between the two modeling approaches is found to be less than a 3% variation in predicted flood depths.

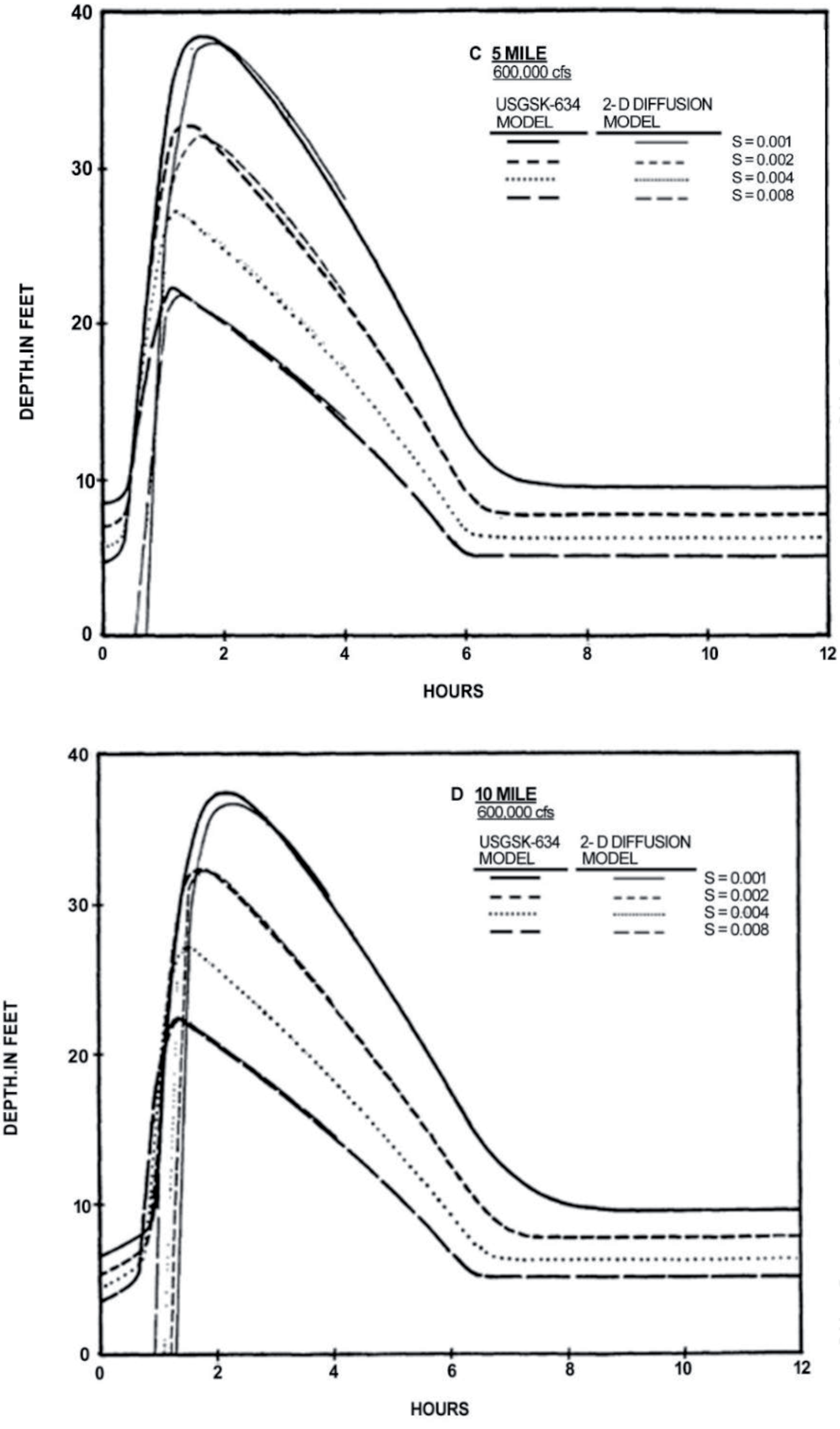

Figure 5.
Comparisons of depths of water at (C) 5 miles and (D) 10 miles downstream from the dam-break site (peak Q = 600,000 cfs).

