

# We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900

Open access books available

186,000

International authors and editors

200M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index  
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?  
Contact [book.department@intechopen.com](mailto:book.department@intechopen.com)

Numbers displayed above are based on latest data collected.  
For more information visit [www.intechopen.com](http://www.intechopen.com)



# Addressing the Pension Decumulation Phase of Employee Retirement Planning

*Jorge Miguel Ventura Bravo*

## Abstract

Longevity increases and population ageing create challenges for all societal institutions, particularly those providing retirement income, healthcare, and long-term care services. At the individual level, an obvious question is how to ensure all retirees have an adequate, secure, stable, and predictable lifelong income stream that will allow them to maintain a target standard of living for, however, long the individual lives. In this chapter, we review and discuss the main pension decumulation options by explicitly modelling consumers' behaviour and objectives through an objective function based on utility theory accounting for consumption and bequest motives and different risk preferences. Using a Monte-Carlo simulation approach calibrated to US financial market and mortality data, our results suggest that purchasing a capped participating longevity-linked life annuity at retirement including embedded longevity and financial options that allow the annuity provider to periodically revise annuity payments if observed survivorship and portfolio outcomes deviate from expected (or guaranteed) values at contract initiation deliver superior welfare results when compared with classical annuitization and non-annuitization decumulation strategies.

**Keywords:** retirement planning, pensions decumulation, longevity-linked life annuity, risk-sharing, income drawdown, financial advice

## 1. Introduction

Longevity increases and population ageing create challenges for all societal institutions, particularly those providing retirement income, healthcare, and long-term care services. Empirical results show that longevity improvements are not homogeneous across all socioeconomic groups (see, e.g., [1–4]). At the individual level, an obvious question is how to ensure all retirees have an adequate, secure, stable, and predictable lifelong income stream that will allow them to maintain a target standard of living for, however, long the individual lives. The answer to this question is not linear and depends on several factors such as the role of occupational/personal pensions, defined benefit (DB) or defined contribution (DC) nature, minimum income guarantees, social networks (e.g., family structure and interconnectedness, informal care networks), institutional and government regulations and interventions (e.g., on the design of the pension system architecture, on the mandatory or optional nature of contributions, on auto-enrolment, on labour

income and pensions taxation), on individual preferences (e.g., regarding continuing to work after retirement, bequest, lifecycle planning), and family background and family shocks (e.g., inheritances, divorce) that prevent accumulation or accelerate decumulation, financial system development (e.g., the existence of efficient capital and insurance markets), or the risks involved in the generation of retirement income (e.g., investment, inflation, contribution, political, longevity, liquidity, behavioural). This also depends on general policy goals and constraints on fiscal policy, old-age poverty, tax neutrality over the life cycle, redistribution objectives, intergenerational fairness, or the political economy of an ageing society.

Planning for retirement requires workers to clearly understand their age-specific needs, vulnerabilities, and preferences at old age, which are certainly not the same during working life. The likelihood of experience disrupting life events such as experiencing changes in physical and mental health that anticipate retirement or generate some impairment, losing a partner that contributed to the households regular budget, caring for a spouse or other family members, changing housing, changing jobs and starting a new career at old age must be part of the retirement planning equation. Individuals must fully understand their financial goals, the expected income (and services) sources they anticipate in the accumulation and decumulating phases and be aware of the risks they are willing to take and the ones they want to insure, diversify, or hedge. Typical old-age financial needs include having a minimum guaranteed income stream that smoothes the transition from working life to retirement and protects from the eroding effect of inflation on the purchasing power of money, having an extra income to guarantee access to health-care and long-term care services (medicines, dental care, care at home, nursing home care), bequeathing (cash inheritance, housing wealth, grandchildren's education, funeral expenses, donations), or paying for life style activities (e.g., travelling). To fund for longer lives, people will ultimately rely on a retirement wallet combining state, employer-based or personal pensions, social institutions, family, own savings (including housing wealth), continued labour income, and insurance sources, with weights determined by both personal and institutional circumstances [5, 6].

The role of funded individual retirement provisions has increased over recent decades, as a result of systemic reforms of public pension schemes (move from NDB schemes towards FDC schemes, introduction of private savings pillars, for example, in Sweden, Poland), the decreasing public generosity of public annuities as a result of fiscally driven public pension reforms (e.g., Portugal, Spain), policies encouraging voluntary supplementary saving and disbursement via life annuity (e.g., Australia and New Zealand), private schemes switching from existing FDB schemes to FDC, and the "Pension freedoms" reform introduced in the UK in April 2015 providing greater flexibility and choice over how to access their defined contribution (DC) pension pots [7, 8]. The way individuals chose to decumulate their assets is one of the most important decisions they will make as they approach and enter into retirement. The decumulation process involves in some cases a one-off decision, made at the time of retirement (e.g., when an individual uses his entire wealth to pay the single premium of a standard life annuity or of a longevity-linked life annuity), but quite often involves a sequence of ad-hoc or programmed decisions spanned throughout the whole retirement period (e.g., when individuals opt to manage their assets and follow some simple or more complex drawdown rules, or when they simply choose not to divest and continue to accumulate savings).

The decumulation strategy may include investment, inflation, longevity, and other biometric risk guarantees, but often requires the individual to decide upon how much to withdrawn periodically from the pension pot to live on, particularly when public or private pension scheme benefits are not enough to pay regular

expenses, considering the remaining lifetime, and deciding on how to allocate the remaining funds during retirement [9]. Traditional disbursement product options include level (nominal or real) life annuity contracts, programmed drawdown schemes and lump sum payments, and several hybrid risk-sharing solutions like longevity-linked life annuities (see, e.g., [10–12]), modern Tontines [13], or a Tonnunity [14]. The different decumulation strategies aim to minimise longevity and investment risks while optimising against a given objective function (maximise consumption, maintain purchasing power of money, eliminate ruin chances, mitigating volatility in income streams, balance between guaranteed income sources and liquid assets, bequest).

For decades, ensuring pension scheme members have saved enough for their retirement was the main concern for employers and pension trustees. Yet, in recent years, there is also growing interest in the payout phase of pensions. This is explained by the fact that the nature of retirement is changing from a one-off decision to a gradual transition with more flexible combinations between continued labour income and pension benefits, by expanding retirement planning horizons due to longevity increases, by the lack of traditional financial instruments like level annuities, and by the number of available options at retirement which demand financial knowledge. Trustees and employers are expected to meet the challenge of preparing members for retirement and actively emphasise the members the advantages of getting financial advice from a suitably qualified adviser if necessary.

The search for the appropriate decumulation option for accumulated individual retirement savings has to take account of a number of particularities in individual preferences and enabling environment, including differences in preferences for annuities, income drawdowns of lump sum payments, including differences in preferences for consumption and bequest, spouses', and dependants' benefits, differences in the exposure to uninsurable risks/shocks that affect decumulation (divorce, death of a partner and effects on wealth level, composition, service access, financial crisis and effects on asset level/composition, health shocks, long-term care), differences in access to financial market institutions and knowledge about them, differences in socioeconomic characteristics, the utility effects of asset conservation, how individuals perceive and quantify their longevity risk exposure and the existence of an efficient and affordable longevity insurance market [15].

In this chapter, we discuss and empirically investigate the welfare enhancing characteristics of alternative annuities (e.g., participating longevity-linked life annuities) and programmed withdrawal decumulation strategies, including simple decision rule methods and actuarial methods. Maximising annuity income is one important issue for many DC scheme members. Providing an efficient risk pooling mechanism that addresses the (individual) uncertainty of death through the provision of a lifetime annuity is one of the main mechanisms pension schemes are considered to redistribute income in a welfare-enhancing manner. Without such an instrument, individuals risk outliving their accumulated (financial, housing, pension) wealth or leaving unintended bequests to his/her dependants. Traditional (fixed, inflation-indexed) life annuities are a key instrument in mandated Defined Benefit (DB) pension schemes, in financial (FDC) and non-financial Notional Defined Contribution (NDC) schemes and in private pensions provided by insurance companies.

