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Improving Heat-Engine Performance via High-Temperature Recharge

Jack Denur

Abstract

Perfect (reversible) cyclic heat engines operate at Carnot efficiency. Perfect (reversible) nonheat engines and noncyclic heat engines operate at unit (100%) efficiency. But a usually necessary, although not always sufficient, requirement to achieve reversibility is that an engine must operate infinitely slowly, i.e., quasi-statically. And infinitely slow operation, which implies infinitesimally small power output, is obviously impractical. Most real heat engines operate, if not at maximum power output, then at least closer to maximum power output than to maximum efficiency. Endoreversible heat engines delivering maximum power output operate at Curzon-Ahlborn efficiency. Irrespective of efficiency, engines' work outputs are in almost all cases totally frictionally dissipated as heat immediately (e.g., an automobile operating at constant speed) or on short time scales. But if a heat engine's work output must be frictionally dissipated, it is best to dissipate it not into the cold reservoir but at the highest practicable temperature. We dub this as high-temperature recharge (HTR). This is not always practicable. But if it is practicable, it can yield improved heat-engine performance. We discuss improvements of the Carnot and Curzon-Ahlborn efficiencies achievable via HTR, and show consistency with the First and Second Laws of Thermodynamics. We reply to criticisms of HTR.

Keywords: Carnot efficiency, Curzon-Ahlborn efficiency, entropy, First Law of Thermodynamics, Second Law of Thermodynamics, heat engines, high-temperature recharge (HTR)

1. Introduction, overview, and general considerations

Perfect (reversible) cyclic heat engines operate at Carnot efficiency [1–7]. Perfect (reversible) nonheat engines and noncyclic (necessarily one-time, single-use) heat engines operate at unit (100%) efficiency. A simple example of a noncyclic heat engine is the one-time expansion of a gas pushing a piston. (If the expansion is isothermal, the heat is supplied from the internal energy of a reservoir; if is adiabatic, the heat is supplied from the internal energy of the gas itself. A polytropic expansion is intermediate between these two extremes.) Other examples include rockets: the piston (payload) is launched into space by a one-time power stroke (but typically most of the work output accelerates the exhaust gases, not the payload) and firearms: the piston (bullet) is accelerated by a one-time power stroke and then discarded (but some, typically less than with rockets, of the work output

accelerates the gases resulting from combustion of the propellant). (Some rocket engines, e.g., those employed in the Space Shuttle and by SpaceX, can be refurbished and reused, but both the first use and each subsequent refurbishment and reuse constitute a necessarily one-time, single-use of a *noncyclic* rocket heat engine.)

But a usually necessary [1–7], although not always sufficient, requirement to achieve reversibility is that a heat engine, whether cyclic or noncyclic—indeed any engine, heat engine or otherwise—must operate infinitely slowly, i.e., quasi-statically [1–7]. And infinitely slow operation, which implies infinitesimally small power output, is obviously impractical [1–7]. Indeed, some types of friction, such as sliding and rolling friction, do *not* vanish as the speed of operation becomes infinitely slow [8–11] [see also Ref. [1], Sections 5-2, 6-1, 6-2, 8-1, 11-1, and 11-2; Ref. [2], Section 4.2 (especially the 3rd and 4th paragraphs) and Figure 4.3; Ref. [3], Problem 4.2-1; and Ref. [4], Section 3.6]. In such cases, reversibility does *not* obtain *even with* infinitely slow, i.e., quasi-static, operation [8–11]. By contrast, for example, in cosmology, reversibility may in some cases obtain even with processes occurring at finite rates [12], but of course this is not relevant with respect to practical heat engines.

Most real heat engines operate, if not at maximum power output, then at least closer to maximum power output than to maximum efficiency. Assuming endoreversibility (irreversible heat flows directly proportional to finite temperature differences but otherwise reversible operation), at maximum power output cyclic heat engines operate at Curzon-Ahlborn efficiency [13–15] (see also Ref. [3], Section 4-9). The work outputs of heat engines—indeed of *all* engines, heat engines or otherwise—are in almost cases totally frictionally dissipated as heat immediately or on short time scales [16, 17]. For example, an automobile’s cyclic heat engine’s work output in initially accelerating the automobile is typically frictionally dissipated only a short time later the next time the automobile decelerates; its work output while the automobile travels at constant speed is immediately and continually frictionally dissipated. [Rare exceptions include, for example, a noncyclic rocket heat engine’s work output being sequestered essentially permanently as kinetic and gravitational potential energy in the launching of a spacecraft (but typically most of the kinetic energy accelerates the exhaust gases, not the payload) and a cyclic heat engine’s work output being sequestered for a long time interval as gravitational potential energy in the construction of a building.]

We note that the *work* output of *any* engine (heat engine or otherwise) can be dissipated *only via friction*. Additional losses can, and almost always if not always, also occur, for example, irreversible heat losses engendered by finite temperature differences (no insulation is perfect). [The Curzon-Ahlborn efficiency [13–15] (see also Ref. [3], Section 4-9) takes into account losses due to irreversible heat flows directly proportional to finite temperature differences but assumes otherwise reversible operation.] But such heat losses are *not* work. An engine’s *work* output per se can be dissipated *only via friction*. This is true because work is a *force* exerted through a distance: thus work can be dissipated *only* by an opposing *force* that is *nonconservative*. And *nonconservative force* is *friction*. [It might be contended that, ultimately, friction is the electromagnetic force, which is conservative. But for all typical macroscopic motions, for which the kinetic energy in any given degree of freedom greatly exceeds $k_B T$ (k_B is Boltzmann’s constant, T is the temperature), friction is *effectively nonconservative*.]

