Chapter

Non-Steady First Matrix Cracking of Fiber-Reinforced Ceramics

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Abstract

Matrix cracking affects the reliability and safety of fiber-reinforced ceramic-matrix composites during operation. The matrix cracking can be divided into two types, that is, steady state crack and non-steady state cracking. This chapter is about the non-steady stable cracking of fiber-reinforced CMCs. The micro stress field of fiber, matrix, and interface shear stress along the fiber direction is analyzed using the shear-lag model. The relationship between the crack opening displacement and the crack surface closure traction is derived. The experimental first matrix cracking stress of different CMCs are predicted.

Keywords: ceramic-matrix composites (CMCs), first matrix cracking stress, brittle matrix, non-steady state, pre-existing defect

1. Introduction

Fiber-reinforced ceramic-matrix composites (CMCs) have greater specific strength and specific stiffness. It will decrease the weight of the aircraft structure when it is applied to the aircraft. However, there are some disadvantages like complex processing and preparation, expansive, and so on. Now, there are some models for first matrix cracking. The MCE model [1] is one of the most famous models which established the relation between A.C.K and crack theory. McCartney model [2] gives a detailed process about the numerical solution. Chiang et al. [3, 4] used a modified shear-lag model considering the matrix deformation and the fiber failure is also considered. This chapter is about the non-steady matrix cracking of fiber reinforced CMCs. We assume that the fiber is strong enough to keep intact when matrix cracking occurs, and the composites with interface debonding are susceptible to weak frictional resistance. The growth characteristics of short cracks are evaluated using the stress intensity method. We will do some analysis about the fiber-matrix stress and solve equations to get the closing traction distribution. Then, the matrix cracking condition is combined to obtain the critical matrix cracking stress. The final results will show how the cracking stress is related to the size of a pre-existing defect and prediction of the threshold stress. Differences between the MCE model and McCartney model are also analyzed.

2. Fiber-matrix stress analysis

All analyses come from McCarteny model [2]. By performing stress analysis, the influence of the fiber can be equivalent to applying a distribution of closing pressure

$p(x_1)$ on the crack surface, and the influence of the applied stress can be evaluated by regarding the stress as a uniform opening pressure σ_∞ acting along the matrix crack surface. Therefore, we can obtain the net pressure on the crack surface, $[\sigma_\infty - p(x)]$. And the relation can be assumed for the continuum model [1]:

$$p(x_1) = \lambda\sqrt{u(x_1)} = p(-x_1) \tag{1}$$

where x_1 represents the location on the crack surface. According to the Sneddon and Lowengrub [5] and the force analysis, we can get the relation between the effective traction $p(x_1)$ and displacement distributions $u(x_1)$ as follows:

$$u(x_1) = \frac{2}{\pi^2}\int_{x_1}^{a} \frac{t}{\sqrt{(t^2 - x_1{}^2)}} \left\{ \int_0^t \frac{\sigma_\infty - p(\xi)}{\sqrt{(t^2 - \xi^2)}} d\xi \right\} dt \; 0 \ll x_1 < a \tag{2}$$

And we can also get the corresponding stress intensity factor [6]:

$$K = 2\sqrt{\frac{a}{\pi}}\int_0^a \frac{\sigma_\infty - p(\xi)}{\sqrt{(a^2 - x^2)}} dx \tag{3}$$

To make the formula more simplified, we can get the following simpler formula [2]:

$$u(x_1) = \frac{1}{\pi^2}\int_0^a \{\sigma_\infty - p(\xi)\} \ln \left| \frac{\sqrt{(a^2 - \xi^2)} + \sqrt{a^2 - x_1{}^2}}{\sqrt{(a^2 - \xi^2)} - \sqrt{a^2 - x_1{}^2}} \right| d\xi \; 0 \ll x_1 < a \tag{4}$$