Yet, contrary to standard Modigliani life-cycle model of savings and consumption prediction, the voluntary market purchase of retirement annuities is in most countries very limited and decreasing and the actual saving/dissaving behaviour after retirement is often at odds with economic theory [16]. Several demand side (e.g., perceived poor value-for-money, the existence of annuity alternatives, bequest motives, behavioural and informational limitations, uncertainty regarding



retirement income, shocks that prevent accumulation (e.g., unemployment spells and scarring effects<sup>1</sup>), precautionary behaviour to face major family shocks) and supply-side (e.g., the regulatory burden of annuity providers, with onerous capital requirements for unhedgeable risks (e.g., longevity risk) within Solvency II, nearly zero or negative interest rate environment and significant interest rate risk exposure and lack of solutions to hedge against (see, e.g., [18]), long-term financial risk, the cost of loss control and loss financing longevity risk management solutions, e.g., pension buy-ins, pension buy-outs, longevity bonds, longevity swaps, q-forwards, S-forwards, longevity options, limited reinsurance capacity to absorb massive exposure-to-risk and reduced risk appetite) arguments have been put forward to explain this “annuity puzzle”, i.e., to explain why the level of annuitization by individuals is much smaller than economic theory would suggest. This has increased the attention towards new contract structures involving financial and longevity risk sharing mechanisms between the annuity provider and annuitants, and increased recommendations towards the use of deferred annuities, that reduce the cost of guarantees and potentially augment their attractiveness to policyholders.

Common drawdown rules are typically derived from subjective judgements, rules of thumb and simple assumptions, for instance, concerning the duration of the payout phase. Simple drawdown rules like the  $1/e_x$  involve splitting the pension pot into equal portions according to a fixed estimate of the life expectancy at age  $x$ ,  $e_x$ , and disbursing one portion out to the retiree annually. The remaining funds are invested in financial assets to ensure a stable income flow. Other rules like the “4% rule” target to maintain the real value of the income flow while also maintaining an account balance that keeps income flowing through retirement. Experts often name this withdrawal rate a safe withdrawal rate (SWR) as the withdrawals will consist primarily of interest and dividends but empirical studies confirm that the rule is suboptimal and inefficient.

Instead of using simple heuristic rules of thumb, an alternative approach to decumulation is to explicitly model consumers’ behaviour and objectives through an objective function based on utility theory or cumulative prospect theory. The objective function should account for preferences towards consumption level, habits and smoothness, time, bequest, investment and longevity risks, health status, and other characteristics. The value of the objective function is controlled by decision variables like the consumption rate or the asset allocation between alternative asset classes including annuity-type structures. Retirement income outcomes can be measured in many ways including absolute dollar amounts, income replacement rates, or as a comparison to benchmark living standards. Several different utility functions have been proposed in the academic literature to investigate decumulation strategies and the optimality of dynamic, integrated consumption, and investment decision problems. To compare the range of possible retirement outcomes from competing decumulation designs, we provide some numerical results on optimal consumption and investment strategies using Monte-Carlo simulation methods and a stochastic mortality and investment risk framework to model biometric and financial market risks. The remaining of this chapter is organised as follows. In Section 2, we analyse the pensioner’s retirement wallet portfolio composition. Section 3 highlights the key risk sources during the payout phase of pensions and the menu of decumulation options available at retirement. Section 4 explains the contract structure and valuation of innovative participating longevity-linked life annuity contracts. Section 5 empirically investigates the welfare enhancing

---

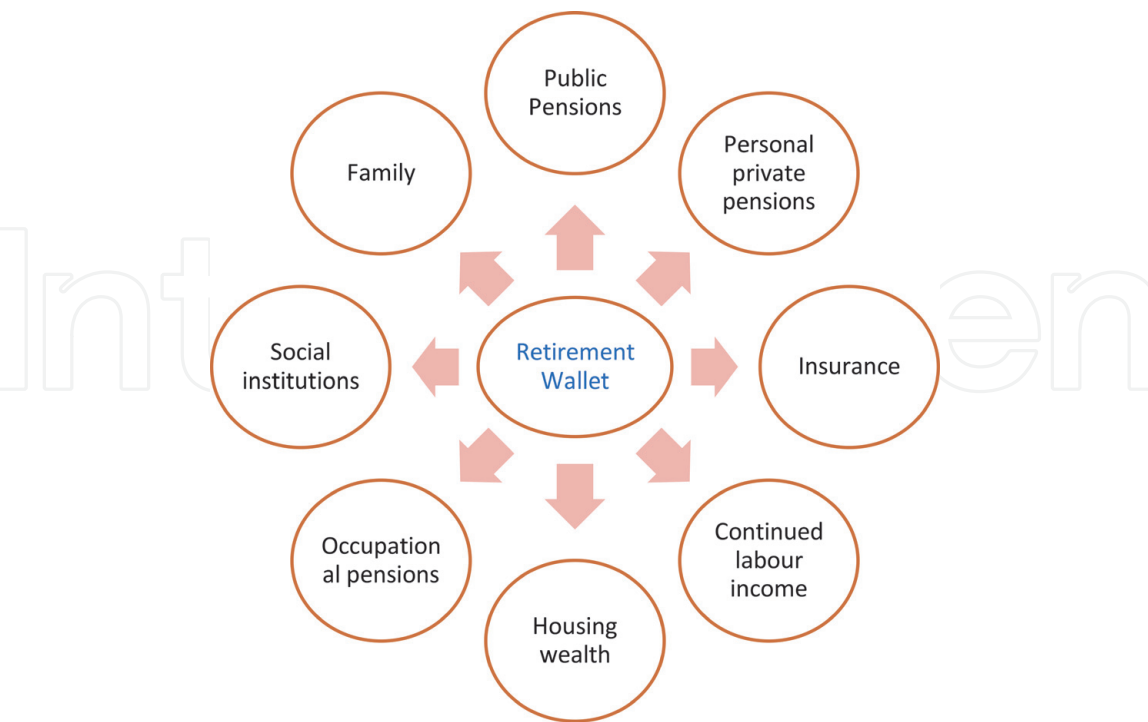
<sup>1</sup> See, e.g., [17] and references therein.

characteristics of alternative pension decumulation strategies using Monte-Carlo simulation methods. Section 6 concludes.

## 2. The retirement portfolio

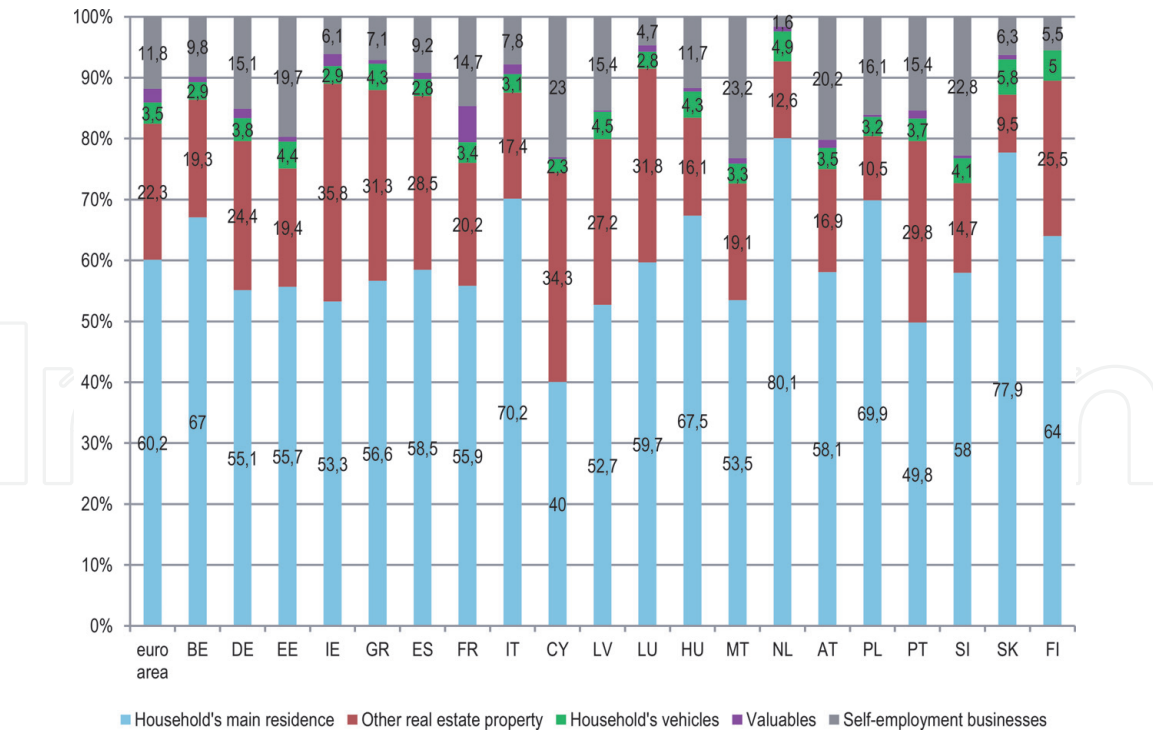
The retirement wallet portfolio of pensioners potentially comprises public and private pensions (state, linked to an employment relationship or occupational, based on contracts between individuals and private pension or insurance life annuity providers), private savings (dividends, coupon payments, cash withdrawals), housing wealth, continued labour income, insurance, (narrow or extended) family, and social institutions (**Figure 1**).