The presented diffusion dam-break model is based upon a straightforward explicit time stepping method which allows the model to operate upon the nodal points without the need to use large matrix systems. Consequently, the model can be implemented on most currently available microcomputers. However, as compared to implicit solution methods, time steps for DHM use are extremely small. Thus, relatively short simulation times must be used.

The diffusion model of Eq. (22) in Chapter 1 can be directly extended to a two-dimensional model by adding the y-direction terms, which are computed in a similar fashion as the x-direction terms. The resulting two-dimensional diffusion model is texted by modeling the considered test problems in the x-direction, the y-direction, and along a 45-degree trajectory across a two-dimensional grid aligned with the x-y coordinate axis. Using a similar two-dimensional model, Xanthopoulos and Koutitas [9] conceptually verify the diffusion modeling technique by considering the evolution of a two-dimensional floodplain which propagates radially from the dam-break site.

From the above conclusions, the use of the diffusion approach (Chapter 1, Eq. 22), in a two-dimensional DHM may be justified due to the low variation in predicted flooding depths (one-dimensional) with the exclusion of the inertial terms. Generally speaking, a two-dimensional model would be employed when the expansion nature of flood flows is anticipated. Otherwise, one of the available one-dimensional models would suffice for the analysis.

3. Two-dimensional analysis

3.1 Introduction

In this section, a two-dimensional DHM is developed. The model is based on a diffusion approach where gravity, friction, and pressure forces are assumed to dominate the flow equations. Such an approach has been used earlier by Xanthopoulos and Koutitas [9] in the prediction of dam-break floodplains in Greece. In those studies, good results were also obtained by using the two-dimensional model for predicting one-dimensional flow quantities. In the preceding section, a one-dimensional diffusion model has been considered, and it has been concluded that for most velocity flow regimes (such as Froude number less than approximately 4), the diffusion model is a reasonable approximation of the full dynamic wave formulation.

An integrated finite difference grid model is developed which equates each cell-centered node to a function of the four neighboring cell nodal points. To demonstrate the predictive capacity of the floodplain model, a study of a hypothetical dam break of the Crowley Lake dam near the city of Bishop, California (**Figure 6**), is considered [18, 19].

The steepness and confinement of the channel right beneath the Crowley Lake dam results in a translation of outflow hydrograph in time. Therefore, the dam-break analysis is only conducted in the neighborhood near the city of Bishop, where the gradient of topography is mild.

3.2 K-634 modeling results and discussion

Using the K-634 model for computing the two-dimensional flow was attempted by means of the one-dimensional nodal spacing (**Figure 7**). Cross sections were obtained by field survey, and the elevation data were used to construct nodal point flow-width versus stage diagrams. A constant Manning's roughness coefficient of 0.04 was assumed for study purposes. The assumed dam failure reached a peak flow rate of 420,000 cfs within 1 h and returned to zero flow 9.67 h later. **Figure 8** depicts the K-634 floodplain limits. To model the flow breakout, a slight gradient was assumed for the topography perpendicular to the main channel. The motivation for such a lateral gradient is to limit the channel flood-way section in order to approximately conserve the one-dimensional momentum equations. Consequently, fictitious channel sides are included in the K-634 model study, which results in artificial confinement of the flows. Hence, a narrower floodplain is delineated in **Figure 8** where the flood flows are falsely retained within a hypothetical channel confine. An examination of the flood depths given in **Figure 9** indicates that at

