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Mathematical Expressions of Radon Measurements

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Abstract

The measurement of radon, thoron and their progeny concentrations also leads to the knowledge of the presence of radioactive elements, which are the sources of these elements such as Uranium-238 and Thorium-232. Using of Solid State Nuclear Tracks Detectors (SSNTDs) it is probably the most widely applied for long term radon measurements. In this chapter, we derived the most important mathematical relationships that researchers need in radon measurements to calculate such as average radon concentration, exhalation rate, equilibrium factor, radon diffusion coefficient and transmission factor to get actual radon concentration in air atmosphere. The relationship between theoretical and experiment calibration drive and other mathematical relationships are given in this chapter.

Keywords: technique radon measurements, SSNTDs, mathematical expressions

1. Introduction

The measurement of radon, thoron and their progeny concentrations also leads to the knowledge of the presence of radioactive elements, which are the sources of these elements. Since Uranium-238 is the parent nuclei of Radon and Thorium-232 that of Thoron, hence with the concentrations of these gases in air, one can predict the presence of high or low concentrations of the source. Radon (chemical symbol, Rn) is a naturally occurring radioactive gaseous element. ^{222}Rn is the decay product of ^{226}Ra , which are part of the long decay chain of ^{238}U . Since uranium is found everywhere in the earth's crust, ^{226}Ra and ^{222}Rn are present in almost all rocks, soil, and water. ^{222}Rn is the decay to short-lived radioactive elements ^{218}Po , ^{214}Pb , ^{214}Bi and ^{214}Po , respectively, which called radon daughters. These daughter products, being the isotopes of heavy metals, get attached to the existing aerosols, suspended particulate matters, in the atmosphere, therefore the inhalations of radon ^{222}Rn progeny are the most important source of irradiation of the human respiratory. The measurement of radon, thoron and their progeny concentrations was done in many countries, with the improvement of experimental apparatus and technical formulation, the same is going on till today. Among the different techniques available for radon measurements, is the method which is based on the use of Solid State Nuclear Tracks Detectors (SSNTDs) it is probably the most widely applied for long term radon measurements. Nuclear track detectors are a plastic detector uses to register alpha particles in the form of tracks. That is will become visible under the optical microscope upon suitable chemical etching of the SSNTDs', the most commonly used CR-39. The closed-can technique (sealed cylindrical plastic container) employs

nuclear tracks detectors to measure the radon concentration, radon exhalation rate, radium content and diffusion coefficient in the soil, building materials and water in laboratory. There are several names for this method, for example, diffusion chamber, accumulation chamber, radon exposure chamber, and time-integrated passive radon dosimeter and emanation container. Different types of cans are being used by different authors, this type is based on geometry (conical, cylindrical, hemispherical, and rectangular), dimensions (radius, height), and material. The can technique has some advantages, simple and efficient method, relatively inexpensive. The technique provides quite reliable measurements. One commonly used design is plastic Poly Vinyl Chloride (PVC) cylindrical can with different dimensions. The aim of this chapter will be to derive mathematical expressions, analysis and discussion, for the most of relationships of measuring the concentration of radon and placed in the summary tables [1–3].

In this chapter, we will derive and discuss the theoretical formalism which is used in the research work and a complete methodology. This chapter will include the detailed description of deriving the most important mathematical relationships used in technique radon measurements. The present work will help in understanding the status of indoor and outdoor radon, thoron and their progeny concentrations and status of the exhalation of these gases from soil. Classification of measurements is also included in this chapter. The necessary procedures and formulae involved in measuring the concentrations of radon, thoron and their progeny, the radioactivity content of samples along with the calculations of, exhalation rate, equilibrium factor, radon diffusion coefficient and other mathematical relationships are given in this chapter.

2. Closed-can technique measurements

2.1 The buildup of radon concentration equation

Since radon is produced continuously from decay of radium in natural decay chains of uranium, the rate of change of the number of radon atoms is determined by radon decay and generation of radon in the decay of radium present in closed can **Figure 1**. Since radium is present as solid and radon as gas, in order to find the rate of change of the number of radon atoms in the air-filled pore space, assuming that [1, 4].

- The radium is only present in soil and decays there.
- The soil column is homogeneous.
- The radium distributes uniformly in the surface soil and does not exist in the air.
- The radon production and decay in the air space.
- Radon transport in the soil is vertical and only due to diffusion and convection in the pore space.
- All radon produced in the solid material will escape (emanate) into the pore-air space.
- Radon-tight containers, no leakage of radon out of the can and no back diffusion effects.

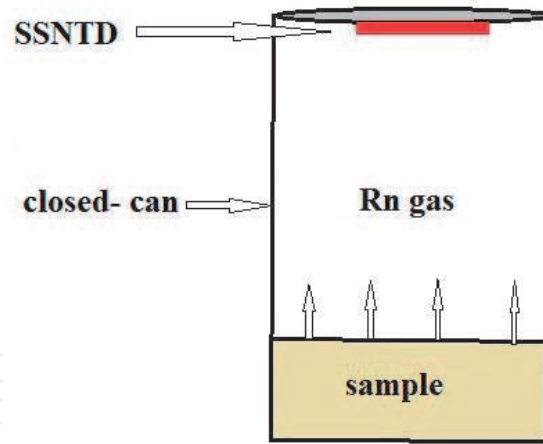


Figure 1.
 Closed-can technique.

Therefore, decay of radium and production of radon can be described by the rate equations for serial radioactive decay chain (Batman equations) [5]:

$$\frac{dN_{Ra}}{dt} = -\lambda_{Ra}N_{Ra} \quad (1)$$

$$\frac{dN_{Rn}}{dt} = \lambda_{Ra}N_{Ra} - \lambda_{Rn}N_{Rn} \quad (2)$$

where λ_{Ra} , λ_{Rn} is decay constant for radium and radon respectively. $T_{1/2}^{Ra} = 1600 \text{ years}$, $T_{1/2}^{Rn} = 3.824 \text{ days}$ are the half-life for radium and radon respectively.

From Eqs. (1) and (2), we can determine the number of undecided radon atoms at a time t . By solving two equations, we will obtain the remaining number of radon atoms without decay at any time is:

$$N_{Rn}(t) = N_{Ra}(0) \lambda_{Ra} \left(\frac{e^{-\lambda_{Ra}t}}{\lambda_{Rn} - \lambda_{Ra}} + \frac{e^{-\lambda_{Rn}t}}{\lambda_{Ra} - \lambda_{Rn}} \right) \quad (3)$$

or

$$N_{Rn}(t) = \frac{\lambda_{Ra}}{\lambda_{Rn} - \lambda_{Ra}} N_{Ra}(0) (e^{-\lambda_{Ra}t} - e^{-\lambda_{Rn}t}) \quad (4)$$

$N_{Ra}(0)$, is the original number of radium atoms. The activity $A(t)$ at time t is defined mathematically:

$$A(t) = \lambda N(t) \quad (5)$$

Eq. (5) becomes:

$$A_{Rn}(t) = \frac{\lambda_{Rn}}{\lambda_{Rn} - \lambda_{Ra}} A_{Ra}(0) (e^{-\lambda_{Ra}t} - e^{-\lambda_{Rn}t}) \quad (6)$$

$A_{Ra}(0)$ is original activity of radium which is constant value.