Public pension schemes (DB, DC, funded, or unfunded) typically provide a basic retirement income, with different layers of pension generosity across countries depending on the pension system structure and economic and financial system development. Private pension plans financed through pension funds, pension insurance contracts, book reserves, and bank or investment companies managed funds are becoming more widespread, but there are still enormous differences in the coverage and significance of private pension provisions across jurisdictions even after accounting for the size of the population or domestic economy.<sup>2</sup> However, most DC scheme members have not contributed enough to receive even a modest income stream in retirement. For the contrary, building up housing wealth through homeownership and mortgage repayment is by far the main way European households set aside for old age [19]. In the Euro area countries, the household's wealth (excluding pension wealth) is primarily (82.2% of total assets) held in the form of real assets, the largest component being the household main residence (HMR), representing 60.2% of total real assets, with only 17.8% in the form of financial



**Figure 1.**  
*Funding for longer lives: the retirement wallet. Source: Author's elaboration based on [6].*

<sup>2</sup> For example, pension funds held assets worth less than 1% of GDP in France or Greece while they held 171% of GDP in the Netherlands, 150.8% in Iceland or 132.6% in Australia.



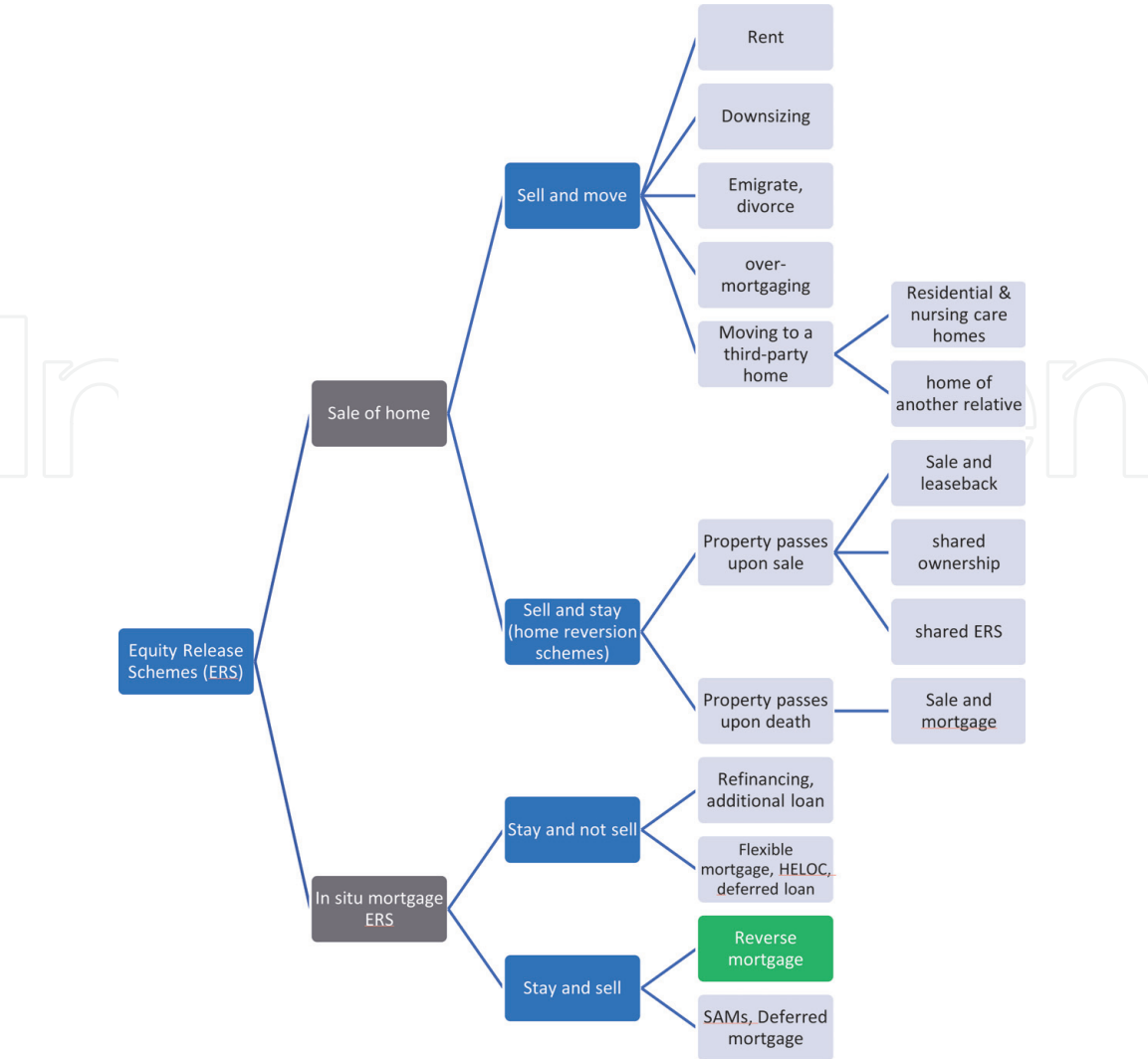
**Figure 2.** *Decomposition of real assets by asset category, euro area. Source: Own composition based on household finance and consumption survey, 2nd wave [19]. Notes: Shares of real assets types on total real assets by euro-area country. The HFCS classifies real assets into five categories: The HMR, other real estate property, vehicles, valuables (valuable jewellery, antiques, or art), and self-employment businesses.*

assets (**Figure 2**). In the EU, roughly 70% of Europeans live in owner-occupied accommodation, ownership is higher in poorer countries and the proportion of homeowners by age band has been steadily increasing with each successive generation. Empirical evidence also shows that homeowners are generally wealthier than their non-home owning counterparts, and this conclusion is valid across the income or net wealth distribution and across countries [20].

If retirees wish to assume responsibility for their welfare needs at retirement, private pensions and private homeownership are the two main assets available to finance them. The two options involve long-term saving and investment decisions over the life cycle, both are motivated by potentially competing and conflicting objectives, but they tend to deliver different outcomes and options in the decumulation phase of pensions [6]. Home homeownership provides a stream of housing services starting at time of house acquisition and represents wealth which could be cashed in later in life, if needed. The asset serves both consumption and investment functions, which are assessed differently by households according to their personal preferences, for example, bequest motives. The main difficulty is currently managing and accessing housing wealth in an efficient way to supplement retirement income. In theory, there is a catalogue of Equity Release Mechanisms (ERS) available to individuals, including contract structures which allow individuals to sell and continue to live in their home (e.g., home reversion schemes), others that involve selling and moving (e.g., rent, downsizing, moving to a third-party home), and in situ mortgage ERS, for instance, reverse mortgages (**Figure 3**). Other design features include the time of equity release, the ownership of the property, or the amount of equity that can be potentially released.<sup>3</sup>

The family will continue to play an important role in the retirement wallet portfolio in most countries and regions, particularly in the form of services and

<sup>3</sup> See [20] for a detailed analysis of ERS schemes.



**Figure 3.**  
*Typology of equity release mechanisms. Source: Own composition based on [20–23] with author’s additions.*

care, although family support is likely to be less relevant in the future due to smaller family sizes, fewer children to provide care, changing family composition, higher children’s mobility, and increasing female labour market participation. Support from local/municipal social institutions (e.g., retirement communities) also has an important role in retirement, especially for low-income groups. Insurance mechanisms must play an increasing role in the retirement wallet portfolio of retirees since some of the most important risks individuals face during retirement (e.g., longevity risk, health-care risk, long-term care risk, investment risk, inflation risk) are insurable risks and traditional and innovative solutions have been developing to address them in a cost-efficient manner [15]. Contrary to traditional models of labour supply, empirical evidence suggests that the share of labour income from continued work after statutory retirement age is increasing and the trend is persistent.