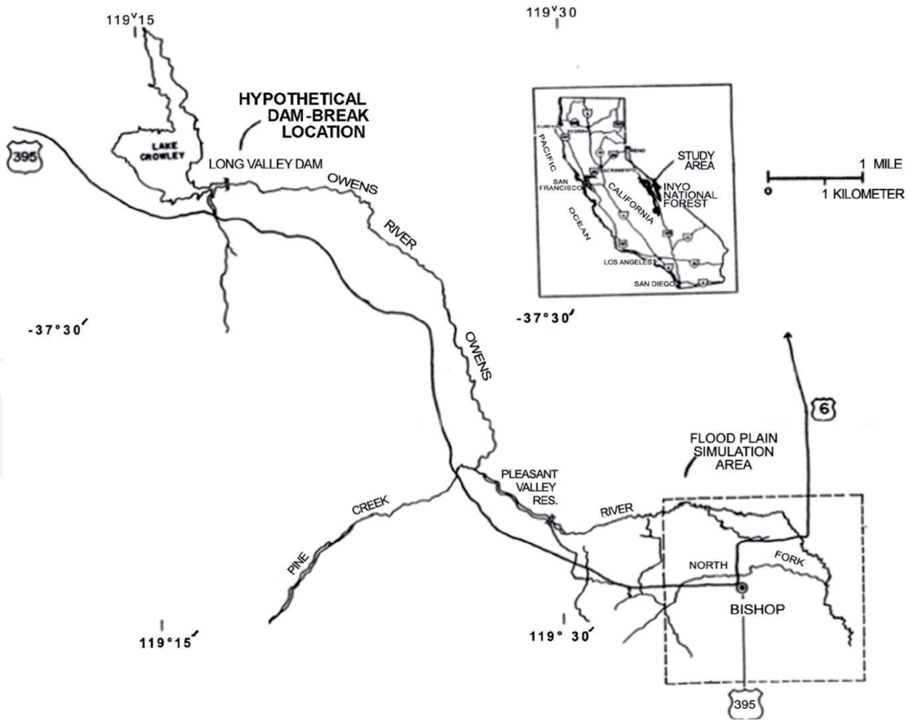

Figure 6.
Dam-break study location.

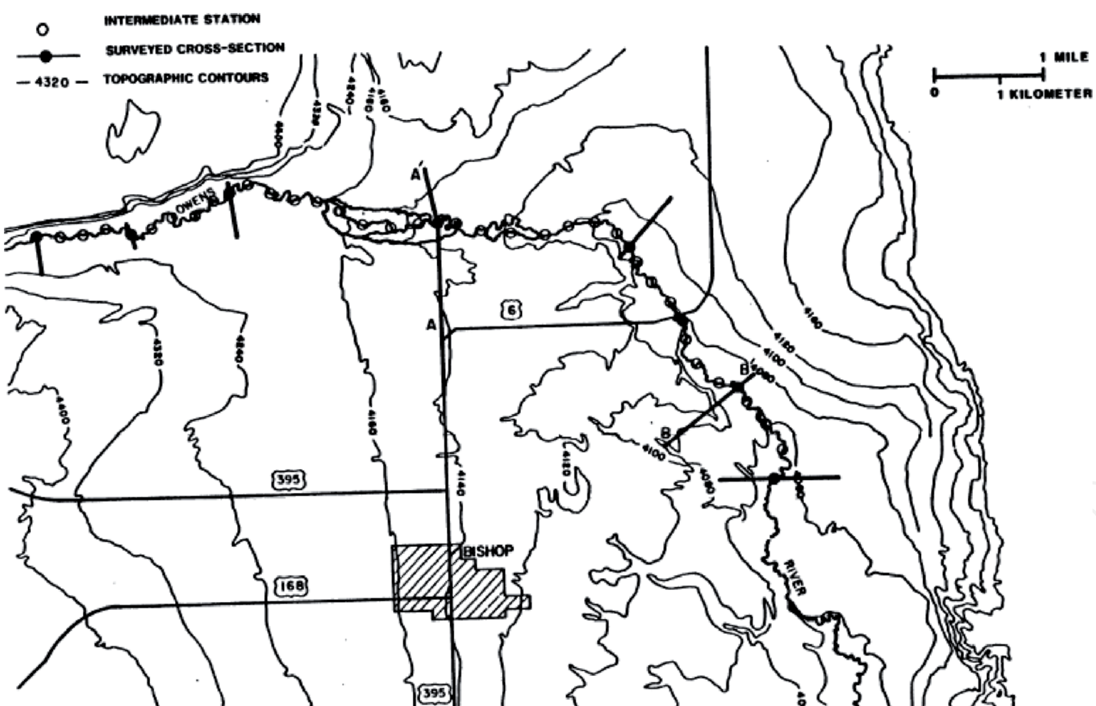

Figure 7.
Surveyed cross section locations on Owens River for use in K-634 model.

the widest floodplain expanse of **Figure 8**, the flood depth is about 6 feet, yet the floodplain is not delineated to expand southerly but is modeled to terminate based on the assumed gradient of the topography toward the channel. Such complications in accommodating an expanding floodplain when using a one-dimensional model are obviously avoided by using a two-dimensional approach.