But if a heat engine’s work output must be frictionally dissipated, it is best to dissipate it not at the temperature of its cold reservoir but instead at the highest practicable temperature. This is consistent with the Second Law of Thermodynamics, which allows frictional dissipation of work into heat at *any temperature* [1–7]

(in Ref. [6], see pp. 11–12, 60–65, and 263–265, especially pp. 263–265). The entropy increase resulting from frictional dissipation of work W at temperature T , namely $\Delta S = W/T$, decreases monotonically with increasing T but is positive for any finite T —and the Second Law requires only that $\Delta S \geq 0$ [1–7].

We should emphasize that the entropy increase $\Delta S = W/T$ associated with work W being frictionally dissipated at temperature T is *always* $\Delta S = W/T$ and hence *always* decreases monotonically with increasing T . Whether the coefficient of friction is small or large *makes no difference* in $\Delta S = W/T$. Of course, all other things being equal, if the coefficient of friction is small, frictional dissipation of W will take longer than if the coefficient of friction is large. But the entropy increase $\Delta S = W/T$ associated with work W being frictionally dissipated at temperature T is the same whether the coefficient of friction is small or large. Even with coefficients of friction in the low range, W will typically be frictionally dissipated immediately or on short time scales.

Of course, efficiency is highest if work W , whether supplied via a heat engine or otherwise, is not frictionally dissipated at all. This would obtain, for example, in perfect (reversible) regenerative braking of an electrically-powered motor vehicle, with the motor operating backward as a generator during braking. It also would obtain, for example, if a noncyclic rocket heat engine's work output is perfectly (reversibly) sequestered as kinetic and gravitational potential energy in the launching of a spacecraft (but typically most of the kinetic energy accelerates the exhaust gases, not the payload) or if a cyclic heat engine's work output is perfectly (reversibly) sequestered as gravitational potential energy in the construction of a building. But of course in practice (as opposed to in principle) *total* avoidance of frictional dissipation of work [and also of additional losses, e.g., due to irreversible heat flows engendered by finite temperature differences (no insulation is perfect)] is not possible.

Although we do not consider them in this chapter, we should note that: (a) There are generalizations of the Curzon-Ahlborn efficiency [13–15] (see also Ref. [3], Section 4-9) at maximum power output both for macroscopic heat engines [18–20] and for microscopic heat engines [21, 22], with irreversible heat flows *not* necessarily directly proportional to temperature differences. (b) There are analyses of maximum heat-engine work output per cycle (as opposed to maximum power output) [23]. Comprehensive discussions concerning the Curzon-Ahlborn efficiency and generalizations thereof are provided in Refs. [24–26]. Some, but not all, such generalized efficiencies [18–26] do not differ greatly from the Curzon-Ahlborn efficiency [13–15] (see also Ref. [3], Section 4-9). In particular, we note that alternative results [26] to the Curzon-Ahlborn efficiency have been derived [26]. But for definiteness and for simplicity, in this chapter, we employ the standard Curzon-Ahlborn efficiency [13–15] (see also Ref. [3], Section 4-9) for cyclic heat engines operating at maximum power output.

A misconception pertaining to the efficiencies of engines (heat engines or otherwise) is discussed and corrected in Section 2.

In Section 3, we review the work outputs, efficiencies, and entropy productions of Carnot (reversible) and Curzon-Ahlborn (endoreversible) heat engines, first without frictional dissipation of a heat engine's work output and then with frictional dissipation thereof into its cold reservoir. In Section 3, we do not consider frictional dissipation of a heat engine's work output at the highest practicable temperature, which we dub as high-temperature recharge (HTR).

In Section 4, we discuss the work outputs, efficiencies, and entropy productions of Carnot and Curzon-Ahlborn heat engines operating with frictional dissipation of a heat engine's work output at the highest practicable temperature—which we dub as high-temperature recharge (HTR)—and the improvements thereof over those

obtainable (as per Section 3) without HTR. Cases wherein HTR is practicable include, but are not necessarily limited to, (a) hurricanes, which via HTR are rendered more powerful than they would otherwise be [27–37], (b) thermoelectric generators [38], and (c) heat engines *powered by a cold reservoir*, employing ambient as the *hot* reservoir, for example, heat engines powered by the evaporation of water [39–51] or by liquid nitrogen [52], ocean-thermal-energy-conversion (OTEC) heat engines [53–56], and heat engines powered by the cold of outer space [57].

Concerning (a) in the immediately preceding paragraph, on the one hand, the importance of HTR (dubbed as “dissipative heating”) has been confirmed in a study of Hurricane Andrew (1992) [36], and, as one might expect, “dissipative heating appears to be a more important process in intense hurricanes, such as Andrew, than weak ones” [36]. But, on the other hand, more recently it has been contended [37] that, while HTR exists in hurricanes, it is of lesser importance than previously supposed [27–36]. [There are occasional speculations concerning extracting useful energy from hurricanes (with or without help from HTR) and also freshwater. But, of course, except for (strongly built!) windmills and ocean-wave-powered generators for extracting energy and reservoirs for extracting freshwater, this is beyond currently available (and perhaps even currently foreseeable) technology. To the extent that HTR increases wind speeds in hurricanes, it increases the power flux density available to (strongly built!) windmills and ocean-wave-powered generators: wind power flux density is proportional to the cube of the wind speed (and directly proportional to the air density). But, at least for the time being, the main (or perhaps even only) employment of HTR in hurricanes is by the hurricanes themselves, to increase their wind speeds, whether as previously supposed [27–36] or to a lesser degree [37].

To the best knowledge of the author, the concept of HTR was first partially and qualitatively broached by Spanner (see Ref. [6], pp. 11–12, 60–65, and 263–265, especially pp. 263–265) and, later, was first fully and quantitatively expounded and developed by Emanuel [27–34] in the course of his research concerning hurricane science. It was subsequently employed by Apertet et al. [38] for increasing efficiencies of thermoelectric generators. In these works [6, 27–34, 38] and in related works [35–37, 58–62], the concept is not dubbed HTR, but of course it is the concept itself, and not the dubbing it with a name, that is important. To the best knowledge of the author, the concept has not been dubbed HTR (dubbed, if at all, as “dissipative heating”) in the previous literature. Heat engines employing it have previously been dubbed “dissipative engines” (see, e.g., Refs. [58–60]).

