Chapter

Review of Injected Oscillators

Ali Reza Hazeri

Abstract

Oscillators are critical components in electrical and electronic engineering and other engineering and sciences. Oscillators are classified as free-running oscillators and injected oscillators. This chapter describes the background necessary for the analysis and design of injected oscillators. When an oscillator is injected by an external periodic signal mentioned as an injection signal, it is called an injected oscillator. Consequently, two phenomena occur in the injected oscillators: (I) pulling phenomena and (II) locking phenomena. For locking phenomena, the oscillation frequency of the injection signal must be near free-running oscillation frequency or its sub-/super-harmonics. Due to these phenomena are nonlinear phenomena, it is tough to achieve the exact equation or closed-form equation of them. Therefore, researchers are scrutinizing them by different analytical and numerical methods for accomplishing an exact inside view of their performances. In this chapter, injected oscillators are investigated in two main subjects: first, analytical methods on locking and pulling phenomena are reviewed, and second, applications of injected oscillators are reviewed such as injection-locked frequency dividers at the latter. Furthermore, methods of enhancing the locking range are introduced.

Keywords: frequency dividers, frequency multipliers, injected oscillators, nonlinear oscillations, locking phenomena, locking range, quadrature oscillators, pulling phenomena

1. Introduction

One of the most important blocks in the electronic systems is undoubtedly the oscillator block. The oscillator converts a direct current (DC) generated by power supply into an alternating current (AC) signal. The design of this block involves many trade-offs between phase noise, oscillation frequency range, power consumption, layout size, etc. Oscillators are employed in many applications such as phase-locked loops (PLL), frequency dividers/multipliers, clock recovery, frequency synthesizers, etc. Nowadays, the demand for compact and portable systems has increased. Hence, the oscillators, amplifiers, mixers, power amplifiers, etc. may be integrated on a chip. For example, FM radio, Bluetooth, GPRS, Wi-Fi, NFC, and GPS are integrated into modern mobile systems. According to the application, the oscillators are classified as the free-running oscillators and injected oscillators. Free-running oscillators have been extremely studied in many electronic engineering books and articles so far [1–6]. In the free-running oscillators, there is no external signal injected into the oscillator. In the injected oscillators, an external periodic signal mentioned as an injection signal is injected to the oscillator, which may be deliberately applied by the designers to make an injected oscillator, or any injection signals are accidentally injected to the oscillator which may be generated from other blocks such as power amplifiers and other oscillators. By injected oscillators, designers can implement many high-performance blocks such as quadrature oscillators and frequency dividers/multipliers, frequency synthesizers without a frequency-locked loop, which are useful in the fast frequency locking-loop systems. Thus, the cost of fabrication is cheaper than a frequency-locked loop. There are many devices operating with different center frequencies. As a result, several oscillators and other devices are placed together for implementing a system called system on chip (SOC). However, when the oscillators are integrated with other devices on the chip, various signals with different center frequencies may leak through the substrate, parasitic elements, or packaging and be injected into the oscillator. Hence, the performance of the oscillators is changed which can be suitable or not dependent on their functionality. When an oscillator is injected with an injection signal, the pulling and locking phenomena occur. Thus, pulling and locking phenomena are important parameters for designers.

The injected locked oscillators designed by engineers are classified into three classes. While the oscillation frequency of the injection signal is near to the free-running oscillator, the first-harmonic injection locking takes place. When the oscillation frequency of the injection signal is near to the sub-/super-harmonic of the oscillation frequency of the free-running oscillator, frequency dividers/multipliers are realized. These phenomena occur since the nonlinear performance of the injected oscillators. So, the size of the layout and complexity of designing frequency dividers/multipliers are lower than the frequency-locked loop because they do not employ many blocks in the frequency-locked loop such as filters, charge pomp, and frequency detector [7]. Consequently, power consumption is reduced. Furthermore, they are fast and may be applied to the high-speed or high clock data recovery and fast-locking systems. At last, the phase noise of the injected oscillator is different from the free-running oscillator and is dependent on the phase noise of the injection signal. Therefore, while the injection signal is generated by an oscillator which owns excellent phase noise, the injected oscillator will have a better phase noise [7, 8]. This chapter tries to disclose all subtleties and challenges encountered during the design of injected oscillators.

The presented chapter aims to investigate the injected oscillators. A summary of the injected oscillator specifications regarding locking and pulling phenomena, previously significantly published papers about the first harmonic injection oscillator, frequency dividers and enhancing locking range are presented. Section 2 covers with introducing pulling and locking phenomena in the first-harmonic injection locking oscillator. Then, a free-running oscillator is implemented for exploring the pulling and locking phenomena by various injection signals. Section 3 will be dedicated to the pulling and locking formula, and beat frequency equation is discussed for injected oscillators for nonharmonic (LC) and harmonic (ring) oscillators. Section 4 will treat the implementation of injected locked frequency dividers and increase the locking range. First, a block of the injection-locked frequency dividers/multiplier is displayed. In a literature overview, two classes of realization will be recognized: the conventional LC-injection-locked frequency dividers where the injection signal is applied to the oscillator by the tail current source or a transistor connected to the output nodes. Furthermore, the previously important published papers are reviewed. Finally, some structures and techniques employed in order to extend the locking range of frequency dividers are exhibited from previously significant published papers. Section 5 will conclude with the main contributions of the presented chapter.