Since $T_{1/2}^{Ra} \gg T_{1/2}^{Rn} \rightarrow \lambda_{Ra} \ll \lambda_{Rn} \rightarrow e^{-\lambda_{Ra}t} \approx 1$ & $\lambda_{Rn} - \lambda_{Ra} \approx \lambda_{Rn}$. Therefore, Eq. (6) becomes:

$$A_{Rn}(t) = A_{Ra}(0) (1 - e^{-\lambda_{Rn}t}) \quad (7)$$

From Eq. (7) the activity of radon grows and becomes exactly the same with original activity of radium when time t passed many of the radon half-life (i.e. ≈ 27 days). In other word, radon atoms are decaying at the same rate at which they are formed. This is called secular equilibrium [5]. The secular equilibrium is important for the calculation of the activity concentration of radon in the can technique. This means that, the radon activity will reach maximum value or steady state value or equilibrium state value after 4 weeks time. This value is called sometimes the final activity or the saturated activity. In other words, we replaced $A_{Ra}(0)$ by steady state (final) activity of radon A_s in Eq. (7) to become:

$$A_{Rn}(t) = A_s(1 - e^{-\lambda_{Rn}t}) \quad (8)$$

Eq. (8) describes the buildup of radon activity through time t . If V is the volume of air-filled space within can, the activity concentration of radon $C(Bq \cdot m^{-3})$ in the air volume of the can given by the following relation [2, 3]:

$$C = \frac{A_{Rn}}{V} = \frac{\lambda_{Rn}N_{Rn}}{V} \quad (9)$$

Eq. (9) becomes:

$$C(t) = C_s(1 - e^{-\lambda_{Rn}t}) \quad (10)$$

Where $C(t)$ is the radon concentration at time t ($Bq \cdot m^{-3}$), C_s is the steady state (final) concentration ($Bq \cdot m^{-3}$). Eq. (10) is the well-known equation which describes the buildup of the concentration of radon emanated from each sample inside the exhalation container with time [1, 6].

2.2 Track density-radon concentration relation

Since the alpha particles emitted by ^{222}Rn and its progeny strike the detectors and leave latent tracks in it, Solid State Nuclear Detector measures the total number of alpha-disintegration in unit volume of the can during the exposure time. The tracks can be visible by chemical or electrochemical etching. The main measured quantity is the track density, which is the total tracks per unit area of detector, i.e. [1].

$$\rho = \frac{N_{total\ track}}{A_D} \quad (11)$$

where ρ is the track density expressed in ($\text{Track} \cdot \text{cm}^{-2}$), A_D is the area of detector in (cm^2). Since the etching track is observed and accounted by using optical microscope, and when looking into a microscope, we will see a lit circular area called field of view. The field of view (FOV) is the maximum area visible through the lenses of a microscope, and it is represented by a diameter. Therefore, we divide the area of detector to n of the field of view A_{FOV} . Eq. (11) becomes:

$$\rho = \frac{N_{total\ track}}{n A_{FOV}} = \frac{N_{avg}}{A_{FOV}} \quad (12)$$

where n the number of fields is, N_{avg} is the average of total tracks and A_{FOV} is the area of the field of view (cm^2). The measured track density rate recorded on the SSNTD is proportional to the radon concentration during the time of exposure [7].

$$\frac{d\rho}{dt} \propto C(t) \quad (13)$$

or

$$\frac{d\rho}{dt} = K C(t) \quad (14)$$

The proportionality constant, is called the calibration factor of the detector or conversional factor or response factor or turned over sensitivity factor [1]. It convert the track density (Track · cm⁻²) to exposure concentration (Bq · m⁻³ · day). By integrate Eq. (14), we obtain with initial condition $\rho(0) = 0$:

$$\rho = K \int_0^T C(t) dt \quad (15)$$

where $C(t)$ is radon concentration in air around the detector (Bq · m⁻³) at time t and T is the total exposure time (day). There are five cases, we can discuss than blow:

1. The radon concentration is proportion to along exposure time. In this state, the track density measured the integrated concentration and not concentration instantaneous or the final concentration. Sometimes, it is called accumulation concentration or exposure concentration. The integrated radon concentration C_I (Bq · m⁻³ · day) after a period of time T is defined mathematically as [1]:

$$C_I = \int_0^T C(t) dt \quad (16)$$

From Eqs. (15) and (16), we obtain:

$$C_I = \frac{\rho}{K} \quad (17)$$

2. We define the average radon concentration (Bq · m⁻³) by the expression of:

$$C_{avg} = \frac{1}{T} \int_0^T C(t) dt \quad (18)$$

From Eqs. (15) and (18), we get:

$$C_{avg} = \frac{\rho}{K T} \quad (19)$$

3. The radon concentration at steady state value C_s is:

$$C_s = \frac{\rho}{K T_{27d}} \quad (20)$$

This means that, the detector should be exposed for at least 27 day to record tracks for radon concentration.

Integrated concentration C_I ($\text{Bq} \cdot \text{m}^{-3} \cdot \text{day}$)	Average concentration C_{avg} ($\text{Bq} \cdot \text{m}^{-3}$)	Steady state concentration C_s ($\text{Bq} \cdot \text{m}^{-3}$)
$\frac{\rho}{K}$	$\frac{\rho}{K T}$	$\frac{\rho}{K T_{27d}}$

Table 1.
Radon concentration in different quantities.

The results lead to the following remarks [1]:

1. The radon concentration reaches the equilibrium state at the same time ($t \cong 7T_{1/2}$) regardless of the volume of the container.
2. There are three different quantities measured by track density, integrated radon concentration, average radon concentration and saturate radon concentration as shown in **Table 1**.
3. The average radon concentration equal to the saturate radon concentration at large exposure time.

2.3 Radon exhalation rate equation

When radium decay in soil, the resulting atoms of radon isotopes first escapes from the mineral to air-filled pores. Being a noble gas, ^{222}Rn can move large distances through rocks and soils. Radon can diffuse through rocks and soil, can move from one place to the other and can leak out in the atmosphere from the soil. The exhalation of radon from soil involves two mechanisms, the emanation and transport. The measurement of radon exhalation rate in soil is helpful to study radon health hazard. The passive measuring techniques “Can Technique” employing a Solid State Nuclear Track Detector (SSNTDs), a simple and efficient method to assess radon exhalation rates besides being relatively inexpensive, the technique provides quite reliable measurements. The exhalation rate is defined as the rate at which radon escapes from soil into the surrounding air. This may be measured by either per unit area or per unit mass of sample. Consider a sealed cylindrical can fitted with a source of radon and a SSNTD dosimeter fixed at the top of the can. Assume that radon, thoron and their daughters are in radioactive equilibrium in the air volume of the can. For diffusion in air, it is expected that all daughters of interest will be deposited except ^{216}Po will be in air volume. Furthermore, ^{220}Rn and ^{216}Po are in homogenously distributed in the air due to their short half-lives. The ^{212}Po and ^{212}Bi formed by the decay of ^{216}Po will be preferentially in homogenously deposited on the wall of the can. It is clear that the track density is registered on the detector which is related to ^{222}Rn , as well as its plated-out daughters. The exhalation of radon from the sample surface represents the source of the number of radon atoms $N(t)$, present in the air between the sample and SSNTD Which is directly proportional to area of surface and life time for radon. The natural decay of radon provides the only removal mechanism. The rate of change of $N(t)$ with time is therefore governed by the following differential Eq. (21) [1–3]:

$$\frac{dN(t)}{dt} = E_A A \tau - \lambda N(t) \tag{21}$$

where $N(t)$ is the total number of radon atoms present in the can at time t , E is the exhalation rate ($\text{Bq} \cdot \text{m}^{-2} \cdot \text{h}^{-1}$), A is cross-sectional area of the can (the surface

area of sample from which the exhalation takes place), λ is the decay constant of radon (h^{-1}) and τ is the live time of radon (h). The quantity τ does not exist in equation of reference [2, 3], that we added to the equation Eq. (21) to unify the units on both sides of the equation. The solution of Eq. (21) with initial condition $N(0) = 0$ is Eq. (22):

$$N(t) = \frac{E_A A \tau}{\lambda} (1 - e^{-\lambda t}) \quad (22)$$

If V is the volume of air (m^3), the activity concentration of radon $C(t)$ in the air volume of the can as a function of time t that can be given by the following relation Eq. (23):

$$C(t) = \frac{A(t)}{V} = \frac{\lambda N(t)}{V} \quad (23)$$

The equation becomes:

$$C(t) = \frac{E_A A \tau}{V} (1 - e^{-\lambda t}) \quad (24)$$

$$\text{But } \tau(h) = \frac{1}{\lambda(h^{-1})} \quad (25)$$

$$C(t) = \frac{E_A A}{\lambda V} (1 - e^{-\lambda t}) \quad (26)$$

By integrating on sides of equation for time, we get:

$$C_I = \frac{E A}{\lambda V} T_{eff} \quad (27)$$

So, the exhalation radon rate as a function radon integrated concentration is given by:

$$E_A = \frac{C_I \lambda V}{A T_{eff}} \quad (28)$$

At steady state (secular equilibrium), $\frac{dN(t)}{dt} = 0$ and $T_{eff} = T$, we get:

$$E_A = \frac{C_s \lambda V}{A} \quad (29)$$

And the exhalation radon rate as a function average radon concentration is given by:

$$E_A = \frac{C_{avg} \lambda V T}{A T_{eff}} \quad (30)$$

From Eqs. (19) and (30), we get the exhalation rate as a function to the tracks density:

$$E_A = \frac{\rho \lambda V}{KA T_{eff}} \quad (31)$$

By the same method we get the mass exhalation rate, where M is the mass of sample in container:

$$\left. \begin{aligned} E_M &= \frac{C_I \lambda V}{M T_{eff}} \\ E_M &= \frac{C_s T \lambda V}{M T_{eff}} \\ E_M &= \frac{C_{avg} \lambda V T}{M T_{eff}} \\ E_M &= \frac{\rho \lambda V}{K M T_{eff}} \end{aligned} \right\} \quad (32)$$