The two-dimensional diffusion hydrodynamic model is now applied to the hypothetical dam-break problem using the grid discretization shown in **Figure 10**. The same

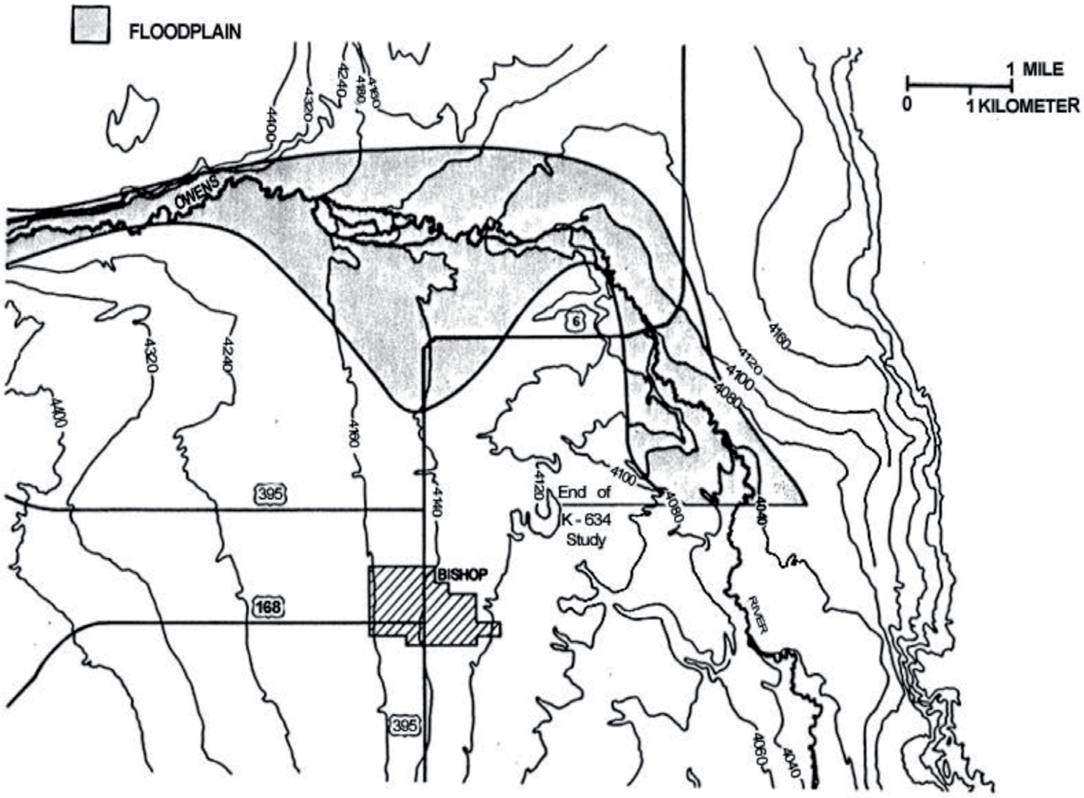

Figure 8.
Floodplain computed from K-634 model.

