Chapter

Characteristics of Radiation of a round Waveguide through a Flat Homogeneous Heat Shield

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Abstract

The problem of obtaining an analytical description of the radiation characteristics of a circular waveguide closed by a flat homogeneous dielectric plate is solved. The radiation characteristics include: the radiation field; the conductivity of the aperture radiation; and the fields of surface, flowing, and side waves, as well as energy characteristics. In such a statement, a strict solution of Maxwell's equations is required. The paper uses the method of integral transformations and the method of eigenfunctions. In this case, the assumption is used that the electrical parameters of the dielectric plate (thermal protection) and the geometric dimensions do not depend on time. The relations describing the directional diagram of a circular waveguide with dielectric thermal protection and taking into account the electrical parameters of thermal protection and its thickness are obtained. Expressions are also obtained for the fields of lateral, surface, and outflow waves, from which it is possible to calculate the power taken away by these fields. Numerical calculations were made for some of the obtained relations. The results showed that the power of the side waves is zero. It also follows from the calculations that the radiation field of surface and flowing waves is absent, that is, their contribution to the directional diagram is not.

Keywords: circular waveguide, a flat, uniform thermal protection, the radiation characteristics

1. Introduction

The onboard antennas of the returned spacecraft are subjected to intensive aerodynamic heating when the spacecraft passes through the dense layers of the atmosphere [1]. In these conditions, radio-transparent heat-resistant thermal protection is used to protect the antennas from external influences. The open ends of the transmission lines are used as the emitter to obtain a wide directional pattern. The most offer used radiation from the open end of the round waveguide. In the first approximation, we consider a flat uniform thermal protection. Under the conditions of aerodynamic heating, the electrical parameters of thermal protection significantly change (relative permittivity ε and tangent of the dielectric loss angle $tg\delta$). These changes lead to a noticeable increase in absorption losses in the heat shield, reflection from its boundaries, as well as to the appearance of surface and side waves. Together, all this leads to a change in the directional pattern and a decrease in the efficiency of the onboard antenna.

Evaluation of these changes is absolutely necessary to determine the radio technical characteristics of the onboard radio equipment. The paper solves the problem of determining the characteristics of the radiation of a circular waveguide through a heat shield subjected to aerodynamic heating.

The problems of calculating the interaction of the onboard antenna with a heat-shielding dielectric insert are very difficult and poorly developed. The development of mathematical models of antenna windows is reduced to solving an external problem of electrodynamics—electromagnetic excitation of bodies or diffraction of radio waves. At the same time, we will use well-known analytical methods of solution. The radio technical characteristics of the antenna window, for which we obtain an analytical description, include a radiation pattern, radiation conductivity, antenna temperature, and a number of other characteristics that describe more subtle electrodynamic effects, as well as energy characteristics.

In theoretical terms, the electrodynamic problem in general can be formulated as follows. There is a radiating, open antenna a, located on an infinite screen, in front of which is a dielectric layer of thickness d with a complex permittivity-bridge $\varepsilon\alpha(x, y, z, t)$ (see **Figure 1**).

In this general formulation, the solution of the electrodynamic problem is associated with significant mathematical difficulties, the main problem being the need to solve the Maxwell equations for an arbitrary law of change in the parameters of media in space and time. With some simplifying assumptions, the problem was solved in the ray approximation. In [2], the wave front method is used to analyze the radiation diagram of an antenna covered by a dielectric layer. In [3–5], the method proposed in [6] is used to find the radiation diagram, according to which the antenna radiation diagram in the presence of an infinite flat dielectric layer is simply multiplied by the diagram in the free space by the flat wave transmission coefficient for the flat layer, taking into account the corresponding angle of arrival and the plane of polarization of the wave.

At their core, all these methods are close to each other and are essentially based on the approximation of geometric optics, which is true in the quasi-optical domain. In relation to the problem under consideration (the resonance region), a strict solution of the Maxwell equations is required. From analytical methods of the solution, it is possible to apply the method of integral transformations and the method of eigenfunctions. Both of these methods will be used in the future. In this case, we assume that the parameters of thermal protection do not depend on time,

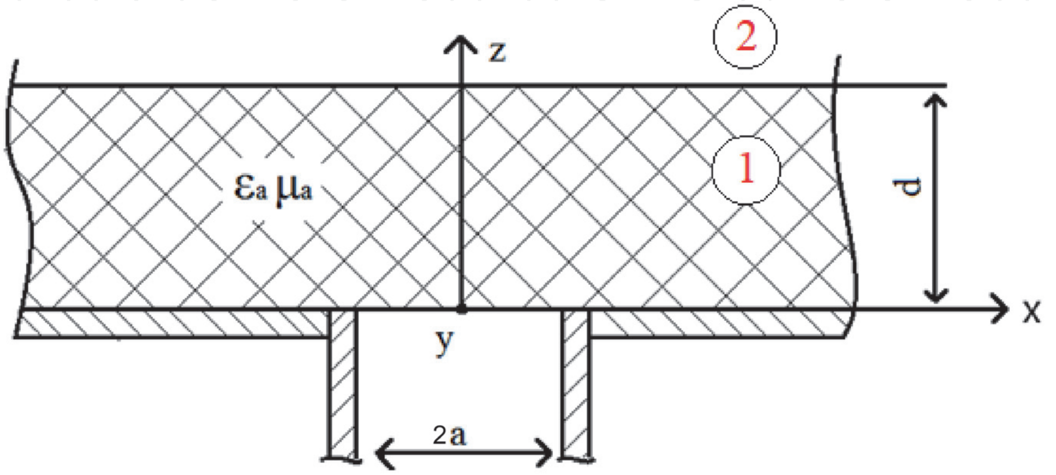

Figure 1.
Electrodynamic model of radiation from a circular waveguide with thermal protection.