2.4 Radium content calculation

Since the half-life of radium is 1600 years and that of radon is 3.82 days, it is reasonable to assume that an effective equilibrium (about 98%) for radium-radon members of the decay series is reached to about three weeks or more. Once the radioactive equilibrium is established, one may use the radon alpha analysis for the determination of steady state activity concentration of radium. The activity concentration of radon begins to increase with time t , after the closing of the can, according to the relation Eq. (7) [1]:

$$A_{Rn}(t) = A_{Ra}(1 - e^{-\lambda_{Rn}t}) \quad (33)$$

By dividing Eq. (33) on the volume of radon, we get on the activity concentration of radon:

$$C_{Rn}(t) = \frac{A_{Ra}}{V}(1 - e^{-\lambda_{Rn}t}) \quad (34)$$

By multi Eq. (34) by dt and integrated for exposure time T , we get:

$$A_{Ra} = \frac{C_I V}{T_{eff}} \quad (35)$$

By dividing Eq. (35) on mass of sample, we get:

$$C_{Ra} = \frac{C_I V}{M T_{eff}} \quad (36)$$

where C_{Ra} is effective radium content in unite (Bq/kg):

$$C_{Ra} = \frac{A_{Ra}}{M} \quad (37)$$

By average radon concentration, we get:

$$C_{Ra} = \frac{C_{avg} T V}{M T_{eff}} \quad (38)$$

From Eqs. (19) and (38) we get the effective radium content as a function to the tracks density:

$$C_{Ra} = \frac{\rho A h}{K M T_{eff}} \tag{39}$$

Since the unit of time in exhalation formula in hour, we should convert units of λ , T_{eff} and K to hour. All the quantities in **Table 2** are known except (C_b, C, C_{avg}) . We can find ρ experimentally by using CR-39 or LR-115 type II based on radon dosimeter and by using continuous radon monitor to determine the value of (C_b, C, C_{avg}) . From **Table 2**, we show that the quantities $(\lambda, T, T_{eff}, K, M, A, V)$ are constants, therefore the relationship between radon exhalation rate and effective radium content with the quantities (C_b, C, C_{avg}, ρ) are linear. From the relations in **Table 2**, we get some relations which show the ship linear between these quantities [1]:

$$\left. \begin{aligned} \frac{E_A}{E_M} &= \frac{M}{A} \\ \frac{E_A}{C_{Ra}} &= \lambda \frac{M}{A} \\ \frac{E_M}{C_{Ra}} &= \lambda \end{aligned} \right\} \tag{40}$$

2.5 Closed-can technique (two different detectors)

In this technique using two different SSNTD detectors (CR-39 & LR-II) were separately placed in close-can, to measurement and discriminate between radon and thoron concentrations which escape from sample, at same time **Figure 2**. During this exposure the α -particles emitted by the nuclei of radon and thoron and its progenies have bombarded the SSNTD films [1, 8].

The global track density rates, due to α -particles emitted by radon, registered on the CR-39 detector ρ_{Rn}^G can be writing mathematical:

$$\rho_{Rn}^G = \rho_{Rn} + \rho_{Po218} + \rho_{Po214} \tag{41}$$

where ρ_i are the track density for radon and its progenies on CR-39 detectors and from Eq. (19) can written by concentration average of radon and its progenies:

$$\rho_{Rn}^G = K_{Rn} C_{Rn} T + K_{Po218} C_{Po218} T + K_{Po214} C_{Po214} T \tag{42}$$

where K_i are the calibration factors for radon and its progenies. By malty and dividing right hand-Eq. (42) C_{Rn} , we obtain:

$$\rho_{Rn}^G = \left(K_{Rn} + K_{Po218} \frac{C_{Po218}}{C_{Rn}} + K_{Po214} \frac{C_{Po214}}{C_{Rn}} \right) C_{Rn} T \tag{43}$$

Quantity	C_I	C_s	C_{avg}	ρ
$E_A (Bq\ m^{-2}\ h^{-1}) =$	$\frac{C_I\ \lambda\ V}{A\ T_{eff}}$	$\frac{C_s\ \lambda\ V}{A}$	$\frac{C_{avg}\ T\ \lambda\ V}{A\ T_{eff}}$	$\frac{\rho\ \lambda\ V}{K A\ T_{eff}}$
$E_M (Bq\ kg^{-1}\ h^{-1}) =$	$\frac{C_I\ \lambda\ V}{M\ T_{eff}}$	$\frac{C_s\ \lambda\ V}{M}$	$\frac{C_{avg}\ T\ \lambda\ V}{M\ T_{eff}}$	$\frac{\rho\ \lambda\ V}{K M T_{eff}}$
$C_{Ra} (Bq\ kg^{-1}) =$	$\frac{C_I V}{M T_{eff}}$	$\frac{C_s V}{M}$	$\frac{C_{avg}\ T\ V}{M T_{eff}}$	$\frac{\rho V}{K M T_{eff}}$

Table 2.
The exhalation radon rate formulize.

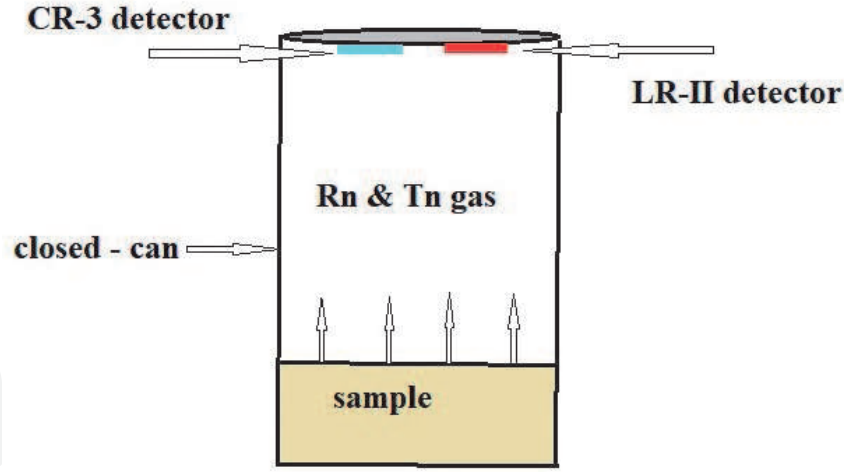


Figure 2.
Closed-can technique (two different detectors).