2. Pulling and clocking phenomena

When an oscillator is injected with an external signal, two phenomena occur for the oscillator. Once the strength of the external signal and frequency difference between the external signal and free-running oscillator are not suitable for locking, the oscillator is perturbed, and the output signal is modulated called pulling phenomenon. In the pulling case, the output spectrum of the injected oscillators has some spurious tones along with the effective oscillation frequency of the external signal. While the strength of the external signal and frequency difference between the external signal and free-running oscillator are suitable, the oscillator is locked, and the frequency of the output signal is locked to the first, sub- or super-harmonic of the oscillation frequency of the external signal called locking phenomenon. The range of the oscillation frequency of the external signal causes a locking phenomenon called the locking range. In **Figures 1** and **2**, these phenomena are simply presented in the time domain and frequency domain.

At first, a free-running oscillator is chosen. Next, the oscillation frequency, the output voltage oscillation amplitude, and the current oscillation amplitude are obtained. Second, the given oscillator is injected with the external signal, which is modeled by a current source and in parallel with the output nodes. Third, the results of the injected oscillator are revealed under locking and pulling phenomena.

The free-running oscillator, implemented by a three-stage differential ring oscillator, is depicted in **Figure 1a** along with the delay cell in **Figure 1b**. In the time and frequency domain, the output voltage of the oscillator is presented in **Figure 1c** and **d**. According to **Figure 1b** and **c**, there is no frequency or phase modulation on the output voltage. It has a center frequency (257 MHz). In addition, the output voltage amplitude is constant (0.481 V), and the oscillation current (I_{osc}) is equal to 0.18 mA. In fact, there is not any amplitude modulation (AM).

Figure 2a shows the given three-stage differential ring oscillator under an injection signal, which its current (I_{inj}) and oscillation frequency (F_{inj}) are equal to $I_{osc}/20$ and 260 MHz, respectively. Assuming locking conditions are covered, the oscillation frequency of the injected oscillator is the same as the injected signal as shown in **Figure 2b** and **c**. **Figure 2b** and **c** displays the output voltage of the injected oscillator under locking phenomenon in the time and frequency domain, respectively. **Figure 2b** and **c** illustrates the output voltage of the injected oscillator locked in F_{inj}. It is clear that there is not any modulation on the output voltage at the time and frequency domain. Then, the oscillation frequency of the injected oscillator is equal to the oscillation frequency of the injected signal.

Figure 2d–**g** illustrates the output voltage of the injected oscillator in the pulling phenomenon when $I_{inj} = I_{osc}/20$ and F_{inj} is equal to 262.5 and 265 MHz. It is obvious that there are modulations on the output voltage at the time and frequency domain. The injected oscillator produces amplitude and frequency (or phase) modulation in the output voltage. According to **Figure 2d** and **f**, the center frequency of the oscillator is pulled to the frequency of the external signal. Furthermore, the beat frequency, instance variation of oscillation frequency, is reduced when F_{inj} becomes near locking range as portrayed in **Figure 2**.

3. Review of the previously significantly published papers

The pulling and locking phenomena have been investigated in previously published papers [2, 8–60]. According to the output waveform, oscillators may be categorized as nonharmonic oscillators, for example, LC oscillators and harmonic oscillators such as ring oscillators. For the nonharmonic oscillators, the output waveform has a center frequency near the resonance frequency of the LC tank. Consequently, the output waveform is almost sinusoidal. For harmonic oscillators, since the output waveform is not sinusoidal, the higher harmonics effect on the output waveform. In fact, nonharmonic oscillators have an LC tank block

