Chapter

Combinatorial Cosmology

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Abstract

In this chapter, a combinatorial model for cosmology is analyzed. We consider each universe as a path in a graph, and the set of all such paths can be made into a finite probability space. We can then consider the probabilities for different kinds of behavior and under certain circumstances argue that a scenario where the behavior of the entropy is monotonic, either increasing or decreasing, should be much more likely than a scenario where the behavior is symmetric with respect to time. In this way we can attempt to construct a model for a multiverse which is completely time symmetric but where the individual universes tend to be time asymmetric, i.e., have an arrow of time. One of the main points with this approach is that this kind of broken symmetry can be studied in very small models using exact mathematical methods from, e.g., combinatorics. Even if the amount of computations needed increases very rapidly with the size of the model, we can still hope for valuable information about what properties more realistic models should have. Some suggestions for further research are pointed out.

Keywords: cosmology, multiverse, graph theory, entropy, time's arrow

1. Introduction

Applications of combinatorics have in recent years invaded many new areas of research. Still, cosmology is probably not the first such area which comes to your mind. Traditional cosmology is usually based on differential geometry and general relativity, often in combination with various ideas from fundamental physics and high-precision astronomical measurements. However, it is very much at the heart of cosmology that any model that we study must be based on rather drastic simplifications. In fact, when the object of study in a sense contains everything, finding the right way to discard nonessential information becomes a fundamental problem. From this point of view, the combinatorial approach is just one of several possible ways to proceed. For a discussion of this question from a more general point of view, see [1].

Different problems may of course call for different kinds of simplifications. As a rather extreme example, I will in this chapter discuss the long open problem of time's arrow, where it can be argued that the best method of attack may be to discard almost everything we know about the universe, just to uncover the underlying combinatorial skeleton. In other words, we should forget almost everything we know about ordinary physics and instead consider all the possible states that a universe could be in as the nodes of a huge graph. Each possible universe then becomes a path in this graph, and our mission becomes to try to decide what kinds of paths are the most common ones.

The ambition here is not to claim any kind of final solution to "the riddle of time." Rather, the ambition is to give a new angle to a well-known problem. And also to show that from this point of view, it may even make sense to study models which are ridiculously small in comparison with the real universe.

After a short introduction to the problem of time's arrow in Section 2, the basic structure of the combinatorial multiverse is presented in Section 3. But in order to study the asymmetry of time, we also need the concept of entropy which is introduced in Section 4, and in Section 5 we then turn to the dynamics. Both the entropy and the dynamics are treated in a very simplified way. However, the essential point here is to try to explain how the time-asymmetric development of the entropy that we observe in our universe can arise from time-symmetric dynamical laws and boundary conditions. In Section 6, the object is to show how standard methods from combinatorics can be used to make computations in the combinatorial multiverse. In Sections 7 and 8, we then consider some very simple probabilistic assumptions which turn the combinatorial multiverse into a probability space and discuss the consequences for time asymmetry.

In the simple model discussed so far, it is not difficult to obtain similar results by heuristic reasoning. However, the approach here should mainly be considered as a preparation for more complicated models, where the same combinatorial methods could be used, but heuristics would be difficult to apply.

Thus, this should more be considered as a starting point for further research than as an endpoint. In Section 9, I will therefore take a step in this direction by suggesting one such possible generalization (out of many), which could be used to obtain a stronger kind of time asymmetry. Finally, some conclusions are then discussed in Section 10.

Many of the ideas presented here have appeared before, e.g., in [2, 3], and in particular [4], although from a somewhat different angle.

2. The arrow of time

The term "time's arrow" was coined by Eddington [5] and refers to the fact that macroscopic time is asymmetric. In fact, we all know that the future is very different from the past. For instance, how does it come that we can remember yesterday but we cannot remember tomorrow? This can also be expressed by saying that we all agree that there is a well-defined direction from the past toward the future. Ever since the time of Ludwig Boltzmann, it has been clear that this has something to do with the growth of entropy and the second law of thermodynamics, although it may still not be quite obvious exactly what the connection is.

What is mysterious about time's arrow is that somehow the macroscopic laws that we observe must emerge from the underlying microscopic laws of motion, and these are in general considered to be essentially time invariant. So how can asymmetric macroscopic laws arise from symmetric microscopic ones?