At radioactive secular equilibrium between radon and its progenies, i.e.:

$$\frac{C_{Po218}}{C_{Rn}} = \frac{C_{Po214}}{C_{Rn}} = 1 \quad (44)$$

Eq. (43) becomes:

$$\rho_{Rn}^G = K_{RnC} C_{Rn} T \quad (45)$$

where

$$K_{RnC} = K_{Rn} + K_{Po218} + K_{Po214} \quad (46)$$

By same procedure, we find ρ_{Th}^G for thoron on CR-39 detector. The total track density ρ_{CR} for radon and thoron become:

$$\rho_{CR} = \rho_{Rn}^G + \rho_{Th}^G = K_{RC} C_{Rn} T + K_{TC} C_{Th} T \quad (47)$$

The final result, we obtain two equations describing the relationship between the radon and thoron concentration and track density on CR-39 and LR-II detectors:

$$\left. \begin{aligned} \rho_{CR} &= K_{RC} T C_{Rn} + K_{TC} T C_{Th} \\ \rho_{LR} &= K_{RL} T C_{Rn} + K_{TL} T C_{Th} \end{aligned} \right\} \quad (48)$$

This are two algebraic linear equations by two variables C_{Rn} and C_{Th} , the general solution for this are:

$$\left. \begin{aligned} C_{Rn} &= \frac{K_{TL} \rho_{CR} - K_{TC} \rho_{LR}}{K_{RC} K_{TL} - K_{RL} K_{TC}} \\ C_{Th} &= \frac{K_{RL} \rho_{CR} - K_{RC} \rho_{LR}}{K_{RL} K_{TC} - K_{RC} K_{TL}} \end{aligned} \right\} \quad (49)$$

After performing a series of mathematical simplifications, we obtain mathematical relationships to calculate concentrations for radon and thoron as function to tracks density ρ_r , radon-thoron concentrations C_r and calibration factors (K_L, K_{Rn}, K_{Th}, K_L) ratio.

$$\left. \begin{aligned} C_{Rn} &= \frac{\rho_{CR}}{(K_{RC} + K_{TC} C_r) T} \\ C_{Tn} &= C_r C_{Rn} \end{aligned} \right\} \quad (50)$$

$$\left. \begin{aligned} C_r &= K_L \left(\frac{\rho_r - K_{Rn}}{K_{Th} - \rho_r} \right) \\ \rho_r &= \frac{\rho_{CR}}{\rho_{LR}} \end{aligned} \right\} \quad (51)$$

$$\left. \begin{aligned} K_{Rn} &= \frac{K_{RC}}{K_{RL}} \\ K_{Th} &= \frac{K_{TC}}{K_{TL}} \\ K_L &= \frac{K_{RL}}{K_{TL}} \end{aligned} \right\} \quad (52)$$

where:

ρ_{CR} : Tracks density registration on CR-39 detector.

ρ_{LR} : Tracks density registration on LR-115(II) detector.

K_{RC} : Calibration factor of radon for CR-39 detector.

K_{RL} : Calibration factor of radon for LR-115(II) detector.

K_{TC} : Calibration factor of thoron for CR-39 detector.

K_{TL} : Calibration factor of thoron for LR-115(II) detector.

Since, C_r is positive quantity, the CR-LR track density ratio should be $K_{Rn} < \rho_r < K_{Th}$ and $K_{Th} > K_{Rn}$. Since, $\rho_{CR} > \rho_{LR}$, because CR-39 records all alpha particles energies, while LR-II detector records window energies (i.e. between E_{min} and E_{max}), this mean $\rho_r > 1$ [9].

3. Indoor radon technique measurements

Radon and thoron present in outdoor and indoor air as they exhaled from soil and building materials of the walls, floor and ceilings. It is critically important that inhalation of radon and their progeny concentrations has been shown experimentally to cause lung cancer in rats and observed to cause lung cancer in men exposed to large amounts in the air of mines. Several techniques have been developed to measure radon indoors. The use of a Solid State Nuclear Track Detector (SSNTD) closed in a cup mode (passive dosimeters) and bare mode has turned out to be the most appropriate for long term measurements. Radon measurement with, bare (open) detector are rough and with rather high uncertainties. Some device of cup mode have single chamber called invented cup. In twin cup mode, the detectors are kept inside a twin cup dosimeter, a cylindrical plastic chamber divided into two equal compartments. The two equal compartments on both sides are filter and pinhole compartments. There is one small compartment at the external middle attached to it which is used for bare mode exposure as shown in **Figure 3** [1, 10, 11].

This design of the dosimeter was well suited to discriminate ^{222}Rn and ^{220}Rn in mixed field situations, where both gases are present. SSNTDs were used as detectors and affixed at the bottom of each cup as well as on the outer surface of the dosimeter. The exposure of the detector inside the cup is termed as cup mode and the one exposed open is termed as the bare mode. One of the cups had its entry covered with a glass fiber filter paper that permits both ^{222}Rn and ^{220}Rn gases into the cup and is called the filter cup. The other cup was covered with a soft sponge and is called the sponge cup. SSNTDs film inside the sponge cup registers tracks contributed by ^{222}Rn only, while that in the filter cup records tracks due to ^{222}Rn

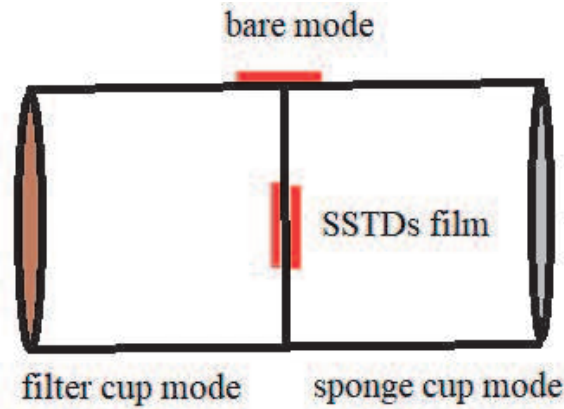


Figure 3.
Twin cups dosimeter.

and ^{220}Rn . The third SSNTDs film exposed in the bare mode registers alpha tracks contributed by the concentrations of both the gases and their alpha emitting progenies. The dosimeters were kept at a height of 1.5 m from the ceiling of the room and care was taken to keep the bare card at least 10 cm minimum away from any surface. This ensured that errors due to tracks from deposited activity from nearby surfaces were avoided, since the ranges of alpha particles from $^{222}\text{Rn}/^{220}\text{Rn}$ are about 10 cm. The global track density rates, due to α -particles emitted by radon, registered on the SSNTDs can write mathematical [11]:

$$\rho_S = K_{RS} T C_{Rn} \quad (53)$$

$$\rho_F = K_{RF} T C_{Rn} + K_{TF} T C_{Th} \quad (54)$$

The general solution for this are:

$$C_{Rn} = \frac{\rho_S}{K_{RS} T} \quad (55)$$

$$C_{Th} = \frac{\rho_S}{K_{TF} T} \left(\frac{\rho_F}{\rho_S} - \frac{K_{RF}}{K_{RS}} \right) \quad (56)$$

where:

ρ_S = Tracks density for sponge cup.

ρ_F = Tracks density for filter piper cup.

K_{RS} = Calibration factor of radon for sponge cup.

K_{RF} = Calibration factor of radon for filter piper cup.

K_{TF} = Calibration factor of thoron for filter piper cup.

Since, C_{Th} is positive quantity, $\frac{\rho_F}{\rho_S}$ track density ratio should be $\frac{\rho_F}{\rho_S} > \frac{K_{RF}}{K_{RS}}$.

4. Equilibrium factor

The equilibrium factor (F) is defined as the ratio of potential alpha energy concentration (PAEC) of actual air-radon mixture (also radon progeny) to the PAEC in secular equilibrium with radon. The equilibrium factor (F) is an important parameter in the determination of radon equivalent dose. A common practice for radon hazard assessment nowadays is to, first, determine the radon gas concentration and then to apply an assumed F with a typical value of about 0.4 [12]. However, in reality, F varies significantly with time and place, and an assumed F cannot reflect the actual conditions. Actually, the radon concentration was very dependent

on the ventilation rate, therefore F was determined by SSNTD based on using can and bare method. In this method, two similar detectors were exposed to radon, one in can-mode configuration (in can detector) and the other in bare-mode configuration (bare detector). F can be found as a function of the track density ratio ρ_B/ρ_S between bare (ρ_B) and in can (ρ_S) detector, respectively. There are three steps for calculating the equilibrium factor [1, 11, 13]:

1. The potential alpha energy concentration of any mixture of (short-live) radon or thoron daughters in air.
2. The working level (WL).
3. The ventilation rate as a function of the track density ratio between bare mode and can mode.
4. The equilibrium factor as a function of the ventilation rate.