After making some substitutions, we can obtain the following equation [2]:

$$P^2(X) = \mu\left\{ \sqrt{1 - X^2} - \frac{1}{\pi}\int_0^1 P(t) \ln \left| \frac{\sqrt{1 - t^2} + \sqrt{1 - X^2}}{\sqrt{1 - t^2} - \sqrt{1 - X^2}} \right| dt \right\} \; 0 \ll X < 1 \tag{5}$$

here

$$\mu = \lambda^2 a/\pi\sigma_\infty \tag{6}$$

$$K = \sigma_\infty\sqrt{(\pi a)}Y \tag{7}$$

here

$$Y = \frac{2}{\pi}\int_0^1 \frac{\{1 - P(X)\}}{\sqrt{1 - X^2}} dX \tag{8}$$

To obtain the parameter λ, equating the energy availability for the continuum and discrete fiber models. For the discrete fiber model, the energy available per unit area for matrix cracking is [2]:

$$E = \frac{R}{6\tau}\frac{E_c}{(1 - v^2)^2}\frac{V_m{}^2 E_m{}^2}{V_f{}^2 E_f}\epsilon^3 \tag{9}$$

For the continuum model, the energy available per unit area for matrix cracking is [2]:

$$E^* = \sigma_\infty \Delta u_2^\infty - \int_0^{\Delta u_2^\infty} p(\Delta u_2) d(\Delta u_2) = \frac{4\pi}{3\lambda^2}\left(\frac{E_c}{1-v^2}\right)^2 \epsilon^3 \tag{10}$$

By equating $E = E^*$, the parameter λ is obtained:

$$\lambda = 2\frac{V_f}{V_m}\left\{\frac{2\pi\tau}{R}\frac{E_f E_c}{{E_m}^2}\right\}^{1/2} \tag{11}$$

this is different with the parameter $\bar{\lambda}$ used by Marshall et al. [1].

$$\bar{\lambda} = 2V_f\left\{\frac{2\pi\tau}{R}\frac{E_f}{V_m E_m}\right\}^{1/2} = \left(\frac{V_m E_m}{E_c}\right)^{1/2}\lambda \tag{12}$$

Now let us compare the parameter λ with $\bar{\lambda}$. The parameter λ is obtained through the energy balance considerations, while the parameter $\bar{\lambda}$ is gotten by doing some mechanical analysis of the discrete fiber model. Therefore, we can know that the parameter λ can account for the energy changes like stored energy and frictional energy dissipation.

What's more, it is necessary to choose a more reasonable matrix cracking condition to obtain the final critical matrix cracking stress. Now two matrix cracking conditions will be on the list. It is noted that all the conditions are not on the physics ground. Firstly, it is about Griffith fracture criterion as follows [2]:

$$K^2 = 2\gamma E_c / \left(1 - v^2\right) \tag{13}$$

According to this, we can derive the cracking condition. Secondly, it is assumed that the matrix and composite stress intensities scale with the stress. So we can derive the relation as follows [2]:

$$K^L = K_c^L = K_c^M E_c / E_m \tag{14}$$

And this chapter adopts the first kind of condition. Finally, the equations concerned with critical cracking condition in this chapter are derived as follows [2]:

$$a/a_0 = \{\mu_c / Y(\mu_c)\}^{\frac{2}{3}} \tag{15}$$

$$\sigma_\infty^c / \sigma_0 = \left\{1/\mu_c Y^2(\mu_c)\right\}^{\frac{1}{3}} \tag{16}$$

here, Y is a function only of μ.

To predict the threshold stress and obtain the relation between the cracking stress and the pre-existing defect, we should firstly solve Eq. (5) and obtain the effective traction distribution. Then, we can obtain many values of $Y(\mu)$ corresponding to a range of values of μ. According to the cracking condition, the curve related to the dependence of critical cracking stress on the pre-existing defect is generated.

