We are IntechOpen,
the world's leading publisher of
Open Access books
Built by scientists, for scientists

6,900
Open access books available

186,000
International authors and editors

200M
Downloads

Our authors are among the

154
Countries delivered to

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities

CLARIVATE ANALYTICS
BOOK CITATION INDEX
INDEXED

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

Chapter

Dynamics of Biostructures on a Fractal/Multifractal Space-Time Manifold

Maricel Agop, Calin Buzea, Decebal Vasincu and Daniel Timofte

Abstract

A theory of space-time is built on a fractal/multifractal variety. Thus, considering that both the spatial coordinates and the time are fractal/multifractal, it is shown that both the energy and the non-differentiable mass of any biostructure depend on both the "state" of the biostructure and a speed limit of constant value. For the dynamics on Peano fractal/multifractal curves and Compton scale resolutions, it is shown that our results are reduced to those of Einstein relativity. In such a context, it has been shown that the "chameleon effect" of cholesterol corresponds to the HDL-LDL state transfer dictated by the spontaneous symmetry breaking through a fractal/multifractal tunnel effect. Then both HDL and LDL become distinct states of the same biostructure as in nuclear physics where proton and neutron are distinct states of the same nucleon.

Keywords: fractal/multifractal tunnel effect, biostructures, cholesterol, spontaneous symmetry breaking, chameleon effect

1. Mathematical model

1.1 Time as a fractal/multifractal

Analyzing the nonrelativistic dynamics of a particle in a fractal/multifractal space [1–4], we observe a big discrepancy between the space coordinates and the temporal one (considered as affine parameter of motion curve). If the space coordinates are fractal/multifractal, the temporal coordinate is not a fractal/multifractal. This discrepancy has an important consequence: the particle travels on an infinite length curve in a finite time span, and so, it has an infinite velocity. In order to eliminate this contradiction, in the following we will assume that not only the space coordinates are fractal/multifractal but also the temporal one is a fractal/multifractal. Practically, we shall build dynamics of biostructures on a non-differentiable space-time manifold. In this framework, the most important elements from the nonrelativistic approach of scale relativity theory with arbitrary constant fractal dimension, as described in [5–7], remain valid, but the time differential element dt is now replaced by the proper time differential element $d\tau$. In this way, not only the space but the entire space-time continuum is considered to be non-differentiable and, therefore, fractal/multifractal.

1.2 Consequences of non-differentiability on a space-time manifold

Let us suppose that on a space-time manifold, the motions of biostructures take place on continuous but non-differentiable curves (in particular fractal/multifractal curves). The non-differentiability of motion curves implies the following [2]:

(i) Any continuous but non-differentiable curve is explicitly scale dependent (which will be referred as $\delta\tau$). In other words, its length tends to infinity when its proper time interval, $\Delta\tau$, tends to zero (an extension of the Lebesgue theorem on a space-time manifold). Consequently, in this limit, a curve in a space-time manifold is zigzagged as one can imagine. Thus, it exhibits the property of self-similarity in all its points of a space-time manifold, which can be translated into an extension property of holography (every part of a space-time manifold reflects the whole of the same space-time manifold).

Then a continuous but non-differentiable space-time is fractal/multifractal in Mandelbrot's sense:

(ii) The differential proper time reflection invariance of any variable is broken. For example, the proper time derivative of four-coordinate X^μ, where $\mu = 0$, 1, 2, 3, can be written two fold:

$$\left[\frac{dX^\mu}{d\tau}\right]_+ = \lim_{\Delta\tau \to 0_+} \frac{X^\mu(\tau + \Delta\tau) - X^\mu(\tau)}{\Delta\tau}$$

$$\left[\frac{dX^\mu}{d\tau}\right]_- = \lim_{\Delta\tau \to 0_-} \frac{X^\mu(\tau) - X^\mu(\tau - \Delta\tau)}{\Delta\tau} \tag{1}$$

These relations are equivalent in the differentiable case, $\Delta\tau \to -\Delta\tau$. In the non-differentiable case, the previous definitions fail since the limits $\Delta\tau \to 0\pm$ are no longer defined. In the approach of the non-differentiable model, the biophysical phenomena are related to the behavior of the function during the "zoom" operation on the proper time resolution $\delta\tau$: then, by means of the substitution principle, $\delta\tau$ will be identified with the differential element $d\tau$, i.e., $\delta\tau \equiv d\tau$, and will be considered as independent variable. Thus, every classical variable $Q(\tau)$ is replaced by the non-differentiable variable $Q(\tau, d\tau)$ explicitly dependent on the proper time resolution interval whose derivative is undefined only in the limit, $\Delta\tau \to 0$. As a consequence, two derivatives of every non-differentiable variable as explicit functions of τ and $d\tau$ will be defined. For example, the two derivatives of the four-coordinate $X^\mu(\tau, \Delta\tau)$ takes the following form:

$$\frac{d_+X^\mu}{d\tau} = \lim_{\Delta\tau \to 0_+} \frac{X^\mu(\tau + \Delta\tau, \Delta\tau) - X^\mu(\tau, \Delta\tau)}{\Delta\tau}$$

$$\frac{d_-X^\mu}{d\tau} = \lim_{\Delta\tau \to 0_-} \frac{X^\mu(\tau, \Delta\tau) - X^\mu(\tau - \Delta\tau, \Delta\tau)}{\Delta\tau} \tag{2}$$

The sign + corresponds to the forward biophysical process and the sign − to the backward one:

(iii) The differential of four-coordinate $dX^\mu(\tau, \Delta\tau)$ can be expressed as the sum of two differentials, one not scale dependent (differentiable part $d_\pm x^\mu(\tau)$) and other scale dependent (non-differentiable part $d_\pm \xi^\mu(\tau, d\tau)$), i.e.,

$$d_\pm X^\mu(\tau, \Delta\tau) = d_\pm x^\mu(\tau) + d_\pm \xi^\mu(\tau, \Delta\tau) \tag{3}$$

(iv) The non-differentiable part of the four-coordinate satisfies the non-differentiable equation

$$d_{\pm}\xi^{\mu}(\tau, \Delta\tau) = \lambda_{\pm}^{\mu}(d\tau)^{1/D_F} \tag{4}$$

where $\lambda_{\pm}^{\mu}$ are constant coefficients whose statistical significance will be given in what follows and D_F is the fractal dimension of the motion curves from the space-time manifold.

In our opinion, the complexity of the biophysical processes implies dynamics on geodesics with various fractal dimensions. Precisely, $D_F = 2$ is a characteristic to the biophysical processes of quantum type, $D_F < 2$ is a characteristic to the biophysical processes of correlative type, while $D_F > 2$ is a characteristic to the biophysical processes of non-correlative type. Since such dynamics simultaneously are operational on a given biophysical system, the space-time manifold will exhibit multifractal type properties [2].

(v) The differential proper time reflection invariance is recovered by combining the derivatives $d_+/d\tau$ and $d_-/d\tau$ in the non-differentiable operator:

$$\frac{\hat{d}}{d\tau} = \frac{1}{2}\left(\frac{d_+ + d_-}{d\tau}\right) - \frac{i}{2}\left(\frac{d_+ - d_-}{d\tau}\right) \tag{5}$$

This specific procedure is called, according to [8], "differentiability by extension in complex on a space-time manifold" (Cresson's theorem). Applying now the non-differentiable operator to the four-coordinate X^{μ} yields the complex velocity:

$$\hat{V}^{\mu} = \frac{\hat{d}X^{\mu}}{d\tau} = \frac{1}{2}\left(\frac{d_+X^{\mu} + d_-X^{\mu}}{d\tau}\right) - \frac{i}{2}\left(\frac{d_+X^{\mu} - d_-X^{\mu}}{d\tau}\right)$$

$$= \frac{1}{2}\left(\frac{d_+x^{\mu} + d_-x^{\mu}}{d\tau} + \frac{d_+\xi^{\mu} + d_-\xi^{\mu}}{d\tau}\right) - \frac{i}{2}\left(\frac{d_+x^{\mu} - d_-x^{\mu}}{d\tau} + \frac{d_+\xi^{\mu} - d_-\xi^{\mu}}{d\tau}\right) = V^{\mu} - iU^{\mu} \tag{6}$$

with

$$V^{\mu} = \frac{1}{2}\left(v_+^{\mu} + v_-^{\mu}\right), U^{\mu} = \frac{1}{2}\left(v_+^{\mu} - v_-^{\mu}\right), v_+^{\mu} = \frac{d_+x^{\mu} + d_+\xi^{\mu}}{d\tau}, v_-^{\mu} = \frac{d_-x^{\mu} + d_-\xi^{\mu}}{d\tau} \tag{7}$$

The real part V^{μ} is differentiable and scale resolution independent, while the imaginary one U^{μ} is non-differentiable and scale resolution dependent.