The potential alpha energy concentration (PAEC) of any mixture of (short-live) radon or thoron daughters in air is the sum of the potential alpha energy of all daughters atoms present per unit volume of air. The usual unit for this quantity is $\text{MeV} \cdot \text{l}^{-1}$. This unit is related to the SI units J and m^3 according to $1 \text{ J} \cdot \text{m}^{-3} = 6.24 \times 10^9 \text{ MeV} \cdot \text{l}^{-1}$ [12]. To express mathematically, we let:

$$PAEC_{Total} = PAEC_1N_1 + PAEC_2N_2 + PAEC_3N_3 + PAEC_4N_4 \quad (57)$$

where $PAEC_{Total}$, is the total potential alpha energy concentration of any mixture of (short -live) radon daughters in air. $PAEC_i$ ($i = 1, 4$) are the potential alpha energy for ^{218}Po , ^{214}Pb , ^{214}Bi and ^{214}Po , respectively. N_i is the number atoms of daughters of radon. The concentration potential alpha energy is defined as:

$$C_{PAET} = \frac{PAET}{V} \quad (58)$$

$$C_i = \frac{A_i}{V} = \frac{\lambda_i N_i}{V} \quad (59)$$

From Eqs. (57), (58) and (59), we get:

$$C_{PAET} = \left[\frac{PAE_1}{\lambda_1} f_1 + \frac{PAE_2}{\lambda_2} f_2 + \frac{PAE_3}{\lambda_3} f_3 + \frac{PAE_4}{\lambda_4} f_4 \right] C_0 \quad (60)$$

$$\left. \begin{array}{l} f_i = \frac{C_i}{C_0} \\ C_0 = C_{Rn} \end{array} \right\} \quad (61)$$

where f_i , is the activity concentration fraction. A special unit for this quantity used for radiation protection purposes is the working level (WL). A WL is defined as corresponding approximately to the potential alpha energy concentration of short-live radon daughters in air which are in radioactive equilibrium with a radon concentration of $3.7 \text{ kBq} \cdot \text{m}^{-3}$ [12].

$$C_1 = C_2 = C_3 = C_4 = C_0 = C_{Rn} = 3.7 \text{ kBq} \cdot \text{m}^{-3} = 3.7 \text{ Bq} \cdot \text{l}^{-1} \quad (62)$$

Eq. (60) become as:

$$C_{PAECT} = WL = \frac{3.7 PE_1}{\lambda_1} + \frac{3.7 PE_2}{\lambda_2} + \frac{3.7 PE_3}{\lambda_3} + \frac{3.7 PE_4}{\lambda_4} = 1.3 * 10^5 \text{ Mev } l^{-1} \quad (63)$$

The numerical calculations of Eq. (63) are shown in **Table 3**.

One (WL) equal to $1.3 \times 10^5 \text{ Mev} \cdot l^{-1}$ of air. To obtain the concentration potential alpha energy by fraction energy, Eq. (63) multi and divide by $3.7 \text{ Bq} \cdot l^{-1}$ and multi and divide by $1.3 \times 10^5 \text{ Mev} \cdot l^{-1}$:

$$C_{PET} = \left[\frac{\frac{3.7PE_1}{\lambda_1}}{1.3 * 10^5} f_1 + \frac{\frac{3.7PE_2}{\lambda_2}}{1.3 * 10^5} f_2 + \frac{\frac{3.7PE_3}{\lambda_3}}{1.3 * 10^5} f_3 + \frac{\frac{3.7PE_4}{\lambda_4}}{1.3 * 10^5} f_4 \right] \frac{1.3 * 10^5}{3.7} C_{Rn} \quad (64)$$

$$C_{PET} = [0.11 f_1 + 0.51 f_2 + 0.38 f_3] \frac{1.3 * 10^5}{3.7} C_{Rn} \quad (65)$$

To obtain the concentration potential alpha energy by unit WL is:

$$C_{PET}(WL) = \frac{F_{Rn} C_{Rn} (Bq \text{ } l^{-1})}{3.7} \quad (66)$$

or

$$C_{PET}(mWL) = \frac{F_{Rn} C_{Rn} (Bq \text{ } m^{-3})}{3.7} \quad (67)$$

where

$$F_{Rn} = 0.11 f_1 + 0.51 f_2 + 0.38 f_3 \quad (68)$$

For Thoron daughters we find the concentration of thoron which give potential energy of alpha its daughters equal to $1.3 \times 10^5 \text{ Mev} \cdot l^{-1}$ is $0.275 \text{ Bq} \cdot l^{-1}$, as shown in **Table 4**.

By the same setup, we drive Eq. (60) for thoron and numerical calculations of it, as shown in **Table 5**.

The concentration potential alpha energy for thoron by unit WL is:

$$C_{PET}(mWL) = \frac{F_{Tn} C_{Tn} (Bq.m^{-3})}{0.275} \quad (69)$$

where

$$F_{Tn} = 0.91 f_2 + 0.09 f_3 \quad (70)$$

Daughters of radon	α-energy (Mev)	Half-life (s)	Ultimate alpha energy (Mev)	Total energy (Mev)	Fraction energy
Po-218	6	183	6 + 7.69	1.34E+04	0.11
Pb-214	zero	1563	zero+7.68	6.41E+04	0.51
Bi-214	zero	1182	zero +7.68	4.85E+04	0.38
Po-214	7.69	1.64E-04	7.68	6.72E-03	zero
$C_{PAECT} =$				1.3E+05	

Table 3.
Numerical calculations of Eq. (63).

Daughters of thoron	α -energy (Mev)	Half-life (s)	Ultimate alpha energy (Mev)	Total energy (Mev)	CTn (Bq · l ⁻¹)
Po-216	6.78	0.15	14.5952	3.16E+00	0.275
Pb-212	zero	38,304	7.8152	4.32E+05	
Bi-212	6.1	3636	7.8152	4.10E+04	
Po-212	8.78	3.04E-07	8.78	3.85E-06	
				4.73E+05	

Table 4.
Numerical calculations of thoron concentration as corresponding to the total energy $1.3 \times 10^5 \text{ Mev} \cdot \text{l}^{-1}$.

Daughters of thoron	α -energy (Mev)	Half-life (s)	Ultimate alpha energy (Mev)	Total energy (Mev)	Fraction energy
Po-216	6.78	0.15	14.5952	8.69E-01	zero
Pb-212	zero	38,304	7.8152	1.19E+05	0.91
Bi-212	6.1	3636	7.8152	1.13E+04	0.09
Po-212	8.78	3.04E-07	8.78	1.06E-06	zero
				1.3E+05	

Table 5.
Numerical calculations of Eq. (63) for thoron.

F_{Rn} and F_{Tn} are the equilibrium factor for radon and thoron, respectively. The maximum value of equilibrium factor is $F = 1$, when the radon or thoron progeny are present in complete equilibrium with radon/thoron that is present. The minimum value of equilibrium factor is $F = 0$, that is mean no-equilibrium between the radon or thoron and its progeny. In our work, we measured the equilibrium factor depending on the ventilation rate. Ventilation rate is one of the parameters used to describe the perturbation caused in radioactive equilibrium of radon/thoron and its descendants in air. Decay of radon and production of radon can be described by the rate equations for serial radioactive decay chain (Batman equations):

$$\frac{dN_i}{dt} = \lambda_{i-1}N_{i-1} - \Lambda_i N_i \quad (i = 1 \dots 4) \tag{71}$$

where

$$\Lambda_i = V + A_i + W_i + \lambda_i \tag{72}$$

The first term on the right is the rate of formation of the i^{th} -member of the progeny by radioactive decay of the $(i-1)^{\text{th}}$ -member, with constant λ_{i-1} ; the second term describes the radioactive leakage rate, owing to ventilation V , to aerosol grains A_i and deposition on the walls W_i to which it added the rate of radioactive decay of the i^{th} -member of the progeny, λ_i . The index i , running from 1 to 4, labels the relevant daughter in the radon family: ^{218}Po , ^{214}Pb , ^{214}Bi , ^{214}Po . ^{222}Rn itself will be label with $i = 0$. For thoron family: ^{216}Po , ^{212}Pb , ^{212}Bi , ^{212}Po . ^{220}Rn (Tn) itself will be labeled with $i = 0$. Ventilation rate affects equally all members of the family. When steady state is reached, radon daughter's activities Eq. (71) become as [13]:

$$\lambda_{i-1}N_{i-1} = \Lambda_i N_i \tag{73}$$

or

$$\frac{C_i}{C_{i-1}} = \frac{\lambda_i}{\Lambda_i} = d_i \quad (74)$$

From Eq. (74), we obtain:

$$\left. \begin{aligned} f_1 &= \frac{C_1}{C_0} = d_1 \\ f_2 &= \frac{C_2}{C_0} = d_1 d_2 \\ f_3 &= \frac{C_3}{C_0} = d_1 d_2 d_3 \\ f_4 &= \frac{C_4}{C_0} = d_1 d_2 d_3 \end{aligned} \right\} \quad (75)$$

where $\frac{C_4}{C_3} = d_4 = 1$ since $C_3 = C_4$ For $\lambda_3 \ll \lambda_4$