that is, in fact, the slow-moving processes of heating of the dielectric thermal protection are considered. This approach and methods of solution were used in a number of works, for example [7].

2. Main part

The problem can be formulated as a boundary with respect to the tangent magnetic field in the material, and with respect to the tangent electric field. The second method is more convenient because of the simple form of boundary conditions for z = 0 [8].

The magnetic component of the electromagnetic field H_y at z ≥ 0 must satisfy the following wave equation in a Cartesian coordinate system x, y, z:

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + \frac{\partial^2 H_y}{\partial z^2} + k^2 \varepsilon H_y = 0, \tag{1}$$

where $\varepsilon = \varepsilon_1$ for 0 ≤ z ≤ d, ε = 1 for z > d, and k is the wave number.

We apply the Fourier transform for x and y coordinates to equation (Eq. (1)). We get

$$\frac{\partial^2 \hat{H}_y}{\partial z^2} + \left(k^2 \varepsilon - k^2{}_x - k^2{}_y\right) \hat{H}_y = 0, \tag{2}$$

where is the direct Fourier transform,

$$\hat{H}_y = \int \int_{-\infty}^{\infty} H_y(x, y, 0) \exp\left(-j\left(k_x x + k_y y\right)\right) dx dy.$$

Solution of equation (Eq. (2)) satisfying the radiation conditions (for $z \geq d$) has the form for area 1, that is, the area occupied by the dielectric plate (0 < z < d),

$$\hat{H}_y^{(1)} = D \exp\left(-jk_{z_1} z\right) + L \exp\left(jk_{z_1} z\right).$$

For region 2, that is, the region behind the plate (z ≥ d), we get

$$\hat{H}_y^{(2)} = M \exp\left(-jk_z z\right),$$

where $k_{z_1} = \sqrt{k^2 \varepsilon - k_x^2 - k_y^2}; k_z = \sqrt{k^2 - k_x^2 - k_y^2}$.

Reasoning in a similar way, with respect to the tangent magnetic component of the field, we obtain the following equations for the spectral component:

$$\hat{H}^{(1)}{}_x = A \exp\left(-jk_{z_1} z\right) + B \exp\left(jk_{z_1} z\right),$$

$$\hat{H}^{(2)}{}_x = C \exp\left(-jk_z z\right).$$

Satisfying the equations arising from the Maxwell equation, we obtain expressions for the spectral components of the electric field

$$\hat{E}_x^{(1)} = -\frac{k_{z_1}}{\omega\varepsilon_0\varepsilon_1}\left(Dexp\left(-jk_{z_1}z\right) - Lexp\left(jk_{z_1}z\right)\right);$$

$$\hat{E}_x^{(2)} = -\frac{k_z}{\omega\varepsilon_0}Mexp\left(-jk_z z\right);$$

$$\hat{E}_y^{(1)} = \frac{k_{z_1}}{\omega\varepsilon_0\varepsilon_1}\left(Aexp\left(-jk_{z_1}z\right) - Bexp\left(jk_{z_1}z\right)\right);$$

$$\hat{E}_y^{(2)} = \frac{k_z}{\omega\varepsilon_0}\left(Cexp\left(-jk_z z\right)\right).$$

Unknown functions A, B, C, D, L, and M are determined from the boundary conditions for z = 0 and z = d. In this case, the boundary conditions for radiation from a circular waveguide, when the field in the aperture is determined by H_{11} waves, have the form

$$-\frac{k_{z_1}}{\omega\varepsilon_0\varepsilon_1}(D - L) = \hat{E}_{x_0}; \tag{3}$$

$$\frac{k_{z_1}}{\omega\varepsilon_0\varepsilon_1}(A - B) = \hat{E}_{y_0}; \tag{4}$$

$$D\exp\left(-jk_{z_1}d\right) - L\exp\left(jk_{z_1}d\right) = \frac{\varepsilon_1 k_z}{k_{z_1}}M\exp\left(-jk_z d\right); \tag{5}$$

$$A\exp\left(-jk_{z_1}d\right) - B\exp\left(jk_{z_1}d\right) = \frac{\varepsilon_1 k_z}{k_{z_1}}C\exp\left(-jk_z d\right); \tag{6}$$

$$\begin{aligned}&-k_x k_y\left(Dexp\left(-jk_{z_1}d\right) + Lexp\left(jk_{z_1}d\right)\right) + \left(k_1^2 - k_x^2\right)\left(Aexp\left(-jk_{z_1}d\right) + Bexp\left(jk_z d\right)\right) = \\ &= \varepsilon_1\left(\left(k^2 - k_x^2\right)Cexp\left(-jk_z d\right) - k_x k_y Mexp\left(-jk_z d\right)\right);\end{aligned} \tag{7}$$