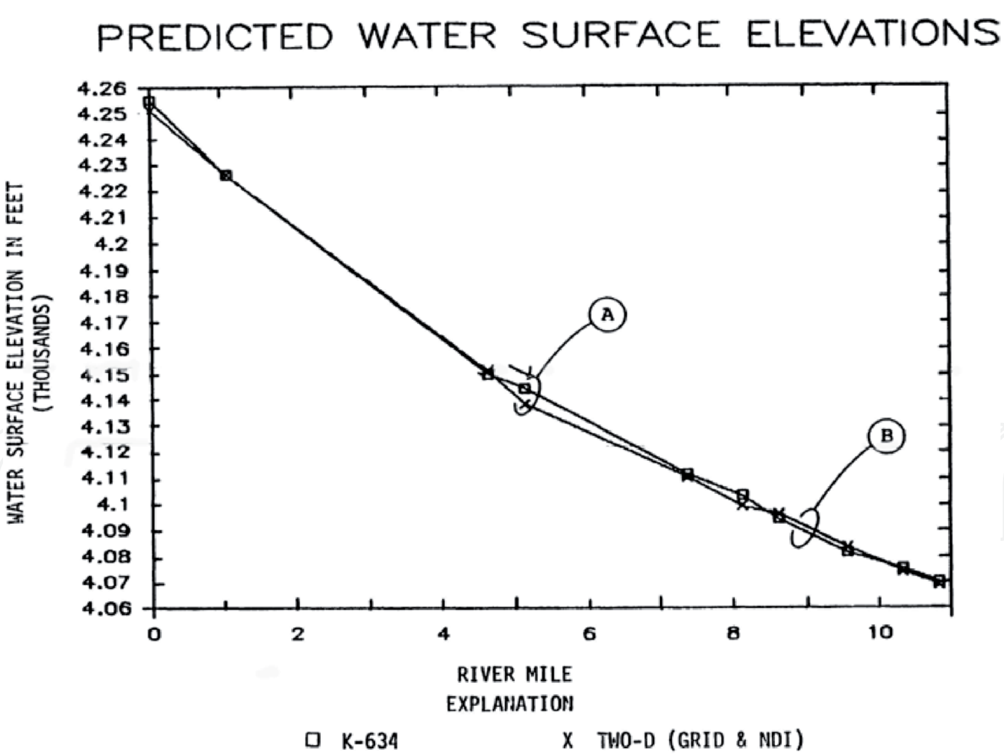

Figure 9.
Comparison of modeled water surface elevations (Points A and B in the figure are selected as example locations where a greater than an average difference between tested model predictions are observed).

inflow hydrograph used in K-634 model is also used for the diffusion hydrodynamic model. Again, Manning's roughness coefficient at 0.04 was used. The resulting floodplain is shown in **Figure 11** for the 1/4 square-mile grid model.

The two approaches are comparable except at cross sections shown as A-A and B-B in **Figure 7**. Cross section A-A corresponds to the predicted breakout of flows

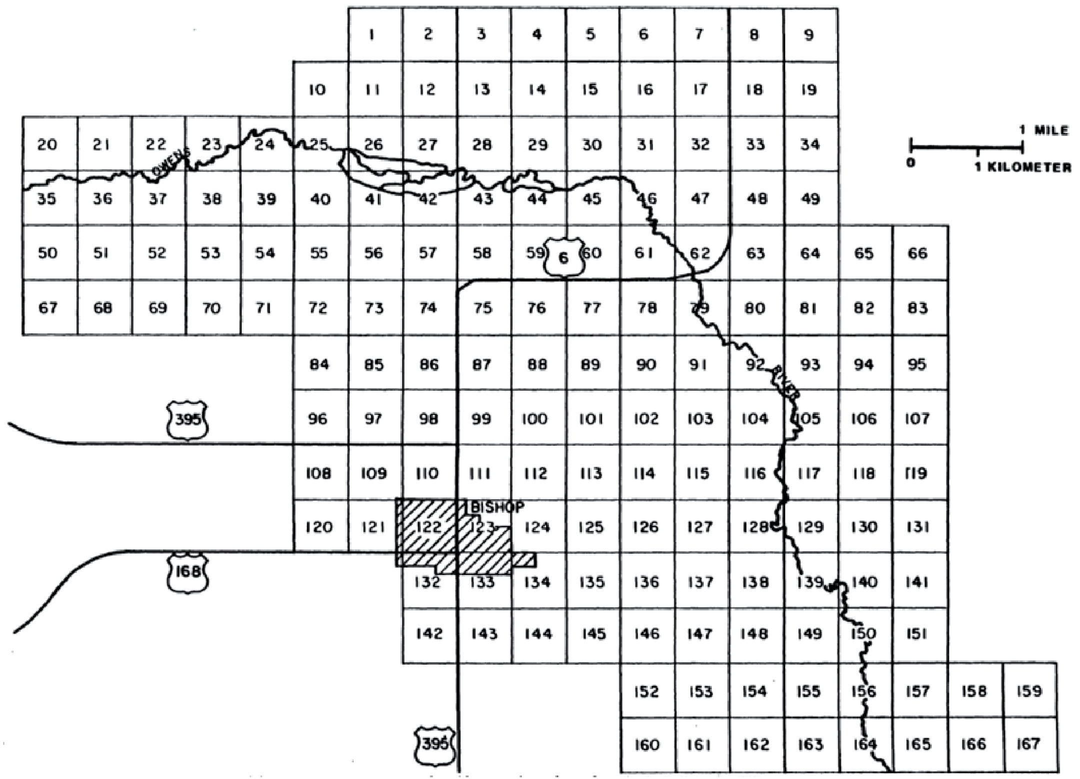

Figure 10.
Floodplain discretization for two-dimensional diffusion hydrodynamic model.