When the ventilation rate is the only environmental affecting disequilibrium or when it is the dominant on, ventilation rates and the equilibrium factor are obtained by track density measurements, so:

$$\Lambda_i = V + \lambda_i \quad (76)$$

The track density of both bare mode and can mode (with sponge filter) detector relates the concentration of radon and its daughters as:

In can mode:

$$\rho_S = \rho_0 + \rho_1 + \rho_4 \quad (77)$$

$$\rho_S = K_0 C_0 T + K_1 C_1 T + K_4 C_4 T \quad (78)$$

$$\rho_S = (K_0 + K_1 f_1 + K_4 f_4) C_0 T \quad (79)$$

In can mode $f_1 = f_2 = f_3 = f_4 = 1$, Eq. (79) become as:

$$\rho_S = K_S C_0 T \quad (80)$$

where

$$K_S = K_0 + K_1 + K_4 \quad (81)$$

In bare mode (in absence thoron)

$$\rho_B = \rho_0 + \rho_1 + \rho_4 \quad (82)$$

$$\rho_B = (\bar{K}_0 + \bar{K}_1 f_1 + \bar{K}_4 f_4) \bar{C}_0 T \quad (83)$$

In bare mode $f_1 \neq f_2 \neq f_3 \neq f_4$ because no equilibrium between the radon and its progenies, but $\bar{K}_0 = \bar{K}_1 = \bar{K}_4 = K_B$, therefore Eq. (83) become as:

$$\rho_B = K_B (1 + f_1 + f_4) \bar{C}_0 T \quad (84)$$

By dividing Eq. (84) on Eq. (80) when $\bar{C}_0 = C_0$, we obtain:

$$1 + f_1 + f_4 = K_{SB} \rho_{BS} \quad (85)$$

$$\left. \begin{aligned} \rho_{BS} &= \frac{\rho_B}{\rho_S} \\ K_{SB} &= \frac{K_S}{K_B} \end{aligned} \right\} \quad (86)$$

From Eqs. (74), (75), (76) and (84), we obtain:

$$\frac{\lambda_1}{(\lambda_1 + V)} + \frac{\lambda_1}{(\lambda_1 + V)} \frac{\lambda_2}{(\lambda_2 + V)} \frac{\lambda_3}{(\lambda_3 + V)} = K_{SB}\rho_{BS} - 1 \quad (87)$$

After some setup, we obtain:

$$V^3 + a_2V^2 + a_1V + a_0 = 0 \quad (88)$$

where

$$\left. \begin{aligned} a_2 &= (1 + B)\lambda_1 + \lambda_2 + \lambda_3 \\ a_1 &= (1 + B)(\lambda_2 + \lambda_3)\lambda_1 + \lambda_2\lambda_3 \\ a_0 &= (1 + 2B)\lambda_1\lambda_2\lambda_3 \\ B &= \frac{1}{1 - K_{SB}\rho_{BS}} \end{aligned} \right\} \quad (89)$$

Ventilation rate is the solution of Eq. (89), obtainable by means of standard algebraic procedures. The equilibrium factor is strongly dependent on the ventilation rate. This dependence was expressed by using Eqs. (68), (70), (75) and (76).

$$F_{Rn} = \frac{\lambda_1}{(\lambda_1 + V)} \left(0.11 + 0.51 \frac{\lambda_2}{(\lambda_2 + V)} + 0.38 \frac{\lambda_2\lambda_3}{(\lambda_2 + V)(\lambda_3 + V)} \right) \quad (90)$$

$$F_{Tn} = \frac{\lambda_1\lambda_2}{(\lambda_1 + V)(\lambda_2 + V)} \left(0.91 + 0.09 \frac{\lambda_3}{(\lambda_3 + V)} \right) \quad (91)$$

Values of the equilibrium factor F , follow from solution of Eq. (88) and combined Eq. (90) and Eq. (91).

5. Determination of the radon diffusion coefficient in porous medium

The diffusion of radon is a process determined by radon gas concentration gradient across the radon gas sources (rocks, soils, building materials, and other different materials) and the surrounding air. The knowledge of radon diffusion coefficient makes it possible to determine other features of a material related to radon. The diffusion of radon through the porous medium can be described by Fick's second laws [1, 14, 15]. The basic concept in measuring radon concentration involves observations of radon diffusion through the porous medium being studied. This medium separates two chambers that have different radon concentrations. In such configurations, radon diffusion comes from one chamber to another, primarily in one direction only. The standard method for the measurement of radon diffusion coefficient beads on the evaluation of radon flux through tested material exposed to high radon concentration and placed between two containers by measuring in both sides of material. For this, one of the containers is connected to radon source (radium source) and the other where the radon concentration is periodically evaluated as shown in **Figure 4**. It is assumed that radon is well mixed in both containers.

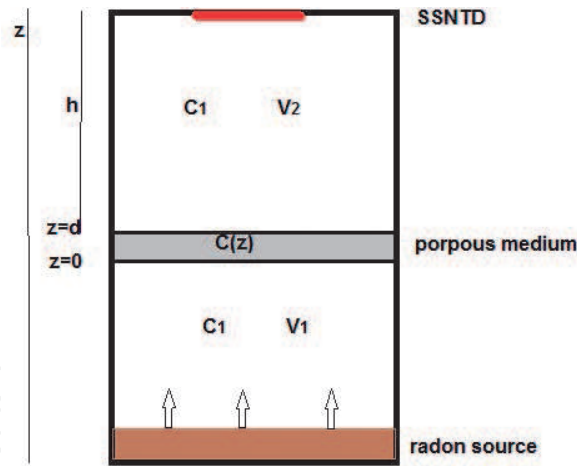


Figure 4.
Diffusion radon chambers.

The determination of the radon diffusion coefficient with this method is normally performed under steady state conditions.

The container 1 has volume V_1 and radon concentration C_1 and container 2 has volume V_2 and radon concentration C_2 . It is assumed that the material represents thickness d and area A produced P is the radon production rate ($\text{Bq} \cdot \text{m}^{-3} \cdot \text{s}^{-1}$) and has diffusion coefficient D . We neglect radon production within the porous material itself and no leakage in containers. Radon transport in porous material is described by a general equation of continuity, which includes four basic processes: generation, decay, diffusion and convection. Under the supposition of the time-dependent one-dimensional differential equation (no convection) describing the radon activity concentration C_0 is given by Fick's second law:

$$\frac{\partial C_0(z, t)}{\partial t} = D_e \frac{\partial^2 C_0(z, t)}{\partial z^2} - \lambda C_0(z, t) + P \quad (92)$$

where $C_0(z, t)$ is the radon concentration within the pores (Bq/m^3), D is the bulk diffusion coefficient (m^2/h , gas flux expressed per unit area of material as a whole), D_e is the effective diffusion coefficient λ is the radon decay constant, ε is the porosity and P is the production rate of radon in the pore air ($\text{Bq}/\text{m}^3 \cdot \text{h}$):

$$D_e = \frac{D}{\varepsilon} \quad (93)$$

$$P = \frac{f \lambda C_{Ra} \rho}{\varepsilon} \quad (94)$$

where, f is the emanation fraction, ρ is the bulk density (kg/m^3) and C_{Ra} is the radium concentration (Bq/kg).