$$\begin{aligned}&\left(k_1^2 - k_y^2\right)\left(Dexp\left(-jk_{z_1}d\right) + Lexp\left(jk_{z_1}d\right)\right) - k_x k_y\left(Aexp\left(-jk_{z_1}d\right) + Bexp\left(jk_{z_1}d\right)\right) = \\ &= \left(\left(k^2 - k_y^2\right)Mexp\left(-jk_z d\right) - k_x k_y Cexp\left(-jk_z d\right)\right)\varepsilon_1\end{aligned} \tag{8}$$

where

$$\hat{E}_{x_0} = \iint_{\Pi} E_x(x', y', 0)\exp\left(-j\left(k_x x' + k_y y'\right)\right)dx'dy'. \tag{9}$$

$$\hat{E}_{y_0} = \iint_{\Pi} E_y(x', y', 0)\exp\left(-j\left(k_x x' + k_y y'\right)\right)dx'dy'. \tag{10}$$

Here, Π is the area of integration on the opening of the waveguide and x',y' are the coordinates counted in the opening of the studied waveguide.

We present in full the expression of the coefficients included in the solution of the wave equations of spectral components. In this case, we use the following variable replacement:

$$k_x = \beta\cos\alpha,\ \ k_y = \beta\sin\alpha, k_z = \sqrt{k^2 - \beta^2},\ \ k_{z1} = \sqrt{k^2\varepsilon_1 - \beta^2}.$$

Since the coefficient expressions are very cumbersome, we use the following notation to obtain a compact form of the record:

$$\sqrt{k^2-\beta^2}=a,\quad \sqrt{k^2\varepsilon_1-\beta^2}=b,\quad \sqrt{k^2\varepsilon_1-\beta^2}\mathrm{d}=\mathrm{c},$$

$$\beta^2\sin\,\alpha\,\cos\,\alpha=\mathrm{e}, k^2-\beta^2\,cos^2\alpha=\mathrm{f}, k^2\varepsilon_1-\beta^2\,cos^2\alpha=\mathrm{g},$$

$$k^2-\beta^2\,sin^2\alpha=\mathrm{h}, k^2\varepsilon_1-\beta^2\,sin^2\alpha=\mathrm{l},\quad \sqrt{k^2-\beta^2}d=p.$$

As a result

$$\begin{aligned}\mathrm{A}=&\frac{\omega\varepsilon_0\varepsilon_1}{b}\frac{\exp{(jc)}}{\Delta}\left[\hat{E}_{x_0}eabk^2(1-\varepsilon_1)(\cos c-j\sin c)+\right.\\&\left.+\hat{E}_{y_0}\left(e^2(a+b)(jb\sin c+a\cos c)+(bf+ag)(-j\sin ch\;nb\cos cla)\right)\right];\end{aligned}\tag{11}$$

$$\mathrm{B}=\mathrm{A}-\hat{E}_{y_0}\frac{\omega\varepsilon_0\varepsilon_1}{b};\tag{12}$$

$$\mathrm{C}=\frac{b}{a}\exp{[j(p+c)]}\left[\hat{E}_{y_0}\frac{\omega}{b}-j\frac{2A}{\varepsilon_1}\sin c\exp{(-jc)}\right];\tag{13}$$

$$\begin{aligned}\mathrm{D}=&\frac{\omega\varepsilon_0\varepsilon_1\exp{(jc)}}{b\Delta}\hat{E}_{x_0}\left[e^2(a+b)(jb\;\sin c+a\;\cos c)-(bh+al)\times\right.\\&\left.\times(jfb\sin c+ag\cos c)+\hat{E}_{y_0}aebk^2(1-\varepsilon_1)(\,j\sin c-\;\cos c)\right];\end{aligned}\tag{14}$$

$$L=D+\hat{E}_{x_o}\frac{\omega\varepsilon_0\varepsilon_1}{b};\tag{15}$$

$$M=-\frac{b}{a}\exp{[j(p+c)]}\left[\hat{E}_{x_0}\frac{\omega\varepsilon_0}{b}+j\frac{2D}{\varepsilon_1}\sin c\;\exp{(-jc)}\right].\tag{16}$$

In the expressions (Eqs. (11)–(16))

$$\begin{aligned}\Delta=&2\sqrt{k^2\varepsilon_1-\beta^2}\sqrt{k^2-\beta^2}k^2\left[j\sqrt{k^2-\beta^2}\sin\left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)+\right.\\&+\sqrt{k^2\varepsilon_1-\beta^2}\cos\left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)\left[\varepsilon_1\sqrt{k^2-\beta^2}\cos\left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)+\right.\\&\left.\left.+j\sqrt{k^2\varepsilon_1-\beta^2}\sin\left(\sqrt{k^2\varepsilon_1-\beta\,d}\right)\right]\right].\end{aligned}$$