The build-up, management, and decumulation of the retirement wallet portfolio are different between individuals and are not guaranteed to be optimal as predicted by the lifecycle hypothesis. This is because uncertainty regarding retirement income, the existence shocks that prevent accumulation (e.g., unemployment spells and scarring effects)<sup>4</sup>, precautionary saving preventing major family shocks (e.g., death of a spouse, divorce), uninsured future health care and long-term expenditures, intended bequests, behavioural and cultural biases, outdated social norms and

<sup>4</sup> See, e.g., [17] and references therein.



psychological barriers, mental-accounting, the design and implementation of mandated earnings-related retirement schemes across countries including minimum income and service guarantees, pensions taxation regimes that penalise accumulating or decumulation, low risk appetite and financial literacy, and the heterogeneity in longevity by income levels [5, 16].

### 3. The decumulation menu of pensions

The decumulation or payout phase of pensions is the process of converting the retirement portfolio into a regular flow of income and services. Decumulation requires individuals to decide upon a retirement strategy, comprising a longevity insurance strategy determining the provisions taken by individuals to protect against longevity risk, that is, to guarantee they do not outlive their retirement portfolio, a withdrawal strategy, stipulating how much to withdraw from the retirement pot to finance regular consumption expenditures, an investment strategy determining how to maximise the portfolio's return considering one's risk profile<sup>5</sup>, and efficient and effective administration.

The key risk sources during decumulation include individual and aggregate longevity, investment, health, liquidity, inflation, retirement timing, bequest, annuitization, life shocks, taxation, political and regulatory interventions, market conduct, and credit risk (**Table 1**). Individuals can manage these risks through, for instance, intra-generational risk pooling mechanisms (e.g., insurance), inter-generational risk sharing vehicles (e.g., pension schemes), proper hedging instruments, risk mitigation and diversification strategies, default investment options, financial education, and effective regulation.

Typical retirement income goals include: (i) maximising the expected retirement income and consumption over one's lifetime; (ii) consumption smoothing, (iii) generate a lifetime retirement income that cannot be outlived, (iv) preserving the ability to bequest unused wealth, (v) liquidity concerns in case of unforeseen expenses (e.g., long-term care), (vi) protecting against common financial and biometric risks (e.g., longevity, inflation, investment, life events, fraud), (vii) preserving the purchasing power of income, (viii) preserving the chance to profit from upward trends in financial markets, and (ix) keeping the investment strategy simple. The main options available for decumulating retirement assets accumulated in DC pension plans include lump sum payments, programmed or phased withdrawals, life annuities, and hybrid solutions.

The possibility of taking accumulated financial savings as a cash lump sum is typically dependent both on the contractual arrangements defined by the pension plan and the tax rules in force in a particular jurisdiction. Lump sum payments offer retirees full flexibility in the use of accumulated savings, including spending on leisure activities, passing on part of their retirement pot to children or other family members, investing in new or additional property, paying off a mortgage on a house or other debts, continuing to pursue an investment strategy, and the ability to "self-annuitize" at a time and on a basis that best suits their financial needs, but also embodies important shortcomings, particularly the lack of protection against (individual and aggregate) longevity risk, and against investment, credit, and inflation risks [7].

---

<sup>5</sup> Note that the benchmark for assessing the investment performance is not in this case in terms of an asset benchmark but in terms of a given liability cash flow stream (consumption expenditures), i.e., this is a liability-driven investing (LDI) strategy (see, e.g., [18, 24–27]).

Risk	Definition
Individual longevity	Risk of outliving the retirement pot or experiencing a substantial reduction in retirement income
Aggregate longevity	Risk that overall population lives longer than anticipated forcing, e.g., a reduction in public pension benefits
Investment	The risk that portfolio investment performance mismatches the desired pattern of consumption in retirement
Inflation	The risk that a generalised rise in prices erodes the purchasing power of pensions benefits and other retirement income
Health/dependency	Risk that a deteriorating health condition significantly increases health-care or long-term care expenditures
Liquidity	Risk arising from the lack of marketability of retirement assets
Retirement timing risk	Uncertainty about when the scheme member will retire from labour market and/or begin to make withdrawals
Bequest	Most parents have an altruistic approach to life and care about their closest relatives, and try to leave them the household's main residence, cash, or her financial and real assets
Annuitization	Mandatory annuitization may take place at a point where interest rates are lower than anticipated
Political and regulatory	The risk that either public or private pension system providers may be forced to reduce their pension payments, because pension systems are financially unsustainable or as a result of a political decision and the risk that regulations change in an adverse way
Taxes	Risk that a variation in the regulatory or tax environment will reduce the disposable retirement income, e.g., an increase in income tax rates or deductions, an increase in VAT taxes, an increase in capital market taxes
Life events	Divorce, death of spouse/partner, etc
Behavioural	Risk that pensioners behave in a way that is not considered to be rational, incapacity to make an 'informed choice' due to insufficient financial literacy and understanding of risks
Market conduct and credit risk	The risk that financial and non-financial service providers act in a way that disadvantages retirees and credit risk referring to the events after which companies or individuals will be unable to make the required payments on their debt or contract obligations

Source: Author's elaboration based on [6, 7, 28].

**Table 1.**  
*Key risk sources during the payout phase of pensions.*

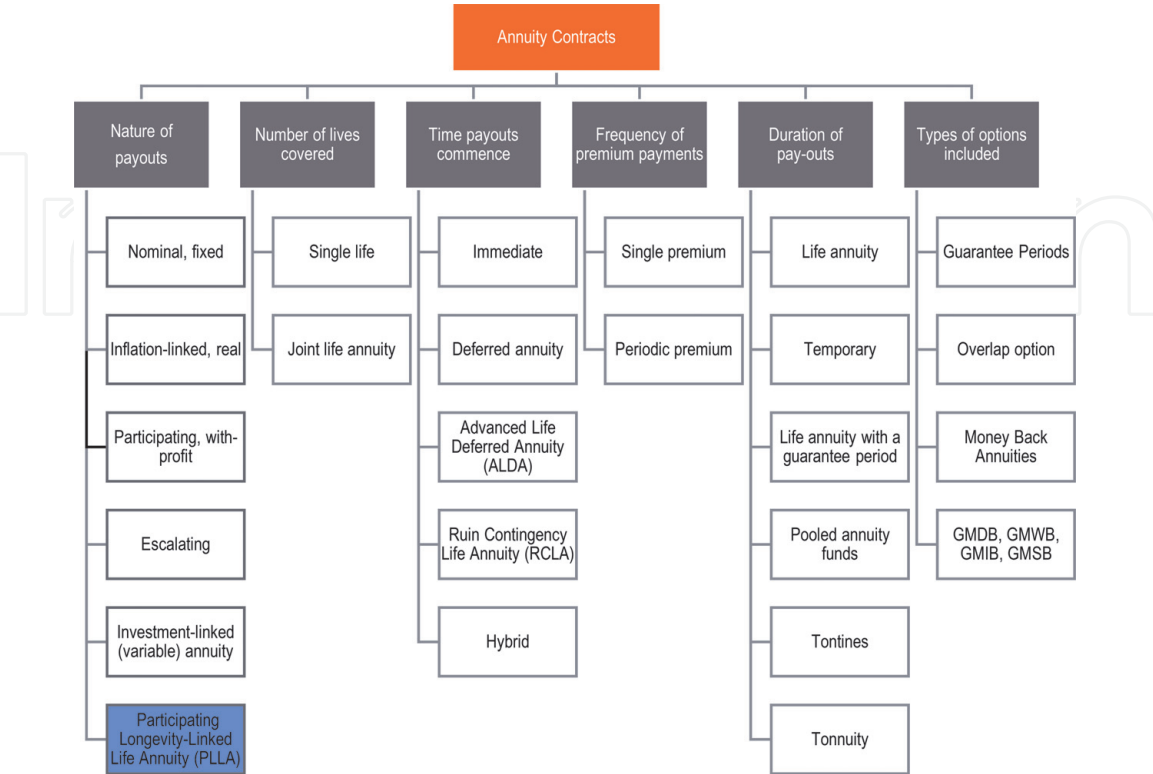
Programmed withdrawal arrangements allow retirees to cash in periodically from their retirement portfolio to cover necessary expenses, allowing individuals to pre-serve control and ownership over their assets, to decide upon the desired investment strategy, but do encompass biometrical risk-pooling that protects, for example, against longevity risk. The regular income flows may be the result of an explicit withdrawal rule or plan (e.g., the so-called 4% sustainable withdrawal rule, a fraction of the remaining life expectancy at the retirement age, possibly with lower and upper bounds, a constant amount) or simply be the result of discretionary actions.<sup>6</sup>

<sup>6</sup> Although self-managed products are available, normally retirement withdrawal products are delegated management retirement products under which the account management activities are allocated to the asset management company.

