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## Chapter

# Combined Gravity or Self-Potential Anomaly Formula for Mineral Exploration

Khalid S. Essa and Mahmoud Elhussein

# Abstract

A combined gravity and/or self-potential anomaly formula is utilized to estimate the model parameters of the buried geologic structures represented by simple geometric. The simple geometric shapes (spheres, cylinders, and sheets) are not really found but often applied to reduce the nonuniqueness in interpreting the gravity and self-potential data. Numerous approaches through the combined formula such as least squares, Werner deconvolution, and the particle swarm optimization method are used. The application of these methods was demonstrated by applying a synthetic gravity and self-potential example without and with 10% random noise to compare their efficiency in estimating the model parameters of the buried structures. Besides, they were applied to two field data for mineral exploration. The appraised model parameter values from each method were compared together and with those published in literature.

**Keywords:** gravity and/or self-potential, model parameters, noise, mineral exploration

## 1. Introduction

Minerals exploration is vital in many countries to increase the income of their people and their economy relies upon discovering minerals. The minerals or ores mined have different variety according to its important in the economy. Geophysical passive method such as gravity and self-potential play an important role in discovering these minerals or ores [1–5]. The gravity method based on measuring the variations in the Earth's gravitational field resulting from the density differences between the subsurface rocks while the self-potential method depended on the electrical potential that develops on the earth's surface due to flow of the natural electrical current on the subsurface [6, 7]. The interpretation of gravity and self-potential data falls on the main two categories as follows: the first category depends on threedimensional and two-dimensional data elucidation [8–13], the second category is depending using the simple geometric-shaped model such as spheres, cylinders, and sheets which are playing a vital role in interpreting the subsurface structures to reach the priors information that help in more investigations [14–20]. In addition, methods depend on the global optimization algorithms such as genetic algorithm [21–24], particle swarm [25, 26], simulated annealing [27–32], flower pollination [33], memory-based hybrid dragonfly [34], differential evolution [35, 36].

Here, a combined formula for both gravity and self-potential [37] is applied to construct this chapter. Moreover, this formula is used to calculate the buried model parameters, for example in case of self-potential data, the parameters are the electric dipole moment or the amplitude coefficient (K), the polarization angle ( $\theta$ ), the depth (z), the shape (q), and the origin location ( $x_o$ ) while in case of gravity data, the parameters are the amplitude coefficient (K), the depth (z), the shape (q), and the origin location ( $x_o$ ) while in case of gravity data, the origin location ( $x_o$ ) for the buried simple-geometric shapes. Three approaches are suggested to interpret the gravity or self-potential anomaly profile through the combined formula. These methods are least squares, Werner deconvolution, and the particle swarm optimization. The advantage of each method is demonstrated by applying a synthetic example for gravity and self-potential data without and with a 10% random noise to compare their efficiency in deducing the buried model parameters. In addition, they tested on two field example for mineral exploration.

#### 2. The suggested combined gravity or self-potential formula

Firstly, the gravity anomaly formula due to simple geometric shapes is [15, 16, 18]

$$g(x_i, z, q) = K \frac{z^m}{\left( (x_i - x_o)^2 + z^2 \right)^q}.$$
 (1)

Secondly, the self-potential anomaly formula for the same simple geometric models is [14]

$$V(x_i, z, \theta, q) = K \frac{x \cos\theta + z \sin\theta}{\left(\left(x_i - x_o\right)^2 + z^2\right)^q}.$$
(2)