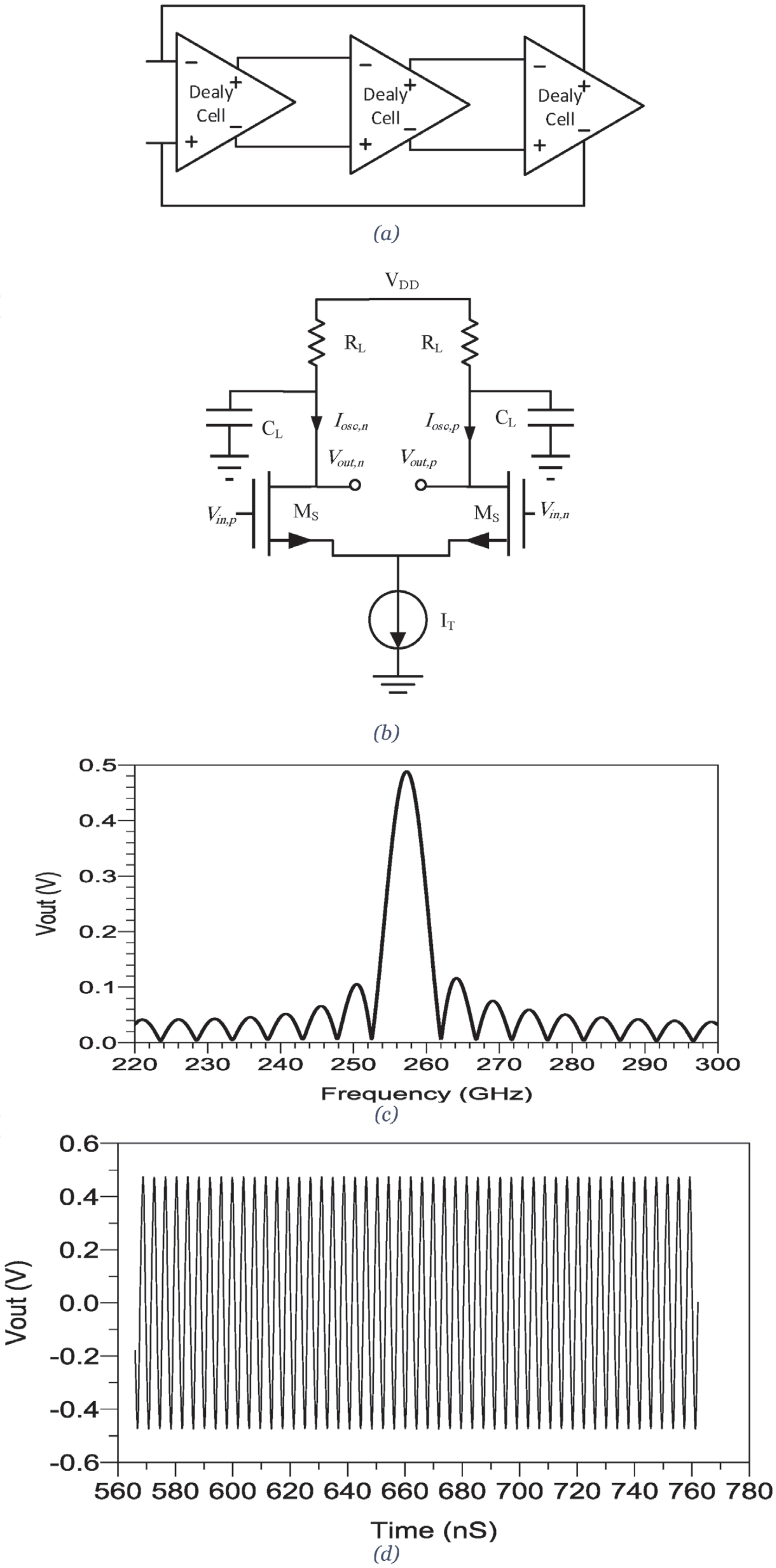

Figure 1.
(a) Three-stage differential ring oscillator, (b) delay cell, (c) output voltage at the time domain, (d) output voltage at the frequency domain.

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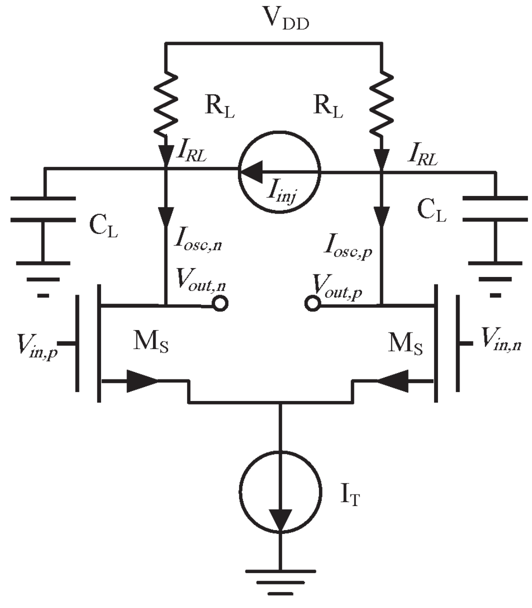
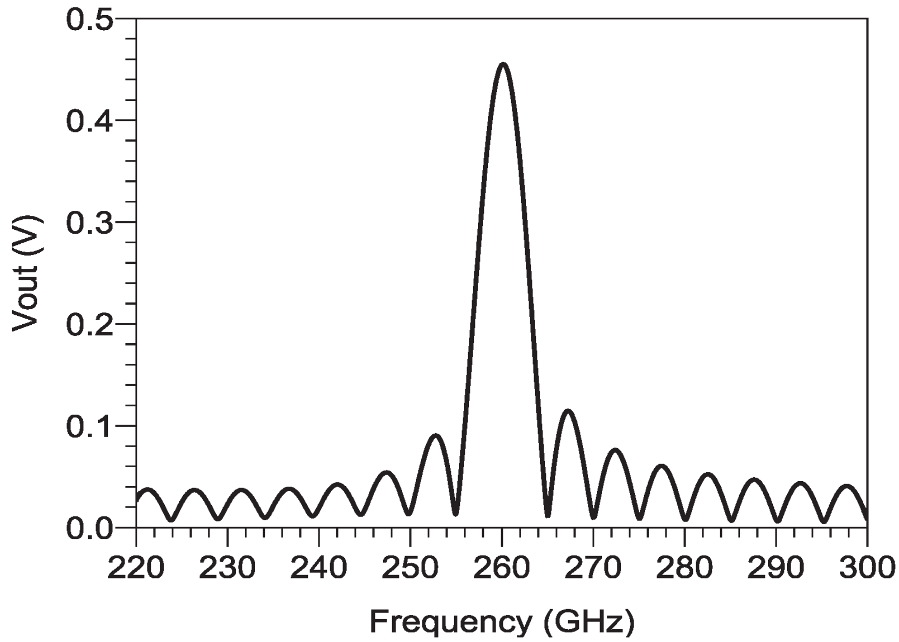
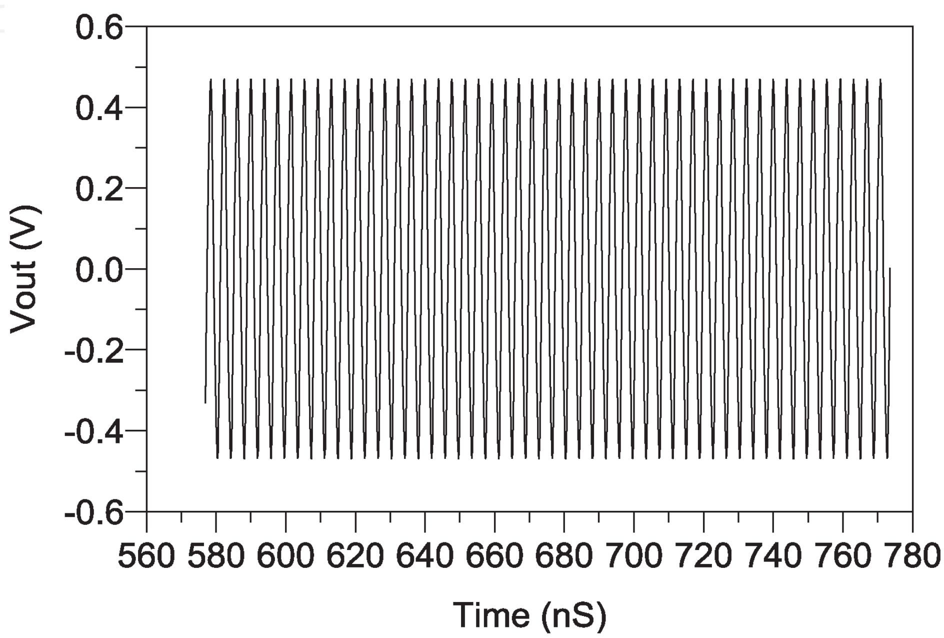