3. Numerical solution to matrix cracking stress

The main problem is to solve the nonlinear integral Eq. (5). The Simpson's integration formula is used here to derive the discrete form of the nonlinear integral equation which makes the substitutions. And finally, we can obtain the following discrete formulation [2]:

$$\left\{g_i^{(N+1)}\right\}^2 - \{2+\mu S_i\}g_i^{(N+1)} + 1 = \mu \sum_{\substack{j=0 \\ j\neq i}}^{n} \left\{g_j^{(N)} - g_i^{(N)}\right\}K_{ij}, i = 0, \dots .n\ N \geq 0 \tag{17}$$

here

$$g_i = g(i/n), S_i = \frac{2(i/n)^2}{1+(i/n)^4}, g_i^{(0)} = 0, i = 0, \dots, n,$$

And,

$$K_{ij} = \frac{8\delta_j}{\pi n^4} \ln\left|\frac{S_j + S_i}{S_j - S_i}\right| \frac{j^3}{\left(1+(j/n)^4\right)^2}, j \neq i,$$

With $\delta_0 = \delta_n = \frac{1}{3}, \delta_j = \begin{cases} \frac{4}{3} & \text{if } j \text{ is odd} \\ \frac{2}{3} & \text{if } j \text{ is even} \end{cases}, j = 1, \dots, n-1.$

And the corresponding discrete form of Eq. (6) for Y [2] is given by

$$Y = \frac{8}{\pi n^2} \sum_{j=0}^{n} \frac{j\delta_j g_j}{1+(j/n)^4} \tag{18}$$

When solving Eq. (15), the parameter μ can be valued like 0.1, 1, 10, 100 and so on. Then we can get the value of g_i^{N+1} by solving a quadratic equation and using the starting value. What's more, $0 \leq g \leq 1$. After getting values of all g_i^{N+1}, by comparing the two adjacent iterations until a group of solution meets the condition [2]:

$$\sqrt{\left\{\frac{1}{n+1}\sum_{i=0}^{n}\left(g_i^{(N+1)} - g_i^{(N)}\right)^2\right\}} < \delta \tag{19}$$

where δ represents the accuracy and can be valued like $10^{-6}, 10^{-8}$.

In this chapter, the value of δ is 10^{-6}, and the values of μ are 0.1, 1, 100.

This is the flow chart of the solution:

4. Results and discussion

By solving Eq. (15), we can get the curve of the distribution of effective tractions acting on the crack surfaces as in **Figure 1**. The horizontal axis represents the location in the crack surface, and the vertical axis represents the continuous effective traction. And it is simple that both the horizontal and vertical axes are in percentage terms. So this figure shows the closing traction distribution in a range of parameter μ. The distributions corresponding to different values of μ are different. And larger the value μ is, more load will the fiber support at the same position.