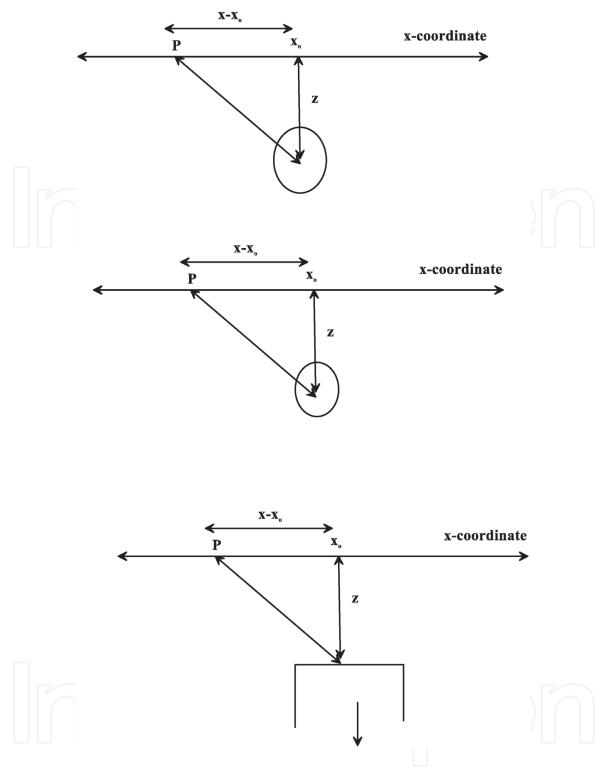
In Refs. [1, 37], Eqs. (1) and (2) were used to join together to produce a combined gravity or self-potential formula for the simple geometric structures such as a semi-infinite vertical cylinder, a dike, a horizontal cylinder, and a sphere (**Figure 1**) as follows:

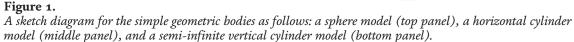
$$J(x_i) = K \frac{cx_i(\cos\theta)^n + z^p(\sin\theta)^m}{\left((x_i - x_o)^2 + z^2\right)^q},$$
(3)

where *K* is the amplitude coefficient, which depends on the shape of the buried model, *z* is the depth,  $\theta$  is the polarization angle,  $x_i$  is the horizontal coordinates,  $x_o$  is the origin location of the buried structure, *q* is the shape (i.e., equals 1.5 for a sphere, 1.0 for a horizontal cylinder, and 0.5 for a semi-infinite vertical cylinder), *c*, *n*, *p*, and m are constants, which depend on the shape [37]. Eq. (3) is the combined formula for interpreting gravity or self-potential data. So, three suggested approaches were applied to estimate the unknown model parameters as follows:

#### 2.1 The least-squares approach

Essa [37] developed this approach, which was relied on solving the problem of finding the depth from the measured data by solving a nonlinear form F(z) = 0 by minimizing it in a least-squares sense. After that, the estimated depth was used in estimating other parameters (the polarization angle and the dipole moment for





self-potential data or the amplitude coefficient for gravity data) via suggesting the shape of the buried structure (the semi-infinite vertical cylinder, the dike, the horizontal cylinder and the sphere) at the lowest root-mean-squared error. This approach is a semiautomatic because that need assuming the shape of the buried structures (a priori information needed) and applied all observed points in estimating the model parameters.

#### 2.2 Werner deconvolution approach

Werner deconvolution was proposed by Werner in 1953 [38]. This approach is used to estimate mainly the origin location and the depth of the buried structures.

Werner proposed to transform the equation of unknown parameters into a rational function. Eq. (3) can be rewritten in linear form follow:

$$J(x_i)\left((x_i - x_o)^2 + z^2\right)^q - Kcx_i(\cos\theta)^n + Kz^p(\sin\theta)^m = 0,$$
(4)

$$J(x_i)\Big((x_i - e_1)^2 + e_2\Big)^q - e_3 x_i + e_4 = 0,$$
(5)

where  $e_1 = x_o$ ,  $e_2 = z^2$ ,  $e_3 = Kc(\cos\theta)^n$ ,  $e_4 = Kz^p(\sin\theta)^m$ .

Eq. (5) is linear form in the four variables  $e_1$ ,  $e_2$ ,  $e_3$ , and  $e_4$ , so that a mathematically unique solution can be found for them from evaluating the equation at four points by assuming the shape of the buried structure.