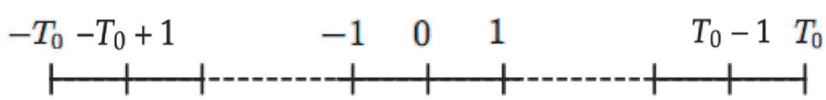
Few questions in physics have generated such a variety of completely different answers (see, e.g., Barbour [6], Halliwell et al. [7], Zeh [8]), and the problem is still wide open. But, as has repeatedly been pointed out by Price [9], most such tentative answers seem to contain some (more or less hidden) asymmetry from the beginning, either in the boundary conditions or in the dynamical laws.

Here I will advocate a different viewpoint. We can consider the set of all possible universes as a probability space, a "multiverse," and this probability space will be completely time symmetric in the sense that reversing the direction of time would generate the same probability space. But for observers, like ourselves, who are by necessity confined to our own universe, it can still be that the symmetry appears to be broken in the sense that in one direction of time the entropy is increasing and in the other it is decreasing. Another way to express this would be to say that all the universes in the multiverse would share the same endpoints, the Big Bang and the Big Crunch. But only half of them would have the same Big Bang as we have. In the other half, our Big Bang would instead be the Big Crunch.

3. The combinatorial multiverse

This is not the place to try to describe all possible combinatorial models for cosmology. Rather, I have chosen to just discuss the simple case of a closed, finite universe. Many cosmologists these days support open models, and it is of course possible to apply combinatorial methods to them too. However, since such models tend to be infinite, they may be considerably more complicated from a probabilistic point of view.

To model the set of all universes in the simplest possible way, let us for each moment of time between the endpoints $-T_0$ and T_0 (i.e., the Big Bang and the Big Crunch) consider the finite set of all possible "states" of a universe. To make everything extremely simple, let us suppose that time is discrete in the sense that we only consider it at a finite number of points as follows:



(1)

Thus, we can measure time just by counting the number of time intervals, which means that time can be viewed as integer valued. At the endpoints $-T_0$ and T_0, there will just be one unique state (with zero volume), but in between, there will be many states for each t. All such states will be the nodes of an enormous graph, and a universe will then be just a path in this graph with the property that there is exactly one state for each moment of time. The dynamics of the model can then be specified by choosing at certain collection of edges between adjacent moments of time, say t and $t+1$, which correspond to those time developments which are possible. A quite schematic picture is displayed in **Figure 1**.

Remark 1. For the readers taking interest in the underlying physics: the word "state" is not referring to quantum states as they are usually interpreted. A better way of thinking of them is to say that they represent "distinguishable configurations." This is in fact a kind of semiclassical approximation (see Tamm [2]).

The important point here is that a given state can lead to different states in the future. This is very much what actually happens when, say, a particle decays: whether or not this happens may, according to the multiverse interpretation, lead to very different futures within a rather short time. And there is no contradiction

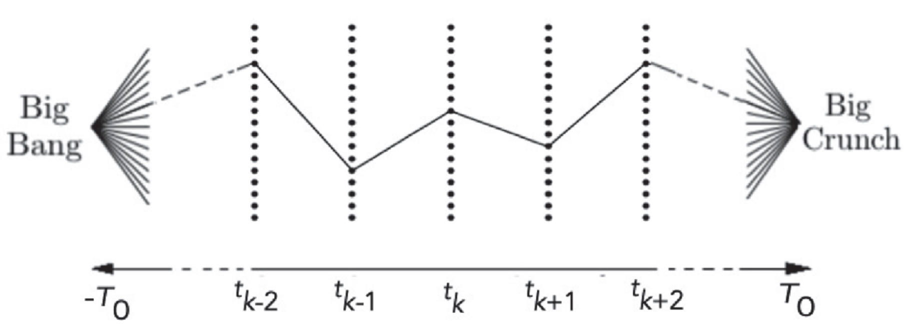


Figure 1.
One universe in the combinatorial multiverse [3].

between this and the fact that the development of the underlying wave function for the whole universe is unique.

Summarizing:

Definition 1. A *universe* U is a chain of states (one state U_t for each moment of time t), with the property that the transition between adjacent states is always possible.

Definition 2. A *multiverse* M is the set of all possible universes U in the sense of Definition 1 together with a probability measure on this set.