Using the obtained expressions of the angular spectrum of plane waves and applying the inverse Fourier transform, taking into account (Eqs. (9) and (10)), we write

$$\begin{aligned}E_x^{(1,2)}=&\iint\limits_{\Pi}F_{x_1}^{(1,2)}(x,y,z,x',y',0)E_x(x',y',0)dx'dy'+\\&+\iint\limits_{\Pi}F_{x_2}^{(1,2)}(x,y,z,x',y',0)E_y(x',y',0)dx'dy';\end{aligned}\tag{17}$$

$$\begin{aligned}E_y^{(1,2)}=&\iint\limits_{\Pi}F_{y_1}^{(1,2)}(x,y,z,x',y',0)E_x(x',y',0)dx'dy'+\\&+\iint\limits_{\Pi}F_{y_2}^{(1,2)}(x,y,z,x',y',0)E_y(x',y',0)dx'dy';\end{aligned}\tag{18}$$

The functions F_x in these expressions are determined based on equations (Eqs. (11)–(16)). After a series of transformations, you can write

$$F_{x_1}^{(2)} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_0^{2\pi}\left\{\frac{j2\ sin\ \sqrt{k^2\varepsilon_1-\beta^2}d}{\Delta}\left(\left[-\beta^4 sin^2\alpha\ cos^2\alpha\left(\sqrt{k^2-\beta^2}+\right.\right.\right.\right.$$
$$\left.+\sqrt{k^2\varepsilon_1-\beta^2}\right)\left(j\sqrt{k^2\varepsilon_1-\beta^2}\right)\sin\left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)+\sqrt{k^2-\beta^2}\cos\left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)\Big]+$$
$$+\left[\sqrt{k^2\varepsilon_1-\beta^2}(k^2-\beta^2 sin^2a)+\sqrt{k^2-\beta^2}\left(k^2\varepsilon_1-\beta^2 sin^2a\right)\right]\Big[j(k^2-\beta^2 cos^2a)\sqrt{k^2\varepsilon_1-\beta^2}\times$$
$$\left.\left.\times\sin\left(\sqrt{k^2\varepsilon_1-\beta^2}d)+(k^2\varepsilon_1-\beta^2 cos^2a)\sqrt{k^2-\beta^2}cos\sqrt{k^2\varepsilon_1-\beta^2}d\right]\right)-1\right\}\times$$
$$\times\exp[-j(kz-(k_z+k_{z1})d]\exp[-j\beta[(x'-x)\cos a+(y'-y)\sin a]]\beta\,d\beta\,da=$$
$$=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_0^{2\pi}\varphi_x^{(2)}\exp(-jk_z z)\exp[-j\beta[(x'-x)\cos a+(y'-y)\sin a]]\beta d\beta\,da; \tag{19}$$

$$F_{x_2}^{(2)} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_0^{2\pi}\frac{j2\ sin\ \left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)}{\Delta}\beta^2 sin\,\alpha\ cos\,\alpha\left(\sqrt{k^2\varepsilon_1-\beta^2}k^2\times\right.$$
$$\times\sqrt{k^2-\beta^2}\exp\left(j\sqrt{k^2-\beta^2}d\right)\exp[-j(k_z z-(k_z+k_{z_1})d)]\times$$
$$\times\exp[-j\beta[(x'-x)\cos a+(y'-y)\sin a]]\beta\,d\beta\,da=$$
$$=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_0^{2\pi}\varphi_y^{(2)}\exp(-jk_z z)[-j\beta[(x'-x)\cos a+(y'-y)\sin a]]\beta d\beta\,da; \tag{20}$$

$$F_{y_1}^{(2)} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_0^{2\pi}\frac{j2\ sin\ \left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)}{\Delta}\beta^2 sin^2\alpha\ cos^2\alpha\left(\sqrt{k^2-\beta^2}\times\right.$$
$$\times\sqrt{k^2\varepsilon_1-\beta^2}k^2(1-\varepsilon_1)\cos\left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)-\mathrm{j}\ \sin\left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)\times$$
$$\times\exp[-j((k_z z)-(k_z+k_{z1})d)]\exp[-j\beta[(x'-x)\cos a+$$
$$+(y'-y)\sin a]]\beta\,d\beta\,da=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_0^{2\pi}\xi_x^{(2)}\exp(-jk_z z)\times$$
$$\times\exp[-j\beta[(x'-x)\cos a+(y'-y)\sin a]]\beta\,d\beta\,da; \tag{21}$$

$$F_{y_2}^{(2)} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_0^{2\pi}\left\{\frac{j2\ sin\ \left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)}{\Delta}\left(\left[\beta^4 sin^2\alpha\ cos^2\alpha\left(\sqrt{k^2-\beta^2}+\right.\right.\right.\right.$$
$$\left.\left.+\sqrt{k^2\varepsilon_1-\beta^2}\right)\right]\left[\left(j\sqrt{k^2\varepsilon_1-\beta^2}\sin\left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)+\right.\right.$$
$$\left.+\sqrt{k^2-\beta^2}\,(k^2\varepsilon_1-\beta^2 cos^2a)\right]\Big[-j\sqrt{k^2\varepsilon_1-\beta^2}(k^2\varepsilon_1-\beta^2 sin^2a)\times$$
$$\times\sin\left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)-\sqrt{k^2-\beta^2}(k^2\varepsilon_1-\beta^2 sin^2a)\times$$
$$\left.\left.\times\cos\left(\sqrt{k^2\varepsilon_1-\beta^2}d\right)\Big]\Big]\right)-1\right\}\exp[-j(k_z z-(k_z+k_{z1})d)]\times$$
$$\times\exp[-j\beta[(x'-x)\cos a+(y'-y)\sin a]]\beta\,d\beta\,da=$$
$$=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_0^{2\pi}\xi_y^{(2)}\exp(-jk_z z)\exp[-j\beta[(x'-x)\cos a+$$
$$+(y'-y)\sin a]]\beta\,d\beta\,da. \tag{22}$$