(vi) An infinite number of geodesics can be found relating any pair of points of a space-time manifold, and this is true on all scale resolutions of the dynamics of biostructures. Then, in the space-time manifold, all the entities of the biostructures are substituted with the geodesics themselves so that any external constraint can be interpreted as a selection of geodesics in the same space-time manifold. The infinity of geodesics in the bundle, their non-differentiability, the two values of the derivative, etc., imply a generalized statistical fluidlike description (fractal/multifractal fluid). In this way, one provides the fractalization/multifractalization type through stochastic processes. From such a perspective, averages, variances, covariances, etc. of the fractal/multifractal fluid variables (by means of which now we can describe the dynamics of the biostructures) must be considered in the sense of the stochastic process associated with fractalization/multifractalization. In such a context, the choice of the average of $d_{\pm}X^i$ in the form

$$\langle d_\pm X^i \rangle \equiv d_\pm x^i \qquad (8)$$

implies through (3)

$$\langle d_\pm \xi^i \rangle = 0 \qquad (9)$$

1.3 Motion non-differentiable operator on a space-time manifold

Let us now consider that the movement curves (continuous and non-differentiable) are immersed in the space-time and that X^μ are the four coordinates of a point on the curve. We also consider a variable $Q(X^\mu, \tau)$ and the following Taylor expansion, up to the second order

$$d_\pm Q(X^\mu, \tau, d\tau) = \partial_\tau Q d\tau + \partial_\mu Q d_\pm X^\mu + \frac{1}{2} \partial_\mu \partial_\nu Q d_\pm X^\mu d_\pm X^\nu \qquad (10)$$

where

$$\partial_\tau = \frac{\partial}{\partial \tau}, \partial_\mu = \frac{\partial}{\partial X^\mu}, \partial_\mu \partial_\nu = \frac{\partial^2}{\partial X^\mu \partial X^\nu}$$

Relations (10) are valid in any point of the space-time manifold and more for the points "X^μ" on the non-differentiable curve which we have selected in relation (10).

From here, forward and backward average values of (10) become

$$\langle d_\pm Q(X^\mu, \tau, d\tau)\rangle = \langle \partial_\tau Q d\tau\rangle + \langle \partial_\mu Q d_\pm X^\mu\rangle + \frac{1}{2}\partial_\mu\partial_\nu Q\langle d_\pm X^\mu d_\pm X^\nu\rangle \qquad (11)$$

We make the following stipulations: the average values of the variables $Q(X^\mu, \tau, d\tau)$ and its derivatives coincide with themselves, and the differentials $d_\pm X^\mu$ and $d\tau$ are independent. Therefore, the average of their products coincides with the product of their averages. In these conditions, (11) takes the form

$$d_\pm Q(X^\mu, \tau, d\tau) = \partial_\tau Q d\tau + \partial_\mu Q\langle d_\pm X^\mu\rangle + \frac{1}{2}\langle \partial_\mu\partial_\nu Q d_\pm X^\mu d_\pm X^\nu\rangle \qquad (12)$$

or using (3), (8), and (9)

$$d_\pm Q(X^\mu, \tau, d\tau) = \partial_\tau Q d\tau + \partial_\mu Q d_\pm x^\mu + \frac{1}{2}\partial_\mu\partial_\nu Q(d_\pm x^\mu d_\pm x^\nu + \langle d_\pm \xi^\mu d_\pm \xi^\nu\rangle) \qquad (13)$$

Even the average values of the 4-non-differentiable coordinate $d_\pm \xi^\mu$ is null, for the higher order of the four-non-differentiable coordinate average, the situation can be different. Let us focus now on the mean $<d_\pm \xi^\mu d_\pm \xi^\nu>$. Using (4), we can write

$$\langle d_\pm \xi^\mu d_\pm \xi^\nu\rangle = \pm\lambda_\pm^\mu \lambda_\pm^\nu (d\tau)^{(2/D_F - 1)} d\tau \qquad (14)$$

using the convention that the sign + corresponds to $d\tau > 0$, while the sign − corresponds to $d\tau < 0$.

Then (13) takes the form:

$$d_\pm Q(X^\mu, \tau, d\tau) = \partial_\tau Q d\tau + \partial_\mu Q d_\pm x^\mu + \frac{1}{2}\partial_\mu\partial_\nu Q d_\pm x^\mu d_\pm x^\nu \pm \frac{1}{2}\partial_\mu\partial_\nu Q\lambda_\pm^\mu \lambda_\pm^\nu (d\tau)^{(2/D_F - 1)} d\tau$$

$$(15)$$

If we divide by $d\tau$ and neglect the terms that contain differential factors, using the method from [5–7], we obtain:

$$\frac{d_{\pm}Q(X^{\mu}, \tau, d\tau)}{d\tau} = \partial_{\tau}Q + \nu_{\pm}^{\mu}\partial_{\mu}Q \pm \frac{1}{2}\lambda_{\pm}^{\mu}\lambda_{\pm}^{\nu}(d\tau)^{(2/D_F - 1)}\partial_{\mu}\partial_{\nu}Q \tag{16}$$

These relations also allow us to define the operators:

$$\frac{d_{\pm}}{d\tau} = \partial_{\tau} + \nu_{\pm}^{\mu}\partial_{\mu} \pm \frac{1}{2}\lambda_{\pm}^{\mu}\lambda_{\pm}^{\nu}(d\tau)^{(2/D_F - 1)}\partial_{\mu}\partial_{\nu} \tag{17}$$

Under these circumstances, let us calculate $\hat{d}/d\tau$. Taking into account (5), (6), and (17), we obtain:

$$\frac{\hat{d}Q}{d\tau} = \frac{1}{2}\left[\left(\frac{d_{+}Q + d_{-}Q}{d\tau}\right) - i\left(\frac{d_{+}Q - d_{-}Q}{d\tau}\right)\right]$$
$$= \partial_{\tau}Q + \hat{V}^{\mu}\partial_{\mu}Q + \frac{1}{4}(d\tau)^{(2/D_F - 1)}D^{\mu\nu}\partial_{\mu}\partial_{\nu}Q \tag{18}$$

where

$$D^{\mu\nu} = d^{\mu\nu} - i\overline{d}^{\mu\nu}$$
$$d^{\mu\nu} = \lambda_{+}^{\mu}\lambda_{+}^{\nu} - \lambda_{-}^{\mu}\lambda_{-}^{\nu}, \overline{d}^{\mu\nu} = \lambda_{+}^{\mu}\lambda_{+}^{\nu} + \lambda_{-}^{\mu}\lambda_{-}^{\nu}, i = \sqrt{-1} \tag{19}$$

The relation also allows us to define the motion non-differentiable operator:

$$\frac{\hat{d}}{d\tau} = \partial_{\tau} + \hat{V}^{\mu}\partial_{\mu} + \frac{1}{4}(d\tau)^{(2/D_F - 1)}D^{\mu\nu}\partial_{\mu}\partial_{\nu} \tag{20}$$

If the non-differentiability of motion curves is realized through Markov type stochastic process [2, 4].

$$\lambda_{+}^{\mu}\lambda_{+}^{\nu} = \lambda_{-}^{\mu}\lambda_{-}^{\nu} = -\lambda\eta^{\mu\nu} \tag{21}$$

where $\eta^{\mu\nu}$ is the Minkowski metric and λ is the coefficient associated with the differentiable-non-differentiable transition, then the motion non-differentiable operator takes the form

$$\frac{\hat{d}}{d\tau} = \partial_{\tau} + \hat{V}^{\mu}\partial_{\mu} + i\frac{\lambda}{2}(d\tau)^{(2/D_F - 1)}\partial_{\mu}\partial^{\mu} \tag{22}$$

If the non-differentiability of motion curves is realized through non-Markov type stochastic process [2, 4].

$$\lambda_{+}^{\mu}\lambda_{+}^{\nu} - \lambda_{-}^{\mu}\lambda_{-}^{\nu} = \lambda_1\eta^{\mu\nu}$$
$$\lambda_{+}^{\mu}\lambda_{+}^{\nu} + \lambda_{-}^{\mu}\lambda_{-}^{\nu} = -\lambda_2\eta^{\mu\nu} \tag{23}$$

where λ_1 and λ_2 are two coefficients associated with the differentiable-non-differentiable transition, then the motion non-differentiable operator takes the form

$$\frac{\hat{d}}{d\tau} = \partial_{\tau} + \hat{V}^{\mu}\partial_{\mu} + \frac{1}{4}(\lambda_1 + i\lambda_2)(d\tau)^{(2/D_F - 1)}\partial_{\mu}\partial^{\mu} \tag{24}$$