It may of course be said that quantum mechanics should allow for transitions between all kinds of states, although the probability for most such transitions may be extremely small. In this extremely simplified treatment, I will assume that for a given state at a given moment of time t, the dynamical laws will only permit transitions to a very limited number of states at the previous and next moments, which will make the probabilistic part of the investigation particularly simple. However, modifications are called for near the endpoints (the Big Bang and the Big Crunch); see Section 5.

As it stands, the model presented so far is too simple to generate any results. In fact, there are no observable differences at all between the states, which mean that there are no measurable variables which could be related to the (so far non-specified) dynamics.

There are of course many different variables which we can choose to enrich this structure, and which ones to choose must depend on what properties we want to explain. For explaining the second law of thermodynamics, the obvious choice is the entropy.

4. Entropy

According to Boltzmann, the total entropy of a certain macro-state at a certain time is given by

$$S = k_B \ln \Omega, \tag{2}$$

or inversely

$$\Omega = W^S, \quad \text{with} \quad W = e^{1/k_B}, \tag{3}$$

where Ω denotes the number of corresponding micro-states and k_B is Boltzmann's constant.

This formula was from the beginning derived for simple cases, like an ideal gas. Nevertheless, it does represent a kind of universal truth in statistical mechanics: the number of possible micro-states corresponding to a given macro-state grows exponentially with the entropy. Although there are many complications when one tries to consider the entropy of the universe as a whole, I will still take it as the starting point for the discussion that the entropy (at a given time t) is an exponential function of the total entropy as in (3). A more difficult question is if and how the constant W may vary with time, but for the purpose of the present paper, I will simply let it be constant.

One may of course argue that this can only be true when the universe is still quite ordered and the entropy is very far from reaching its maximum. But this is certainly what the situation is like in our universe today, and according to the computations in [10, 11], it would take an almost incredibly long time to reach such a state of maximal entropy. Thus, it will in the following be taken for granted that this time is much longer than the life-span of our universe.

5. The dynamics

The next step is to construct a model for the dynamics. The idea, which essentially goes back to Boltzmann (see [12]), is that any given macro-state at any given time is extremely likely to develop into a state with higher entropy at the next moment of time, simply because there are so many more states with higher entropy than with lower entropy (compare with (3)). The problem with this in the present situation, however, is that this way of thinking in fact presupposes a preferred direction of time. Otherwise, given that the dynamical laws are time symmetric, why can we not similarly argue that the entropy should also grow when we go backward in time? (compare [9]).

There have been many attempts to avoid this problem by looking for defects in the symmetries. But my conclusion here is that we must actually accept Boltzmann's argument in both directions of time and hence we are led to the following:

Principle 1. At every moment of time t and for every state with entropy S, there are very many "accessible states" with higher entropy, both at the previous moment of time $t-1$ and at the next one $t+1$. On the other hand, the chance for finding such accessible states with lower entropy, both at times $t-1$ and $t+1$, is extremely small.

This principle also implies a shift of perspective in the search for time's arrow. Rather than trying to find the reason for the asymmetry, we must concentrate on understanding why we cannot observe the symmetric structure of the multiverse as a whole.

As still one more simplification, let us assume that the entropy can only change by ± 1 during each unit of time. This assumption, however, has to be modified near the endpoints (BB and BC) for the following reason: it is a very important aspect of this approach to assume that physics during the first and last moments is very different from the rest of the time, since at these moments quantum phenomena can be expected to become global. To model this in a simple way, we can split the life-span of our multiverse up into three parts:

$$[-T_0, -T_1] \cup [-T_1, T_1] \cup [T_1, T_0]. \tag{4}$$

Here the first and last parts may be called "the extreme phases," which are characterized by the property that transition between very different states can be possible. During the "normal phase" in between on the other hand, physics is supposed to behave more or less as we are used to.