The obtained expressions (Eqs. (19)–(22)) together with (Eqs. (17) and (18)) determine the radiation field of a circular waveguide with uniform thermal protection through the tangent components of the electric field in the opening of the waveguide.

To calculate the integrals (Eqs. (20) and (22)) by the saddle method, it should be taken into account that, when the integration contour is deformed, it is necessary to bypass the branch points of the integrand and that the saddle path intersects the poles of the integrand. We determine which branch points and poles of the integrands must be taken into account in the said approximation. Integrals (Eqs. (20) and (22)) can be written as

$$I = \int_{-\infty}^{\infty}\int_{0}^{2\pi} f(\beta)\exp\left(-j(d-z)\sqrt{k^2-\beta^2}\right)\exp\left(-j\beta\rho\cos\left(\alpha-\varphi_0\right)\right)d\beta d\alpha =$$

$$= 2\pi\int_{-\infty}^{\infty} f(\beta)\exp\left(\ j(d-z)\sqrt{k^2-\beta^2}\right)J_0(\beta\rho)d\beta.$$

where $\varphi_0 = arctg(x'-x)/(y'-y), \rho = \sqrt{(x'-x)^2+(y'-y)^2}$,

$J_0(\beta\rho)$ is the Bessel function.

We use the asymptotic expression of the Bessel function for large arguments

$$J_0(\beta\rho) \approx \sqrt{\frac{2\pi}{\pi\beta\rho}}\left(\exp\left(\ j(\beta\rho-\pi/4)\right)+\ \exp\left(-j(\beta\rho-\pi/4)\right)\right).$$

Then, passing to the plane of the complex angle τ by replacing β = ksinτ, we obtain

$$I = \int_{-\infty}^{\infty} Af(\beta)\exp\left(jkL\cos\left(\tau\pm\varphi\right)\right)d\tau \tag{23}$$

where $A = 2k\sqrt{\frac{2\pi}{\beta\rho}}\cos\tau\exp\left(-j\pi/4\right), d-z = L\cos\varphi, \rho = L\sin\varphi$.

Considering the obtained expression of the integral I, we find the analytical expression of the saddle path from the following equation:

$J_m(\ j\cos\left(\tau\pm\varphi\right)) = \text{const}, \text{sciliset}\, J_m(\ j\cos\left(\tau\pm\varphi\right)) = j$. Means $\cos\left(\tau\pm\varphi\right) = 1$. Because the $\tau = \tau_r + j\tau_j$,

then $\cos\left(\tau_r\pm\varphi\right) = \cos\left(\tau_r\pm\varphi\right)\text{ch}\tau_j - j\sin\left(\tau_r\pm\varphi\right)\text{sh}\tau_j$, $\cos\left(\tau_r\pm\varphi\right) = \frac{1}{ch\tau_j} = \text{sch}\tau_j$.

Finally, we get the expression of the saddle path

$$\tau_r\pm\varphi = \arccos\left(sch\tau_j\right). \tag{24}$$

Satisfying the radiation condition at infinity, we obtain regions on the complex plane τ in which the saddle path defined by (Eq. (24)) lies:

1. $0 < \tau_r < \pi,\ \tau_i > 0$;

2. $-\pi < \tau_r < 0, \tau_j < 0$;

moreover, the second area of determination of the transit path in our case makes sense, since from (Eqs. (20) and (22)) it is clear that the lower limit of β is 0. Denoting the coordinates of the branch points τ_B and the pole τ_p, we obtain from

(Eq. (24)) the condition that determines the branch points and poles intersected during deformation of the initial path of integration into the saddle,

$$\tau_{z_в, z_р} \pm \varphi > \arccos\left(\mathrm{sch}\tau\ _{j_в, j_р}\right).$$

In the general case, in accordance with the Cauchy theorem, the integrals for functions can be represented in the following form:

$$F = \frac{1}{4\pi^2}\left(\int\limits_{l} \dots d\beta + U(C_в)\int\limits_{l_в} \dots d\beta + U\left(C_p\right)\int\limits_{l_p} \dots d\beta\right), \tag{25}$$

where $U\left(C_{в,\ p}\right)$is the only Heaviside function; $C_в, C_p$ are values determined on the basis of Eq. (24) as follows on the complex plane:

$$C_{в,\ p} = Re\left(\arcsin\frac{\beta_{в,\ p}}{k}\right) \pm \varphi - \arccos\left(sch\, J_m\left(\arcsin\frac{\beta_{в,\ p}}{k}\right)\right), \tag{26}$$