1.4 Non-differentiable geodesics on a space-time manifold

In what follows, let us consider the functionality of the scale covariance principle [5–7]: the biophysics laws are simultaneously invariant both with respect to the four-coordinate transformation and with respect to scale transformations. Then the passage from differentiable biophysics in a space-time manifold to the non-differentiable biophysics in a same space-time, manifold which is considered here, can be implemented by replacing the standard derivative $d/d\tau$ by the non-differentiable operator $\hat{d}/d\tau$. This operator plays the role of a "covariant derivative operator," namely, it is used to write the fundamental equations of dynamics of biostructures under the same form as in the classical (differentiable) case. Thus, applying the operator (20) to the complex velocity (6), the geodesics equation becomes:

$$\frac{\hat{d}\hat{V}^{\mu}}{d\tau} = \partial_{\tau}\hat{V}^{\mu} + \hat{V}^{\nu}\partial_{\nu}\hat{V}^{\mu} + \frac{1}{4}(d\tau)^{(2/D_F - 1)}D^{\alpha\beta}\partial_{\alpha}\partial_{\beta}\hat{V}^{\mu} \equiv 0 \tag{25}$$

or, also using (6), through separation of motions on scale resolutions (the real part from the imaginary one):

$$\frac{\hat{d}V^{\mu}}{d\tau} = \partial_{\tau}V^{\mu} + V^{\nu}\partial_{\nu}V^{\mu} - U^{\nu}\partial_{\nu}U^{\mu} + \frac{1}{4}(d\tau)^{(2/D_F - 1)}d^{\alpha\beta}\partial_{\alpha}\partial_{\beta}V^{\mu}$$

$$- \frac{1}{4}(d\tau)^{(2/D_F - 1)}\overline{d}^{\alpha\beta}\partial_{\alpha}\partial_{\beta}U^{\mu} = 0$$

$$\frac{\hat{d}U^{\mu}}{d\tau} = \partial_{\tau}U^{\mu} + V^{\nu}\partial_{\nu}U^{\mu} + U^{\nu}\partial_{\nu}V^{\mu} + \frac{1}{4}(d\tau)^{(2/D_F - 1)}d^{\alpha\beta}\partial_{\alpha}\partial_{\beta}U^{\mu}$$

$$+ \frac{1}{4}(d\tau)^{(2/D_F - 1)}\overline{d}^{\alpha\beta}\partial_{\alpha}\partial_{\beta}V^{\mu} = 0 \tag{26}$$

For motions on non-differentiable curves realized through Markov type stochastic process [1, 2, 4], the geodesics equation takes the form

$$\frac{\hat{d}\hat{V}^{\mu}}{d\tau} = \partial_{\tau}\hat{V}^{\mu} + \hat{V}^{\nu}\partial_{\nu}\hat{V}^{\mu} - i\frac{\lambda}{2}(d\tau)^{(2/D_F - 1)}\partial^{\nu}\partial_{\nu}\hat{V}^{\mu} = 0 \tag{27}$$

or through separation of motions on scale resolutions:

$$\frac{\hat{d}V^{\mu}}{d\tau} = \partial_{\tau}V^{\mu} + V^{\nu}\partial_{\nu}V^{\mu} - \left(U^{\nu} - \frac{\lambda}{2}(d\tau)^{(2/D_F - 1)}\partial^{\nu}\right)\partial_{\nu}U^{\mu} = 0$$

$$\frac{\hat{d}U^{\mu}}{d\tau} = \partial_{\tau}U^{\mu} + V^{\nu}\partial_{\nu}U^{\mu} + \left(U^{\nu} - \frac{\lambda}{2}(d\tau)^{(2/D_F - 1)}\partial^{\nu}\right)\partial_{\nu}V^{\mu} = 0 \tag{28}$$

For motions on non-differentiable curves realized through non-Markov type stochastic process [1, 2, 4], the geodesics equation becomes

$$\frac{\hat{d}\hat{V}^{\mu}}{d\tau} = \partial_{\tau}\hat{V}^{\mu} + \hat{V}^{\nu}\partial_{\nu}\hat{V}^{\mu} + \frac{1}{4}(\lambda_1 + i\lambda_2)^{(2/D_F - 1)}\partial_{\nu}\partial^{\nu}\hat{V}^{\mu} = 0 \tag{29}$$

or through separation of motions on scale resolutions:

$$\frac{\hat{d}V^\mu}{d\tau} = \partial_\tau V^\mu + V^\nu \partial_\nu V^\mu - \left(U^\nu - \frac{\lambda_2}{4}(d\tau)^{(2/D_F-1)}\partial^\nu \right)\partial_\nu U^\mu + \frac{\lambda_1}{4}(d\tau)^{(2/D_F-1)}\partial_\nu \partial^\nu V^\mu = 0$$

$$\frac{\hat{d}U^\mu}{d\tau} = \partial_\tau U^\mu + V^\nu \partial_\nu U^\mu + \left(U^\nu - \frac{\lambda_2}{4}(d\tau)^{(2/D_F-1)}\partial^\nu \right)\partial_\nu V^\mu + \frac{\lambda_1}{4}(d\tau)^{(2/D_F-1)}\partial_\nu \partial^\nu U^\mu = 0$$

(30)

1.5 Non-differentiable geodesics in terms of the scalar complex field on a space-time manifold

Let us choose $\hat{V}^\mu$ in terms of the scalar complex field Ψ:

$$\hat{V}^\alpha = i\lambda(d\tau)^{(2/D_F-1)}\partial_\alpha \ln \Psi \tag{31}$$

Then the geodesics equation (27) becomes

$$\frac{\hat{d}\hat{V}_\alpha}{d\tau} = \lambda(d\tau)^{(2/D_F-1)}\partial_\tau \partial_\alpha \ln \Psi$$

$$+ \left[i\lambda(d\tau)^{(2/D_F-1)}\partial^\mu \ln \Psi + i\frac{\lambda}{2}(d\tau)^{(2/D_F-1)}\partial^\mu \right]\partial_\mu \partial_\alpha \left[i\lambda(d\tau)^{(2/D_F-1)}\ln \Psi \right] = 0 \tag{32}$$

Since

$$\partial_\alpha\left(\partial_\mu \ln \Psi \partial^\mu \ln \Psi\right) = 2\partial^\mu \ln \Psi \partial_\alpha \partial_\mu \ln \Psi$$

$$\partial_\alpha \partial_\mu \partial^\mu \ln \Psi = \partial^\mu \partial_\mu \partial_\alpha \ln \Psi$$

$$\partial_\alpha\left(\partial_\mu \ln \Psi \partial^\mu \ln \Psi + \partial_\mu \partial^\mu \ln \Psi\right) = \partial_\alpha\left(\frac{\partial_\mu \partial^\mu \Psi}{\Psi}\right) \tag{33}$$

Equation (32) takes the form:

$$i\lambda(d\tau)^{(2/D_F-1)}\partial_\tau \partial_\alpha \ln \Psi + \lambda^2(d\tau)^{(4/D_F-2)}\partial_\alpha\left(\frac{\partial_\mu \partial^\mu \Psi}{\Psi}\right) = 0 \tag{34}$$

By integrating the above relation, we obtain:

$$\lambda^2(d\tau)^{(4/D_F-2)}\partial_\mu \partial^\mu \Psi + i\lambda(d\tau)^{(2/D_F-1)}\partial_\tau \Psi + F^2(\tau)\Psi = 0 \tag{35}$$

where $F^2(\tau)$ is an arbitrary function depending on τ.

Consequently, the non-differentiable geodesics (35) in terms of Ψ are well defined up to an arbitrary function $F^2(\tau)$ depending on τ. A particular form of $F^2(\tau)$ can be obtained, for instance, based on a correspondence with the standard Klein-Gordon equation.