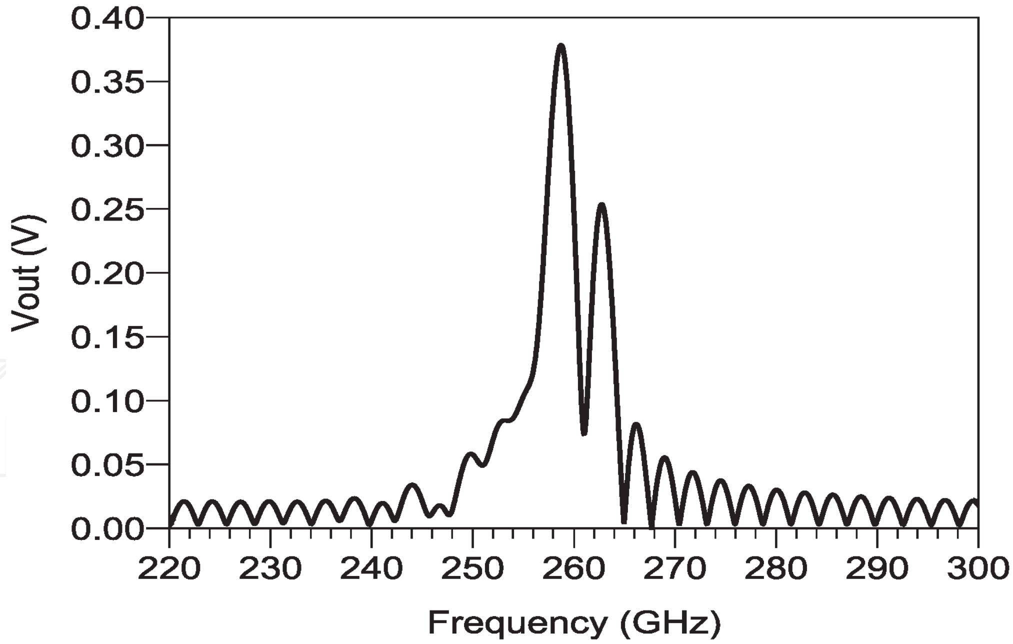
(a)



(b)



(c)


0.40
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Vout (V)
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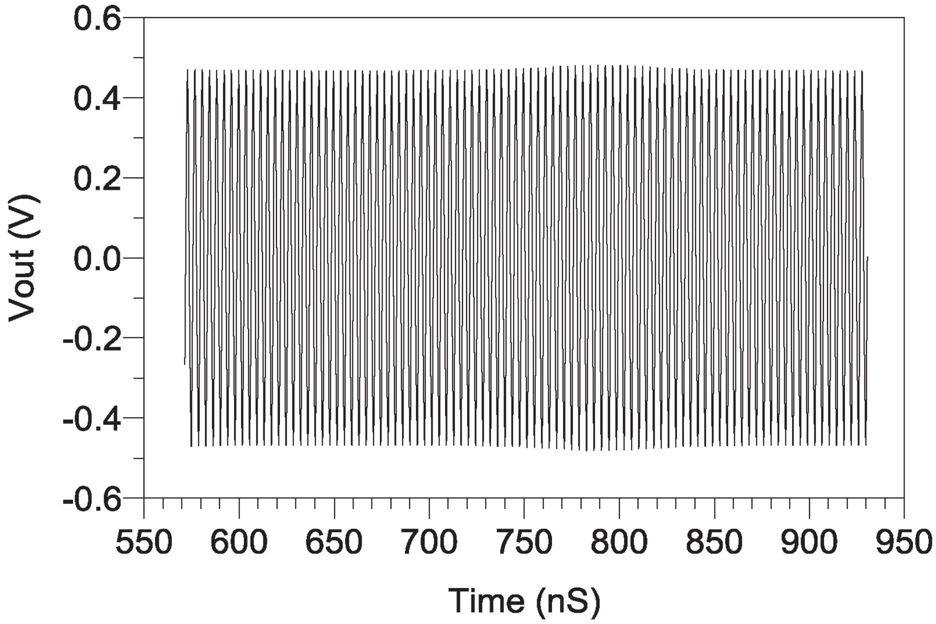
260
270
280
290
300
Frequency (GHz)

(d)