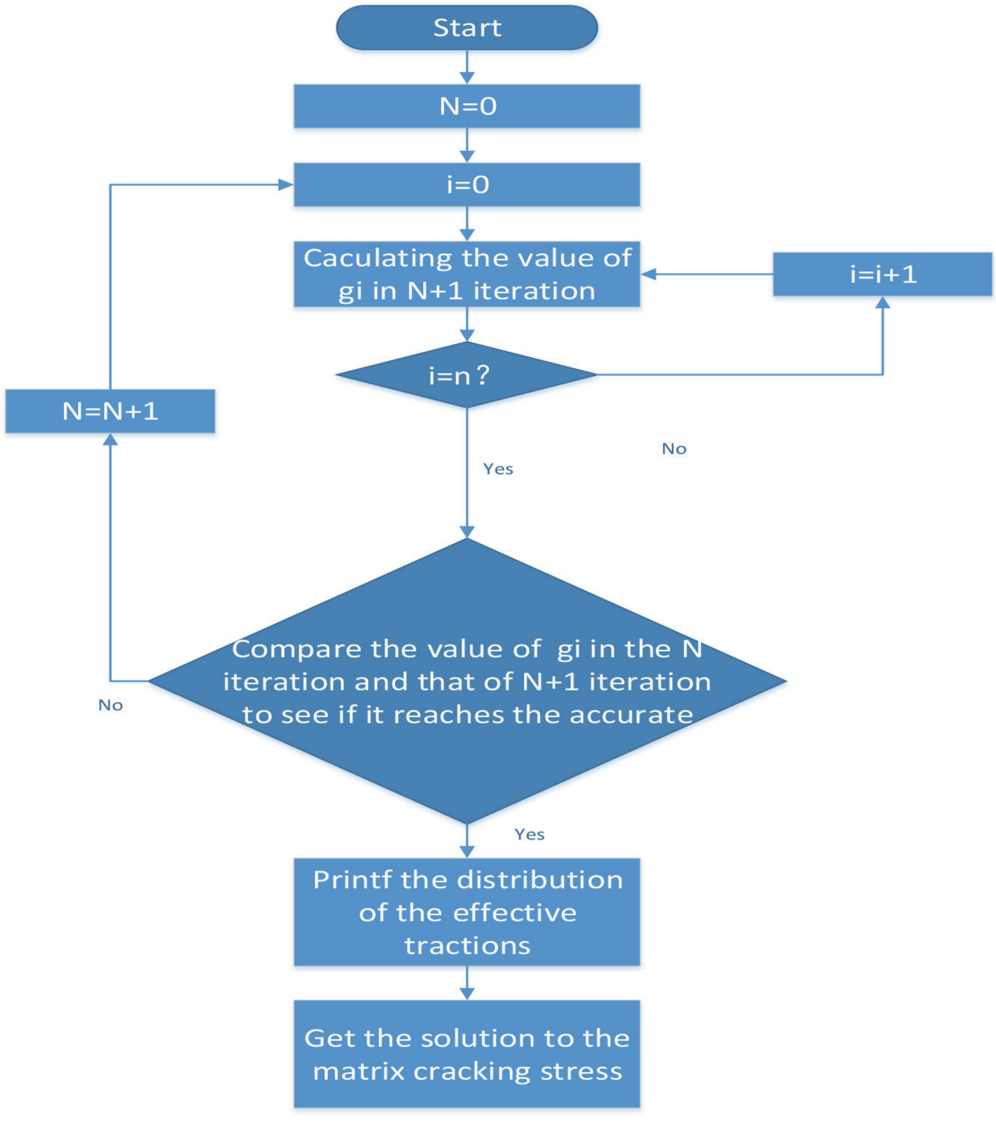


Figure 1.
The flow chart of the solution to the effective fiber tractions.

When μ comes to 100, the fiber supports the applied load for most of the crack length. As is mentioned above, this figure gives the accurate result for Eq. (1). And according to this result, we can get the critical matrix cracking stress.

And the corresponding curve which shows the relation between the critical cracking stress and the pre-existing defect is gotten with the cracking condition in **Figure 2**. And the value of δ is 10^{-6}, and the values of n is 60. The result can be obtained by choosing different values of a/a0 and getting the corresponding value of μ. And the values σ^c_∞/σ_0 varies with a/a_0. a/a_0 represents the length of pre-existing defect, and σ^c_∞/σ_0 represents the corresponding matrix cracking stress. Both a/a_0 and σ^c_∞/σ_0 are standard forms. In this curve, it is noted that the critical stress decreases with the length of pre-existing defect increasing when the length of pre-existing defect is short. And the stress tends to be constant with the length rising when the length is over a value. When the length of pre-existing defect is below a value, the corresponding critical stress is decided by the length of crack, while the critical stress will be independent of the total pre-existing crack length when the length of the defect is over the characteristic distance (**Figure 3**).

To obtain the threshold matrix cracking stress by choosing $a/a_0 \to \infty$, we choose the parameter μ like in the following table. It is shown that $\sigma^c_\infty/\sigma_0 \to 1.331$ when $a/a_0 \to \infty$. We can know that it is close to the theoretical value by calculation (**Table 1**).