1.6 Non-differentiable geodesics in terms of Klein-Gordon equation of fractal/ multifractal type

If Ψ is independent on τ, i.e., $\partial_\tau \Psi = 0$ and $F^2(\tau) = V_0^2$ = const., with V_0 a limit velocity with constant value, the geodesics (35) become the Klein-Gordon equation of fractal/multifractal type

$$\partial_\mu \partial^\mu \Psi + \frac{1}{\overline{\Lambda}^2} \Psi \equiv 0 \tag{36}$$

with

$$\overline{\Lambda} = \overline{\Lambda}_0 (d\tau)^{(2/D_F - 1)}, \Lambda_0 = \frac{\lambda}{V_0} \tag{37}$$

From (37) it results in a scale resolution dependence of the fundamental length $\overline{\Lambda}$, where $\overline{\Lambda}_0$ is the fundamental unscaled length. For relativistic motions on Peano curves, $D_F = 2$, at Compton scale $\overline{\Lambda}_0 = \lambda/V_0 \equiv /(m_0 c), \lambda = /m_0, V_0 \equiv c$ with $\hbar$ the reduced Planck constant, m_0 the rest mass of the biophysical system entity, and c the velocity light in vacuum, (37) takes the usual form of Klein-Gordon equation:

$$\partial_\mu \partial^\mu \Psi + \left(\frac{m_0 c}{\hbar} \right)^2 \Psi \equiv 0$$

1.7 Non-differentiable specific potential force and energy

Using the explicit form of the function, $\Psi = \sqrt{\rho} e^{iS}$, where $\sqrt{\rho}$ is an amplitude and S is a phase, the expression of U_α becomes

$$U_\alpha = -\lambda \partial_\alpha \ln \sqrt{\rho} \tag{38}$$

Thus it results in

$$\left[U_\mu - \frac{\lambda}{2} (d\tau)^{(2/D_F - 1)} \partial_\mu \right] \partial^\mu U_\alpha = \lambda^2 (d\tau)^{(4/D_F - 2)} \left(\partial^\mu \ln \sqrt{\rho} \partial_\mu \partial_\alpha \ln \sqrt{\rho} + \frac{1}{2} \partial^\mu \partial_\mu \partial_\alpha \ln \sqrt{\rho} \right) \tag{39}$$

Since the identities from (33) work in variable $\ln \sqrt{\rho}$, Eq. (39) becomes

$$\left[U_\mu - \frac{\lambda}{2} (d\tau)^{(2/D_F - 1)} \partial_\mu \right] \partial^\mu U_\alpha = \frac{\lambda^2}{2} (d\tau)^{(4/D_F - 2)} \partial_\alpha (\partial^\mu \ln \sqrt{\rho} \partial_\mu \ln \sqrt{\rho} + \partial^\mu \partial_\mu \ln \sqrt{\rho})$$

$$= \lambda^2 (d\tau)^{(4/D_F - 2)} \partial_\alpha \left(\frac{\partial^\mu \partial_\mu \sqrt{\rho}}{\sqrt{\rho}} \right) \tag{40}$$

which implies through the specific non-differentiable potential

$$Q = \frac{\lambda^2}{2} (d\tau)^{(4/D_F - 2)} \frac{\partial^\mu \partial_\mu \sqrt{\rho}}{\sqrt{\rho}} = \frac{1}{2} U^\mu U_\mu - \lambda (d\tau)^{(2/D_F - 1)} \partial^\mu U_\mu =$$

$$= \lambda^2 (d\tau)^{(4/D_F - 2)} \partial_\alpha \left(\frac{\partial^\mu \partial_\mu \sqrt{\rho}}{\sqrt{\rho}} \right) \tag{41}$$

the specific non-differentiable force

$$F_\alpha = \frac{\lambda^2}{2} (d\tau)^{(4/D_F - 2)} \partial_\alpha \left(\frac{\partial^\mu \partial_\mu \sqrt{\rho}}{\sqrt{\rho}} \right) = \left[U^\mu - \frac{\lambda}{2} (d\tau)^{(2/D_F - 1)} \partial^\mu \right] \partial^\mu U_\alpha \tag{42}$$

Thus, the first equation (28) takes the form

$$\frac{\hat{d}V_\alpha}{d\tau} = \partial_\tau V_\alpha + V^\mu \partial_\mu V_\alpha = \frac{\lambda^2}{2}(d\tau)^{(4/D_F-2)}\partial_\alpha\left(\frac{\partial^\mu \partial_\mu \sqrt{\rho}}{\sqrt{\rho}}\right) \tag{43}$$

If

$$V_\alpha = \lambda(d\tau)^{(2/D_F-1)}\partial_\alpha S \tag{44}$$

which implies

$$V^\nu \partial_\nu V_\alpha = V^\nu \partial_\alpha V_\nu \tag{45}$$

the relation (43) becomes

$$\frac{\hat{d}V_\alpha}{d\tau} = \partial_\tau V_\alpha + V^\nu \partial_\nu V_\alpha - \frac{\lambda^2}{2}(d\tau)^{(4/D_F-2)}\partial_\alpha\left(\frac{\partial^\nu \partial_\nu \sqrt{\rho}}{\sqrt{\rho}}\right) \tag{46}$$

and more, for $\partial_\tau V_\alpha = 0$:

$$\partial_\alpha\left[V^\nu V_\nu - \frac{\lambda^2}{2}(d\tau)^{(4/D_F-2)}\frac{\partial^\mu \partial_\mu \sqrt{\rho}}{\sqrt{\rho}}\right] = 0 \tag{47}$$

Now, by a suitable choice of the constant integration and knowing that [2]:

$$V^\nu V_\nu = \left(\frac{E}{m_0 V_0}\right)^2 - \left(\frac{\boldsymbol{p}}{m_0}\right)^2 = V_0^2 + \lambda^2(d\tau)^{(4/D_F-2)}\frac{\Box\sqrt{\rho}}{\sqrt{\rho}}$$

we obtain the non-differentiable energy expression in the form

$$E = \pm V_0\left[(m_0 V_0)^2 + \boldsymbol{p}^2 + (m_0\lambda)^2(d\tau)^{(4/D_F-2)}\frac{\Box\sqrt{\rho}}{\sqrt{\rho}}\right]^{1/2} \tag{48}$$

where

$$\Box = -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} + V_0^{-2}\frac{\partial^2}{\partial t^2}$$

For relativistic motions on Peano curves, $D_F = 2$ at Compton scale, $\overline{\Lambda}_0 = \lambda/V_0 \equiv /(m_0 c)$, $\lambda = /m_0$, $V_0 = c$, the fractal energy (48) is reduced to the de Broglie's relation:

$$E = \pm c\left[(m_0 c)^2 + \boldsymbol{p}^2 + \hbar^2\frac{\Box\sqrt{\rho}}{\sqrt{\rho}}\right]^{1/2} \tag{49}$$

Relation (48) specifies the following: (i) information propagates with a limit speed V_0 which differs from one biophysical structure to another; (ii) energy, through $\Box\sqrt{\rho}/\sqrt{\rho}$, depends on the state of the biophysical structure; and (iii) the non-differentiable mass

$$M = \pm m_0\left[1 + \frac{\boldsymbol{p}^2}{(m_0 V_0)^2} + \left(\frac{\lambda}{V_0}\right)^2(d\tau)^{(4/D_F-2)}\frac{\Box\sqrt{\rho}}{\sqrt{\rho}}\right]^{1/2} \tag{50}$$

depends also on the state of the biophysical structure, through $\Box\sqrt{\rho}/\sqrt{\rho}$.

1.8 Non-differentiable state density conservation law

Let us consider Eq. (35) and its complex conjugate:

$$\lambda^2(d\tau)^{(4/D_F-2)}\partial_\mu\partial^\mu\overline{\Psi} - i\lambda(d\tau)^{(2/D_F-1)}\partial_\tau\overline{\Psi} + \mathrm{F}^2(\tau)\overline{\Psi} = 0 \tag{51}$$

Multiplying (35) by $(i\lambda)^{-1}(d\tau)^{1-2/D_F}\overline{\Psi}$, (51) by $(i\lambda)^{-1}(d\tau)^{1-2/D_F}\Psi$ and subtracting the results, one obtains the state density conservation law:

$$\partial_\tau\rho + \partial_\mu j^\mu = 0 \tag{52}$$

where

$$\rho = \Psi\overline{\Psi}, j^\mu = i\lambda(d\tau)^{(2/D_F-1)}\left(\Psi\partial^\mu\overline{\Psi} - \overline{\Psi}\partial^\mu\Psi\right) \tag{53}$$

In the above relations, ρ defines the state density, while j^μ defines the state density 4-current. If Ψ does not depend on τ, which implies $\partial_\tau\rho \equiv 0$, then for relativistic motions on Peano curves, $D_F = 2$ at Compton scale $\overline{\Lambda}_0 = /(m_0c)$, and relation (52) reduces to the state density standard conservation law:

$$\partial_\mu j^\mu = 0 \tag{54}$$

2. Applications of the mathematical model

2.1 Stationary dynamics of the cholesterol at fractal/multifractal scale resolutions

Since cholesterol in any of its forms (principally LDL and HDL) is a fundamental component of blood, its dynamics will be dictated by those of the blood at fractal/multifractal scale resolutions having in view the average dimensions of the cholesterol particles (9–10 nm for HDL and 20–27 nm for LDL [9–12]).

In such a framework, nonrelativistic equations of the non-differentiable hydrodynamics at fractal/multifractal scale resolutions for the stationary case write like

$$f^i = \left(U^i + \lambda(dt)^{(2/D_F-1)}\partial_l\right)\partial^l U^i = 0 \tag{55}$$

$$\partial_l U^l = 0, i = 1, 2, 3 \tag{56}$$

results obtained from Eq. (28) under the conditions $V^i \equiv 0$ and $|U^i| << V_0$.