0.6
0.4
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-0.2
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Vout (V)
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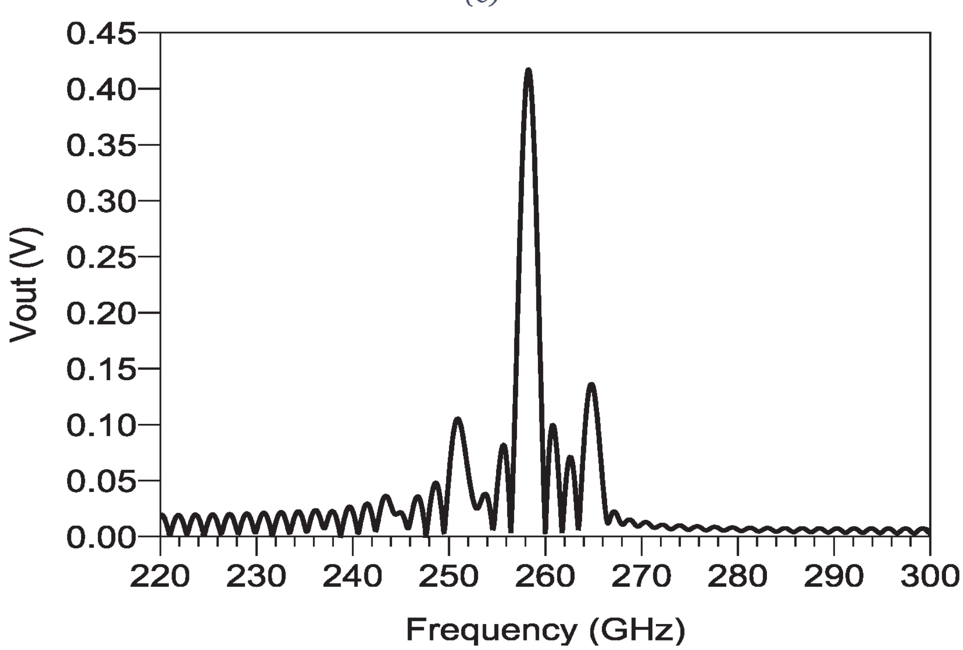
900
950
Time (nS)

(e)


0.45
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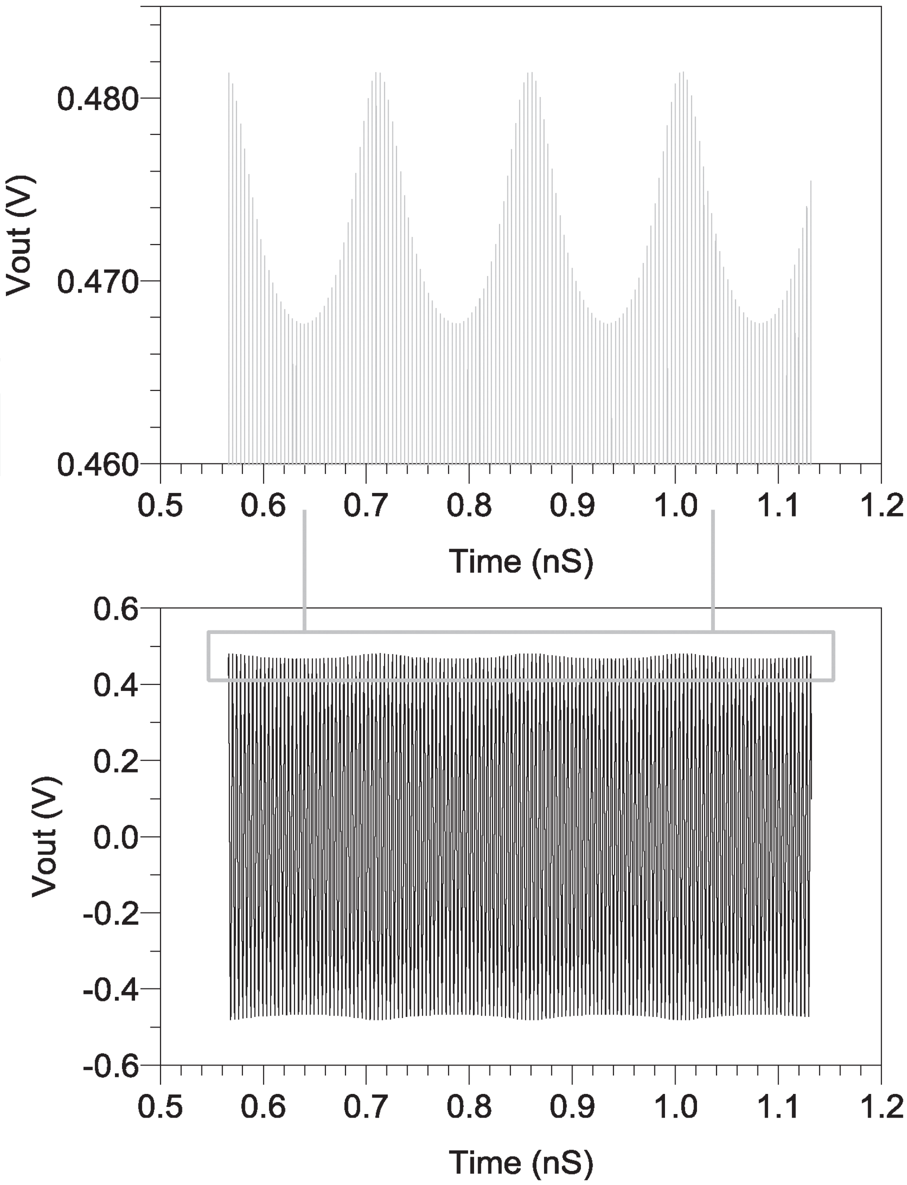
Frequency (GHz)

(f)



(g)