#### 2.3 The particle swarm approach

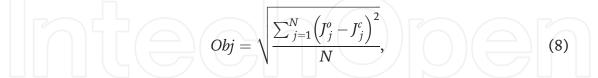
The particle swarm was suggested by [39] and has many various applications, for example, in geophysics [40–42]. For more detail in this approach, you find it many published literature [43, 44]. The model parameters values of the unknowns are relied upon the objective function, so that every problem can be resolved. In this approach, the particles represent the parameter which we are invert. In the beginning, each particle has a location and velocity. After that each particle changes its location ( $P_{best}$ ) at every iteration until reach the optimum location ( $J_{best}$ ). This operation is done by using the following forms:

$$V_i^{k+1} = c_3 V_i^k + c_1 rand (P_{best} - x_i^{k+1}) + c_2 rand (J_{best} - x_i^{k+1}),$$
(6)

$$x_i^{k+1} = x_i^k + V_i^{k+1}, (7)$$

where  $v_i^k$  is the velocity of the particle *i* at the *k*th cycle,  $x_i^k$  is the current *i* modeling at the *k*th cycle, rand is the random number between [0, 1],  $c_1$  and  $c_2$  are positive constant numbers and equal 2,  $c_3$  is the inertial coefficient which control the velocity of the particle and usually taken less than 1,  $x_i^k$  is the positioning of the particle *i* at the *k*th cycle.

The five source parameters (K, z,  $\theta$ ,  $x_o$ , and q) can be assessed by using the particle swarm approach on the subsequent objective function (*Obj*):



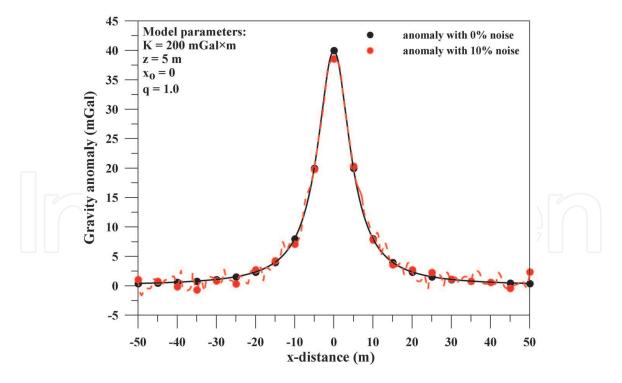
where *N* is the data points number,  $J_j^o$  is the observed gravity or self-potential anomaly, and  $J_i^c$  is the estimated anomaly at the point  $x_j$ .

#### 3. Synthetic example

To test the ability of each suggested approach in assessing the buried model parameters for the simple geometric shapes such as spheres, cylinders, and sheets. Two synthetic examples are suggested for these interpretation. First one is belonging to use the gravity data and second is applying the self-potential data.

#### 3.1 Gravity anomaly model

A gravity anomaly of a horizontal cylinder model is generated using the following parameters  $K = 200 \text{ mGal} \times \text{m}$ , z = 5 m,  $x_o = 0$ , q = 1.0, and profile length = 100 m



#### Figure 2.

A gravity model due to horizontal cylinder without and with a 10% of random noise (K = 200 mGal×m, z = 5 m,  $x_0 = 0$ , q = 1, and profile length = 100 m).

(**Figure 2**). The procedures of interpreting the forward model are done using three steps as follows:

First step: using the least-squares approach to interpret the gravity anomaly yielding from the above mentioned parameters for different s-values for the three suggested shape bodies, i.e., q = 0.5, q = 1.0, and q = 1.5, after that the RMS is

s-values	J(x <sub>j</sub> )	q = 0.5		q = 1.0		q = 1.5	
(m)		K	z	K	Z	K	z
(III)	(mGal)	(mGal×m)	(m)	(mGal×m)	(m)	(mGal×m <sup>2</sup> )	(m)
		For s	ynthetic da	ata with 0% noise			
+1	38.46	50	2.4	200	5	1100	6.5
-1	38.46	50	2.4	200	5	1100	0.5
+2	34.48	50	2.4	200	5	1100	6.5
-2	34.48	50	2.4	200	5	1100	0.5
+3	29.41	50	2.4	200	5	1100	6.5
-3	29.41	50	2.4	200	5	1100	0.5
+4	24.39	50	2.4	200	5	1100	6.5
-4	24.39	50	2.4	200	5	1100	0.5
Aver	rage	50	2.4	200	5	1100	6.5
RMS		5.7		0		4.1	
(mC	Gal)					4.1	
		For sy	nthetic da	ta with 10% noise	e		
+1	38.01	48.6	2.3	195.6	4.9	1086.6	6.4
-1	37.92	40.0	2.5	175.0	<del>т</del> .)	1080.0	0.4
+2	34.64	48.5	2.3	196.1	4.9	1082.3	6.5
-2	34.15	40.5	2.3	190.1	4.2	1082.5	0.5
+3	29.15	47.9	2.4	195.8	5.0	1085.4	6.4
-3	28.75	47.9	2.4	195.0	5.0	1085.4	0.4
+4	24.68	48.9	2.3	195.9	5.0	1084.1	6.4
-4	24.40	40.7	2.3	193.9	5.0	1064.1	0.4
Aver	rage	48.5	2.3	195.8	4.9	1084.6	6.4
RM (mC		5.8		1.0		4.2	