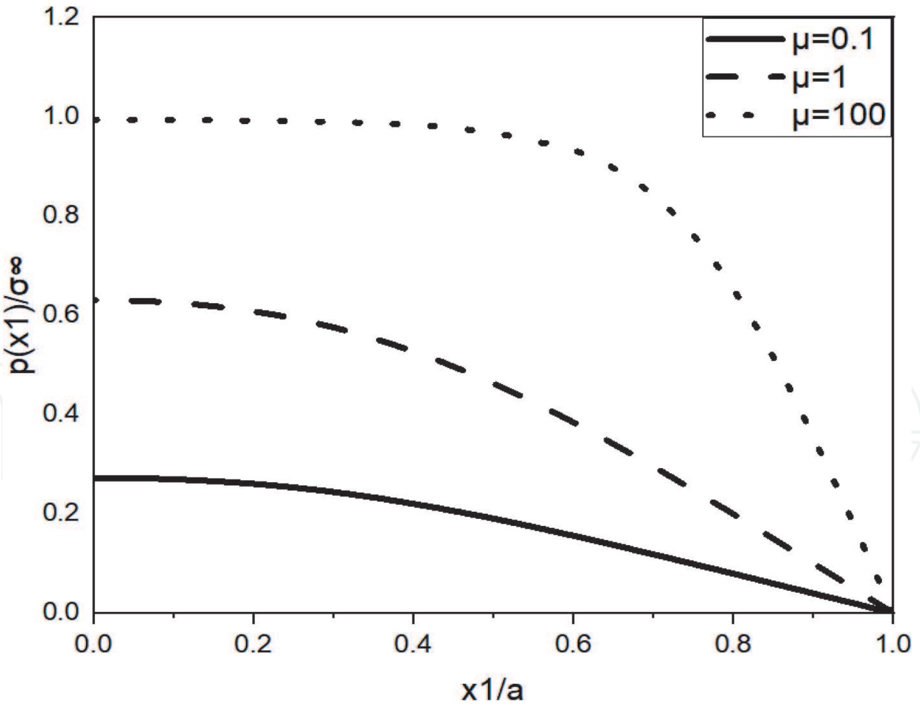


Figure 2.
The distribution of effective traction acting on the crack surface.

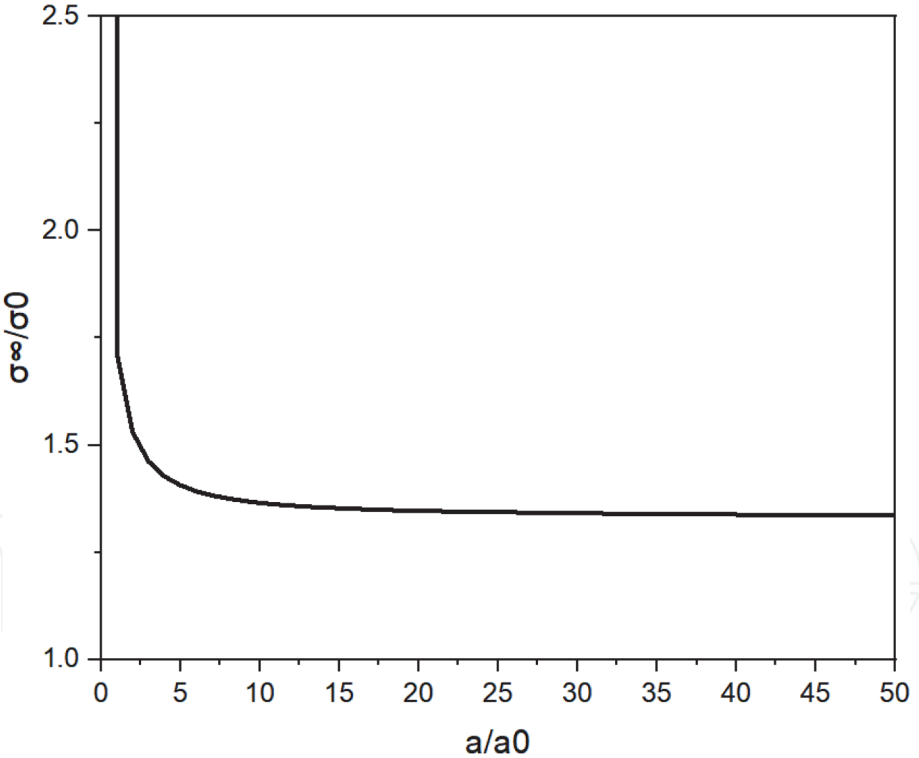
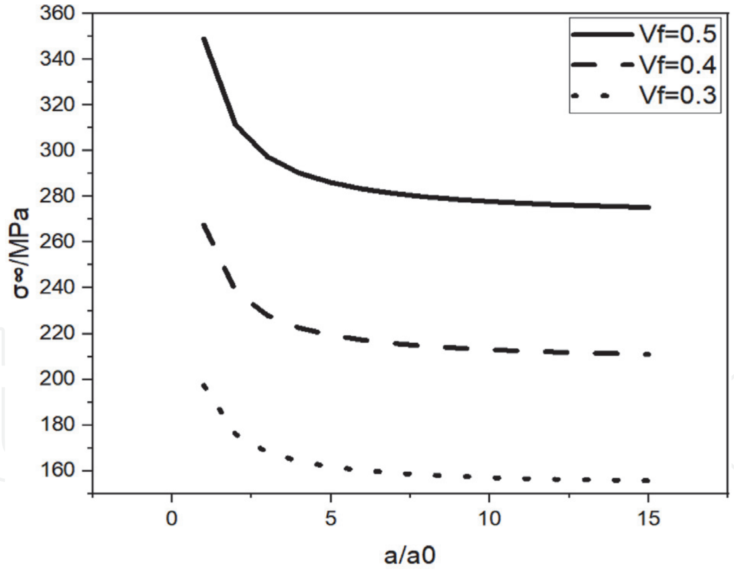