Figure 2.
*(a) Injected delay cell, (b) locking phenomenon in frequency domain (*Finj *= 260 MHz and* $I_{inj} = I_{osc}/20$*), (c) locking phenomenon in time domain (*Finj *= 260 MHz and* $I_{inj} = I_{osc}/20$*), (d) pulling phenomenon in frequency domain (*Finj *= 262.5 MHz and* $I_{inj} = I_{osc}/20$*), (e) pulling phenomenon in time domain (*Finj *= 262.5 MHz and* $I_{inj} = I_{osc}/20$*), (f) pulling phenomenon in frequency domain (*Finj *= 265 MHz and* $I_{inj} = I_{osc}/20$*), (g) pulling phenomenon in the time domain (*Finj *= 265 MHz and* $I_{inj} = I_{osc}/20$*).*

operated similarly to a band-pass filter, and harmonic oscillators have a low-pass filter. The pulling and locking equations are different in these classes. Most of the methods can be used for a class. The locking phenomena have been detected in pendulum clocks in 1629–1695. At first, Adler explained pulling and locking phenomena for LC oscillators for a weak injection signal [9, 54]. Then, case studies that are more special have been reported such as [8, 10–17]. In [10], Adler's equation was extended for applications that the strength of the injection signal is not weak. In [8], by vector diagram of the instantaneous currents, the locking range (LR) equation was improved. The same locking range equation was obtained by another method in [16] and partly in [24, 29]. In [22] using nonlinear feedback analysis, injection locking was investigated. A cross-coupled oscillator under the injection signal is depicted in **Figure 3**. By writing differential equations on the output nodes of the oscillator and solving them in nonlinear, vector diagram or empirical methods, the locking range of the cross-coupled oscillator is achieved.

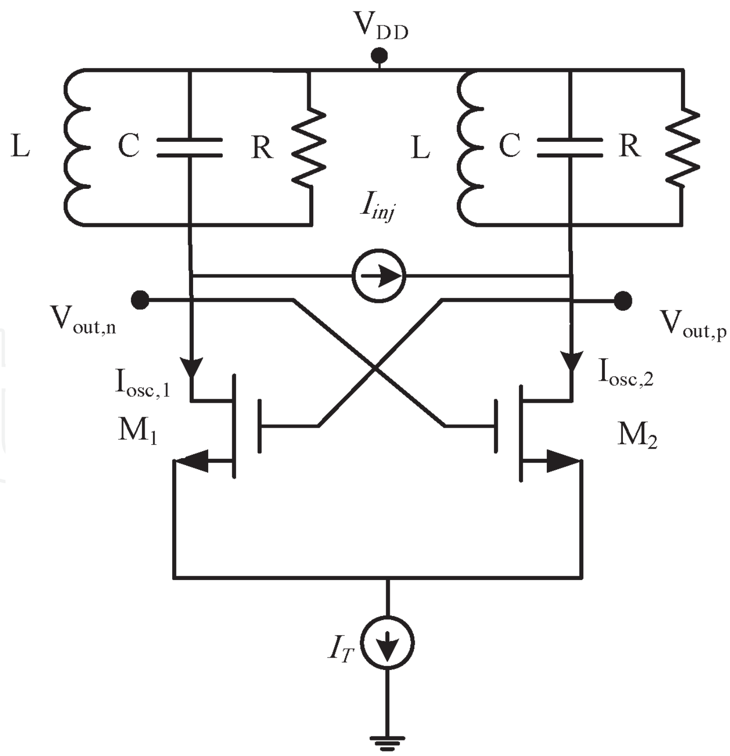


Figure 3.
Cross-coupled LC oscillator.

In **Table 1**, previously significantly published locking ranges are offered, which Q is the quality factor of the LC tank circuit. When $I_{inj}/I_{osc} \ll 1$, it is clear that the locking range is proportional to I_{inj} and inversely proportional Q and I_{osc}.

The pulling phenomena for LC oscillators were completely investigated in [8, 25, 27, 28, 33, 39, 41, 47–49]. When $I_{inj}/I_{osc} \ll 1$, the beat frequency mathematical formula (ω_b) for LC oscillators is as follows:

$$\omega_b|_{LC} = \sqrt{\Omega^2 - K^2}, K = (\omega_0 I_{inj})/(2QI_{osc}), \Omega = |\omega_{osc} - \omega_{inj}| \tag{1}$$

According to Eq. (1), ω_b is increased by increasing Ω or I_{osc} when other parameters are constant. Eq. (1) was improved for strong injection in [41, 47]. The multi-injection signals with different total phases have been reported for the LC oscillators in [24, 29, 56].

Nonharmonic injected oscillators, for instance, ring oscillators and relaxation oscillators, have been studied in several papers [21, 23, 26, 30, 46, 50, 52]. In [21], by using approach of the LC oscillators, a three-stage single-ended ring oscillator was studied. In [23, 30], a locking range equation was obtained at the time domain analysis. In [26], by using current phase diagrams, a locking range was calculated. The locking range of [26] was improved in [50] for a larger injection level. In [52], various locking range equations for ring oscillators were introduced. In addition, comprehensive and exact analyses of the ring oscillators were presented to both

Refs.	Locking range
[9, 44, 54]	$\omega_{osc} I_{inj}/(2QI_{osc})$
[8, 24, 47]	$\omega_{osc} I_{inj}/\left(2QI_{osc}\sqrt{1-(I_{inj}/I_{osc})^2}\right)$

Table 1.
Previously significant published locking ranges for LC oscillators.