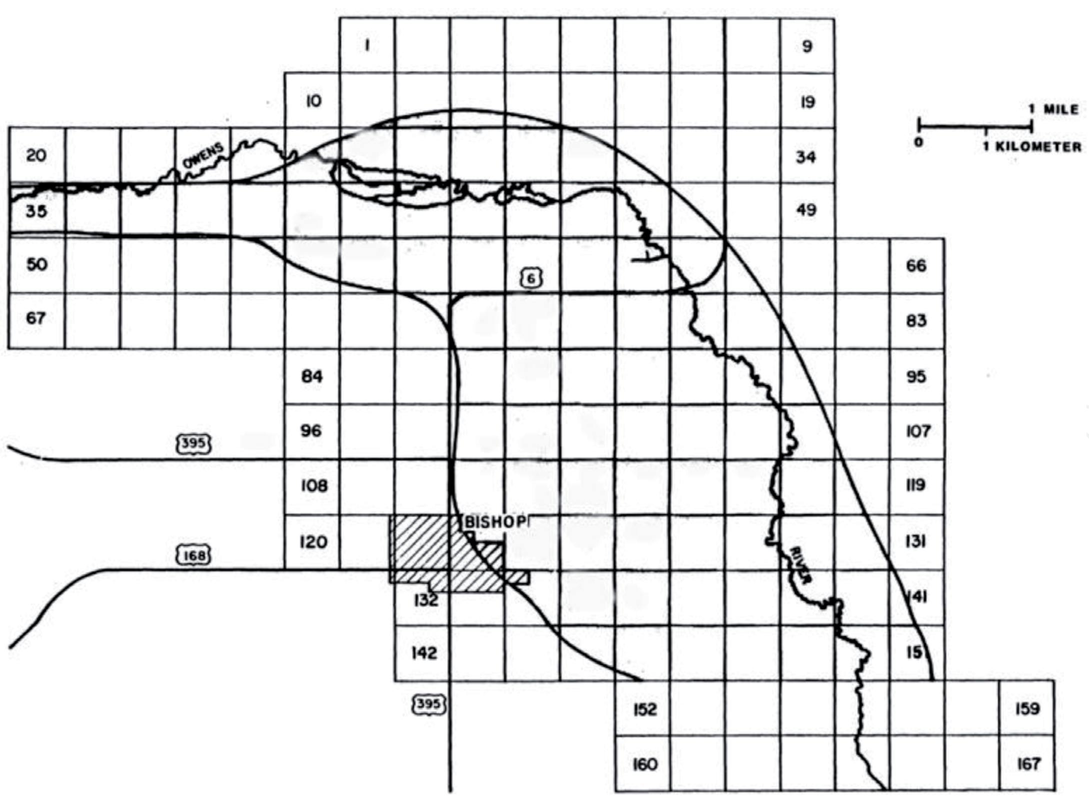

Figure 11.
Floodplain for two-dimensional diffusion hydrodynamic model.

away from the Owens River channel with flows traveling southerly toward the city of Bishop. As discussed previously, the K-634 predicted flood depth corresponds to a flow depth of 6 feet (above natural ground) which is actually unconfined by the channel. The natural topography will not support such a flood depth, and, consequently, there should be southerly breakout flows such as predicted by the two-dimensional model. With such breakout flows included, it is reasonable that the two-dimensional model would predict a lower flow depth at cross section A-A.

At cross section B-B, the K-634 model predicts a flood depth of approximately 2 feet less than the two-dimensional model. However, at this location, the K-634 modeling results are based on cross sections, which traverse a 90-degree bend. In this case K-634 model will overestimate the true channel storage, resulting in an underestimation of flow depths.

4. Conclusions

The contribution of inertial terms for one-dimensional flows resulting from a dam break was investigated by comparing the results of the DHM with the K-634 model, which includes inertial terms. The close agreement between the two models predicted results justifies the use of the DHM for these applications.