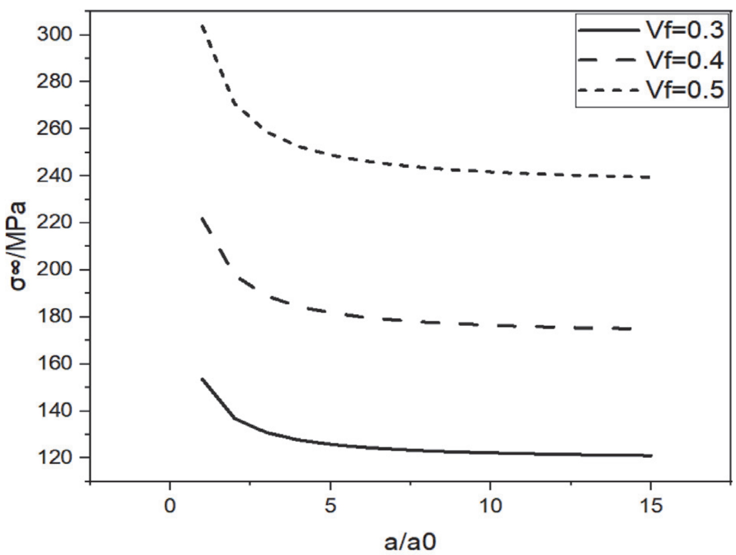
Figure 3.
The critical stress for matrix cracking on the length of the pre-existing matrix crack.

u	0.1	1	10	100	1000	1500	2000
$\sigma_{\infty}^{c}/\sigma_0$	2.52038	1.582594	1.355636	1.332882	1.331029	1.331012	1.331025

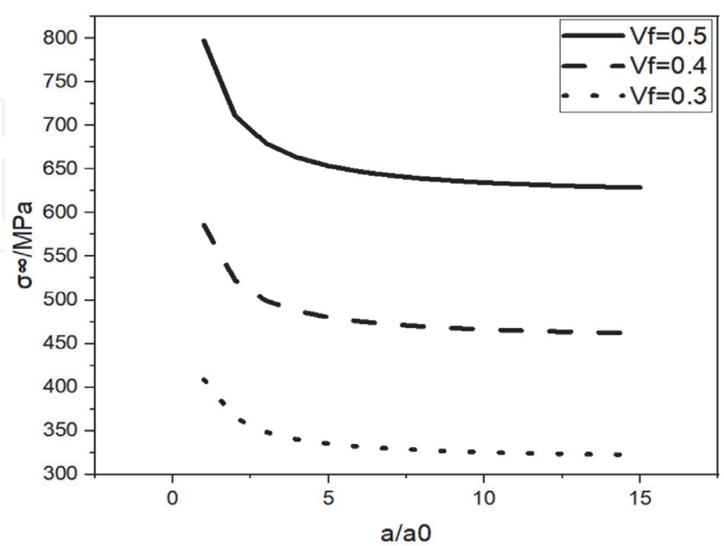
Table 1.
Values of the critical cracking stress for various values of μ *[2].*