The increases in efficiency attainable via HTR are not practicable if frictional dissipation of work into other than the cold reservoir is not practicable. Thus they are *never* practicable for noncyclic (necessarily one-time, single-use) heat engines: however the work output of a noncyclic (necessarily one-time, single-use) heat engine might be frictionally dissipated, the heat thereby generated *cannot* restore the engine to its initial state. Moreover in many cases the work outputs of noncyclic (necessarily one-time, single-use) heat engines are not frictionally dissipated *at all*, at least not during practicable time scales, for example, a noncyclic rocket heat engine’s work output is sequestered essentially permanently as kinetic and gravitational potential energy in the launching of a spacecraft (but typically most of the kinetic energy accelerates the exhaust gases, not the payload). They also are *never* practicable for reverse operation of cyclic heat engines as refrigerators or heat pumps, because for both refrigerators and heat pumps, the *total* energy output (the work W , plus the heat Q_C extracted from a cold reservoir at the expense of W as required by the Second Law of Thermodynamics) *always* is deposited as heat Q_H into the hot reservoir ($Q_H = Q_C + W$): thus there is *never* any *additional* energy to be deposited into the hot reservoir (as there is from frictional dissipation of work

done via forward operation of cyclic heat engines). {See Ref. [1], Section 20-3; Ref. [2], Sections 4.3, 4.4, and 4.7 (especially Section 4.7); Ref. [3], Sections 4-4, 4-5, and 4-6 (especially Section 4-6); Ref. [5], Section 5.12 and Problem 5.22; Ref. [7], pp. 233–236 and Problems 1, 2, 4, 6, and 7 of Chapter 8; Ref. [16], Chapter XXI; Ref. [17], Sections 6.7, 6.8, 7.3, and 7.4; and Ref. [54], Sections 5-7-2, 6-2-2, 6-9-2, and 6-9-3, and Chapter 17. [Problem 2 of Chapter 8 in Ref. [7] considers absorption refrigeration, wherein the entire energy output is into an *intermediate*-temperature (most typically ambient-temperature) reservoir, and hence for which HTR is *even more strongly never practicable*.]} They also are not practicable for cyclic heat engines in cases wherein a cyclic heat engine's work output is not frictionally dissipated immediately or on short time scales [16, 17], for example, as gravitational potential energy sequestered for a long time interval in the construction of a building. For a building once erected typically remains standing for a century or longer. Even if, when it is finally torn down, its gravitational potential energy were to be totally frictionally dissipated into a hot reservoir, it is simply impracticable to wait that long. Thus HTR is not practicable in all cases. But in the many cases wherein cyclic heat engines' work outputs are frictionally dissipated immediately or on short time scales [16, 17], practicability obtains: improved conversion—and reconversion—of frictionally dissipated heat into work, and hence improved cyclic heat-engine performance, can then obtain.

Since HTR is *never* practicable for noncyclic (necessarily one-time, single-use) heat engines such as rockets or firearms, or for reverse operation of cyclic heat engines as refrigerators or heat pumps, henceforth we will (except where otherwise mentioned) focus exclusively on forward operation of cyclic heat engines.

Note that the primarily relevant time scale pertaining to “in the many cases wherein cyclic heat engines' work outputs are frictionally dissipated immediately or on short time scales [16, 17]” is (i) the time interval between a cyclic heat engine's work output and frictional dissipation of this work output [16, 17], *not* (ii) the time interval required for frictional dissipation per se. The time interval (ii) is *zero* in all cases wherein work is done against the nonconservative force of friction and hence frictionally dissipated *immediately*. Indeed, in *all* cases of steady-state engine operation against friction (e.g., an automobile traveling at constant speed) *both* time intervals are *zero*. This is by far the most common mode of engine operation. Even work output sequestered when an engine is started or when an automobile accelerates is typically frictionally dissipated only a short time later, when the engine is turned off or when the automobile decelerates. Work output sequestration for a century or longer can obtain (as gravitational potential energy) in the construction of buildings and essentially permanently in the launchings of spacecraft—but these are rare exceptions. Thus we focus on time interval (i): a necessary (but not sufficient) condition for HTR to be practicable is that the time interval (i) be zero or at most short. (This condition is automatically met in steady-state engine operation, wherein work is frictionally dissipated *immediately* and hence *both* time intervals are *zero*.)

But for cyclic heat engines whose work outputs typically are frictionally dissipated immediately or on short time scales [16, 17], HTR often *is* practicable. For cyclic heat engines employing ambient as the *cold* reservoir, the existent hot reservoir is likely already at the practicable upper temperature limit. Hence for these cyclic heat engines, HTR at the temperature of the hot reservoir could increase efficiency, but HTR at a still higher temperature probably would not be practicable. By contrast, consider cyclic heat engines *powered by a cold reservoir*, employing ambient as the *hot* reservoir, for example, cyclic heat engines powered by the evaporation of water [39–51] or by liquid nitrogen [52], ocean thermal-energy-conversion (OTEC) heat engines [53–56], and heat engines powered by the cold of outer space [57]. For *these* cyclic heat engines, HTR at a higher temperature than ambient probably

would be practicable. For *these* cyclic heat engines, employment of HTR could boost the temperature of the hot reservoir from ambient to the highest practicable temperature for HTR.

Henceforth if HTR is employed we construe the terms “the highest practicable temperature for HTR” and “the hot reservoir” to be synonymous.

Recapitulation and generalization are provided in Section 5. A reply to criticisms [58, 59] of HTR is provided in Section 6 and in references cited therein. Concluding remarks are provided in Section 7.