The first of these equations corresponds to the canceling of specific multifractal force at a differentiable scale resolution, while the second equation corresponds to the incompressibility of the blood at non-differentiable scale.

Generally, it is difficult to obtain an analytical solution for our previous equation system, taking into account its nonlinear nature (induced both by means of non-differentiable convection $U^l\partial_l U^i$ and by the non-differentiable dissipation $\lambda(dt)^{2/D_F-1}\partial_l\partial^l U^i$).

We can still obtain an analytic solution in the case of a plane symmetry (in x, y coordinates) of the dynamics of the blood. For this purpose, let us consider the equation system (55) and (56) in the form:

$$u\partial_x u + v\partial_y u = \nu\partial_{yy}^2 u \tag{57}$$

$$\partial_x u + \partial_y v = 0 \tag{58}$$

where we substituted

$$U_x = u(x,y), U_y = v(x,y), \upsilon = \lambda(dt)^{2/D_F - 1} \tag{59}$$

Using the similarities method given in [6, 7] to solve the equation system (57) and (58) with limit conditions

$$\lim_{y\to 0} v(x,y) = 0, \lim_{y\to 0}\frac{\partial u}{\partial y} = 0, \lim_{y\to\infty} u(x,y) = 0 \tag{60}$$

and a constant flux momentum per unit of depth,

$$q = \rho \int_{-\infty}^{+\infty} u^2 dy = const., \tag{61}$$

we obtain the field of velocities as solutions of the equation system (57) and (58) in the form:

$$u = \frac{1.5\left(\frac{q}{6\rho}\right)^{\frac{2}{3}}}{(\nu x)^{\frac{1}{3}}} sech^2\left[\frac{0.5y\left(\frac{q}{6\rho}\right)^{\frac{1}{3}}}{(\nu x)^{\frac{2}{3}}}\right] \tag{62}$$

$$v = \frac{1.9\left(\frac{q}{6\rho}\right)^{\frac{2}{3}}}{(\nu x)^{\frac{1}{3}}} \left\{ \frac{y\left(\frac{q}{6\rho}\right)^{\frac{1}{3}}}{(\nu x)^{\frac{2}{3}}} sech^2\left[\frac{0.5y\left(\frac{q}{6\rho}\right)^{\frac{1}{3}}}{(\nu x)^{\frac{2}{3}}}\right] - tanh\left[\frac{0.5y\left(\frac{q}{6\rho}\right)^{\frac{1}{3}}}{(\nu x)^{\frac{2}{3}}}\right] \right\} \tag{63}$$

The above equations are simplified greatly if we introduce both non-dimensional variables and non-dimensional parameters:

$$X = \frac{x}{x_0}, Y = \frac{y}{y_0}, U = \frac{u}{w_0}, V = \frac{v}{w_0}, \tag{64}$$

$$\xi = \frac{\upsilon}{\upsilon_0}, \upsilon_0 = \frac{y_0^{\frac{3}{2}}}{x_0}\left(\frac{q}{6\rho}\right)^{\frac{1}{2}}, w_0 = \frac{1}{(y_0)^{\frac{1}{2}}}\left(\frac{q}{6\rho}\right)^{\frac{1}{2}}, \tag{65}$$

where x_0, y_0, w_0, and υ_0 are lengths, velocity, and "multifractality degree" specific to the blood. The normalized velocity field is obtained:

$$U = \frac{1.5}{(\xi X)^{\frac{1}{3}}} sech^2\left[\frac{0.5Y}{(\xi X)^{\frac{2}{3}}}\right], \tag{66}$$

$$V = \frac{1.9}{(\xi X)^{\frac{1}{3}}}\left\{ \frac{Y}{(\xi X)^{\frac{2}{3}}} sech^2\left[\frac{0.5Y}{(\xi X)^{\frac{2}{3}}}\right] - tanh\left[\frac{0.5Y}{(\xi X)^{\frac{2}{3}}}\right] \right\}, \tag{67}$$

Any of Eqs. (62)–(65) specifies the nonlinearity of the velocity fields: a multifractal soliton for the velocity field across the Ox axis, respectively, "mixtures" of multifractal soliton-multifractal kink for the velocity fields across the Oy axis.

U (a.u.)

(a)

U (a.u.)

(b)

U (a.u.)

(c)
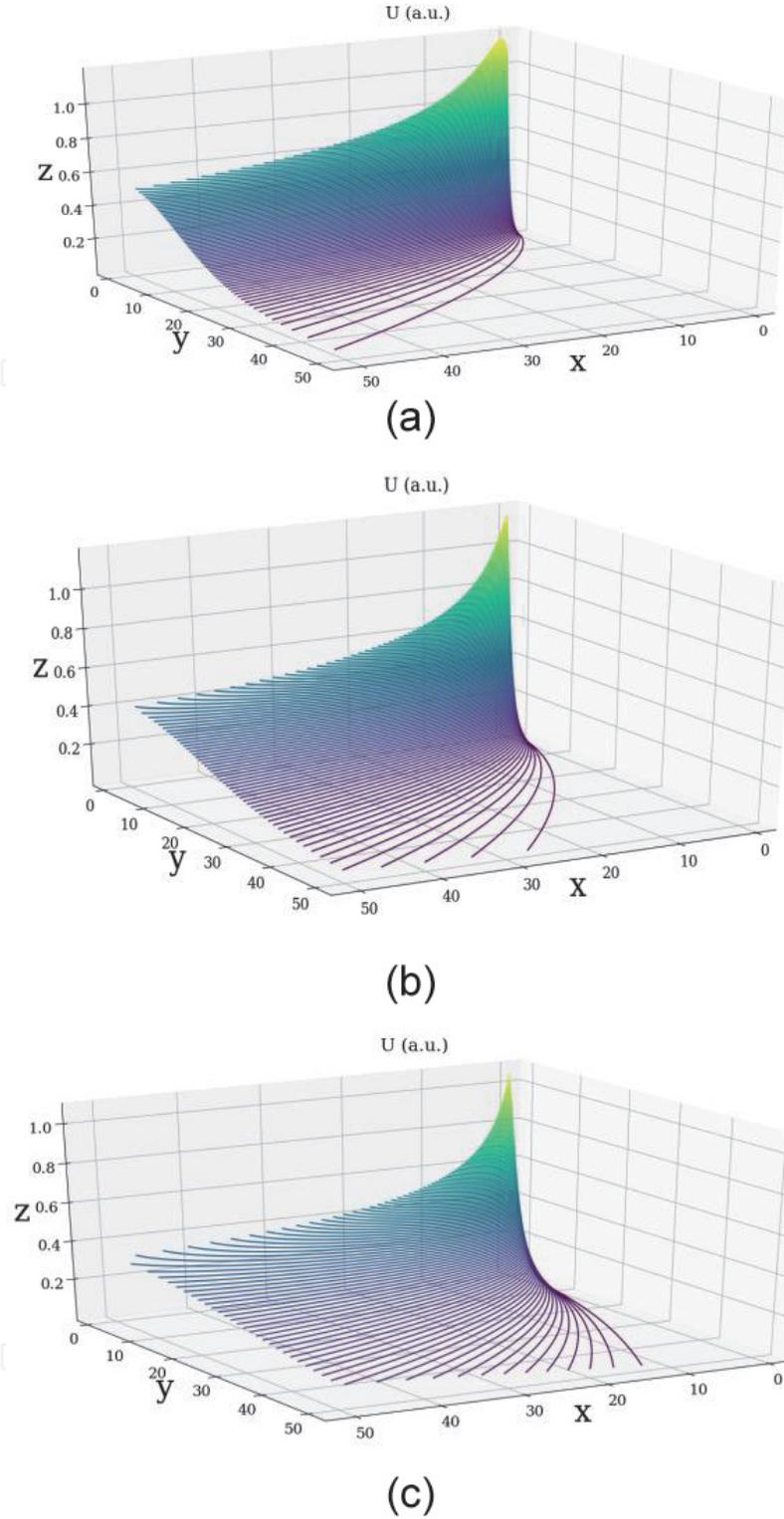
Figure 1.
Normalized velocity field U *for various fractal degrees: (a)* ξ = 0.4; *(b)* ξ = 1.0; *(c)* ξ = 1.9.

The multifractality of the system is "explained" through its dependence from scale resolutions [**Figures 1a–c** and **2a–c**].