The first integrals over the circuit l are calculated by the saddle method. Finally, by the saddle method, we get

$$F^{(2)}_{x1\ sad} = \frac{jk\ \exp(jkr)}{2\pi r^2} z\varphi^{(2)}_x \Big|_{\substack{k_x = k_{x'} \\ k_y = k_{y'}}}, \tag{27}$$

$$F^{(2)}_{x2\ sad} = \frac{jk\ \exp(jkr)}{2\pi r^2} z\varphi^{(2)}_y \Big|_{\substack{k_x = k_{x'} \\ k_y = k_{y'}}}, \tag{28}$$

$$F^{(2)}_{y1\ sad} = \frac{jk\ \exp(jkr)}{2\pi r^2} z\xi^{(2)}_y \Big|_{\substack{k_x = k_{x'} \\ k_y = k_{y'}}}, \tag{29}$$

$$F^{(2)}_{y2\ sad} = \frac{jk\ \exp(jkr)}{2\pi r^2} z\xi^{(2)}_y \Big|_{\substack{k_x = k_{x'} \\ k_y = k_y}}. \tag{30}$$

In the expressions (Eqs. (27)–(30))

$$k'_x = \frac{k_x}{r},\ k'_y = \frac{k_y}{r},\ r = \sqrt{(x' - x)^2 + (y' - y)^2 + z^2}.$$

After a series of transformations for even E modes and odd H modes, we obtain

$$F^{(2)}_{x_{1E}} = \frac{i}{2\pi}\sum_{i=1}^{n}\frac{U\left(C_{p_i}\right)\int_0^{2\pi}\varphi_x^2(\beta_i, a)N\left(\beta_i, a\right)da\psi_1(\beta_i)}{\psi_1{}'(\beta_i)} \Big| \beta_i = \beta_i^E, \tag{31}$$

$$F^{(2)}_{x_{2E}} = \frac{i}{2\pi}\sum_{i=1}^{n}\frac{U\left(C_{p_i}\right)\int_0^{2\pi}\varphi_y^2(\beta_i, a)N\left(\beta_i, a\right)da\psi_1(\beta_i)}{\psi_1{}'(\beta_i)} \Big| \beta_i = \beta_i^E, \tag{32}$$

$$F^{(2)}_{y_{1E}} = \frac{i}{2\pi}\sum_{i=1}^{n}\frac{U\left(C_{p_i}\right)\int_0^{2\pi}\varphi_x^2(\beta_i, a)N\left(\beta_i, a\right)da\psi_1(\beta_i)}{\psi'(\beta_i)} \Big| \beta_i = \beta_i^E, \tag{33}$$

$$F^{(2)}_{y_{1E}} = \frac{i}{2\pi}\sum_{i=1}^{n}\frac{U\left(C_{p_i}\right)\int_0^{2\pi}\varphi_y^2(\beta_i, a)N\left(\beta_i, a\right)da\psi_1(\beta_i)}{\psi_1{}'(\beta_i)} \Big| \beta_i = \beta_i^E. \tag{34}$$

In these expressions

$$N = -2\sqrt{k^2\varepsilon_1 - \beta^2}\sqrt{k^2 - \beta^2}k^2 \exp\left(-jk_z z\right)\times$$

$$\times \exp\left[-j\beta[(x' - x)\cos a + (y' - y)\sin a]\right]\beta;$$

$$\psi_1(\beta) = j\, ctg\left(\sqrt{k^2\varepsilon_1 - \beta^2 d}\right)k^2\varepsilon_1\sqrt{k^2 - \beta^2} - k^2\sqrt{k^2\varepsilon_1 - \beta^2},$$

C_{pi} is calculated according to (Eq. (26)). The expressions for the H modes will be characterized by equations similar to equations (Eqs. (31)–(34)), in which ψ_1 (β) is replaced by ψ_2 (β) and ψ_1 (β) by ψ_2 (β) and which will be calculated for the values of β corresponding to the poles, and

$$\psi_2(\beta) = \sqrt{k^2 - \beta^2} - j\sqrt{k^2k^2\varepsilon_1 - \beta^2}ctg(\sqrt{k^2\varepsilon_1 - \beta^2}d.$$

Expressions $F^{(1)}_{x_{1E}},\ F^{(1)}_{x_{2E}},\ F^{(1)}_{x_{yE}},\ F^{(1)}_{y_{1E}}$ are written similar to expressions with replacement, $\varphi^{(2)}_{x_N}$ by $\varphi^{(1)}_{x_N}$, $\varphi^{(2)}_{y_N}$ by $\varphi^{(1)}_{y}$, $\xi^{(2)}_{x_N}$ by $\xi^{(1)}_{x_N}$, and $\xi^{(2)}_{y_N}$ by $\xi^{(1)}_{y}$, respectively.