2. Correcting a misconception pertaining to the efficiencies of engines

The efficiency of any engine in general is its work output (force-times-distance output) divided by its energy input, and the efficiency of a heat engine in particular it is its work output (force-times-distance output) divided by its heat-energy input. What happens to an engine’s work output after the work has been done is an entirely different issue.

Work can be done either against a conservative force, in which case it is sequestered, or against the nonconservative force of friction, in which case it is dissipated as heat. To re-emphasize, in either case—whether the opposing force is conservative or nonconservative—the efficiency of any engine in general is its work output (force-times-distance output) divided by its energy input, and the efficiency of a heat engine in particular it is its work output (force-times-distance output) divided by its heat-energy input. What happens to an engine’s work output after the work has been done is an entirely different issue.

In this Section 2, we wish to correct a misconception that is sometimes made, according to which an engine’s efficiency can exceed zero only if its work output is done against a conservative force. This misconception is erroneous.

In the vast majority of cases, for almost all engines on Earth, work is done against the nonconservative force of friction, and hence instantaneously and continually dissipated as heat. The engines work at steady state, and while working, their internal energy and the internal energy of any equipment they might be operating do not change. Consider, for example, the engine of any automobile, train, ship, submarine, or aircraft traveling at constant speed, any factory or workshop engine such as a power saw operating at constant speed, or any domestic appliance engine such as that of a dishwasher, refrigerator, etc., operating at constant speed. According to the erroneous misconception that an engine’s efficiency is zero if its work output is done against the nonconservative force of friction, the efficiency of all of these engines—indeed of almost all engines on Earth—would falsely be evaluated at zero. If their efficiencies were truly zero, they could do zero work against *any* opposing force, conservative or nonconservative, i.e., they could not operate *at all*. A specific example is the following: If the efficiency of an engine (heat engine or otherwise) attempting to maintain an automobile at constant speed was zero, the engine could do zero work against friction, and the automobile’s speed would also be zero.

Only in rare cases, such as the construction of buildings and the launchings of spacecraft, is the work done even against conservative forces (e.g., gravity, inertia, etc.) sequestered for any significant lengths of time. Even in most cases wherein work is done against a conservative force, it is frictionally dissipated a short time later. For example, the work done in accelerating an automobile against its own inertia is typically frictionally dissipated as heat a short time later the next time the automobile decelerates. The net effect of the acceleration/deceleration process is frictional dissipation of the automobile’s temporarily sequestered kinetic energy, the same as the instantaneous and continual frictional dissipation of its kinetic energy while it operates at constant speed.

In general, when an engine (heat engine or otherwise) is turned on, part of its work output is sequestered as its own kinetic energy and the kinetic energy of any equipment that it might be operating. But this kinetic energy is frictionally dissipated as heat when the engine is turned off, so the net effect of the on/off process is frictional dissipation of this temporarily sequestered kinetic energy, the same as the instantaneous and continual frictional dissipation of the engine's work output while it operates at constant speed between the time it is turned on and the time it is turned off.

3. Carnot and Curzon-Ahlborn efficiencies without high-temperature recharge (HTR)

The standard (without high-temperature recharge or HTR) Carnot efficiency $\epsilon_{\text{Carnot, std}}$ and Curzon-Ahlborn efficiency $\epsilon_{\text{CA, std}}$ corresponding to heat-engine operation between a hot reservoir at temperature T_H and a cold reservoir at temperature T_C are given by the respective well-known formulas:

$$\epsilon_{\text{Carnot, std}} = \frac{W}{Q_H} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H} \equiv 1 - R_T \quad (1)$$

and

$$\epsilon_{\text{CA, std}} = \frac{W}{Q_H} = \frac{T_H^{1/2} - T_C^{1/2}}{T_H^{1/2}} = 1 - \left(\frac{T_C}{T_H}\right)^{1/2} \equiv 1 - R_T^{1/2}. \quad (2)$$

We define the *temperature* ratio between a heat engine's cold and hot reservoirs as $R_T \equiv T_C/T_H$. Obviously in all cases $0 \leq R_T \leq 1$. The case $R_T = 1$ is of no interest because it corresponds to zero efficiency of any heat engine. The case $R_T = 0$ is unattainable because there is no cold reservoir at absolute zero (0 K) [63], and even if there was [63], it would no longer be at 0 K the instant after a heat engine began operating and exhausting its waste heat into it [see also Ref. [2], Chapter 10 (especially Section 10.4); Ref. [3], Chapter 11 (especially Section 11-3); and Ref. [4], Chapter 14 (especially Sections 14.3–14.5 and 14.7)]. Hence we confine our attention to the range $0 < R_T < 1$.

The *efficiency* ratio $R_{\epsilon 1}$ between these efficiencies is

$$R_{\epsilon 1} = \frac{\epsilon_{\text{Carnot, std}}}{\epsilon_{\text{CA, std}}} = \frac{1 - R_T}{1 - R_T^{1/2}}. \quad (3)$$

$R_{\epsilon 1}$ increases monotonically with increasing R_T . In the limit $R_T \rightarrow 0$, it is obvious that $R_{\epsilon 1} \rightarrow 1$. In the limit $R_T \rightarrow 1$, $R_{\epsilon 1} \rightarrow 2$. The latter limit is most easily demonstrated by setting $R_T = 1 - \delta$, letting $\delta \rightarrow 0$, and applying the binomial theorem. This yields

$$\lim_{R_T \rightarrow 1} R_{\epsilon 1} = \lim_{\delta \rightarrow 0} R_{\epsilon 1} = \lim_{\delta \rightarrow 0} \frac{1 - (1 - \delta)}{1 - (1 - \delta)^{1/2}} = \frac{\delta}{1 - (1 - \frac{1}{2}\delta)} = \frac{\delta}{\frac{1}{2}\delta} = 2. \quad (4)$$