In steady state $\frac{\partial C_0(z, t)}{\partial t} = 0$ and Eq. (92) becomes:

$$\frac{d^2 C_0(z)}{dz^2} - \frac{\lambda}{D_e} C_0(z) + \frac{P}{D_e} = 0 \quad (95)$$

The general solution is:

$$C_0(z) = A e^{z/l} + B e^{-z/l} + \frac{P}{\lambda} \quad (96)$$

$$l = \sqrt{\frac{D_e}{\lambda}} \quad (97)$$

where A, B are integral constant obtained by boundary conditions, l is the diffusion length of radon (m). The boundary conditions are formed by, it is assumed that the radon concentration at the boundary conditions is specified by the radon concentration of incoming air and the radon concentration is set to a constant value at these boundary conditions. Two boundary conditions at equilibrium rate, we needed:

$$\left. \begin{aligned} C_0(z=0) &= C_1 \\ C_0(z=d) &= C_2 \end{aligned} \right\} \quad (98)$$

We get two equations:

$$C_1 = A + B + \frac{P}{\lambda} \quad (99)$$

$$C_2 = A e^{\beta} + B e^{-\beta} + \frac{P}{\lambda} \quad (100)$$

where

$$\beta = \frac{d}{l} \quad (101)$$

By solving Eqs. (99) and (100) and some mathematical step, we get:

$$A = \frac{-e^{-\beta}}{e^{\beta} - e^{-\beta}} C_1 + \frac{1}{e^{\beta} - e^{-\beta}} C_2 - \frac{1 - e^{-\beta}}{e^{\beta} - e^{-\beta}} \frac{P}{\lambda} \quad (102)$$

$$B = \frac{e^{\beta}}{e^{\beta} - e^{-\beta}} C_1 - \frac{1}{e^{\beta} - e^{-\beta}} C_2 + \frac{1 - e^{\beta}}{e^{\beta} - e^{-\beta}} \frac{P}{\lambda} \quad (103)$$

By using the relations:

$$\left. \begin{aligned} e^{\beta} + e^{-\beta} &= 2 \cosh \beta \\ e^{\beta} - e^{-\beta} &= 2 \sinh \beta \end{aligned} \right\} \quad (104)$$

And mathematical steps, we get the general solution:

$$C_0(z) = \frac{\sinh\left(\frac{d-z}{l}\right)}{\sinh\left(\frac{d}{l}\right)} C_1 + \frac{\sinh\left(\frac{z}{l}\right)}{\sinh\left(\frac{d}{l}\right)} C_2 + \left(\frac{\sinh\left(\frac{d-z}{l}\right) + \sinh\left(\frac{z}{l}\right)}{\sinh\left(\frac{d}{l}\right)} - 1 \right) \frac{P}{\lambda} \quad (105)$$

From steady state of exhalation radon Eq. (29), we get:

$$E = \frac{\lambda V C_2}{A} \quad (106)$$

From Fick's first law:

$$E = -D_e \left. \frac{dC_0(z)}{dz} \right|_{z=d} \quad (107)$$

We get:

$$\frac{dC_0(z)}{dz} = \frac{-\cosh\left(\frac{d-z}{l}\right)}{l \sinh\left(\frac{d}{l}\right)} C_1 + \frac{\cosh\left(\frac{z}{l}\right)}{l \sinh\left(\frac{d}{l}\right)} C_2 + \left(\frac{\cosh\left(\frac{z}{l}\right) - \cosh\left(\frac{d-z}{l}\right)}{l \sinh\left(\frac{d}{l}\right)} \right) \frac{P}{\lambda} \quad (108)$$

$$\left. \frac{dC_0(z)}{dz} \right|_{z=d} = \frac{-1}{l \sinh \left(\frac{d}{l} \right)} C_1 + \frac{\cosh \left(\frac{d}{l} \right)}{l \sinh \left(\frac{d}{l} \right)} C_2 + \left(\frac{\cosh \left(\frac{d}{l} \right) - 1}{l \sinh \left(\frac{d}{l} \right)} \right) \frac{P}{\lambda} \quad (109)$$

Substitute Eq. (109) in Eq. (107), we obtain:

$$E = \frac{-D_e}{l \sinh \left(\frac{d}{l} \right)} \left\{ -C_1 + \cosh \left(\frac{d}{l} \right) C_2 - \left(\cosh \left(\frac{d}{l} \right) - 1 \right) \frac{P}{\lambda} \right\} \quad (110)$$

From Eq. (106) and Eq. (110), we get:

$$C_2 = \frac{C_1}{\cosh \left(\frac{d}{l} \right) + \frac{\lambda V_2}{D_e A} \sinh \left(\frac{d}{l} \right)} + \frac{1 - \cosh \left(\frac{d}{l} \right)}{\cosh \left(\frac{d}{l} \right) + \frac{\lambda V_2}{D_e A} \sinh \left(\frac{d}{l} \right)} \frac{P}{\lambda} \quad (111)$$

Since,

$$\left. \begin{aligned} V_2 &= A h \\ l^2 &= \frac{D_e}{\lambda} \\ \frac{\lambda l V_2}{D_e A} &= \frac{h}{l} \end{aligned} \right\} \quad (112)$$

Substitute set Eq. (112) in Eq. (111), we get:

$$C_2 = \frac{C_1}{\cosh \left(\frac{d}{l} \right) + \frac{h}{l} \sinh \left(\frac{d}{l} \right)} + \frac{1 - \cosh \left(\frac{d}{l} \right)}{\cosh \left(\frac{d}{l} \right) + \frac{h}{l} \sinh \left(\frac{d}{l} \right)} \frac{P}{\lambda} \quad (113)$$

Rewrite Eq. (113):

$$\left(1 - \frac{P}{\lambda C_2} \right) \cosh \left(\frac{d}{l} \right) + \frac{h}{l} \sinh \left(\frac{d}{l} \right) - \left(\frac{C_1}{C_2} - \frac{P}{\lambda C_2} \right) = 0 \quad (114)$$

From Eqs. (101) and (114), we get final relation:

$$\left(1 - \frac{P}{\lambda C_2} \right) \cosh \beta + \frac{h}{d} \beta \sinh \beta - \left(\frac{C_1}{C_2} - \frac{P}{\lambda C_2} \right) = 0 \quad (115)$$

Eq. (115) is nonlinear equation; therefore, the radon diffusion coefficient D can be numerically calculated by using the Newton-Raphson method.

IF β is small, and by using set relation in Eq. (116), we can make approximation to evolution radon diffusion coefficient by simple relation as:

$$\left. \begin{aligned} \sinh \beta &= \frac{e^\beta - e^{-\beta}}{2} = \frac{1 + \beta - 1 - \beta}{2} = \beta \\ \cosh \beta &= \frac{e^\beta + e^{-\beta}}{2} = \frac{1 + \beta + 1 - \beta}{2} = 1 \end{aligned} \right\} \quad (116)$$

We find simple relation to calculate the effective diffusion coefficient Eq. (117).

$$D_e = \frac{\lambda d h}{\frac{c}{C_2} - 1} \quad (117)$$

6. Transmission factor

The purpose of using covers of cup (sponge or paper filter) to allow entry radon and thoron gases only without its daughters for filter paper and radon only for

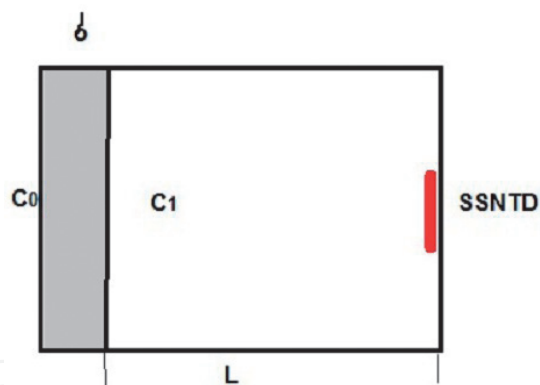


Figure 5.
 Diffusion radon cup.

sponge. In actual terms it means that the covers are reduced from concentration inter cup to detection [16]. Therefore, we define transmission factor as the ratio between the concentrations inter cup and outside cup. The purpose from calculate diffusion coefficient and transmission factor to get actual radon concentration in air atmosphere. It is assumed that the gas enters the chamber through porous filter by the process of diffusion with diffusion coefficient D_1 and D_2 in air. If C_1 is the average radon/thoron gas concentration in the cup and C_o is out the cup. Let the radon diffusion in x-direction **Figure 5** [1, 14, 16].