where

$$\varphi^{(1)}_{x_N} = \varphi^{(1)}_{x_{(-)}}\left[N_{(-)} - N_{(+)}\right] - N_{(+)}; \tag{35}$$

$$\varphi^{(1)}_{y_N} = \varphi^{(1)}_{y_{(-)}}\left[N_{(-)} - N_{(+)}\right]; \tag{36}$$

$$\xi^{(1)}_{x_N} = \xi^{(1)}_{x_{(-)}}\left[N_{(-)} - N_{(+)}\right]; \tag{37}$$

$$\xi^{(1)}_{y_N} = \xi^{(1)}_{x_{(-)}}\left[N_{(-)} - N_{(+)}\right] - N_{(+)}. \tag{38}$$

In expressions (Eqs. (35)–(38)), the expression N (−) corresponds to the expression N with the replacement of exp ($-jk_z z$) by exp ($-jk_1 z$);

expression N (+) corresponds to expression N replacing exp ($-jk_z z$) with exp ($-jk_1 z$),

$$\varphi^{(1)}_{x_{(-)}} = \frac{\exp\,(jc)}{\Delta}\left[e^2(a+b)(jb\sin c + a\cos c) - (bh + al)(jb\sin c + ag\cos c)\right];$$

$$\varphi^{(1)}_{(-)} = \xi^{(1)}_{x(-)}\frac{aebk^2(\varepsilon_1 - 1)}{\Delta};$$

$$\xi^{(1)}_{y(-)} = \frac{\exp\,(jc)}{\Delta}\left[(bf + ag)(jb\sin c + al\cos c) - l^2(a+b)(jb\sin c + a\cos c)\right].$$

The relation for functions includes expressions of spectral components. Considering the main type of oscillations in the waveguide H_{11}, we get

$$\hat{E}_{x1} = \pi\omega\mu_0 H_0 a^6 k_x k_y\left[0,1 + 6,5 + 10^3 a^2\left(3,4 + k_x^2 + k_y^2\right) + 1,45\cdot 10^3 a^4\left(k_x^2 + k_y^2\right)\right];$$

$$\hat{E}_{y1} = 0,85\pi\omega\mu_0 H_0 a^2[1 - 1,25a^2\left(b^2 + k_x^2 + 0,5k_y^2 + 3,125\cdot 10^{-2}\right)\times$$

$$\times a^4\left(0,33\,k_x^2k_y^2 + 0,33b^2k_x^2 + 0,5b^2k_y^2\right) - 0,5\cdot 10^{-4}a^6b^2k_x^2k_y^2];$$

The second integral (Eq. (25)) is calculated along the contour l_B, which covers the cut and is carried out so that the integrand is unique. From the form of the integrands, it is obvious where the first-order branch points are located. From the form of the integrands of the integrals (Eq. (25)) it is obvious that the first-order branch points are located at $\beta_{в_1} = \pm k$ and $\beta_{в_2} = \pm k\sqrt{\varepsilon_1}$. From these branch points, it is necessary to take $\beta_{в_1} = k$ and $\beta_{в_2} = k\sqrt{\varepsilon_1}$, in order to satisfy the radiation conditions. A calculation according to (Eq. (26)) shows that $C_B > 0$ for named branch points and, with the exception of the case when ε1 is complex and the losses are sufficiently large (tanδ > o.5). Thus, we find that the side wave can contribute to the radiation field, and this should be taken into account. An analysis of the integrands (Eqs. (20) and (22)) showed that the developed and well-known methods for the asymptotic estimation of integrals along the banks of a section covering branch points that are valid for a saddle point turn out to be inapplicable in this case. For this reason, the contribution of the side wave to the radiation diagram can be determined only by numerical integration (Eq. (25)) along the contour l_B. Moreover, from the analysis of integrands, it follows that it is advisable to choose the section so that it is a straight line parallel to the imaginary axis of the complex plane β. Then the integral along the banks of the section $\int\limits_{l_в} \dots d\beta$ will take the form

$$\int\limits_{\mathrm{Re}\,k+j\infty}^{k} v_1(\beta)d\beta + \int\limits_{k}^{\mathrm{Re}\,k+j\infty} v_2(\beta)d\beta,$$

where $v_1(\beta)$ and $v_2(\beta)$ are sub-integral expressions with signs in front of $\sqrt{k^2 - \beta^2}$.

The conditions for the existence of surface and leaky waves are determined from the location of the poles of the integrands (Eqs. (20) and (22)), and the poles correspond to the equality of the denominators of the marked expressions to zero.

By the Cauchy theorem, integral (Eq. (25)) along the contour lp is defined as follows:

$$\int\limits_{l_p} \dots d\beta = \sum_{i=1}^{n} \mathrm{Re}\,s(\beta_i),$$

where β_i are the roots of the denominators (Eq. (25)).

In order to separate the singular points into poles corresponding to surface and outgoing waves, it is advisable to again go from the complex plane β to the plane of the complex angle τ. Moreover, from the analysis of the exponent of expression (23), it follows that the surface wave will take place at

$$\tau_r = {}^{\pi}\!/_{2}, \tau_j > 0$$

and the outgoing wave corresponds to the following region of complex angles:

$$0 < \tau < \pi, (\text{except for } \tau r = \pi/.2), \tau ji > 0.$$

Moreover, the region $0 < \tau r < \phi$ determines the backward wave that does not satisfy the conditions at infinity.