The velocity fields (66) and (67) induce the multifractal minimal vortex (**Figure 3a–c**).

$$\Omega = \left(\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial Y} \right) = \frac{0.57Y}{(\xi X)^2} + \frac{0.63\xi}{(\xi X)^{\frac{4}{3}}} tanh \left[\frac{0.5Y}{(\xi X)^{\frac{2}{3}}} \right] + \frac{1.9Y}{(\xi X)^2} sech^2 \left[\frac{0.5Y}{(\xi X)^{\frac{2}{3}}} \right] -$$

$$-\frac{0.57Y}{(\xi X)^2} \, tanh^2\left[\frac{0.5Y}{(\xi X)^{\frac{2}{3}}}\right] - \left[\frac{1.5}{\xi X} + \frac{1.4Y^2}{X(\xi X)^{\frac{5}{3}}}\right] sech^2\left[\frac{0.5Y}{(\xi X)^{\frac{2}{3}}}\right] tanh\left[\frac{0.5Y}{(\xi X)^{\frac{2}{3}}}\right], \qquad (68)$$

Since the fractal degree depends on the dimensions of the cholesterol particle (the bigger, the lower the fractal degree), from the analysis of both the velocity field and the vortex field, it results that the LDL particles will deposit at the wall, while the HDL particles will not deposit themselves at the wall.

2.2 On the chameleonic behavior of cholesterol

Cholesterol fractions, especially LDL and HDL cholesterol, are frequently analyzed biomarkers in clinical laboratories [9]. Observational studies have shown that LDL and HDL have opposing associations with the risk of myocardial infarction, with LDL cholesterol being a positive factor and HDL cholesterol being a negative (protective) factor [10]. Observational studies cannot separate the causal role in the pathological process from the role of a marker of the underlying pathophysiology. The results of both randomized trials of LDL-cholesterol-lowering treatments [11] and from human Mendelian diseases [12] are suggesting that plasma LDL
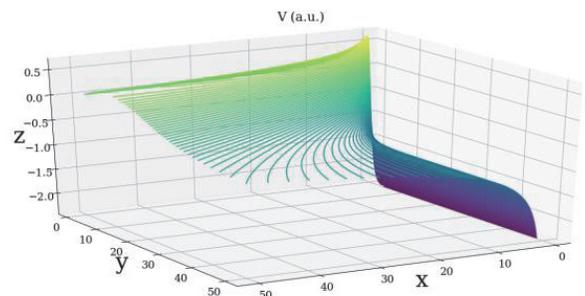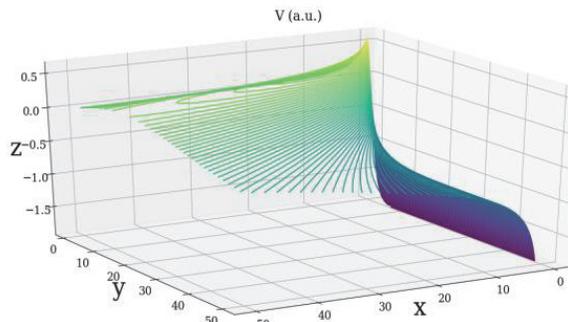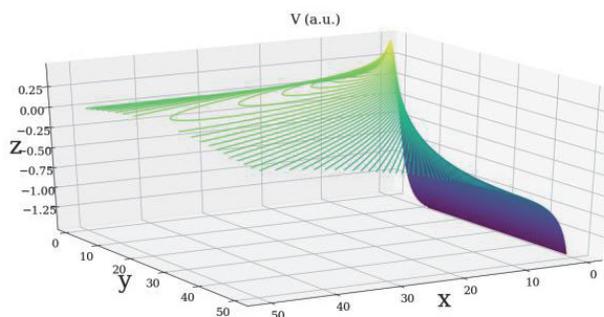
Figure 2.
Normalized velocity field V for various fractal degrees: (a) ξ = 0.4; (b) ξ = 1.0; (c) ξ = 1.9.

(a)

(b)

(c)
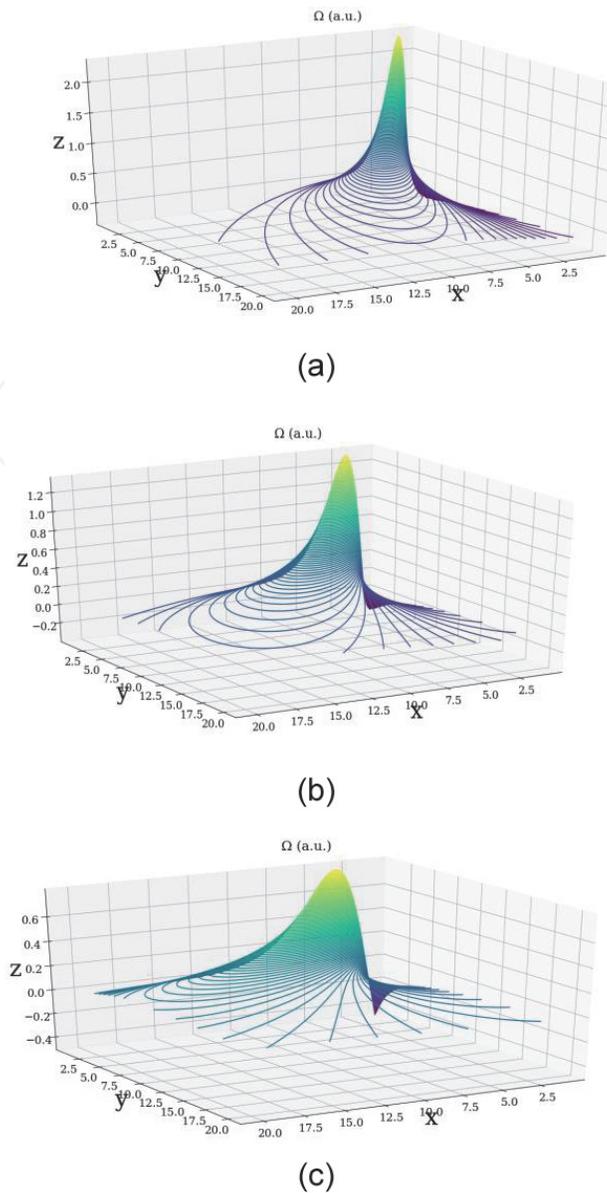
Figure 3.
Multifractal minimal normalized vortex field Ω for various fractal degrees: (a) ξ = 0.4; (b) ξ = 1.0; (c) ξ = 1.9.

cholesterol is related to the risk of myocardial infarction. However, few proofs are available for the causal relevance of HDL cholesterol from randomized trials or Mendelian diseases, and the existing ones are inconsistent [10, 11]. Moreover, more and more studies are starting to oppose the idea that raising plasma HDL cholesterol will surely translate into a risk reduction of myocardial infarction [9–12]. Therefore both LDL and HDL cholesterol can constitute risk factors for myocardial infarction. Such a behavior has been called by experts in the field the "chameleonic effect" of cholesterol [9–12]. In the present paragraph, using our previous mathematical model, LDL and HDL cholesterol dynamics is proposed. In such a context, a fractal/multifractal tunneling effect for biostructures with spontaneous symmetry breaking is analyzed. If the spontaneous symmetry breaking is assimilated to an inflammation (in the form of a specific scalar potential), then two fractal/multifractal states can be observed. In these conditions, these two states, which have been associated with biostructures such as LDL and HDL, transfer their states through a fractal/multifractal tunneling effect. As a result, in our opinion, the widely used notions of "good" and "bad" cholesterol must be redefined as two different states of the same biostructure named "cholesterol," such as in nuclear physics the neutron and proton are two different states of the same particle named nucleon.

With this aim in view, let us reconsider the differential equation (35) with $F^2(\tau) = 0$ subjected to an external constraint independent on τ given as a scalar potential U. One gets

$$\lambda^2(d\tau)^{(4/D_F-2)}\partial_\mu\partial^\mu\Psi + i\lambda(d\tau)^{(2/D_F-1)}\partial_\tau\Psi - \frac{U}{2}\Psi = 0 \qquad (69)$$

For nonrelativistic dynamics, Eq. (69) in the one-dimensional case admits the fractal/multifractal stationary solution:

$$\psi(z,t) = \theta(z)\exp\left[-\frac{i}{m_0\lambda(dt)^{(2/D_F)-1}}Et\right] \qquad (70)$$

where E is the fractal/multifractal energy of the fractal/multifractal stationary cholesterol state $\theta(x)$ and m_0 is the rest mass of the cholesterol particle. Then $\theta(x)$ becomes a fractal/multifractal solution of the fractal/multifractal space equation:

$$\partial_{zz}\theta(z) + \frac{1}{m_0\lambda^2(dt)^{(4/D_F)-2}}(E-U)\theta(z) = 0 \qquad (71)$$

If, in such a context, we suppose that the state transfer between LDL and HDL cholesterol implies spontaneous symmetry breaking [13], then $U = V(z)$ from (71) must have the form of an effective potential, as shown in **Figure 4**.