（a）SiC-glass ceramic



（b）SiC/borosilicata



（c）C/borosilicata

Figure 4.
The distribution of effective tractions with different values of parameter V_f *in three kinds of material. (a) SiC-glass ceramic. (b) SiC/borosilicata. (c) C/borosilicata.*

a. The condition that the fiber is strong enough to keep intact and it does not take into account the shear deformation in the matrix above the slipping region is considered. But obviously the fiber failure and the deformation above the slipping region influences the matrix cracking stress. This chapter gives the numerical solution. MCE model gives the approximate analytical solution [1] for short crack. And we can see that the two methods show the same trends, and the numerical solution is lower than the analytical solution. It is noted that the MCE model assumed the cracking condition instead of deriving the condition. And the result of MCE model do not establish the threshold matrix cracking stress below which is impossible to make the matrix crack. The model will be valid when the ratio a/R is large enough, as the fiber radius R was not quoted. It is not possible to determine that part of the curve in **Figure 5** satisfying the validity condition $a/R > 10$. So it is of vital importance to derive numerical results. As **Figure 1** shows, the distribution of effective tractions $p(x_1)/\sigma_\infty$ increases with the parameter μ rising at the same position. By changing the value of V_f, we can finally change the standard value σ_0. As a result, the value of matrix cracking stress changes. And the relation between the matrix cracking stress and V_f is shown in **Figure 4**. The parameters of three kinds of material are listed in **Tables 2**–**4**. They are SiC-glass ceramic, SiC/borosilicate, and C/borosilicate. And the formula can be derived according to Eqs. (6) and (11):

$$\mu = \frac{8\tau a {V_f}^2 E_f (E_f V_f + E_m V_m)}{R {V_m}^2 {E_m}^2 \sigma_\infty} \tag{20}$$

The relation that the parameter μ increases with the V_f rising can be gotten by taking the derivative of the equation to determine its monotonicity. As a result, the distribution of effective tractions $p(x_1)/\sigma_\infty$ acting on the crack surface rises with parameter V_f increasing. The parameter a represents the matrix cracking length, and σ_∞ represents the applied stress. And as we all know, the distribution of

Parameter	**E_f/GPa**	**E_m/GP$_a$**	**$K_{IC}^m(MPa-\sqrt{m})$**	**R/μm**	**τ/MPa**
Value	200	85	2	8	2

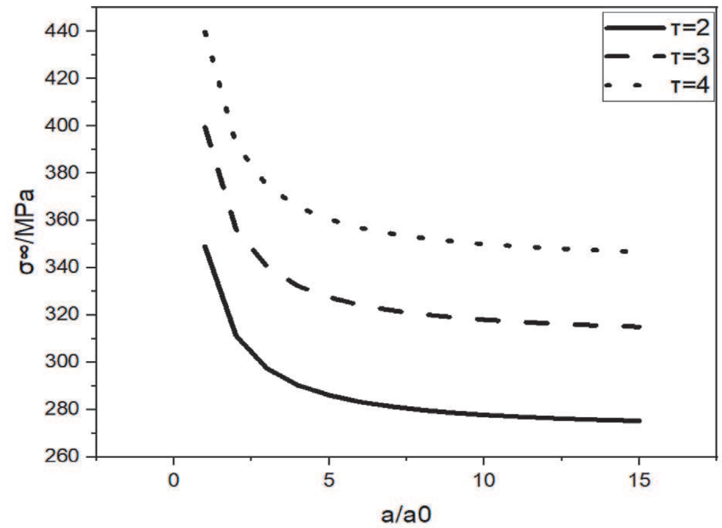
Table 2.
The parameters of SiC-glass ceramic.