#### Table 1.

Numerical results using the least-squares approach for a gravity model due to horizontal cylinder without and with a 10% of random noise (K = 200 mGal×m, z = 5 m,  $x_0 = 0$ , q = 1, and profile length = 100 m).

Type of body	Parameters	Used ranges	Result	RMS (mGal)				
	For synthetic data with 0% noise							
	K (mGal×m)	K (mGal×m) 100-700 2						
	z (m)	2-12	5	0				
	q	0-3	1	0				
Horizontal	$x_{o}\left(m ight)$	-20-50	0					
cylinder model	For synthetic data with 10% noise							
	K (mGal×m)	100-700	197					
	z (m)	2-12	4.9	0.12				
	q	q 0-3		0.12				
	x <sub>o</sub> (m)	-20-50	0.01					

#### Table 2.

Numerical results using the particle swarm approach for a gravity model due to horizontal cylinder without and with a 10% of random noise (K = 200 mGal×m, z = 5 m,  $x_0 = 0$ , q = 1, and profile length = 100 m).

calculated to execute the best-fit parameters (happens at the lowest RMS) (**Table 1**). Second step: Werner deconvolution approach is utilized to infer the same gravity data. An 11 clustered solutions to determine in the average evaluated depth (4.9 m) (**Figure 2**). Third step: the particle swarm method is applied to obtain the parameters (**Table 2**).

Moreover, a 10% random noise added to the synthetic gravity data mentioned above (**Figure 2**) to test the efficiency of the suggested approaches in interpreting the gravity data. Also, the three approaches are used for this data as mentioned in

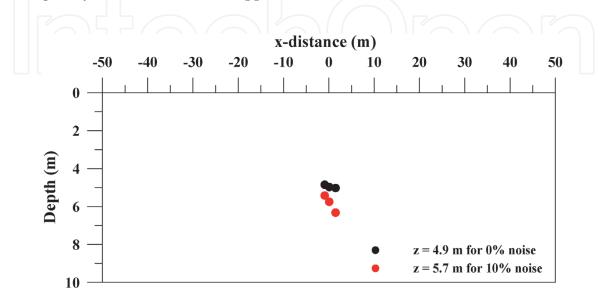
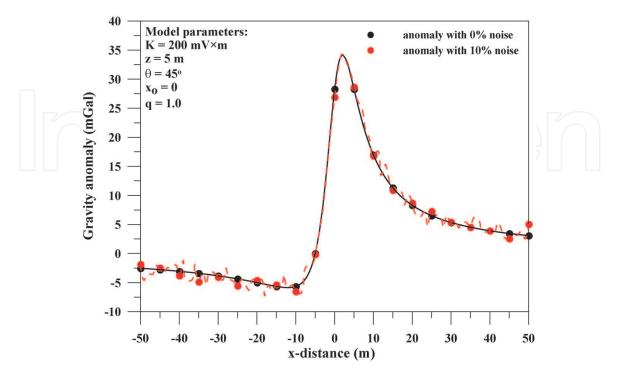


Figure 3.

Werner deconvolution solutions for a gravity model due to horizontal cylinder without and with a 10% of random noise (K = 200 mGal×m, z = 5 m,  $x_0 = 0$ , q = 1, and profile length = 100 m).

**Table 1** (the least-squares approach results), **Figure 3** (Werner deconvolution results), and **Table 2** (the particle swarm results). Finally, the estimated parameters are in all case are in good agreement with the true parameters.



#### Figure 4.

A self-potential model due to horizontal cylinder without and with a 10% of random noise (K = 200 mV×m, z = 5 m,  $\theta = 45^{\circ}$ ,  $x_0 = 0$ , q = 1, and profile length = 100 m).