The fixed percentage, the constant (inflation-adjusted), and the floor-and-ceiling withdrawals rules were introduced by [29, 30]. The first spending rule considers fixed-percentage withdrawals (users spend a constant percentage of their assets in each year of retirement) and the second one considers an annual adjustment for inflation. The third method considers establishes a floor and a ceiling to cash withdrawals. According to [31, 32], this strategy allows greater spending when markets do well and spending reductions when markets do poorly. The target Percentage Adjustment rule was introduced in [33] and defines whether spending adjusts for inflation given a fixed-return assumption and a 45-year time horizon.

Programmed withdrawal has some advantages compared with annuity purchase (e.g., higher liquidity and flexibility to respond to unexpected consumption expenditures, retaining control over retirement assets, potentially higher payouts due to enhanced investment returns, possibility to allocate assets to inflation-linked investments, compatibility with the bequest motive, death benefit options), but also several drawbacks, particularly the lack of protection against longevity risk, no mortality cross subsidy, and significant exposure to investment risk [6].

The classical payout option for generating a predictable income stream in retirement is a life annuity. Annuity products offer protection against individual and aggregate longevity risk, a mortality cross subsidy from pooling risks but imply the loss of control over assets and no flexibility to address unexpected expenditures or the bequest motive. There are many types of annuities that can be differentiated, for instance, by the nature of payment, by the number of people covered, by the duration of payments, by the time that payouts commence, by the frequency of premium payments, by the distribution channel and types of options included, among other features (**Figure 4**). Recent developments in this area include participating longevity-linked life annuities in which annuitants share both investment



**Figure 4.** Types of life annuities. Source: Author’s elaboration based on [6, 7].

and longevity risk but profit from risk-pooling [11, 12, 34], variable annuities<sup>7</sup>, modern Tontines [13], and Tonnuity [14].

According to [35–37], a deferred annuity is suitable for risk-averse individuals, because it insures against increases in annuity prices and provides a smooth income. The group self-annuitization scheme (GSA) proposed by [39, 40] was analysed in [38] where pool participants are insured against individual longevity risk and share systematic mortality and longevity risks. They put forward that the GSA can outperform inflation-linked annuities when there are loadings. When individuals have a bequest motive, portfolios with phased withdrawals improve individuals' welfare. Their results follow previous literature which suggests that individuals should not fully annuitize their wealth but hold an equity portfolio and match their consumption expenditures with regular cash withdrawals.

#### **4. Sharing investment and longevity risk: participating longevity-linked life annuities**

Contrary to standard life-cycle theory, the voluntary market purchase of retirement annuities is very limited and decreasing in most countries and the actual saving/dissaving behaviour after retirement is often at odds with economic theory [16]. A number of demand side (e.g., perceived poor value-for-money, the existence of annuity alternatives, bequest motives, behavioural and informational limitations) and supply-side (e.g., the regulatory burden of annuity providers) with onerous capital requirements for unhedgeable risks (e.g., longevity risk) within Solvency II, nearly zero interest rate environment and significant interest rate risk exposure, long-term financial risk, the cost of loss control and loss financing longevity risk management solutions, limited reinsurance capacity to absorb massive exposure-to-risk) arguments have been put forward to explain this “annuity puzzle”, that is, to explain why the level of annuitization by individuals is much smaller than economic theory would suggest [6, 41, 42]. Several index-type and indemnity-type mechanisms have been proposed in the literature to directly or indirectly share financial and longevity risks between annuity providers and individuals [10, 11, 43].

Participating longevity-linked life annuities (PLLAs) are life insurance contracts in which benefits are updated periodically based on the dynamics of both a longevity index, defined as the ratio between the expected survival probability and the survival rate observed in a reference population, and of an interest rate adjustment factor, defined as the ratio between observed and guaranteed financial returns

---

<sup>7</sup> Variable annuities can adopt different forms. Annuity benefits can rise (or fall) at a prescribed fixed nominal rate that escalates with the age of the annuitant (escalating annuity); they can be indexed to inflation, thus providing a guaranteed income in real terms (inflation linked or real annuity); they can be linked to observed survival probability (longevity-linked life annuity); they can depend on the insurance company's surplus (participating or with profit annuity); or even reflect the performance of an underlying investment portfolio, usually represented by a family of mutual funds (investment-linked or variable annuity). In some annuities, pay-outs can also participate in mortality risk. The most common guarantees embedded in variable annuities are: (i) guaranteed minimum death benefit (GMDB); (ii) guaranteed minimum income benefit (GMIB); (iii) guaranteed minimum withdrawal benefit (GMWB); (iv) guaranteed lifetime withdrawal benefit (GLWB); (v) guaranteed minimum accumulation benefit (GMAB).



[10, 11].<sup>8</sup> In this section, we briefly describe the benefit structure and risk sharing design of immediate PLLAs. We then introduce the valuation setup via embedded longevity option decomposition.

Consider an index-type participating longevity-linked life annuity (PLLA) along the lines proposed by [11, 12]. Under this arrangement, the annuity benefit is periodically updated according to both the observed survival experience of a reference pool and the investment performance of the financial assets backing the contract. Without loss of generality, let us assume a single cohort product in which annuitants contribute equal amounts into the annuity fund and, in return, receive equal annuity benefit payments  $b_t$  at time  $t$ . Under this contract, the annual benefit at some future date  $t_0 + k$ ,  $b_{t_0+k}$ , will depart from the initial benefit,  $b_{t_0}$ , according to the dynamics of both a longevity  $I_{t_0+k}$  and an interest rate adjustment (IRA)  $R_{t_0+k}$  factors as follows:

$$b_{t_0+k} = b_{t_0} \times I_{t_0+k} \times R_{t_0+k}, \quad k = 1, \dots, \omega - x \quad (1)$$

where  $I_{t_0+k}$  is a ratio between the expected survival probability and the survival rate observed in a reference population, defined by

$$I_{t_0+k} = \frac{{}_k p_{x_0}^{[F_0]}(t_0)}{{}_k p_{x_0}^{[F_k]}(t_k)} = \prod_{j=0}^{k-1} \frac{p_{x_0+j}^{[F_0]}(t_0+j)}{p_{x_0+j}^{[F_k]}(t_0+j)} \quad (2)$$

with

$${}_k p_{x_0+j}^{[F_0]}(t_0+j) = \prod_{j=0}^{k-1} [1 - q_{x_0+j}(t_0+j)] \quad (3)$$

denoting the  $k$ -year survival probability of some reference population cohort aged  $x_0$  at time  $t_0$  (computed at contract inception on a market or national population life table) and  ${}_k p_{x_0}^{[F_k]}(t_k)$  is the corresponding  $k$ -year survival probability observed at time and the highest attainable age. In Eq. (3),  $q_{x_0+j}(t_0+j)$  is the 1-year death probability of an individual aged  $x_0 + j$  at time  $t_0 + j$ . The IRA factor  $R_{t_0+k}$  is defined by

$$R_{t_0+k} = \frac{\prod_{j=0}^{k-1} (1 + R_t)}{(1 + i_{t_0})^k} \quad (4)$$

where  $R_t$  denotes the observed net investment return in year  $t$  and  $i_{t_0}$  is the (generally non-negative) guaranteed minimum interest rate set at time 0.

If investment and mortality improvements are as expected, the arrangement resembles a classical nominal fixed life annuity. If  $R_t = i_{t_0}$  and observed longevity improvements are higher (lower) than predicted, that is,  $I_{t_0+k} < 1$  ( $I_{t_0+k} > 1$ )  $\forall k$ , annuity payments will decline (increase) along with the dynamics of  $I_{t_0+k}$ . If mortality improvements are as expected and investments perform above the guaranteed interest rate, the extra return is returned to participants in the form of a higher benefit payment. This risk-sharing nature of this contract contrasts with classical

<sup>8</sup> A related approach is found in [34] in which annuity payments are updated only if observed survivorship rates exceed a given threshold.

fixed annuities in which all risks (financial and biometric) are transferred to the annuity provider. Benefit volatility may be mitigated by introducing bounds to the longevity and IRA adjustment factors (or to the benefit amount).