Parameter	**E_f/GPa**	**E_m/GP$_a$**	**$K_{IC}^m(MPa-\sqrt{m})$**	**R/μm**	**τ/MPa**
value	400	63	0.77	70	6–8

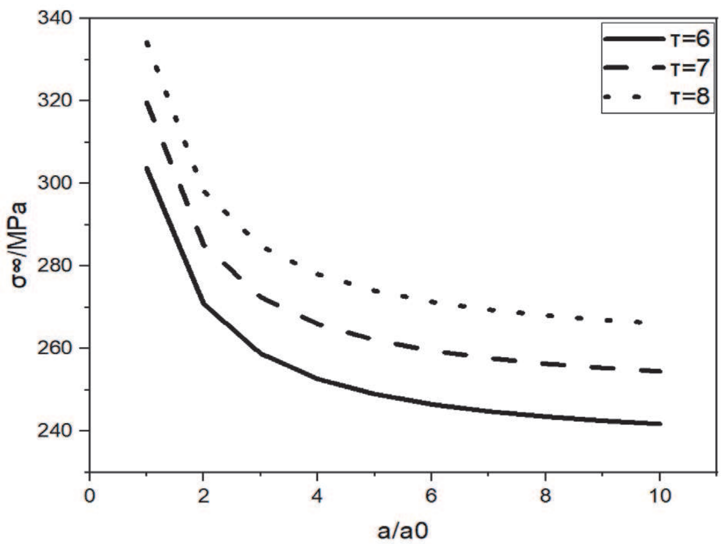
Table 3.
The parameters of SiC/borosilicata.

Parameter	**E_f/GPa**	**E_m/GP$_a$**	**$K_{IC}^m(MPa-\sqrt{m})$**	**R/μm**	**τ/MPa**
value	380	70	0.75	4	10

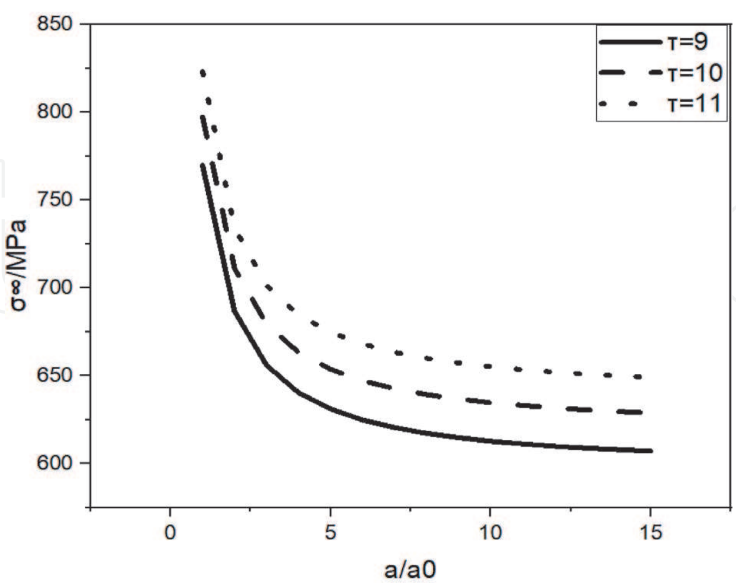
Table 4.
The parameters of C/borosilicata.



(a) SiC-glass ceramic



(b) SiC/borosilicata



(c) C/borosilicata

Figure 5.
The distribution of effective tractions with different values of parameter τ in three kinds of material. (a) SiC-glass ceramic. (b) SiC/borosilicata. (c) C/borosilicata.

effective traction which is equivalent to the effect of fiber traction can make the fiber ends displace to be rejoined. So the greater value of $p(x_1)/\sigma_\infty$, the harder the matrix to crack which means the value of the matrix cracking stress is bigger. And the parameter τ is in positive correlation with the parameter μ. In the same way, the matrix cracking stress will increase with the parameter τ rising. The curve can be seen in **Figure 5** with three kinds of material which show the same tendency.

5. Conclusion

McCartney model gives a more reasonable parameter λ which is gotten by equating the available energy of fiber discrete model and continuum model, while the MCE model do some stress analysis of fiber discrete model to get this parameter. So the McCartney model can explain the energy changes. McCartney model derived rather than assuming the matrix cracking condition. The cracking condition is gotten through the Griffith fracture criterion in McCartney model. MCE model assumed that the matrix and composite stress intensities scale with the stresses. And McCartney gives the threshold stress to verify the theoretical value.

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