S-	I(m)	q =	= 0.5			q = 1.0			q = 1.5	
values	$J(x_j)$ (mV)	Κ	z	θ	K	z	θ	K	z	θ
(m)	$(\mathbf{m} \mathbf{v})$	(mV)	(m)	(°)	$(mV \times m)$	(m)	(°)	$(mV \times m^2)$	(m)	(°)
				For syn	thetic data wi	th 0% noise		-	-	
+1	38.46	48.9	1.6	67.2	200	5	45	1565.3	6.4	29.7
-1	38.46	40.7	1.0	07.2	200		45	1505.5	0.4	29.1
+2	34.48	48.9	1.6	67.2	200	5	45	1565.3	6.4	29.7
-2	34.48	+0.7	1.0	07.2	200			1505.5	0.7	27.1
+3	29.41	48.9	1.6	67.2	200	5	45	1565.3	6.4	29.7
-3	29.41	+0.9	1.0	07.2	200	5	75	1505.5	0.4	27.1
+4	24.39	48.9	1.6	67.2	200	5	45	1565.3	6.4	29.7
-4	24.39									
	erage	48.9	1.6	67.2	200	5	45	1565.3	6.4	29.7
1	MS	12.8			0			4.1		
(m	nV)					-				
				For synt	hetic data wit	h 10% noise	2	1		
+1	38.01	46.5	1.5	64.8	192.3	4.8	42.1	1512.7	5.9	27.6
-1	37.92									
+2	34.64	47.1	1.6	65.3	195.4	4.8	42.5	1523.9	6.0	27.4
-2	34.15									
+3	29.15	46.7	1.6	66.4	194.3	5.0	43.6	1514.8	6.1	27.1
-3	28.75									
+4	24.68	47.4	1.4	65.9	195.6	4.8	42.8	1531.4	5.8	28.1
-4	24.40							1500 5		
	rage	46.9	1.5	65.6	195.8	4.9	4.8	1520.7	5.9	27.6
1	MS iV)	1	3.0		1.1		4.2			

#### Table 3.

Numerical results using the least-squares approach for a self-potential model due to horizontal cylinder without and with a 10% of random noise (K = 200 mV×m, z = 5 m,  $\theta$  = 45°, x<sub>0</sub> = 0, q = 1, and profile length = 100 m).

## 3.2 Self-potential anomaly model

A self-potential anomaly of a horizontal cylinder model is generated using the following parameters  $K = 200 \text{ mV} \times \text{m}$ , z = 5 m,  $\theta = 45^{\circ}$ , q = 1.0, and profile length = 100 m (**Figure 4**). The similar interpretation procedures mentioned above are used

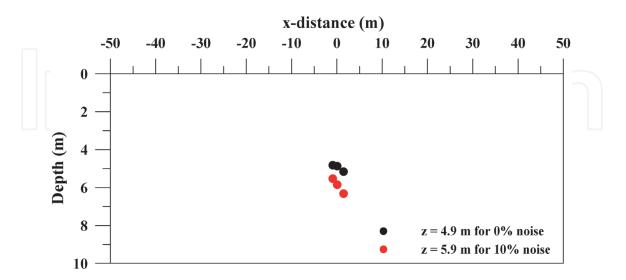


Figure 5.

Werner deconvolution solutions for a self-potential model due to horizontal cylinder without and with a 10% of random noise (K = 200 mV×m, z = 5 m,  $\theta$  = 45°,  $x_0$  = 0, q = 1, and profile length = 100 m).

	Type of body	Parameters	Used ranges	Result	RMS (mV)	]			
		For synthetic data with 0% noise							
		K (mV×m)	100-700	200		1			
		z (m)	2-12	5					
		θ (°)	5-85	45	0				
	Horizontal	q	q 0-3 1						
		x <sub>o</sub> (m)	-20-50	0					
	cylinder model	For synthetic data with 10% noise							
		K (mV×m)	100-700	195		1			
		z (m)	2-12	4.9					
		θ (°)	5-85	43.5	0.22				
		q		0.98					
		x <sub>0</sub> (m)	-20-50	-0.02					

#### Table 4.

Numerical results using the particle swarm approach for a self-potential model due to horizontal cylinder without and with a 10% of random noise (K = 200 mV×m, z = 5 m,  $\theta$  = 45°, x<sub>0</sub> = 0, q = 1, and profile length = 100 m).