The valuation of a PLLA at time  $t_0$  can be obtained via longevity option decomposition (see [11] for details), that is, by decomposing the PLLA into a long position in a fixed annuity  $a_{x_0}^{[F_0]}(t_0)$  and a short position in an embedded European-style longevity floor  $L^{F_0}(t_0)$  with underlying  $I_{t_0+k}$ , constant strike equal to one unit of currency and maturity  $\omega - x_0$ ,

$$a_{x_0}^{PLLA}(t_0) = a_{x_0}^{[F_0]}(t_0) - L^F(t_0) \quad (5)$$

with

$$L^F(t_0) = \sum_{k=1}^{\omega-x_0} E^Q \left( B(0, k) \times {}_k p_{x_0}^{[F_0]}(t_0) \times (1 - I_{t_0+k})^+ | F \right) \quad (6)$$

where  $B(t, T)$  is the interest rate discount factor,  $a^+ := \max(a, 0)$  and, without loss of generality, we consider an immediate PLLA contract with initial benefit  $b_{t_0} = 1$  and a scenario in which observed longevity improvements are higher than predicted and investment performance matches the guaranteed interest rate.

## 5. On the welfare of pension decumulation strategies

In this section, we empirically investigate the welfare enhancing characteristics of alternative pension decumulation strategies. Specifically, we test eight alternative strategies tested:

- i. A fixed withdrawal rule based on life expectancy observed at age in the retirement year  $t_0$ , that is, a  $1/e_{x,t_0}$  rule
- ii. A fixed withdrawal rule based on the life annuity factor estimated at age in the retirement year  $t_0$ ,  $1/a_{x,t_0}$ , with no annuitization
- iii. A withdrawal rule based on life expectancy observed at age  $x$  in year  $t$ ,  $1/e_{x,t}$
- iv. The “4%” SWR rule
- v. Buying a PLLA at retirement age
- vi. Buying a classical single premium nominal life annuity (SPLA) at retirement age
- vii. Investing 80% of the initial wealth in the “4%” SWR rule and the remaining 20% in a ALDA
- viii. Buying an Inflation-Protected Annuity (IPA) at retirement age

The welfare analysis considers a time-separable utility function including life-time consumption and bequest motives [44]. We assume individuals want to maximise the expected present value of utility derived from consumption through their remaining lifetime,

$$U_t = \max_{c_1, c_1, \dots, c_T} E_t \left[ \sum_{k=1}^{\omega-x_0} \beta^k \left\{ k p_{x_0}^{[F_0]} u(c_t) + {}_{t-1}q_{x_0}^{[F_0]} \gamma(W_t) \right\} \right] \quad (7)$$

with

$$u(c_t) = \frac{c_t^{1-\rho}}{1-\rho}, \quad c_t \geq 0 \quad (8)$$

and

$$\gamma(W_t) = \left( \frac{\varphi}{1-\varphi} \right) \frac{\rho \left( \frac{\varphi}{1-\varphi} c_b + W_t \right)^{1-\rho}}{1-\rho}, \quad W_t \geq 0 \quad (9)$$

where  $\beta$  is the subjective utility discount factor,  $\rho > 0$  the level of risk aversion,  $c_b$  measures the degree to which bequests are considered as luxury goods,  $\varphi[0,1]$  measures the strength of the member's bequest motive when bequest has kicked in (i.e., when  $W_t > c_b$ ).

The utility function expresses the preferences of individuals by assigning higher values on favourable outcomes (higher consumption and bequest) while marking down poor outcomes such as a consistently lower level of retirement income. The utility function separately considers the risk aversion over consumption and bequest, allowing for subjective adjustments to how the individuals value different retirement outcomes. The dynamics of wealth is

$$W_{t+1} = (W_t - c_t + P_t)(1 + R_{t+1}), \quad (10)$$

where  $R_{t+1}$  is the stochastic (post-tax) rate of return.

To compare the performance of alternative decumulation strategies, we compute a utility score,  $S_0$ , defined as the constant level of income which delivers an equivalent level of expected utility to that delivered by each strategy,

$$S_0 = \left[ U_0 \times \frac{1-\rho}{\left( \sum_{k=1}^{\omega-x_0} \beta^k \left\{ k p_{x_0}^{[F_0]} + {}_{t-1}q_{x_0}^{[F_0]} + \left( \frac{\varphi}{1-\varphi} \right) \right\} \right)} \right]^{\frac{1}{1-\rho}}, \quad (11)$$

The valuation framework comprises a risk-neutral, frictionless and continuous financial market in which the annuity provider invests the insurance premium in a portfolio of dividend-paying stocks (30%) and coupon bonds (70%), and a risk-free interest rate. We assume the yield curve dynamics is well captured by a two-factor equilibrium model [45] and the stock market index follows a standard geometric Brownian motion diffusion process.

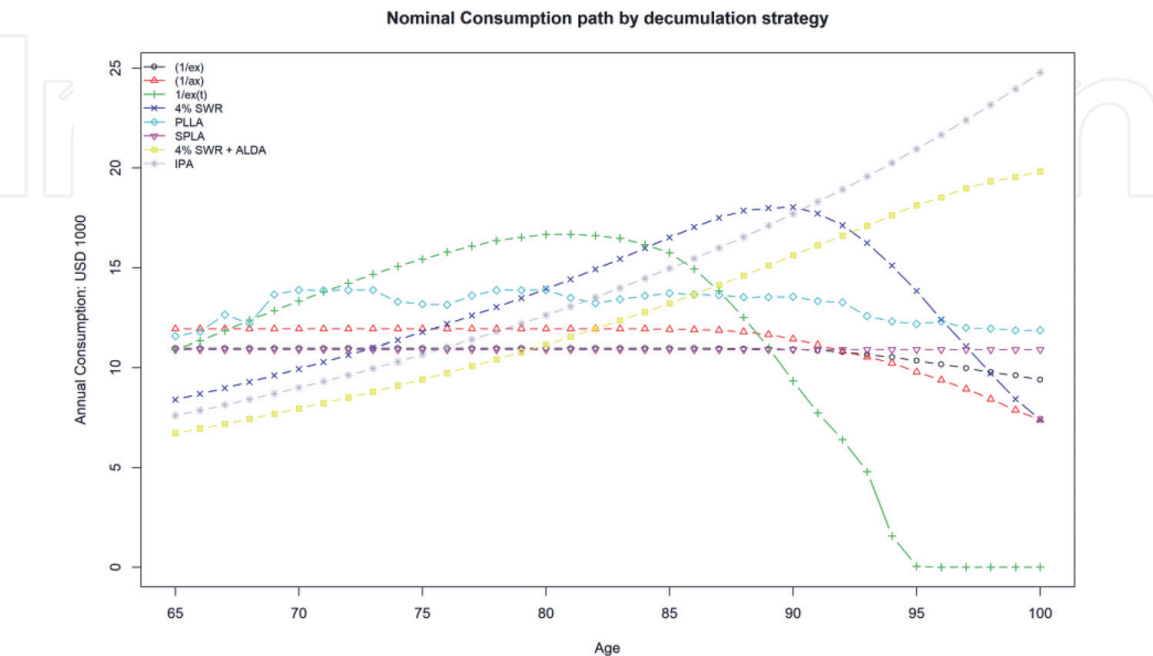
To account for the longevity risk premium in pricing the contracts, we compute cohort-specific risk-adjusted survival probabilities by using a risk-neutral simulation approach assuming the dynamics of mortality rates is well represented by the log bilinear Lee-Carter model under a Poisson setting, with time trend parameter modelled using a general ARIMA(p,d,q) model and risk neutral distribution of the innovations obtained using the Wang transform. The results are generated through 10,000 independent sample paths for both the survival probability of a cohort aged  $x$  in at time 0 and the portfolio returns. We calibrate the models to historical (monthly) data on US 3-month and 10-year maturity bond yields from January 1, 2010 to September 1, 2019 and to US mortality data from 1960 to 2016 obtained from the human mortality database [46]. The baseline parameters are given in **Table 2**.