6. Modeling the dynamics

To construct a miniature multiverse for computational purposes, one can proceed as follows: first of all, in the very small multiverses studied here, the extreme phases will only last for one single unit of time. Also, for ease of notation, let us put $T_1 = m$, so that the moments of time can in this context be denoted as

$$-m-1,\ -m,\ -m+1,\ \dots, m-1, m, m+1. \tag{5}$$

The dynamics is specified by randomly choosing for each state at time t with entropy S, K edges to states at time $t+1$ with entropy $S+1$, and similarly K edges to states at time $t-1$ with entropy $S+1$ (with obvious modifications at the endpoints). In this section, again to make everything as simple as possible, K will be set equal to 2. These random choices are in practice carried out by the random number generator in, e.g., Mathematica or MATLAB. But once these are chosen, they specify a model for the dynamics for the miniature multiverse, and we are faced with the problem of computing the number of paths of different kinds. It should be observed that if $K \ll W$, then only a small fraction of all states will be connected to states with lower entropy at the next or previous moment, in spite of the fact that all states are connected to several states with higher entropy, just as in the Principle 1 in Section 5.

As an illustration, a schematic picture of the set of the possible states in the case of a very small multiverse with only 5 moments of time between the Big Bang and the Big Crunch and with $W = 4$ is shown in **Figure 2**.

Note that due to the way we have set up the dynamics, the entropy can grow with at most one unit during each unit of time. This means that if we start from an ordered state with $S = 0$ at one end of the normal phase, then only values of S less than or equal to four can occur during the life-span of the corresponding universe. This means that the part of the multiverse graph displayed in **Figure 2** is sufficient for computing the number of all possible universes with zero entropy at one end. To actually carry out the computation, we can proceed as follows: it is easy to compute the number of paths with monotonically increasing entropy. According to the above assumptions, each state with entropy S is connected to exactly two states with entropy $S + 1$, both at the next and at the previous moment (with an obvious restriction to just one side at times $-m$ and m). This clearly implies that for each unit of time, the number of paths doubles: from the state with $S = 0$ at time $-m$, there are precisely two edges to states with $S = 1$ at time $-m + 1$, and for each of these, there will also be precisely two edges to states with $S = 2$ at time $-m + 2$ which gives in total four paths. At the next step, there will then be eight paths to states with $S = 3$ at time $-m + 3$ and so on.

In the case $m = 2$, we obtain $2^4 = 16$ such universes, since there are in this case four unit intervals of time. For $m = 3$, we get in an analogous way $2^6 = 64$ universes since there are in this case six unit intervals of time.

One has to work harder to compute the number of paths with zero entropy at both ends, at least if we want exact results and not just heuristic ones. The number of such universes must be considered as a statistical variable which depends on the random choices of the edges which defines the dynamics. In fact, the most

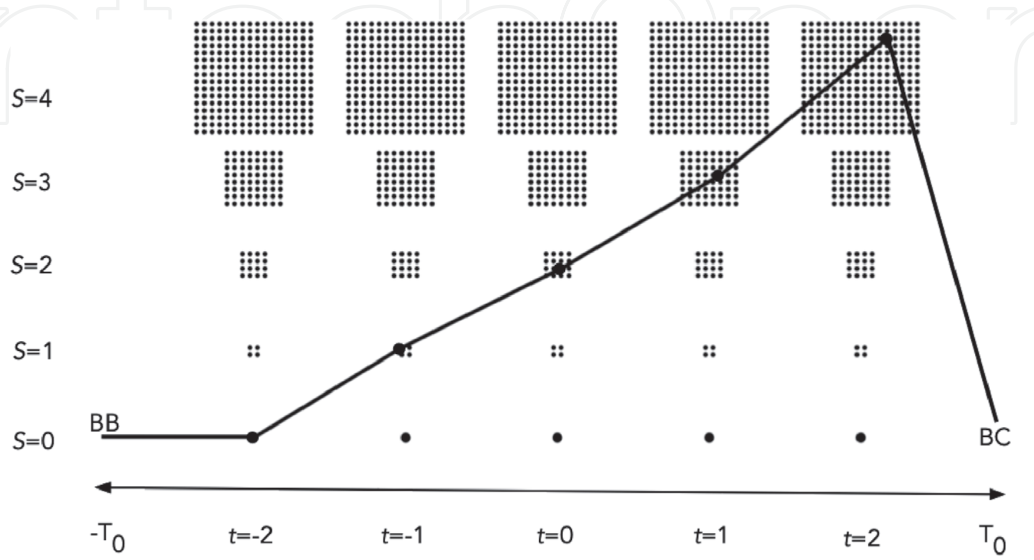


Figure 2.
A schematic picture of a universe in a very small multiverse with only five moments of time between the endpoints (i.e., $m = 2$). In this case, the universe has a monotonically increasing entropy [4].

significant variable will be the average number of universes when we consider many graphs at the same time.