Then, the steady state equations for described the radon gas diffusion internal porous filter and in air space in cup may be written as Eq. (95), ($P = 0$, because of the lack of a source of radium) respectively:

$$\frac{d^2 C_1(x)}{dx^2} - \frac{\lambda}{D_1} C_1(x) = 0 \quad (118)$$

$$\frac{d^2 C_2(x)}{dx^2} - \frac{\lambda}{D_2} C_2(x) = 0 \quad (119)$$

The general solutions are

$$C_1(x) = A_1 e^{x/l_1} + B_1 e^{-x/l_1} \quad (120)$$

$$C_2(x) = A_2 e^{x/l_2} + B_2 e^{-x/l_2} \quad (121)$$

$$\left. \begin{aligned} l_1 &= \sqrt{\frac{D_1}{\lambda}} \\ l_2 &= \sqrt{\frac{D_2}{\lambda}} \end{aligned} \right\} \quad (122)$$

where A_1, B_1, A_2, B_2 , are integral constants obtained by boundary conditions, l_1, l_2 are the diffusion length of radon (m), D_1, D_2 are diffusion coefficients for porous filter and air respectively.

1. The boundary conditions at equilibrium at sides of porous filter are:

$$\left. \begin{aligned} A_1 e^{\delta/l_1} + B_1 e^{-\delta/l_1} &= C_o & \text{at } x &= -\delta \\ A_1 + B_1 &= C_i & \text{at } x &= 0 \end{aligned} \right\} \quad (123)$$

The solutions of these equations are:

$$\left. \begin{aligned} A_1 &= \frac{e^{\delta/l_1}}{e^{\delta/l_1} - e^{-\delta/l_1}} C_i - \frac{1}{e^{\delta/l_1} - e^{-\delta/l_1}} C_o \\ B_1 &= \frac{-e^{-\delta/l_1}}{e^{\delta/l_1} - e^{-\delta/l_1}} C_i + \frac{1}{e^{\delta/l_1} - e^{-\delta/l_1}} C_o \end{aligned} \right\} \quad (124)$$

From Eqs. (120) and (124), we get the general solution is:

$$C_1(x) = \frac{\sinh\left(\frac{x-\delta}{l_1}\right)}{\sinh\left(\frac{\delta}{l_1}\right)} C_i + \frac{\sinh\left(\frac{x}{l_1}\right)}{\sinh\left(\frac{\delta}{l_1}\right)} C_o \quad (125)$$

2. The boundary conditions at equilibrium in air space enter cup are:

$$\left. \begin{aligned} C_2(x=0) &= C_i \\ \frac{dC_2(x)}{dx} \Big|_{x=L} &= 0 \end{aligned} \right\} \quad (126)$$

We get two equations:

$$\left. \begin{aligned} A_2 + B_2 &= C_i & \text{at } x=0 \\ \frac{A_2}{l_2} e^{L/l_2} - \frac{B_2}{l_2} e^{-L/l_2} &= 0 & \text{at } \frac{dC_2(x)}{dx} \Big|_{x=L} = 0 \end{aligned} \right\} \quad (127)$$

The solutions of these equations are:

$$\left. \begin{aligned} A_2 &= \frac{e^{-L/l_2}}{e^{L/l_2} + e^{-L/l_2}} C_i \\ B_2 &= \frac{e^{L/l_2}}{e^{L/l_2} + e^{-L/l_2}} C_i \end{aligned} \right\} \quad (128)$$

From Eqs. (121) and (128), we get the general solution as:

$$C_2(x) = \frac{\sinh\left(\frac{x-L}{l_2}\right)}{\sinh\left(\frac{L}{l_2}\right)} C_i \quad (129)$$

The average concentration radon inter cup, we get from relation:

$$\bar{C} = \frac{1}{L} \int_0^L C_2(x) dx \quad (130)$$

The average concentration radon inter cup:

$$\bar{C} = \frac{l_2}{L} \tanh\left(\frac{L}{l_2}\right) C_i \quad (131)$$

3. The boundary condition number 3 is:

$$E_1|_{x=0} = E_2|_{x=0} \quad (132)$$

or

$$\left. \begin{aligned} -D_1 \frac{dC_1(x)}{dx} \Big|_{x=0} &= -D_2 \frac{dC_2(x)}{dx} \Big|_{x=0} \\ \frac{dC_1(x)}{dx} \Big|_{x=0} &= \frac{1}{l_1} \left\{ \frac{\cosh\left(\frac{\delta}{l_1}\right)}{\sinh\left(\frac{\delta}{l_1}\right)} C_i - \frac{1}{\sinh\left(\frac{\delta}{l_1}\right)} C_o \right\} \end{aligned} \right\} \quad (133)$$

and

$$\frac{dC_2(x)}{dx} \Big|_{x=0} = -\frac{1}{l_2} \tanh\left(\frac{L}{l_2}\right) C_i \quad (134)$$

Substitute Eq. (134) in Eq. (133), we get:

$$C_i = \frac{C_o}{\cosh\left(\frac{\delta}{l_1}\right) + \frac{D_2 l_1}{D_1 l_2} \sinh\left(\frac{\delta}{l_1}\right) \tanh\left(\frac{L}{l_2}\right)} \quad (135)$$

IF $\frac{\delta}{l_1}$ is small $\sinh\left(\frac{\delta}{l_1}\right) = \frac{\delta}{l_1}$ and $\cosh\left(\frac{\delta}{l_1}\right) = 1$, we get:

$$\bar{C} = \frac{\frac{l_2}{L}}{\frac{\lambda l_2 \delta}{D_1} + \coth\left(\frac{L}{l_2}\right)} C_o \quad (136)$$

From define transmission factor we get as:

$$Transmission (\%) = \frac{\bar{C}}{C_o} = \frac{\frac{l_2}{L}}{\frac{\lambda l_2 \delta}{D_1} + \coth\left(\frac{L}{l_2}\right)} \quad (137)$$

7. The calibration factor

Quantitative measurements with both single and multi-SSNT detectors device can be performed only if the calibration factor is known. It can be measured experimentally or estimated theoretically. Moreover, the theoretically derived calibration factor relations may supply reasonable basic design criteria. There are many method to deriving formulas of calibration factor such as Mont- Carlo, mean critical angle by unit cm. At the same time, the radon chamber is used to determine the calibration factors for different dosimeter geometry configuration [1, 17–19]. The theoretical calibration factor is calculated by unit (cm), while the experiment calibration factor is measured by $\left(\frac{Track \text{ cm}^{-2}}{Bq \cdot m^{-3} \cdot day}\right)$. Since 1 Bq = Disintegration/second = track/second, 1 day = 86,400 sec and $1 \text{ m}^3 = 10^6 \text{ cm}^3$, so, $1 \text{ cm} = 0.0864 \left(\frac{Track \text{ cm}^{-2}}{Bq \cdot m^{-3} \cdot day}\right)$ [20]. The relation between $K_{theoretical}$ and $K_{experement}$ in the can technique is:

$$K_{experement} \left(\frac{Track \text{ cm}^{-2}}{Bq \cdot m^{-3} \cdot day}\right) = 0.0864 \sum_i K_i^{theoretical} (cm) \quad (138)$$

8. Conclusion remarks

1. The concentration of radon emanated from each sample inside the closed container reach to the steady state after a fixed period of time without leakage out of the container Eq. (10).
2. The detector should be exposed for at least 27 day to record tracks for radon concentration.
3. The radon concentration reaches the equilibrium state at the same time ($t \cong 7T_{1/2}$) regardless of the volume of the container.
4. There are three different quantities measured by track density, integrated radon concentration, average radon concentration and saturate radon concentration as shown in **Table 1**.
5. From the relations in **Table 2**, we get some relationship between radon concentration, area and mass radon exhalation and effective radium content are liner equations **Table 2**.
6. The radon/thoron concentrations equations which used in technique two different SSNTDs are very critical because it's a mathematical relationship as a fraction.
7. For more accurate estimation of the effective dose from radon/thoron exposure, one should measure the equilibrium factor at each site.
8. High dose does not necessarily mean there is a high concentration of radon, there may be high equilibrium factor (bad ventilation), and so we recommend interest in ventilation factor when houses and buildings design.
9. It is necessary to correct the radon concentration when using covers (sponge or filter paper) in measurements of radon concentration by transmission factor.

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