In the presence of losses in the dielectric plate k1 = kr-jkj, analysis of the exponent (Eq. (23)) shows that the poles corresponding to the relations for the complex angle τ

$$\tau_r = \arccos\left(-\frac{\varepsilon tg\delta}{sh\tau_j}\right), \varepsilon - \sqrt{1 - \frac{\varepsilon^2 tg^2\delta}{sh^2\tau_j}} ch\tau_j < 0$$

determine the surface wave field.

If

$$\cos\tau_r sh\tau_j \neq -\varepsilon tg\delta,$$

then the poles that satisfy the last relation determine the field of the leaky wave. The conductivity of the aperture is determined by the following expression:

$$Y_a = \int\int_{-\infty}^{\infty} E_x^{(1)^*} H_y^{(1)} dx\, dy.$$

Applying the Parseval theorem, after a series of transformations, we obtain the expression for the conductivity referred to as the conductivity of the open end of the waveguide in the form

$$\begin{aligned} Y_{an} = {} & \frac{\omega\varepsilon_0}{2\pi^2 a}\int_0^{\infty}\int_0^{\pi}\frac{\beta\hat{E}_{x_0}^2 \exp(jc)}{\Delta}((e\hat{E}_{x_0} + e\hat{E}_{y_0} eabk^2(1-\varepsilon_1)\times \\ & \times(\cos c - j\sin c) - \left(e\hat{E}_{x_0} - e\hat{E}_{y_0}\right)e^2(a+b)(jb\sin c + a\cos c) + \\ & + \frac{\Delta}{2\exp(jc)} + e\hat{E}_{x_0}(hb+la)(jfb\sin c + ag\cos c) - \\ & - e\hat{E}_{y0}(bf+ag))d\beta\, da = \frac{\omega\hat{E}_0}{2\pi^2 b}\int_0^{\infty}\int_0^{2\pi}\frac{G(\beta a)}{\Delta}d\beta\, da. \end{aligned} \tag{39}$$

The contribution of surface and leaky waves to the conductivity of a circular waveguide with thermal protection is determined using the Cauchy theorem. As a result, we get

$$Y_{sur} = \frac{j\omega\varepsilon_0}{2\pi^2 a}\sum_{i=1}^{n}\int_0^{2\pi}\frac{G(\beta_i,\alpha)}{\Delta}d\alpha. \tag{40}$$

We will conduct a quantitative assessment of various loss mechanisms by means of transmission coefficients χ, attenuation υ, and reflection ρ, which are defined as follows:

$$\begin{aligned} & \eta = \eta_1 + \eta_2 + \eta_3 + \eta_4 = \frac{P_{rad}}{P_f} + \frac{P_{rad.sur}}{P_f} + \frac{P_{rad.res}}{P_f} + \frac{P_{rad.sid}}{P_f}, \\ & \nu = \nu_1 + \nu_2 + \nu_3 + \nu_4 = \frac{P_{rad} - P_{rad.sur}}{P_f} + \frac{P_{rad} - P_{rad.res}}{P_f} + \frac{P_{rad} - P_{rad.sid}}{P_f}, \\ & |R| = \left|\frac{1 - Y_n}{1 + Y_n}\right|. \end{aligned} \tag{41}$$

These expressions take into account the orthogonality of surface, leaky, and side waves and P_f is the power incident (supplied to the emitter), P_{rad}-radiated power,

$P_{rad.sur}$, $P_{rad.res}$, $P_{rad.sid}$. waves, respectively, P_{sur}, P_{res}, and P_{sid} are the power of surface, leaky, and side waves, respectively; υ_1 is the attenuation coefficient of radiated power, determined by losses in dielectric thermal protection.

$$\nu_1 = 1 - \eta - \nu_2 - \nu_3 - \nu_4 - |R|^2.$$

In expression (Eq. (41))

$$Y_n = Y_{an} - Y_{sur}.$$

3. Conclusion

Expressions (Eqs. (17), (18), (27)–(30), (39), and (40)) characterize the radiation field of a circular waveguide with uniform thermal protection and its input conductivity. The theoretical relations obtained, along with an estimate of the apparent loss of radiated power due to absorption in and reflection from thermal protection, allow us to estimate more subtle effects, such as losses on surface, leaky, and side waves. It is also possible to assess the influence of these waves on the radiation pattern.

Further development of the obtained theoretical relations should consist of taking into account the probable inhomogeneity of the heat-shielding layer in the direction of the z axis. For this, the method of approximate solution of the wave equation can be applied—the WKB method (Wentzel, Kramer, Bruellen) [9]. If we consider a round1 waveguide in the form of an onboard antenna of the returned spacecraft, then due to aerodynamic heating, a melt layer appears on the outer surface of the heat shield, which has electrical characteristics different from the characteristics of the material in the solid phase. Then the radiation from the waveguide should be considered through a two-layer dielectric thermal protection [10].

Unfortunately, all the obtained relations turned out to be very cumbersome and their use becomes only with numerical integration. For some of the obtained ratios, numerical calculations were performed. The results showed that the power of the side waves is zero. It also follows from the above calculations that the radiation field of surface and leaky waves is absent, that is, their contribution to the radiation pattern is not. Further research in this area should be directed to the development of computer calculation programs for the basic radiation characteristics.

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