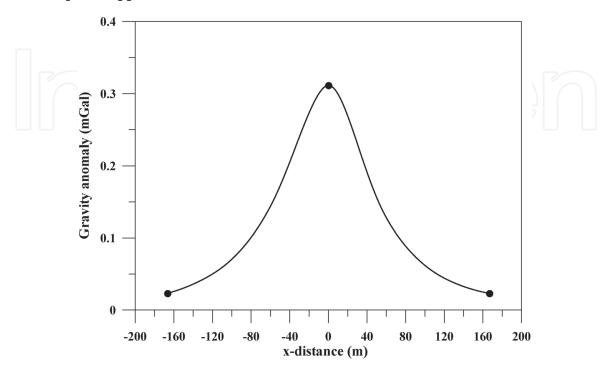
as follows: first, the least-squares approach is applied to interpret the self-potential data using various s-values for the three suggested shape bodies, i.e., q = 0.5, q = 1.0, and q = 1.5, after that the RMS is calculated to execute the best-fit parameters (happens at the lowest RMS) (**Table 3**). Secondly, the Werner deconvolution approach is utilized to infer the same self-potential data using 11 clustered solutions to determine in the average evaluated depth (4.9 m) (**Figure 5**). Third step: the particle swarm method is applied to obtain the parameters (**Table 4**). Besides, a 10% random noise was added to this data (**Figure 3**) to test the efficiency of the suggested approaches in interpretation. Furthermore, the results from applying the three approaches are mentioned in **Table 3** (the least-squares approach results), **Figure 5** (Werner deconvolution results), and **Table 4** (the particle swarm results). Finally, the estimated parameters are in all case are in good agreement with the actual parameters.

# 4. Field examples

The three suggested approaches have been organized to inspect the gravity and self-potential anomalies due to three simple bodies of various structures, e.g., sheets, cylinders, and spheres. Two mineral field examples from India and Turkey have been interpreted to study the reliability of the suggested approaches. The relevant model parameters (K, z,  $\theta$ ,  $x_o$ , and q) are evaluated in an integrated way with the existing geological and geophysical results.

## 4.1 Gravity anomaly of manganese ore body

**Figure 6** shows a gravity anomaly was collected over a manganese deposit near Nagpur, India [45]. This gravity profile has a length of 333 m and digitized with an interval of 27 m. This gravity anomaly is subjected to the three interpretation approaches as discussed earlier. Firstly, the interpreted results due to applying the least-squares approach are shown in **Table 5** for various s-values. Besides, the use of



**Figure 6.** *A gravity anomaly due to a manganese ore body, India.* 

s-values	I(w)	q = 0.5		q = 1.0		q = 1.5			
(m)	J(x <sub>j</sub> ) (mGal)	K	Z	К	Z	K	Z		
(111)	(mour)	(mGal×m)	(m)	(mGal×m)	(m)	(mGal×m <sup>2</sup> )	(m)		
+27	0.225	124.6	18.4	24.8	63.2	342.4	112.8		
-27	0.233	124.0	10.4	24.0	03.2	342.4	112.0		
+54	0.155	156 1	156.1 21.9	25.3	64.3	448.4	123.4		
-54	0.139	150.1							
+81	0.095	178.3	.3 25.7	22.1	57.4	456.2	124.1		
-81	0.107	178.5		22.1					
Aver	age	153	22	24.1	61.6	415.7	120.1		
RMS (mGal)		40.3		2.7		6.2			

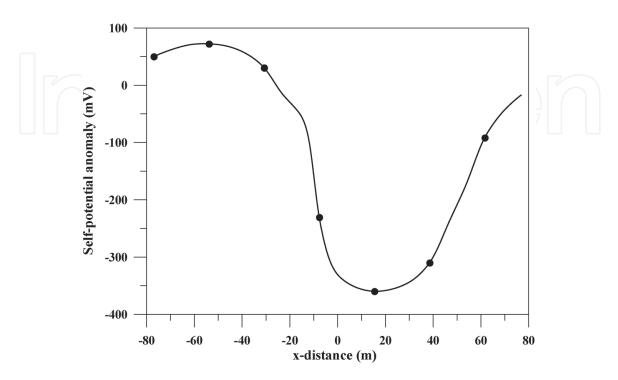
 Table 5.