By the First and Second Laws of Thermodynamics, for a standard reversible heat engine operating (without HTR) at Carnot efficiency, the heat input Q_H from its

hot reservoir, the work output W , the waste heat Q_C exhausted to its cold reservoir, the efficiency $\epsilon_{\text{Carnot, std}}$, the entropy change $\Delta S_{H, \text{Carnot, std}}$ of its hot reservoir, the entropy change $\Delta S_{C, \text{Carnot, std}}$ of its cold reservoir, and the total entropy change $\Delta S_{\text{total, std}} = \Delta S_{H, \text{Carnot, std}} + \Delta S_{C, \text{Carnot, std}}$ are related in accordance with

$$\begin{aligned} W &= Q_H - Q_C = Q_H \epsilon_{\text{Carnot, std}} = Q_H(1 - R_T) \\ &\Rightarrow Q_C = R_T Q_H. \end{aligned} \quad (5)$$

and

$$\Delta S_{\text{total, Carnot, std}} = \Delta S_{H, \text{Carnot, std}} + \Delta S_{C, \text{Carnot, std}} = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C} = -\frac{Q_H}{T_H} + \frac{R_T Q_H}{R_T T_H} = 0. \quad (6)$$

We note that, in most derivations (in textbooks or elsewhere) of $\epsilon_{\text{Carnot, std}}$, $\Delta S_{\text{total, Carnot, std}} = 0$ is invoked as the initial Second Law assumption or postulate and is employed along with the initial First Law postulate $W = Q_H - Q_C$ [1–7].

Similarly, by the First and Second Laws of Thermodynamics, for a standard endoreversible heat engine operating (without HTR) at Curzon-Ahlborn efficiency, the heat input Q_H from its hot reservoir, the work output W , the waste heat Q_C exhausted to its cold reservoir, the efficiency $\epsilon_{\text{CA, std}}$, the entropy change $\Delta S_{H, \text{CA, std}}$ of its hot reservoir, the entropy change $\Delta S_{C, \text{CA, std}}$ of its cold reservoir, and the total entropy change $\Delta S_{\text{total, CA, std}} = \Delta S_{H, \text{CA, std}} + \Delta S_{C, \text{CA, std}}$ are related in accordance with

$$\begin{aligned} W &= Q_H - Q_C = Q_H \epsilon_{\text{CA, std}} = Q_H(1 - R_T^{1/2}) \\ &\Rightarrow Q_C = R_T^{1/2} Q_H. \end{aligned} \quad (7)$$

and

$$\begin{aligned} \Delta S_{\text{total, CA, std}} &= \Delta S_{H, \text{CA, std}} + \Delta S_{C, \text{CA, std}} = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C} = -\frac{Q_H}{T_H} + \frac{R_T^{1/2} Q_H}{R_T T_H} \\ &= \frac{Q_H}{T_H} \left(\frac{1}{R_T^{1/2}} - 1 \right) = \frac{Q_H}{T_H} \left(R_T^{-1/2} - 1 \right) \\ &> \Delta S_{\text{total, Carnot, std}} = 0. \end{aligned} \quad (8)$$

Note that for any R_T in general and as $R_T \rightarrow 0$ in particular, $\Delta S_{\text{total, Carnot, std}} = 0$; by contrast, for any R_T in general $\Delta S_{\text{total, CA, std}} > 0$ ($\Delta S_{\text{total, CA, std}} \rightarrow 0$ only in the limit $R_T \rightarrow 1$), and as $R_T \rightarrow 0$, $\Delta S_{\text{total, CA, std}} \rightarrow \infty$.

As we have already noted, heat engines' work outputs are, in almost all cases, totally frictionally dissipated as heat immediately or on short time scales [16, 17]. For example, an automobile heat engine's work output in initially accelerating the automobile is typically frictionally dissipated only a short time later the next time the automobile decelerates; its work output while the automobile travels at constant speed is immediately and continually frictionally dissipated. [Rare exceptions include, for example, a noncyclic rocket heat engine's work output being sequestered essentially permanently as kinetic and gravitational potential energy in the launching of a spacecraft (but typically most of the kinetic energy accelerates the

exhaust gases, not the payload) and a cyclic heat engine's work output being sequestered for a long time interval as gravitational potential energy in the construction of a building.] Apart from such rare exceptional cases, in the operation of *any* cyclic heat engine operating at *any* efficiency without HTR—whether reversible at Carnot efficiency, endoreversible at Curzon-Ahlborn efficiency, or otherwise—not only is the waste heat Q_C immediately exhausted into the cold reservoir, but also the heat engine's work output W is frictionally dissipated into the cold reservoir immediately or on short time scales [16, 17]. Thus, apart from such rare exceptional cases, the *ultimate* total entropy increase $\Delta S_{\text{total,ultimate}}$ resulting from the operation of *any* heat engine without HTR at *any* efficiency occurs as soon as the work is done or shortly thereafter and is [1–7, 16, 17]:

$$\begin{aligned}\Delta S_{\text{total,ultimate}} &= -\frac{Q_H}{T_H} + \frac{Q_C}{T_C} + \frac{W}{T_C} = -\frac{Q_H}{T_H} + \frac{Q_C + W}{T_C} = -\frac{Q_H}{T_H} + \frac{Q_H}{T_C} \\ &= -\frac{Q_H}{T_H} + \frac{Q_H}{R_T T_H} = \frac{Q_H}{T_H} \left(\frac{1}{R_T} - 1 \right) = \frac{Q_H}{T_H} (R_T^{-1} - 1) \quad (9) \\ &> \Delta S_{\text{total,CA,std}} = \frac{Q_H}{T_H} \left(R_T^{-1/2} - 1 \right) \\ &> \Delta S_{\text{total,Carnot,std}} = 0.\end{aligned}$$

Note that for any $R_T > 0$, the inequality $\Delta S_{\text{total,ultimate}} > 0$ is stronger than the inequality $\Delta S_{\text{total,CA,std}} > 0$. In particular, note that as $R_T \rightarrow 1$, $\Delta S_{\text{total,ultimate}} \rightarrow 0$ more slowly than $\Delta S_{\text{total,CA,std}} \rightarrow 0$, while as $R_T \rightarrow 0$, $\Delta S_{\text{total,ultimate}} \rightarrow \infty$ faster than $\Delta S_{\text{total,CA,std}} \rightarrow \infty$.