For two-dimensional flows, comparing the various model predicted flood depths and delineated plains, it is seen that the two-dimensional diffusion hydrodynamic model predicted more reasonable floodplain boundary, which is associated with broad, flat plains such as those found at the study site. The model approximates channel bends, channel expansions and contractions, flow breakouts, and the general area of inundation. Additionally, the diffusion hydrodynamic model approach allows for the inclusion of return flows (to the main channel), which were the result of upstream channel breakout, and other two-dimensional flow effects, without the need for special modeling accommodations that would be necessary with using a one-dimensional model.

Author details

Theodore V. Hromadka II [1*] and Chung-Cheng Yen[2]

1 Department of Mathematical Sciences, United States Military Academy, West Point, NY, USA

2 Tetra Tech, Irvine, CA, USA

*Address all correspondence to: ted@phdphdphd.com

References

[1] Land LF. Mathematical simulations of the Toccoa Falls, Georgia, Dam-Break Flood. Water Resources Bulletin. 1980;**16**(6):1041-1048

[2] Land LF. Evaluation of selected dam-break flood-wave models by using field data. U.S.G.S. Water Resources Investigations. 1980;**80**(4):54

[3] Ponce VM, Tsivoglou AJ. Modeling gradual dam breaches. ASCE Journal of Hydrauic Division. 1981;**107**(7):829-838

[4] Rajar R. Mathematical simulation of dam-break flow. ASCE Journal of Hydrauic Division. 1978;**104**(7):1011-1026

[5] Sakkas JG, Strelkoff T. Dam-break flood in a prismatic dry channel. ASCE Journal of Hydrauic Division. 1973;**99**(12):2195-2216

[6] Chen C. Laboratory verification of a dam-break flood model. ASCE Journal of Hydrauic Division. 1980;**106**(4):535-556

[7] Chen C, Armbruster JT. Dam-break wave model: Formulation and verification. ASCE Journal of Hydrauic Division. 1980;**106**(5):747-767

[8] Katopodes N, Strelkoff T. Computing two-dimensional dam-break flood waves. ASCE Journal of Hydrauic Division. 1978;**104**(9):1269-1288

[9] Xanthopoulos T, Koutitas C. Numerical simulation of a two-dimensional flood wave propagation due to dam failure. ASCE Journal of Hydraulic Research. 1976;**14**(4):321-331

[10] Akan AO, Yen BC. Diffusion-wave flood routing in channel networks. ASCE Journal of Hydrauic Division. 1981;**107**(6):719-732

[11] Hunt T. Asymptotic solution for dam-break problem. ASCE Journal of Hydrauic Division. 1982;**108**(1):115-126

[12] Ponce VM, Li RM, Simons DB. Applicability of kinematic and diffusion models, in verification of mathematical and physical models in hydraulic engineering. In: American Society of Civil Engineers, Hydraulic Division, Special Conference College Park, Maryland: University of Maryland; 1978. pp. 605-613

[13] Ponce VM. Nature of wave attenuation in Open Channel flow. ASCE Journal of Hydrauic Division. 1982;**108**(2):257-262

[14] Miller WA, Cunge JA. Simplified Equations of Unsteady Flow. Chapter 5 of Unsteady Flow in Open Channels. Vol. 1. Fort Collins, Colorado: Water Resources Publications; 1975. pp. 183-257

[15] Morris EM, Woolhiser DA. Unsteady one-dimensional flow over a plane: Partial equilibrium and recession hydrographs. Water Resources Research, AGU. 1980;**16**(2):356-360

[16] Henderson FM. Flood waves in prismatic channels. ASCE Journal of Hydrauic Division. 1963;**89**(4):39-67

[17] Basco DR. Introduction to numerical method—Part I and II. Verification of mathematical and physical models in hydraulic engineering. In: American Society of Civil Engineers, Hydraulic Division, Special Conference College Park, Maryland: University of Maryland; 1978. pp. 280-302

[18] Hromadka II TV, Lai C. Solving the two-dimensional diffusion flow model. In: Proceedings of ASCE Hydraulics Division Specialty Conference, Orlando, Florida. 1985

[19] Hromadka II TV, Nestlinger AB. Using a two-dimensional diffusional dam-break model in engineering planning. In: Proceedings of ASCE Workshop on Urban Hydrology and Stormwater Management, Los Angeles County Flood Control District Office, Los Angeles, California. 1985