The basic combinatorial tool for making these computations is the adjacency matrix (see [13] or [14]) of the graph.

Thus, recall that a (directed) path $[v_1, v_2, \dots, v_m]$ from v_1 to v_m is a sequence of nodes such that for each $j = 1, 2, \dots, m-1$, the pair v_j, v_{j+1} belongs to the set of (directed) edges of the graph. A lot of work has been done in combinatorics to calculate the number of paths of a given length between two nodes. In general, this is a hard problem or at least a time-consuming one. But for graphs with the special time-related properties in this chapter, the task may be somewhat easier. In particular, in this case all paths joining two given nodes all have the same length.

Definition 3. The *adjacency matrix* of the (directed) graph G with nodes $v_1, v_2, \dots, v_m$ is the $m \times m$-matrix $A = (a_{ij})$, where $a_{ij} = 1$ if the pair v_i, v_j determines a (directed) edge in G and $a_{ij} = 0$ otherwise.

The reason why this matrix is useful to us lies in the following classical result:

Theorem 1. The element at position ij in the k$^{\text{th}}$ power of the adjacency matrix, A^k, equals the number of paths of length k starting at v_i and ending at v_j.

Remark 2. The fact that I have chosen to work with directed graphs here should not be confused with some kind of preferred direction of time. It would in fact be possible to work with two-sided paths as well. This would however introduce more elements different from zero in the adjacency matrix and hence slow down the computations. In other words, the choice to work with directed graphs is just for technical reasons. In fact, when considering the universes in this chapter, the number of directed paths from $t = -m$ to $t = m$ is precisely the same as the number of nondirected path between $t = -m$ and $t = m$.

When considering powers of the adjacency matrix below, everything we need to know about paths starting with $S = 0$ at $-m$ can be obtained from the first row of A^{2m}. Thus, this can all essentially be done by simple linear algebra. Although simple in principle, the size of A grows very fast with the size of the model, i.e., primarily with m and W.

In view of our simple choice for the dynamics and in particular of the fact the entropy can only change by ± 1 at each step during the normal phase, it suffices to consider nodes in the graph with $S \leq t + m$.

Starting from $S = 0$ at time $-m-2$, we observe that at time $t = -1$, only states which have $S \leq 1$ have to be considered, which gives 1 + 4 = 5 states. In the same way, we get for $t = 0$, 1 + 4 + 16 = 21 states; for $t = 1$, 1 + 4 + 16 + 64 = 85 states; and finally for $t = 2$, 1 + 4 + 16 + 64 + 256 = 341 states.

The adjacency matrix can now be written as a block matrix in the following way:

$$A = \left(\begin{array}{c|c|c|c|c} & B_{12} & & & \\ \hline & & B_{23} & & \\ \hline & & & B_{34} & \\ \hline & & & & B_{45} \\ \hline & & & & \end{array} \right). \tag{6}$$

Here empty blocks should be understood as containing just zeros. Each of the five-block rows/columns correspond to a moment of time, i.e., to $-2, -1, 0, 1, 2$ as in **Figure 2**. Inside each such row/column, the states are ordered according to entropy: the first element is the unique state with $S = 0$. Then (if $t \geq -1$) the four elements with $S = 1$ follow, thereafter (if $t \geq 0$) the 16 elements with $S = 2$, and so on. The internal order between all the elements with equal S and t is not at all important in the following.

With this setup and the random dynamics introduced earlier, each B-matrix contains all the information about the edges from all the states at one moment of time to the states at the next one. For example, B_{12} contains the information about all edges from the single state with $S = 0$ at time $t = -2$ to the five states with $S \leq 1$ when $t = -1$. In the same way, B_{23} gives a complete description of the edges from the 5 states with $S \leq 1$ at time $t = -1$ to the 21 states with $S \leq 2$ when $t = 0$.