4. Carnot and Curzon-Ahlborn efficiencies with high-temperature recharge (HTR)

If, as is almost always the case, a cyclic heat engine's work output W is totally frictionally dissipated as heat immediately or on short time scales [16, 17], the engine's efficiency can be increased if this dissipation is not at the temperature of its cold reservoir but instead at the highest practicable temperature. For cyclic heat engines employing ambient as the *cold* reservoir, the existent hot reservoir is likely already at the practicable upper temperature limit. Hence for these cyclic heat engines, HTR at the temperature of the hot reservoir could increase efficiency, but HTR at a still higher temperature probably would not be practicable. By contrast, for cyclic heat engines *powered by a cold reservoir*, employing ambient as the *hot* reservoir [39–57], frictional dissipation at a higher temperature than ambient probably *would* be practicable. For *these* cyclic heat engines, employment of HTR could boost T_H , the temperature of the hot reservoir, from ambient to the the highest practicable temperature for HTR. In this Section 4, we take T_H , the temperature of the hot reservoir, to be the highest practicable temperature for frictional dissipation of a cyclic heat engine's work output into heat.

Consider first a reversible heat engine operating at Carnot efficiency. If the engine's work output $W = Q_H(1 - R_T)$ is frictionally dissipated into its hot reservoir (at temperature T_H), then the *net* heat input $Q_{H,\text{net}}$ required from its hot reservoir is reduced from Q_H to $Q_H - W = Q_H - Q_H(1 - R_T) = Q_H - Q_H R_T = Q_C$. Hence, with the help of Eqs. (1) and (5), the efficiency $\epsilon_{\text{Carnot,HTR}}$ of a Carnot engine operating with HTR is

$$\begin{aligned}\epsilon_{\text{Carnot,HTR}} &= \frac{W}{Q_{H,\text{net}}} = \frac{W}{Q_H - W} = \frac{W}{Q_C} = \frac{Q_H - Q_C}{Q_C} = \frac{Q_H}{Q_C} - 1 = \frac{T_H}{T_C} - 1 = R_T^{-1} - 1 \\ \Rightarrow R_{\epsilon 2} &= \frac{\epsilon_{\text{Carnot,HTR}}}{\epsilon_{\text{Carnot,std}}} = \frac{R_T^{-1} - 1}{1 - R_T} = R_T^{-1} > 1.\end{aligned}\quad (10)$$

Note that not only is $R_{\epsilon 2} > 1$ in all cases, but also that in some cases also $\epsilon_{\text{Carnot,HTR}} > 1$. The efficiency $\epsilon_{\text{Carnot,HTR}}$ increases monotonically from zero in the limit $R_T \rightarrow 1$ to ∞ in the limit $R_T \rightarrow 0$. And the efficiency ratio $R_{\epsilon 2}$ increases monotonically from unity in the limit $R_T \rightarrow 1$ to ∞ in the limit $R_T \rightarrow 0$. If $R_T < \frac{1}{2} \Leftrightarrow R_{\epsilon 2} > 2$, $\epsilon_{\text{Carnot,HTR}} > 1$. Yet the First and Second Laws of Thermodynamics are *not* violated. The First Law is not violated because no energy is created (or destroyed): $\epsilon_{\text{Carnot,HTR}} > 1$ if $R_T < \frac{1}{2} \Leftrightarrow R_{\epsilon 2} > 2$ obtains via recycling and reusing the same energy, not via the creation of new energy. The Second Law is not violated because the change in total entropy is positive:

$$\begin{aligned}\Delta S_{\text{total,Carnot,HTR}} &= -\frac{Q_H}{T_H} + \frac{W}{T_H} + \frac{Q_C}{T_C} \\ &= \frac{Q_C}{T_C} - \frac{Q_H - W}{T_H} = \frac{Q_C}{T_C} - \frac{Q_C}{T_H} \\ &= Q_C \left(\frac{1}{T_C} - \frac{1}{T_H} \right) = R_T Q_H \left(\frac{1}{R_T T_H} - \frac{1}{T_H} \right) \\ &= \frac{R_T Q_H}{T_H} \left(\frac{1}{R_T} - 1 \right) = \frac{R_T Q_H}{T_H} \left(\frac{1 - R_T}{R_T} \right) \\ &= \frac{Q_H}{T_H} (1 - R_T) \\ &> \Delta S_{\text{total,Carnot,std}} = 0.\end{aligned}\quad (11)$$

In Eq. (11), we applied Eqs. (5) and (6). Yet, also applying Eq. (9), $\Delta S_{\text{total,Carnot,HTR}}$, while positive and hence greater than $\Delta S_{\text{total,Carnot,std}} = 0$, is smaller than $\Delta S_{\text{total,ultimate}}$ by the *entropy ratio*

$$R_{S1} = \frac{\Delta S_{\text{total,Carnot,HTR}}}{\Delta S_{\text{total,ultimate}}} = \frac{1 - R_T}{R_T^{-1} - 1} = R_T. \quad (12)$$

The entropy ratio R_{S1} decreases monotonically from unity in the limit $R_T \rightarrow 1$ to zero in the limit $R_T \rightarrow 0$.