**Figure 5** plots the dynamics of consumption over time for the eight strategies tested. Except for level fixed SPIA annuities that payoff a constant nominal benefit for life, all other strategies experience some benefit volatility and different trends. Inflation-protected annuities guarantee constant real benefits, which means nominal steadily increase with inflation over time. The 4% SWR rule generates increasing consumption over time but only up to some point, when the portfolio value is exhausted and consumption levels must be reduced. When combined with the acquisition of an ALDA annuity, the 4% delivers increasing and sustainable

Parameter	Value	Notes
Subjective utility discount factor: $\beta$	0.975	
Risk aversion coefficient: $\rho$	5	
Strength of bequest motive: $\varphi$	0.83	
Bequests as luxury goods level: $c_b$	25.8	$\times 10^3$ USD
Initial wealth (1000 s): $W_0$	210	$\times 10^3$ USD
Tax rate: $s$	20%	
ALDA deferment period: $k$	20	years
Pension benefits: $P_t$	0	
Life annuity loading	10%	
PLLA bounds	(80%; 120%)	
Number of simulations	1000	
Investment horizon	35	years
Guaranteed annuity interest rate (GIR)	1%	
Minimum consumption level	6	$\times 10^3$ USD

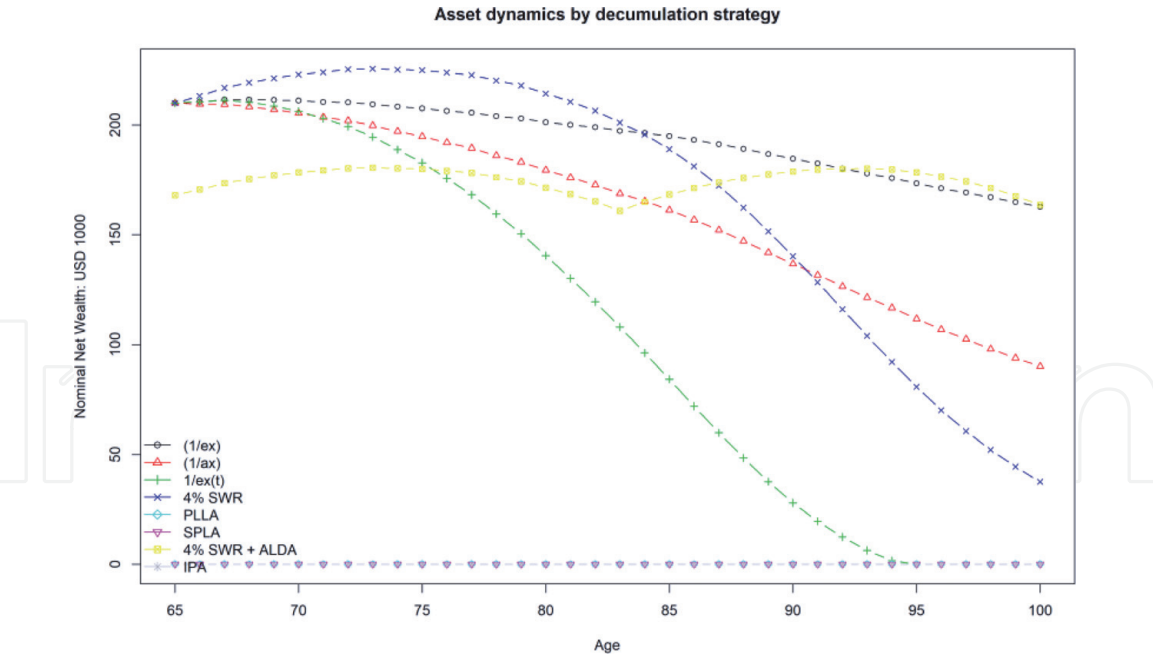
Source: Author's elaboration.

**Table 2.**  
Baseline parameters.



**Figure 5.**  
Dynamics of consumption level by decumulation strategy. Source: Author's elaboration.





**Figure 6.** Wealth dynamics by decumulation strategy. Source: Author’s elaboration.

consumption levels and longevity insurance. The dynamic life expectancy strategy generates consumption levels that cannot be sustained over time, quickly converging towards deprivation and personal bankruptcy. The PLLA structure with 20% maximum benefit volatility pays higher benefits compared with a classic SPIA annuity or self-annuitization strategies and offers longevity insurance for life, but at the expense of additional benefit volatility.

**Figure 6** plots the wealth dynamics by decumulation strategy over a 35-year period. For strategies involving full annuitization, the remaining wealth is, by definition, zero since all assets are used to pay for the insurance premium. For the remaining, the dynamics of the asset portfolio depends on regular consumption levels and portfolio annual returns. We can observe that for all strategies the portfolio value reduces with time, in some cases until its complete exhaustion (e.g.,  $1/ex,t$  strategy).

We can observe also that the classical 4% SWR rule is not sustainable for long planning horizons, contrary to the conservative static life expectancy strategy  $1/ex,0$  that delivers significant bequest utility. When combined with advanced age longevity insurance, the strategy is likely to improve individuals’ welfare when they have a bequest motive since additional portfolio value is preserved. Ceteris paribus, the higher the wealth level at death (if positive), the higher the utility from leaving values in the residual benefit. Individuals pursuing a self-annuitization strategy  $1/ax,t$  are also likely to guarantee an inheritance to their heirs.

**Table 3** provides illustrative simulation-based welfare results for the decumulation strategies tested in this study in which individuals are exposed to market and longevity risk. The results include the mean consumption and wealth levels over the planning horizon and the asset balance at age 100, consumption volatility (standard deviation), expected utility computed using Eq. (7) and utility score values.<sup>9</sup>

We can observe that the decumulation strategy involving purchasing a participating longevity-linked annuity at retirement age provides the highest utility score,

<sup>9</sup> Detailed results can be obtained from the author upon request.

Metric	Strategy							
	1/ex,0	1/axe,0	1/ex,t	4% rule	PLLA	SPLA	4% + ALDA	IPA
Mean $C_t$	9.38	8.77	6.98	9.06	12.53	10.90	14.57	22.24
Ct stdev	2.43	4.13	7.19	8.23	1.30	0.00	6.80	11.92
Mean $W_t$	180,98	129.72	68.62	112.77	0.00	0.00	150.22	0.00
$W_t$ (age 100)	162.70	90.18	0.00	37.54	0.00	0.00	163.54	0.00
$E(U_t)$	-4.0	-263,7	-744.078	-318.4	-0.00012	-0.00025	-0.00071	-0.00043
$S_0$	11.29	11.78	9.29	10.47	13.69	11.45	8.82	9.99
Source: Author's elaboration. Notes: Market price of longevity risk set by $\lambda = 0.3$ (Wang Transform parameter).								

**Table 3.**  
*Simulation-based welfare results: baseline scenario.*

that is, the highest (certain equivalent) constant level of income (considering the trade-off against residual benefit) which delivers an equivalent level of expected utility. The self-annuitization decumulation strategy comes second in terms of utility score, followed by classical SPIA annuity purchase. The good performance of the PLLA strategy is explained essentially by the higher consumption levels it delivers in early retirement years which are less discounted (actuarially and intertemporally) by individuals since, by construction, the strategy offers no utility from bequest. The strategy provides longevity risk insurance and relatively low consumption volatility when compared with most alternatives.

6. Conclusions

In this chapter, we review and discuss the main pension decumulation options by explicitly modelling consumers' behaviour and objectives though an objective function based on utility theory accounting for consumption and bequest motives and different risk preferences. We conclude that purchasing a capped participating longevity-linked life annuity at retirement including embedded longevity and financial options that allow the annuity provider to periodically revise annuity payments if observed survivorship and portfolio outcomes deviate from expected (or guaranteed) values as contract initiation deliver superior welfare results when compared with classical annuitization and non-annuitization decumulation strategies. Contrary to standard fixed annuities in which the insurer bears all risk, PLLAs offer an efficient and transparent way of sharing biometric and financial market risks between annuity providers and policyholders. They are an interesting and promising product for the payout phase of pension schemes since the contract tackles some of the demand- and supply-side constraints that prevent individuals from annuitizing their retirement wealth and may contribute to help insurers writing new annuity policies. By linking the annuity benefit to the survival experience of a given underlying population and to the performance of the asset portfolio backing the contract PLLAs provide a direct mechanism to share financial and longevity risk and are an interesting alternative to manage systematic longevity risk in markets in which alternative risk management solutions (longevity-linked securities, reinsurance arrangements, capital allocation) are scarce and/or expensive.

IntechOpen

IntechOpen

### **Author details**

Jorge Miguel Ventura Bravo  
NOVA IMS & MagIC & CEFAGE-UE & Université Paris-Dauphine,  
Nova University of Lisbon, Lisbon, Portugal

\*Address all correspondence to: [jbravo@novaims.unl.pt](mailto:jbravo@novaims.unl.pt)

### **IntechOpen**

---

© 2020 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/3.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. 