 Numerical results using the least-squares approach for a manganese field example, India.

Type of body	Parameters	Used ranges	Result	RMS (mGal)	
Horizontal cylinder model	K (mGal×m)	1-100	17.3		
	z (m)	) 1-100 57.8		0.06	
	q	0-3	1.0	0.00	
	x <sub>o</sub> (m)	-10-10	0.8		

## Table 6.

Numerical results using the particle swarm approach for a manganese field example, India.



**Figure 7.** A self-potential anomaly due to a Weiss copper ore body, Turkey.

a values	s-values J(x <sub>i</sub> )		q = 0.5			q = 1.0			q = 1.5		
(m)	$J(x_j)$ (mV)	K (mV)	z (m)	θ (°)	K (mV×m)	z (m)	θ (°)	${ m K} \ ({ m mV} imes{ m m}^2)$	z (m)	θ (°)	
+7.7	-352.81	-1148.3	18.4	72.4	-9876.2	63.2	54.6	-472589.6	28.7	38.4	
-7.7	-230.57	-1146.5	10.4 72.	72.4	-9870.2	03.2	54.0	-472389.0	20.7	30.4	
+15.4	-359.75	-1192.1	21.9	65.8	-10056.4	64.3	50.9	-471236.1	29.6	37.8	
-15.4	-52.78	-1192.1	21,7	05.0	-10050,4	04.5	50.7	+/1250.1	27.0	57.0	
+23.1	355.58	-1181.7	25.7	64.9	-9986.3	57.4	49.0	-470893.4	30.1	37.2	
-23.1	-15.28	-1101.7	1.7 23.7	5.7 04.9	-7780.5	57.4	49.0	470893.4	50.1	31.2	
Ave	rage	-1174.0	22.0	67.7	-9972.9	61.6	51.5	-471236.1	29.5	37.8	
RMS (mV)		107.9		64.3		38.0					
		$\left( \begin{array}{c} - \end{array} \right)$						())(4)	$ \rightarrow 1 $	$\left( \right)$	

Table 7.

Numerical results using the least-squares approach for a Weiss copper field example, Turkey.

Type of body	Parameters	Used ranges	Result	RMS (mV)
	K (mV×m <sup>2</sup> )	-200000500000	-471200.3	
	z (m)	10-50	29.7	
Spherical model	θ (°)	5-85	37.6	34.2
	q	0-3	1.48	
	$x_{o}\left(m ight)$	-10-10	-2.3	

## Table 8.

Numerical results using the particle swarm approach for a Weiss copper field example, Turkey.

Werner deconvolution approach, the interpretive results obtained are z = 56.8 m and  $x_o = 0.6$  m. Finally, the depth and the other model parameters evaluated by using the particle swarm approach are presented in **Table 6**.

# 4.2 Self-potential anomaly of manganese ore body

**Figure** 7 demonstrates a self-potential anomaly over a Weiss copper ore body in the Ergani copper district, Turkey [46]. The Weiss self-potential anomaly profile has a length of 144 m and digitized with an interval of 7.7 m. This anomaly has subjected to the three interpretation approaches as discussed earlier. Firstly, the interpreted results due to applying the least-squares approach are shown in **Table** 7 for various s-values. Also, the applying of the Werner deconvolution approach, the interpretive results obtained are z = 36.9 m and  $x_o = -2.1$  m. Finally, the depth and the other model parameters evaluated by using the particle swarm approach are presented in **Table 8**.

# 5. Conclusions

The three geophysical approaches (the least-squares approach, Werner deconvolution approach, and the particle swarm approach) discussed here to interpret gravity or self-potential data using a combined formula for the simple

geometric models (spheres, cylinders, and dikes) are stable and give a good results. The stability of these approaches has been confirmed and tested applying two synthetic examples with a 10% and without random noise and two field data for mineral explorations. The estimated parameters in all cases demonstrated the importance of these approaches in interpreting the gravity or self-potential data.

# Acknowledgements

The authors would like to thank and express appreciation to Ms. Dolores Kuzelj, Author Service Manager, for her assistance and cooperation in this issue.

# **Conflict of interest**

There is no conflict of interest.

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