In these conditions, the stationary fractal/multifractal equation becomes

$$\frac{d^2\theta_\alpha}{dz^2} + \frac{1}{m_0\lambda^2(dt)^{(4/D_F)-2}}[E-V_\alpha]\theta_\alpha = 0, \alpha = \overline{1,3} \qquad (72)$$
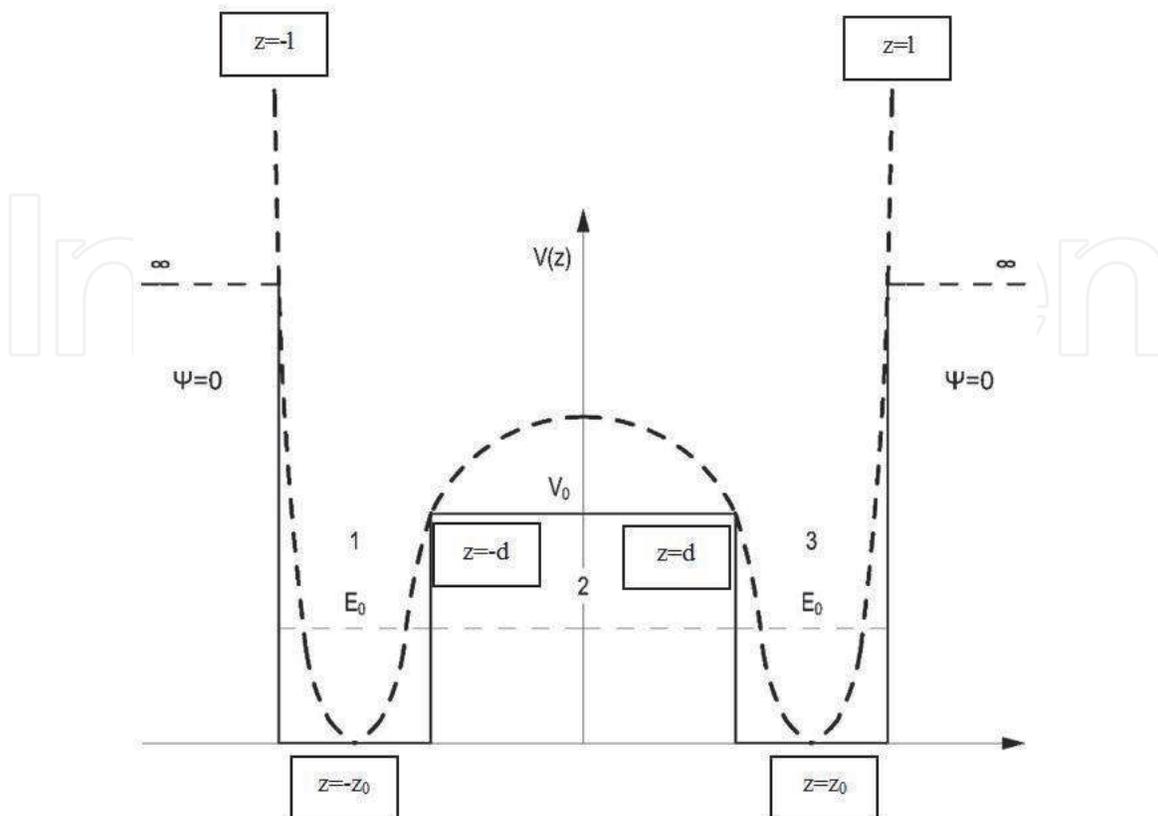
Figure 4.
The effective potential of a fractal/multifractal tunneling effect in the dynamics of biostructures with spontaneous symmetry breaking.

For each of the three regions, the solutions of the equations are

$$
\begin{aligned}
\theta_1(z) &= C_+ e^{ikz} + C_- e^{-ikz} \\
\theta_2(z) &= B e^{qz} + C e^{-qz} \\
\theta_3(z) &= D_+ e^{ikz} + D_- e^{-ikz}
\end{aligned}
\tag{73}
$$

with

$$
\begin{aligned}
k &= \left[\frac{E}{m_0 \lambda^2 (dt)^{(4/D_F)-2}} \right]^{1/2} \\
q &= \left[\frac{V_0 - E}{m_0 \lambda^2 (dt)^{(4/D_F)-2}} \right]^{1/2}
\end{aligned}
\tag{74}
$$

and

$$
C_+, C_-, B, C, D_+, D_-
$$

integration constants.

Due to the infinite potential in the two extreme regions, $|z| > l$, the fractal/multifractal state function continuity in $z = \pm l$ implies

$$
\begin{aligned}
\theta_2(-l) &= C_+ e^{-ikl} + C_- e^{ikl} = 0 \\
\theta_3(l) &= D_+ e^{ikl} + D_- e^{-ikl} = 0
\end{aligned}
\tag{75}
$$

Since the state density $|\Psi|^2$ is not altered by the multiplication of the fractal/multifractal state function in the form of a constant phase factor, the two equations for $C_\pm$ and $D_\pm$ can be immediately solved by imposing the forms:

$$
\begin{aligned}
C_+ &= \frac{A}{2i} e^{ikl}, \quad C_- = -\frac{A}{2i} e^{-ikl} \\
D_+ &= \frac{D}{2i} e^{-ikl}, \quad D_- = -\frac{D}{2i} e^{ikl}
\end{aligned}
\tag{76}
$$

so that $\theta_{1,3}$ are given through simple expressions:

$$
\begin{aligned}
\theta_1(z) &= A \sin\left[k(z+l)\right] \\
\theta_3(z) &= D \sin\left[k(z-l)\right]
\end{aligned}
\tag{77}
$$

These, along with θ_2, lead to the concrete form of "alignment conditions" in $z = \pm d$

$$
\begin{aligned}
&\theta_1(-d) = \theta_2(-d), \ \theta_2(d) = \theta_3(d) \\
&\frac{d\theta_1}{dz}(-d) = \frac{d\theta_2}{dz}(-d), \ \frac{d\theta_2}{dz}(d) = \frac{d\theta_3}{dz}(d)
\end{aligned}
\tag{78}
$$

namely

$$
\begin{aligned}
e^{-qd} B + e^{qd} C &= A \sin\left[k(l-d)\right] \\
q e^{-qd} B - q e^{qd} C &= kA \cos\left[k(l-d)\right] \text{ in } z = -d \\
e^{qd} B + e^{-qd} C &= -D \sin\left[k(l-d)\right] \\
q e^{qd} B - q e^{-qd} C &= kD \cos\left[k(l-d)\right] \text{ in } z = d
\end{aligned}
\tag{79}
$$

Due to the algebraic form of the two equation pairs, in order to establish the actual expression of the "secular equation" (for eigenvalues E of the energy), $\Delta[E] = 0$, we avoid calculating the 4th order determinant, $\Delta[k(E), q(E)]$, formed with the fractal/multifractal amplitude coefficients A, B, C, D, by employing the following: we note with ρ the ratio C/B, and we divide the first equation to the second one, for each pair. It results in

$$\frac{e^{2qd}\rho + 1}{e^{2qd}\rho - 1} = -\frac{q}{k}\tan[k(l - d)]$$

$$\frac{e^{-2qd}\rho + 1}{e^{-2qd}\rho - 1} = \frac{q}{k}\tan[k(l - d)] \tag{80}$$

which leads to the equation for ρ:

$$\frac{e^{2qd}\rho + 1}{e^{2qd}\rho - 1} + \frac{e^{-2qd}\rho + 1}{e^{-2qd}\rho - 1} = 0 \tag{81}$$

We find

$$\rho^2 = 1$$

which implies

$$\rho_- = -1, \rho_+ = 1 \tag{82}$$

For $\rho_+ = 1$, the amplitude function, $\theta_2(z) \cong \coth(qz)$, is symmetric just as the fractal/multifractal states of cholesterol with regard to the (spatial) reflectivity against the origin. Then the permitted value equation of the energy of these states, E_s, has the actual form:

$$\tan[k_S(l - d)] = -\frac{\coth(q_S d)}{q_S}k_S \tag{83}$$

where

$$k_S = \left[\frac{E_S}{m_0\lambda^2(dt)^{(4/D_F)-2}}\right]^{1/2}$$

$$q_S = \left[\frac{V_0 - E_S}{m_0\lambda^2(dt)^{(4/D_F)-2}}\right]^{1/2} \tag{84}$$

For $\rho_- = -1$, the amplitude function $\theta_2(z) \cong \sinh(qz)$, so that the states will be antisymmetric and permitted values equation, E_A, becomes

$$\tan[k_A(l - d)] = -\frac{\tanh(q_A d)}{q_A}k_A \tag{85}$$

where

$$k_A = \left[\frac{E_A}{m_0\lambda^2(dt)^{(4/D_F)-2}}\right]^{1/2}$$

$$q_A = \left[\frac{V_0 - E_A}{m_0\lambda^2(dt)^{(4/D_F)-2}}\right]^{1/2} \tag{86}$$

It results in, for now, at least qualitatively that the presence of the barrier (of finite height V_0) between *-d* and *d* leads to the splitting of the fundamental level E_0 into two sublevels E_s and E_A accounting for the two types of fractal/multifractal states, symmetric and antisymmetric, respectively, in which the system can be found (both states can be associated to LDL and HDL). Because both eigenvalue equations are strongly transcendent, a direct estimation of solutions $E_{s,A}$ could be possible only by means of numerical methods, which in our case is not necessary. More precisely, we can see here a process of coupling between two different fractal/multifractal (LDL and HDL) states, made possible through a fractal/multifractal tunneling effect.