The number of rows and columns in the B-matrices are now given as follows:

$$B_{12} : 1 \times 5, \qquad B_{23} : 5 \times 21, \qquad B_{34} : 21 \times 85, \qquad B_{45} : 85 \times 341. \tag{7}$$

For the quadratic adjacency matrix A, this gives the format 453×453. The matrices $B_{k,k+1}$ can also be described as block matrices in the following way: $B_{12} = (0|0101)$ (the first element is always a 0 and among the other four, two randomly chosen elements will be one instead of zero). For the following matrix, we obtain (with certain random choices of ones as before)

$$B_{23} = \left(\begin{array}{c|c|c} & C_1 & \\ \hline C_2 & & C_3 \end{array} \right) =$$

$$= \left(\begin{array}{c|cccc|cccccccccccccccc} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right). \tag{8}$$

Both C_1 and C_3 have rows containing only zeros, except for two randomly chosen positions where there are ones instead (these are the edges which connect to states with higher entropy one unit of time later), and C_2 is a column of zeros with two randomly chosen ones instead (these are the edges which connect to states with lower entropy one unit of time later).

The structures of B_{34} and B_{45} are similar:

$$B_{34} = \left(\begin{array}{c|cc|c} & D_1 & & \\ \hline D_2 & & D_3 & \\ \hline & D_4 & & D_5 \end{array} \right), \qquad B_{45} = \left(\begin{array}{c|c|c|c|c} & E_1 & & & \\ \hline E_2 & & E_3 & & \\ \hline & E_4 & & E_5 & \\ \hline & & E_6 & & E_7 \end{array} \right), \tag{9}$$

where now all D:s and E:s with odd indices have rows with two randomly chosen ones and those with even indices have columns with two randomly chosen ones.

7. Modeling the combinatorial multiverse as a probability space

Now when we have specified the dynamics of the model, i.e., decided which paths (universes) can occur, it is time to attribute to each such path its probability weight so that the multiverse becomes a probability space. Following the tradition in statistical mechanics, I will frequently make use of un-normalized probabilities. This means that summing up all (un-normalized) probabilities will give the "state sum," which in general is not equal to one. To obtain the usual probabilities, one has to divide by the state sum. This may seem unnatural at first but turns out to be very practical in situations where only the relative sizes of the probabilities are needed.

As for the normal phase, the choice will, to start with, be the simplest possible one: each path is either possible or not, corresponding to the probability weights 1 and 0. During the extreme phases, this assumption is no longer reasonable. Again the model will be extremely simplified, but still it is based on physical intuition and, most importantly, completely time symmetric. Assume that the only types of edges having a non-neglectable chance of occurring during the extreme phase $[-m-1, -m]$ are of the following two kinds: The first scenario is that the universe passes through the extreme phase into a state of zero entropy. The other scenario is that it passes into a state with high entropy (equal to $2m$). Universes of one of these two types will be given the (un-normalized) probability 1 or p, respectively. Here $p > 0$ should be thought of as a very small number, at least when the size of the model becomes large. During the other extreme phase $[m, m+1]$, near the Big Crunch, we make the completely symmetric assumption.

Remark 3. These assumptions may perhaps seem somewhat arbitrary. And to a certain extent, this may be so. However, they do represent the following viewpoint of what may happen at the full cosmological scale: we may think of the Big Bang and the Big Crunch as states of complete order with zero volume and entropy. Such states can very well be metastable, very much like an oversaturated gas at a temperature below the point of condensation. If no disturbance takes place, such metastable states can very well continue to exist for a substantial period of time. In particular, a low-entropy state can have a very good chance of surviving the intense but extremely short extreme phase. On the other hand, if a sufficiently large disturbance occurs, then the metastable state may almost immediately decay into a very disordered state of high entropy.

It is not my intension to further argue in favor of this viewpoint here. The main thing in this chapter is to show that completely symmetric boundary conditions at the endpoints may give rise to a broken time symmetry.

The multiverse now splits up into four different kinds of paths:

- LL: The entropy is low (=0) at both ends ($-m$ and m).
- LH: The entropy is 0 at $-m$ and $2m$ at m.
- HL: The entropy is $2m$ at $-m$ and 0 at m.
- HH: The entropy is high ($= 2m$) at both ends ($-m$ and m).