It may be of interest to note that $\epsilon_{\text{Carnot,HTR}}$ of Eq. (10) is the multiplicative inverse of the Carnot coefficient of performance of a refrigerator, whereas, by contrast, $\epsilon_{\text{Carnot,std}}$ of Eq. (1) is the multiplicative inverse of the Carnot coefficient of performance of a heat pump. These relations may be of interest. But, as we have noted in Section 1, HTR is *never* practicable for reverse operation of cyclic heat engines as refrigerators or heat pumps.

Consider next an endoreversible heat engine operating at Curzon-Ahlborn efficiency. If the engine's work output $W = Q_H \left(1 - R_T^{1/2} \right)$ is frictionally dissipated into its hot reservoir (at temperature T_H), then the *net* heat input $Q_{H,\text{net}}$ required from its hot reservoir is reduced from Q_H to $Q_H - W = Q_H - Q_H \left(1 - R_T^{1/2} \right) = Q_H R_T^{1/2} = Q_C$. Hence, with the help of Eqs. (2) and (7), the efficiency $\epsilon_{\text{CA,HTR}}$ of a Curzon-Ahlborn engine operating with HTR is

$$\begin{aligned}\epsilon_{CA,HTR} &= \frac{W}{Q_{H,net}} = \frac{W}{Q_H - W} = \frac{W}{Q_C} = \frac{W}{Q_H} \frac{Q_H}{Q_C} = \frac{Q_H(1 - R_T^{1/2})}{Q_H} \frac{Q_H}{Q_C} \\ &= (1 - R_T^{1/2}) \frac{Q_H}{Q_C} = (1 - R_T^{1/2}) \frac{Q_H}{R_T^{1/2} Q_H} = \frac{1 - R_T^{1/2}}{R_T^{1/2}} \\ &= R_T^{-1/2} - 1\end{aligned}\quad (13)$$

$$\Rightarrow R_{\epsilon_3} = \frac{\epsilon_{CA,HTR}}{\epsilon_{CA,std}} = \frac{R_T^{-1/2} - 1}{1 - R_T^{1/2}} = R_T^{-1/2}.$$

The efficiency $\epsilon_{CA,HTR}$ increases monotonically from zero in the limit $R_T \rightarrow 1$ to ∞ in the limit $R_T \rightarrow 0$. And the efficiency ratio R_{ϵ_3} increases monotonically from unity in the limit $R_T \rightarrow 1$ to ∞ in the limit $R_T \rightarrow 0$. If $R_T < \frac{1}{4} \Leftrightarrow R_{\epsilon_3} > 2$, $\epsilon_{CA,HTR} > 1$. Yet the First and Second Laws of Thermodynamics are *not* violated. The First Law is not violated because no energy is created (or destroyed): $\epsilon_{CA,HTR} > 1$ if $R_T < \frac{1}{4} \Leftrightarrow R_{\epsilon_3} > 2$ obtains via recycling and reusing the same energy, not via the creation of new energy. The Second Law is not violated because the change in total entropy is positive:

$$\begin{aligned}\Delta S_{total,CA,HTR} &= -\frac{Q_H}{T_H} + \frac{W}{T_H} + \frac{Q_C}{T_C} \\ &= \frac{Q_C}{T_C} - \frac{Q_H - W}{T_H} = \frac{Q_C}{T_C} - \frac{Q_C}{T_H} \\ &= Q_C \left(\frac{1}{T_C} - \frac{1}{T_H} \right) = R_T^{1/2} Q_H \left(\frac{1}{R_T T_H} - \frac{1}{T_H} \right) \\ &= \frac{R_T^{1/2} Q_H}{T_H} \left(\frac{1}{R_T} - 1 \right) \\ &= \frac{Q_H}{T_H} \left(R_T^{-1/2} - R_T^{1/2} \right) \\ &> \Delta S_{total,CA,std} = \frac{Q_H}{T_H} \left(R_T^{-1/2} - 1 \right) \\ &> \Delta S_{total,Carnot,std} = 0.\end{aligned}\quad (14)$$

In Eq. (14), we applied Eqs. (7) and (8). Yet, also applying Eq. (9), $\Delta S_{total,CA,HTR}$, while not merely positive but greater than $\Delta S_{total,CA,std}$, is smaller than $\Delta S_{total,ultimate}$ by the entropy ratio

$$R_{S2} = \frac{\Delta S_{total,CA,HTR}}{\Delta S_{total,ultimate}} = \frac{R_T^{-1/2} - R_T^{1/2}}{R_T^{-1} - 1} = R_T^{1/2}.\quad (15)$$

But, as one would expect, $\Delta S_{total,CA,HTR}$ is larger than $\Delta S_{total,Carnot,HTR}$. Applying Eqs. (11), (14), and (15), it is larger by the entropy ratio

$$R_{S3} = \frac{\Delta S_{total,CA,HTR}}{\Delta S_{total,Carnot,HTR}} = \frac{R_T^{-1/2} - R_T^{1/2}}{1 - R_T} = R_T^{-1/2} = R_{S2}^{-1}.\quad (16)$$

There is one more efficiency ratio that is of interest. Applying Eqs. (10) and (13):

$$R_{\epsilon 4} = \frac{\epsilon_{\text{Carnot,HTR}}}{\epsilon_{\text{CA,HTR}}} = \frac{R_T^{-1} - 1}{R_T^{-1/2} - 1}. \quad (17)$$

The efficiency ratio $R_{\epsilon 4}$ increases monotonically from 2 in the limit $R_T \rightarrow 1$ to ∞ in the limit $R_T \rightarrow 0$. The latter limit is obvious. The former limit is most easily demonstrated by setting $R_T = 1 - \delta$, letting $\delta \rightarrow 0$, and applying the binomial theorem. This yields

$$\lim_{R_T \rightarrow 1} R_{\epsilon 4} = \lim_{\delta \rightarrow 0} R_{\epsilon 4} = \lim_{\delta \rightarrow 0} \frac{(1 - \delta)^{-1} - 1}{(1 - \delta)^{-1/2} - 1} = \frac{(1 + \delta) - 1}{(1 + \frac{1}{2}\delta) - 1} = \frac{\delta}{\frac{1}{2}\delta} = 2. \quad (18)$$