## References

- [1] Ayuso M, Bravo JM, Holzmann R. On the heterogeneity in longevity among socioeconomic groups: Scope, trends, and implications for earnings-related pension schemes. *Global Journal of Human Social Sciences—Economics*. 2017;17(1):31-57
- [2] Ayuso M, Bravo JM, Holzmann R. Addressing longevity heterogeneity in pension scheme design. *Journal of Finance and Economics*. 2017;6(1): 1-21
- [3] Ayuso M, Bravo JM, Holzmann R. Getting life expectancy estimates right for pension policy period versus cohort approach. *Journal of Pension Economics and Finance*. 2019. Accepted, (in press)
- [4] Bravo JM, Ayuso M, Holzmann R, Palmer E. Addressing the life expectancy gap in pension policy. 2019. Preprint submitted for publication
- [5] Bravo JM. Taxation of pensions in Portugal: A semi-dual income tax system. CESifo DICE Report—Journal for Institutional Comparisons. 2016; 14(1):14-23
- [6] Bravo JM. Funding for longer lives: Retirement wallet and risk-sharing annuities. *EKONOMIAZ Basque Economic Review*. 2019;96(II-2019): 268-291
- [7] Bravo JM, Holzmann R. The payout phase of funded pensions plans: Risks and payment options. In: OECD PISA Financial Education Program. Madrid, Spain: BBVA Pensions Institute; 2014. Available from: <https://www.jubilaciondefuturo.es/es/blog/la-fase-de-percepcion-de-los-pagos-de-los-planes-de-pensiones.html>
- [8] Herce J, Bravo JM. Las pensiones en España y Portugal: Descripción de los esquemas y evolución reciente comparada. In: *Es posible planificar la jubilación? Dos años del Instituto BBVA de Pensiones en España*. Madrid, Spain: Instituto BBVA de Pensiones; 2015. pp. 89-126
- [9] Bernhardt T, Donnelly C. Pension Decumulation Strategies: A State-of-the-Art Report. WP Actuarial Research Centre (ARC), The Institute and Faculty of Actuaries (IFoA); 2018
- [10] Alho J, Bravo JM, Palmer E. Annuities and life expectancy in NDC. In: Holzmann R, Palmer E, Robalino D, editors. *Non-Financial Defined Contribution Pension Schemes in a Changing Pension World, Volume 2: Gender, Politics and Financial Stability*. Washington: World Bank Publications; 2012. pp. 395-436
- [11] Bravo JM, El Mekkaoui N. Valuation of longevity-linked life annuities. *Insurance: Mathematics and Economics*. 2018;78:212-229
- [12] Bravo JM. Pricing participating longevity-linked life annuities: A Bayesian model ensemble approach. Preprint submitted for publication. 2019
- [13] Milevsky MA, Salisbury TS. Optimal retirement income tontines. *Insurance: Mathematics and Economics*. 2015; 64(1):91-105
- [14] Chen A, Hieber P, Klein J. Tonuity: A novel individual-oriented retirement plan. *ASTIN Bulletin*. 2017; 49(1):5-30
- [15] Bravo JM, Díaz-Giménez J. La longevidad es un riesgo asegurable? Cubriendo lo incubible? In: *Es posible planificar la jubilación? Dos años del Instituto BBVA de Pensiones en España*. Madrid, Spain: Instituto BBVA de Pensiones; 2015. pp. 205-240
- [16] Holzmann R, Ayuso M, Alaminos E, Bravo JM. Life cycle saving and



dissaving revisited across three-tiered income groups: Starting hypotheses, refinement through literature review, and ideas for empirical testing. In: IZA Discussion Paper No. 12655. 2019

[17] Bravo JM, Herce JA. Career breaks, Broken pensions? Long-run effects of early and late-career unemployment spells on pension entitlements. 2019. Preprint submitted for publication in the Journal of Pension Economics and Finance

[18] Bravo JM, Silva C. Immunization using a stochastic process independent multifactor model: The Portuguese experience. Journal of Banking and Finance. 2006;**30**(1):133-156

[19] ECB. The household finance and consumption survey: Results from the second wave. ECB Statistics Paper No. 18. December – Household Finance and Consumption Network; 2016

[20] Bravo JM, Ayuso M, Holzmann R. Making use of home equity: The potential of housing wealth to enhance retirement security. In: IZA Discussion Paper Series No. 12656. Germany: IZA Institute of Labour Economics; 2019

[21] Ong R, Haffner M, Wood G, Jefferson T, Austen S. Assets, debt and the drawdown of housing equity by an ageing population. In: Positioning Overmortgage Paper No. 153. Melbourne: Australian Housing and Urban Research Institute; 2013

[22] OECD. Pensions at a Glance 2013: OECD and G20 Indicators. OECD Publishing; 2013. DOI: 10.1787/pension\_glance-2013-en

[23] French D, McKillop D, Sharma T. Analysis of Housing Equity Withdrawal by its Forms. CHARMS—Centre for Health Research at the Management School; 2017. Working Paper Series 2017-04. Available from: [https://ideas.](https://ideas.repec.org/p/qub/charms/1704.html)

[repec.org/p/qub/charms/1704.html](https://ideas.repec.org/p/qub/charms/1704.html). [December 16, 2018]

[24] Simões C, Oliveira L, Bravo JM. Immunization strategies for funding multiple inflation-linked liabilities. 2019. Preprint submitted for Journal publication

[25] Chamboko R, Bravo JM. On the modelling of prognosis from delinquency to normal performance on retail consumer loans. Risk Management. 2016;**18**(4):264-287

[26] Chamboko R, Bravo JM. Modelling and forecasting recurrent recovery events on consumer loans. International Journal of Applied Decision Sciences. 2019;**12**(3):271-287

[27] Chamboko R, Bravo JM. Frailty correlated default on retail consumer loans in developing markets. International Journal of Applied Decision Sciences. 2019;**12**(3):257-270

[28] Blake D. We need a national narrative: Building a consensus around retirement. The Report of the Independent Review of Retirement Income; 2016. Available from: <http://www.pensions-institute.org/IRRIReport.pdf>

[29] Bengen WP. Determining withdrawal rates using historical data. Journal of Financial Planning. 1994; **17**(3):172-180

[30] Bengen WP. Conserving client portfolios during retirement, Part IV. Journal of Financial Planning. 2001; **14**(5):110-119

[31] Pfau WD. An international perspective on safe withdrawal rates from retirement savings: The demise of the 4 percent rule? Journal of Financial Planning. 2010;**23**:52-61

[32] Pfau WD. Making sense out of variable spending strategies for retirees.

Journal of Financial Planning. 2015;  
 28(10):42-51

[33] Zolt D. Achieving a higher safe withdrawal rate with the target percentage adjustment. Journal of Financial Planning. 2013:51-59

[34] Bravo JM, Corte-Real P, Silva C. Participating life annuities incorporating longevity risk sharing arrangements. Working Paper. 2009. Available from: [rdpc.uevora.pt](http://rdpc.uevora.pt)

[35] De Waegenare A, Melenberg B, Stevens R. Longevity risk. De Economist. 2010;158(2):151-192

[36] Horneff W, Maurer R, Rogalla R. Dynamic portfolio choice with deferred annuities. Journal of Banking Finance. 2010;34(11):2652-2664

[37] Post T. Individual welfare gains from deferred life-annuities under stochastic mortality. Asia-Pacific Journal of Risk and Insurance. 2012;6(2)

[38] Hanewald K, Piggott J, Sherris M. Individual post-retirement longevity risk management under systematic mortality risk. Insurance: Mathematics and Economics. 2013; 52(1):87-97

[39] Piggott J, Valdez E, Detzel B. The simple analytics of pooled annuity funds. The Journal of Risk and Insurance. 2005;72(3):497-520

[40] Valdez E, Piggott J, Wang L. Demand and adverse selection in a pooled annuity fund. Insurance: Mathematics and Economics. 2006; 39(2):251-266

[41] Milevsky M. Life Annuities: Optimal Products for Retirement Income. The Research Foundation of the CFA Institute; 2013. Available from: [www.cfapubs.org](http://www.cfapubs.org)

[42] Holzmann R. Addressing longevity risk through private annuities: Issues

and options. In: Paper Prepared for the 22nd Annual Colloquium of Superannuation Researchers, CEPAR and ABS' School of Risk and Actuarial Studies, New South Wales University, Sydney. 2015

[43] Olivieri A, Pitacco E. Longevity-linked annuities how to preserve value creation against longevity risk. 2019. DOI: 10.2139/ssrn.3458441

[44] Bell D, Liu E, Shao A. Member's Default Utility Function for Default Fund Design Version 1 ("MDUF v1"). Sydney: Working Paper; 2017

[45] Vasicek O. An equilibrium characterization of the term structure. Journal of Financial Economics. 1977;5: 177-188

[46] Human Mortality Database. USA/ Germany: University of California, Berkeley/Max Planck Institute for Demographic Research; 2019. Available from: [www.mortality.org](http://www.mortality.org) or [www.humanmortality.de](http://www.humanmortality.de)