If we now denote by N_{LL}, N_{LH}, N_{HL} and N_{HH} the number of paths of the indicated kinds, then with the above assumptions we also get the corresponding probability weights for the corresponding types as

$$P_{LL} = N_{LL}, \quad P_{LH} = pN_{LH}, \quad P_{HL} = pN_{HL}, \quad P_{HH} = p^2 N_{HH}. \tag{10}$$

We can now consider the following two types of broken time symmetry:

Definition 4. A multiverse is said to exhibit a *weak* broken time symmetry if

$$P_{LL} \ll P_{LH} + P_{HL}. \tag{11}$$

Definition 5. A multiverse is said to exhibit a *strong* broken time symmetry if

$$P_{LL} + P_{HH} \ll P_{LH} + P_{HL}. \tag{12}$$

Both these definitions should of course be made more precise when applied to specific models for the multiverse, e.g., by showing that the corresponding limits

$$\lim \frac{P_{LL}}{P_{LH} + P_{HL}} \quad \text{and} \quad \lim \frac{P_{LL} + P_{HH}}{P_{LH} + P_{HL}} \tag{13}$$

equal zero when certain parameters tend to infinity in some well-defined way. However, it is worthwhile at this stage to note their implications for cosmology.

The strong broken symmetry in Definition 5 actually means that a monotonic behavior of the entropy is far more probable than a non-monotonic one. In the case of a weak broken symmetry, this is not necessarily so; it could very well be that the most probable scenario would be high entropy at both ends. Thus, this is definitely a weaker statement, but it can nevertheless be argued that it can be used to explain the time asymmetry that we observe, referring to a kind of anthropic principle: it is an obvious observational fact that we live in a universe with low entropy at at least one end. If the statement in Definition 4 is fulfilled, then clearly among such scenarios, the monotonic ones (LH and HL) are the by far most probable ones. Thus, since universes with high entropy at both ends would seem to be quite uninhabitable, one can argue that given the existence of an observer, then with almost certainty he must live in a universe with monotonic entropy.

Summing up, both limits above can be used to argue in favor of time asymmetry. Nevertheless, at least to the mind of the author, the strong broken symmetry is the preferable one. This alternative will be further studied in Section 9.

8. Numerical computations in the combinatorial multiverse

With the setup in Sections 6 and 7, we can now use Mathematica or MATLAB to generate instances of the combinatorial multiverse for small values of m and W and then compute the corresponding probability weights P_{LL}, P_{LH}, P_{HL} and P_{HH}. It is important to note that the matrices here can be treated as sparse, rather than as full matrices, which make the computations considerably faster.

In particular, in the case $m = 2$ in Section 6 and with a randomly generated dynamics which is manifested by an adjacency matrix A, we can compute the power A^4 and read of the first row, which contains all the information we need about the paths from the state at $t = -2$ with $S = 0$. So what do we find?

In **Figure 3**, I have plotted the ratio $N_{LL}/(N_{LH} + N_{HL})$ for the cases $m = 2$ (light gray) and $m = 3$ (dark gray) for values of W ranging from 3 to 30. What is actually displayed are the mean values of 1000 randomly generated matrices as above for each value of W. Although the picture clearly supports the claim that

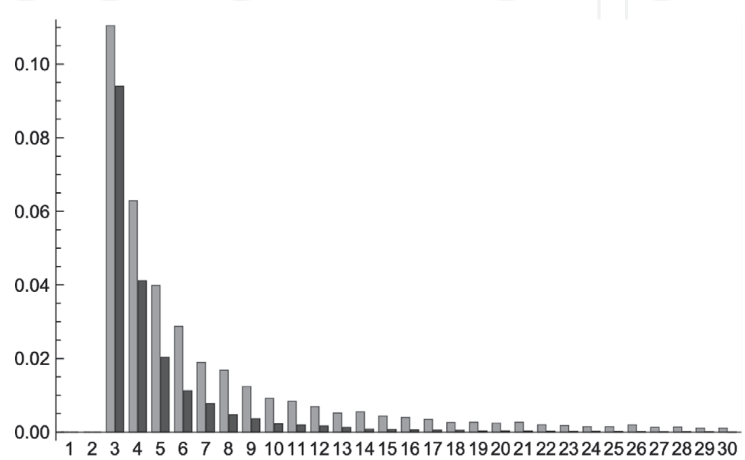


Figure 3.
The ratio $N_{LL}/(N_{LH} + N_{HL})$ as a function of W for the cases $m = 2$ (light gray) and $m = 3$ (dark gray) [4].

$N_{LL}/(N_{LH} + N_{HL}) \to 0$ when $W \to \infty$, there is not really enough support for a firm prediction about the more precise asymptotic behavior for large W. Having said this, the behavior seems to be rather close to a relationship of the form $\rho \sim 1/W$.