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Refs.	Locking range
[5]	$\frac{I_{inj}}{NC_L \sum_{i=1}^{\infty} (-1)^{i+1} (2i-1) A_{2i-1}}$
[23, 30]	$\frac{2\omega_{osc} I_{inj}}{NI_{osc} \sin(2\pi/N)}$
[26]	$\frac{\omega_{osc}}{N} \frac{1+\tan^2(\pi/N)}{\tan(\pi/N)\sqrt{1-\left(\frac{I}{I_{osc}}\right)^2}} \frac{I_{inj}}{I_{osc}}$
[50]	$\frac{\frac{\omega_{osc}}{N} \frac{1+\tan^2(\pi/N)}{\tan(\pi/N)}}{\sqrt{1-\left(\frac{I_{inj}}{I_{osc}}\right)^2} - \tan(\pi/N)\frac{I_{inj}}{I_{osc}}} \frac{I_{inj}}{I_{trans.}}$
A_i *is the amplitude of the* ith *output voltage harmonic and* N *is the number of stages.*	

Table 2.
Previously significant published locking ranges for ring oscillators.

pulling case and locking case considering higher-order harmonics. Moreover, the multi-injection signals with the same frequency and different initial phase (δ) have been achieved for the ring oscillators. Like locking range for LC oscillators, the locking range for rings oscillators are calculated by solving nonlinear differential equations in the output nodes and proportional to I_{inj}/I_{osc} and inversely proportional to the number of stages (N) [52]. In **Table 2**, previously significantly published locking ranges are exhibited. Moreover, the beat frequency mathematical formula for ring oscillators is expressed as below [5]:

$$\omega_b|_{ring} = \sqrt{\Omega^2 - k^2}, k = I_{inj}/(NA_1C_L), \Omega = |\omega_{osc} - \omega_{inj}| \tag{2}$$

Furthermore, the relaxation oscillators with injection signals were presented in [46]. In addition, a perturbation projection vector (PPV) was employed for analyzing both harmonic and nonharmonic oscillators [18–20]. However, using PPV is complicated, and most of the time it needs preprocessing by simulators such as Cadence.

4. Review of the frequency multipliers/dividers

At first, fractional frequency generators utilizing regenerative modulation have been introduced by [61]. A block diagram of the injection frequency divider/multiplier is displayed in **Figure 4**. By subharmonic ($\omega_{inj} \approx \omega_0/M$) or super-harmonic ($\omega_{inj} \approx M\omega_0$) injection signals, frequency dividers and multipliers were reported in many papers such as [13, 15, 17], where, M is a positive number. Frequency dividers were explored in [36, 38, 40, 43–45, 49, 60]. Studies on

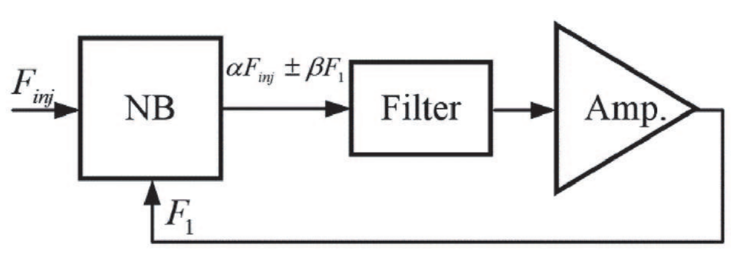

Figure 4.
Block diagram of a frequency divider/multiplier.

frequency multipliers/dividers are generally in two sections. The first one is to obtain an approximation equation of the locking and pulling phenomena [36, 38, 40, 43–45, 49]. The second one is increasing the locking range. In the frequency multipliers/dividers, injection signals may be injected from tail node or output node called direct injection as demonstrated in **Figure 5**. In [13, 15, 17], the general models of the frequency multiplier/dividers have been proposed. The transistor is modeled as a nonlinear block (NB). By using a summer and nonlinear block, a conceptual model was introduced [13]. Nevertheless, once an oscillator behaves similar mixers for the injection signal, for example, when the injection signal is parallel with the tail current source, this model is not correct. In order to achieve a general conceptual model, the summer block is replaced with a multiplier block [17]. However, this model is dependent on SPICE parameters and preprocessing. By phase-domain macromodel, injection-locked frequency dividers were analyzed. Nonetheless, the phase-domain macromodel requests preprocessing and time-consuming. For ÷ 2, an injection signal has been paralleled to the tail current source of the cross-coupled oscillator, and then, the injection signal is modeled as an equivalent injection signal at output nodes whose oscillation frequency and amplitude are $\omega_{inj}/2$ and $2I_{inj}/\pi$, respectively [8]. Therefore, the locking range was