Taking the above into account, we can thus state that LDL and HDL are two different states of the same biostructure, like in the case of neutron and proton which are two different states of the same particle, named nucleon. The state transfer between LDL and HDL occurs by means of a fractal/multifractal tunneling effect (**Figure 5**).

The fact presented above is in accordance with the latest study results. Thus, we can unequivocally state that the role of cholesterol fractions must be clearly reconsidered. Our model could offer an explanation of why high values of HDL cholesterol can be "toxic" or why, in certain conditions, LDL cholesterol can be a protective factor. We can practically discuss about different states of the same entity, HDL and LDL being expressions of a unique entity—cholesterol—with a pro- or antiatherogenic effect modeled by the instant state and the alternation between the two possible sides. As a consequence, as long as cholesterol fractions maintain a continuous "fluidity," the maximum benefit will be attained if the total cholesterol, in absolute value, is decreased. Our mathematical model only enforces the recent medical findings in the field, which are more and more frequent. At the same time, in our opinion, the present mathematical model confirms and explains the apparent paradoxes from clinical studies.
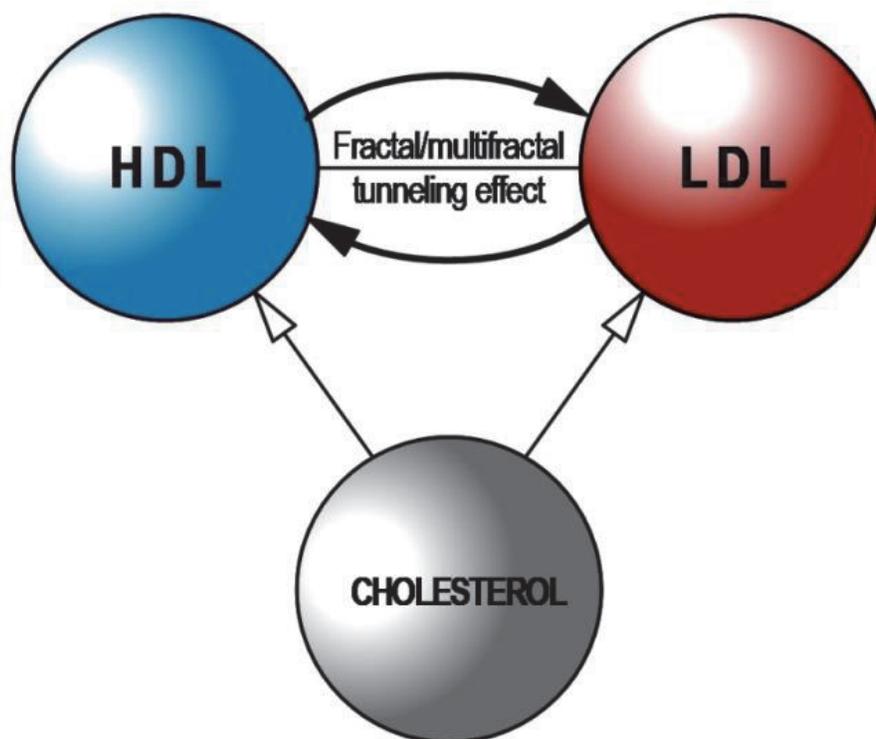
Figure 5.
Schematic representation of the chameleonic behavior of cholesterol.

The mathematical model developed here allows also some numerical evaluations on both the time of transfer between the LDL and HDL states and on the probability of achieving such a transfer. Thus, having in view the nonrelativistic relations,

$$E = 2m_0\lambda(dt)^{(2/D_F-1)}\frac{1}{\tau} \tag{87}$$

$$E = \frac{m_0 v_0^2}{2}$$

one gets through $\lambda = \alpha v_0$, in the case of motion on Peano curves of the cholesterol particles, a time of transfer τ, of the state, of the form

$$\tau = \frac{4\alpha}{v_0} \tag{88}$$

In the relations (87) and (88), α is the dimension of the cholesterol particle and v_0 the blood flow speed, e.g., knowing that in the arteries the average speed of the blood flow is $v_0 \approx 12$ cm/s [10, 11] and the average dimensions of the cholesterol particles are $\alpha_{HDL} \approx 9$ nm and $\alpha_{LDL} \approx 25$ nm, then through (88) we get $\tau_{HDL} \approx 0.189$ μs and $\tau_{LDL} \approx 0.526$ μs. Accordingly, the HDL → LDL transition is faster than the inverse one.

3. Conclusions

The main conclusions of the present work are as follows: (i) we develop a dynamics of the biological systems on a fractal space-time manifold. In such a context, we build the motion operator and the equations of geodesics for rotational and irrotational motions on non-differentiable curves induced by Markov and non-Markov type *stochasticities*, and we establish correlations with known theories of motion (relativity theory, de Broglie relativistic model, etc.). (ii) In the two-dimensional relativistic case, we determine both the velocity field and the vortex one of the cholesterol type biological structure. Based on these we show that the process of wall deposition of the LDL cholesterol is much more accentuated than the HDL cholesterol; (iii) using a multifractal Schrödinger-type equation, we show that by spontaneous symmetry breaking HDL transforms into LDL and vice versa by means of a fractal tunneling effect. We calculate the time transfer probability HDL ↔ LDL, and we show that the HDL → LDL process is more probable than the inverse one.

Author details

Maricel Agop[1,2], Calin Buzea[3]*, Decebal Vasincu[4] and Daniel Timofte[5]

1 Technical University "Gheorghe Asachi", Iasi, Romania

2 Academy of Romanian Scientists, București, Romania

3 National Institute of Research and Development for Technical Physics, Iasi, Romania

4 Biophysics Department, University of Medicine and Pharmacy "Grigore T. Popa", Iasi, Romania

5 Surgical Department, University of Medicine and Pharmacy "Grigore T. Popa", Iasi, Romania

*Address all correspondence to: calinb2003@yahoo.com

IntechOpen

References

[1] Nottale L. Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity. Singapore: World Scientific; 1993

[2] Nottale L. Scale Relativity and Fractal Space-Time—A New Approach to Unifying Relativity and Quantum Mechanics. London: Imperial College Press; 2011

[3] Nottale L. Scale relativity: A fractal matrix for organization in nature. Electronic Journal of Theoretical Physics. 2007;**4**(16(III)):15-102

[4] Mandelbrot B. The Fractal Geometry of Nature (Updated and Augm. Ed.). New York: W.H. Freeman; 1983

[5] Merches I, Agop M. Differentiability and Fractality in Dynamics of Physical Systems. Singapore: World Scientific; 2016

[6] Agop M, Paun VP. On the New Perspectives of Fractal Theory. Fundaments and Applications. Bucuresti: Romanian Academy Publishing House; 2017

[7] Agop M, Merches I. Operation Procedures Describing Physical Systems. Boca Raton, Florida, United States: CRC Press, Taylor and Francis Group; 2019

[8] Cresson J. Scale calculus and the Schrödinger equation. Journal of Mathematical Physics. 2003;**44**(11): 4907-4938

[9] Fogelman AM. Further evidence that high-density lipoprotein is a chameleon-like lipoprotein. European Heart Journal. 2015;**36**(43):3017-3019

[10] Zewinger S, Drechler C, Kleber ME, Dressel A, Riffel J, Triem S, et al. Serum amyloid A (SAA): High-density lipoproteins (HDL) interaction and cardiovascular risk. European Heart Journal. 2015;**36**:3007-3016

[11] Kathiresan S, Melander O, Anevski D, et al. Polymorphisms associated with cholesterol and risk of cardiovascular events. The New England Journal of Medicine. 2008;**358**: 1240-1249

[12] Ravnskov U, Diamond DM, Hama R, et al. Lack of an association or an inverse association between low-density-lipoprotein cholesterol and mortality in the elderly: a systematic review. BMJ Open 2016;**6**:e010401. DOI: 10.1136/bmjopen-2015-010401

[13] Chaichian M, Nelipa NF. Introduction to Gauge Field Theories. Berlin, Heidelberg: Springer-Verlag; 1984