It should be possible, although perhaps not so easy, to prove exact limit theorems to confirm these kinds of predictions. The problem is that we use a large number of instances to model something much more complicated, namely, the full quantum mechanical development of the multiverse. For very special unlikely choices of these instances, the ratio $N_{LL}/(N_{LH} + N_{HL})$ may behave quite differently.

9. Can the dynamics be modified to generate a strong broken symmetry?

Obviously, the above model represents an extreme simplification. But from the point of view of the author, most of the simplifications can be said to be rather harmless for the purpose of explaining time's arrow.

However, there is one assumption which is somewhat problematic in the dynamics that we have discussed so far: the model can be said to exhibit a kind of Markov property in the sense that the probability for the entropy to go up or down at a certain step is completely independent of the prehistory of the state; it just depends on the state itself. This does not appear to be what is happening in our own universe: for instance, light emitted from (more or less) pointlike sources like stars continues to spread out concentrically for billions of years, and in this way it preserves a memory of the prehistory for a very long time.

A very interesting research project is therefore to try to find better models which do not exhibit this property. We can, for instance, attempt to construct models where the behavior of the entropy not only depends on the previous (or following) step but on a larger part of the prehistory (or post-history). As a particularly simple example one could let the probabilities for an increase (or decrease) of the entropy at a certain step, depend not only on the previous and following step but on the two previous (and following) steps. In fact, such dynamics would not only be more realistic but would in general also have a much better chance to exhibit a strong broken time symmetry.

I will now briefly discuss an example of such a modified model. In Section 6 it was noted that the number of paths between a state i at time $-m$ and another state j at time m can be computed using the adjacency matrix A as

$$\left(A^{2m}\right)_{ij} = \sum_{q_1}\sum_{q_2}\cdots\sum_{q_{2m-1}} a_{iq_1}a_{q_1q_2}\cdots a_{q_{2m-1}j}. \tag{14}$$

This sum can now be modified by introducing various weights depending on the path. An example of such a weight can be constructed as follows: given a path U with vertices $v_{-m}, v_{-m+1}, v_{-m+2}, \dots, v_m$, we let $S_{-m}, S_{-m+1}, S_{-m+2}, \dots, S_m$ denote the corresponding entropies. We can now define

$$\xi = \sum_{k=-m+1}^{m} (S_k - S_{k-1})(S_{k+1} - S_k), \tag{15}$$

and note that periods of monotonic growth or decrease of the entropy will tend to make ξ positive, whereas switches between growth and decrease tend to make it negative. In fact, if S is monotonic on $[k-1, k+1]$, then $(S_k - S_{k-1})(S_{k+1} - S_k) = 1$ and if not, then $(S_k - S_{k-1})(S_{k+1} - S_k) = -1$.

Given a real number $\mu \geq 0$, we can then consider the probability measure which to each path U assigns the (un-normalized) probability $\exp\{\mu\xi\}$ and replace the sum in (14) by

$$\sum_{q_1}\sum_{q_2}\cdots\sum_{q_{2m-1}} e^{\mu\xi} a_{iq_1} a_{q_1 q_2} \cdots a_{q_{2m-1}j}. \tag{16}$$

With this definition, it is now again possible to compute the probability weights P_{LL}, P_{LH}, P_{HL} and P_{HH}, and we can note that for $\mu = 0$, these will be exactly the same as in the case without weights in Section 8. Thus, this model is really a generalization of the previous theory.

Conjecture 1. If $\mu > 0$, then we have a strong broken time symmetry in the limit $m \to \infty$ (for a suitable fixed choice of p, K, and W with $K \ll W$).

10. Conclusions

Clearly, there is a large gap between the extremely simplified dynamics in this paper and more realistic dynamics based on, say, ordinary Newtonian physics or quantum mechanics. This is, for better or for worse, both the strength and the weakness of the combinatorial method presented here: extreme simplification may be the price we have to pay in order to see the forest in spite of all the trees.

In any case, the few simple examples in this paper should only be considered as a first step toward more realistic models. And in fact, when the object of study is something as enormously large as the multiverse, one should not expect a single method of attack to give all the answers. Rather, it can be expected that future developments will have to combine computer computations, heuristics, and exact mathematical methods in completely new ways.

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