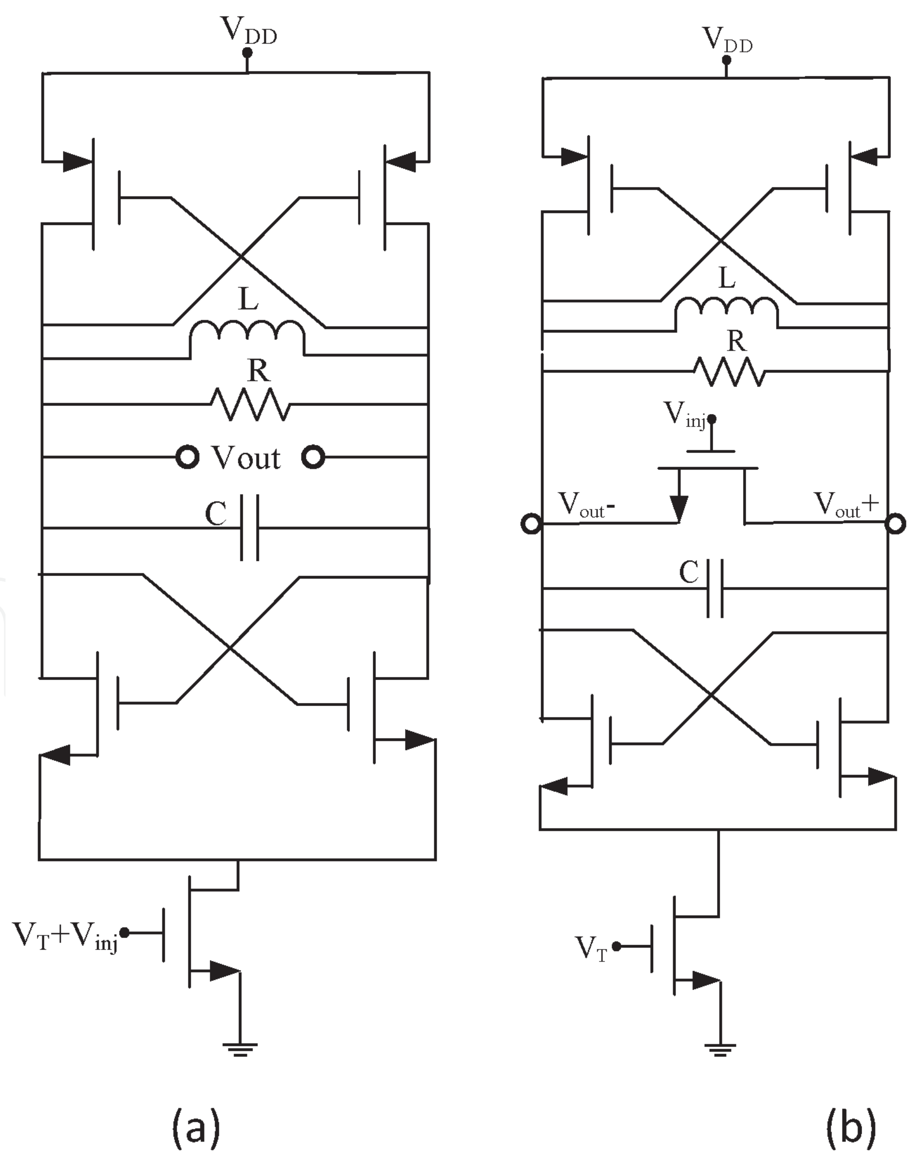

Figure 5.
Conventional injection locking frequency divider/multipliers, (a) injection in the tail, (b) injection in the drain.

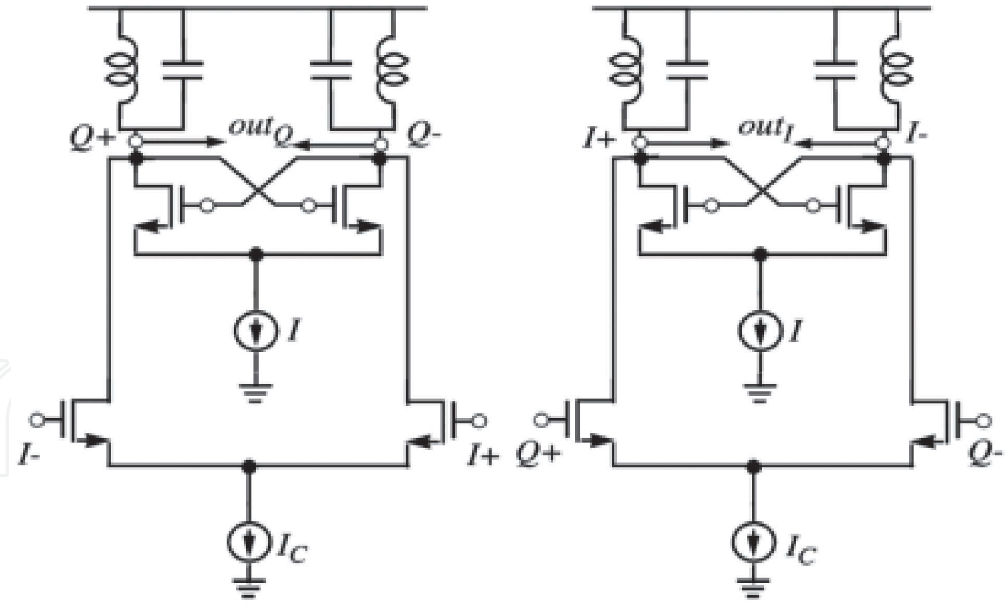

Figure 6.
The quadrature LC oscillator [29].

accomplished similar to the first-harmonic injection locking. This locking range has been acquired by a slightly different analysis in [62]. The asymptotic analysis, or slowly varying amplitude, averaging method, and phase analysis have been utilized to analyze the injected oscillators which make frequency dividers [43–45]. In [45], the locking range has been obtained when the injection signal is applied to the tail current source. Moreover, by the numerical bifurcation analysis using continuation software such as AUTO, they have been analyzed [36, 40]. In [60], an exact analysis for the locking range in injection-locked frequency dividers has been proposed by phasor diagrams and differential equations.

Due to the small locking range of injected LC oscillators, various techniques have been realized to enhance the locking range. Passive and active structures are explored for improving the injection efficiency such as combining inductors in series or parallel with the injection mixer to enhancing its transconductance, body biasing, transformer feedback, dual-resonance RLC resonators, dual injection for increasing the voltage and current injection paths, tapped resonators, switched resonators, harmonic suppression, and distributed injection to distribute the injection signals; in other words, the injection component is divided to several smaller components; input-power-matching and inductive input-matching network is located to the gate of the NMOS switch to heighten the injection power [63–72]. **Figure 6** discloses a quadrature LC oscillator employed in the injection signal.

5. Conclusions

Some basic concepts and definitions have been presented in this chapter. First, pulling and locking phenomena have been introduced which both contain for injection oscillators. Next, previously significantly published papers have been explored. Furthermore, locking range and beat frequency formula have been studied for both first-harmonic injected LC and ring oscillators. Finally, previously significantly published papers about injected locked frequency dividers have been reviewed. Moreover, some previously important published papers about increasing the locking range of the injected locked frequency dividers have been introduced.

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