Note that: (a) Applying Eqs. (3), (4), (17), and (18), in the limit $R_T \rightarrow 1$, $R_{\epsilon 1} \rightarrow 2$ from below; by contrast $R_{\epsilon 4} \rightarrow 2$ from above. (b) Applying Eqs. (3), (4), (17), and (18), in the limit $R_T \rightarrow 0$, $R_{\epsilon 1} \rightarrow 1$ but $R_{\epsilon 4} \rightarrow \infty$. (c) Applying Eqs. (10) and (13), in the limit $R_T \rightarrow 0$, $\epsilon_{\text{Carnot,HTR}} \rightarrow \infty$ as R_T^{-1} but $\epsilon_{\text{CA,HTR}} \rightarrow \infty$ only as $R_T^{-1/2}$; thus, while both approach ∞ , $\epsilon_{\text{Carnot,HTR}}$ does so at higher order—hence as $R_T \rightarrow 0$, $R_{\epsilon 4} \rightarrow \infty$.

As an aside, it may be of interest to note, applying Eqs. (3) and (17), that

$$\begin{aligned} \frac{R_{\epsilon 4}}{R_{\epsilon 1}} &= \frac{\frac{R_T^{-1} - 1}{R_T^{-1/2} - 1}}{\frac{1 - R_T}{1 - R_T^{1/2}}} = \frac{R_T^{-1} - 1}{1 - R_T} \times \frac{1 - R_T^{1/2}}{R_T^{-1/2} - 1} \\ &= R_T^{-1} \times R_T^{1/2} = R_T^{-1/2}. \end{aligned} \quad (19)$$

5. Recapitulation and generalization

Efficiency is of course highest if work W , whether supplied via a heat engine or otherwise, is not frictionally dissipated at all. This would obtain, for example, in perfect (reversible) regenerative braking of an electrically-powered motor vehicle, with the motor operating backward as a generator during braking. It also would obtain, for example, if a noncyclic (necessarily one-time, single-use) rocket heat engine's work output is perfectly (reversibly) sequestered as kinetic and gravitational potential energy in the launching of a spacecraft (but typically most of the kinetic energy accelerates the exhaust gases, not the payload) or if a cyclic heat engine's work output is perfectly (reversibly) sequestered as gravitational potential energy in the construction of a building. But of course in practice (as opposed to in principle) *total* avoidance of frictional dissipation of work [and also of additional losses, e.g., due to irreversible heat flows engendered by finite temperature differences (no insulation is perfect)] is not possible.

The Second Law of Thermodynamics allows frictional dissipation of work into heat at *any temperature* [1–7] (in Ref. [6], see pp. 11–12, 60–65, and 263–265, especially pp. 263–265). The entropy increase resulting from frictional dissipation of work W at temperature T , namely, $\Delta S = W/T$, decreases monotonically with increasing T but is positive for any finite T —and the Second Law requires only that $\Delta S \geq 0$ [1–7]. The diminution of $\Delta S = W/T$ at higher T is, ultimately, what yields increased efficiency via HTR, within the restriction of frictional dissipation of W being unavoidable. The diminution of $\Delta S = W/T$ via frictional dissipation of W at T_H as opposed to at $T_C = R_T T_H$ is

$$\begin{aligned} \text{diminution of } \Delta S &= \frac{W}{T_C} - \frac{W}{T_H} = W \left(\frac{1}{T_C} - \frac{1}{T_H} \right) = W \left(\frac{1}{R_T T_H} - \frac{1}{T_H} \right) \\ &= \frac{W}{T_H} (R_T^{-1} - 1). \end{aligned} \quad (20)$$

And the corresponding saving of exergy or free energy is

$$\Delta X = T_C \times (\text{diminution of } \Delta S) = T_C W \left(\frac{1}{T_C} - \frac{1}{T_H} \right) = W \left(1 - \frac{T_C}{T_H} \right) = W(1 - R_T). \quad (21)$$

Note that in the limit $R_T \rightarrow 1$, both (diminution of ΔS) $\rightarrow 0$ and $\Delta X \rightarrow 0$, while in the limit $R_T \rightarrow 0$, (diminution of ΔS) $\rightarrow \infty$ and $\Delta X \rightarrow W$.

Consider work W from *any* source, heat engine or otherwise, frictionally dissipated into heat at temperature T_H [6]. The temperature T_H could be generated via the friction itself [6]. Thus frictional dissipation of work W from *any* source, heat engine or otherwise, could, in at least in principle, generate an arbitrarily high temperature T_H [6]. In the limit $T_H \rightarrow \infty$ (with T_C fixed) or equivalently in the limit $R_T \rightarrow 0$, not only the standard (non-HTR) Carnot efficiency $\epsilon_{\text{Carnot, std}}$ but even the standard (non-HTR) Curzon-Ahlborn efficiency $\epsilon_{\text{CA, std}}$ approaches unity, and entropy production even given the standard (non-HTR) Curzon-Ahlborn efficiency $\epsilon_{\text{CA, std}}$ approaches zero. And both HTR efficiencies, $\epsilon_{\text{Carnot, HTR}}$ and $\epsilon_{\text{CA, HTR}}$, approach infinity ($\epsilon_{\text{Carnot, HTR}}$ approaching infinity at higher order), with entropy production corresponding to both approaching zero (that corresponding to $\epsilon_{\text{Carnot, HTR}}$ approaching zero at higher order). Thus in the limit $R_T \rightarrow 0$, work frictionally dissipated into heat can completely be reconverted back into work by a heat engine. When this work is, in turn, frictionally dissipated, the process can be repeated over and over again—indeinitely in the limit $R_T \rightarrow 0$. We emphasize again: no energy is created (or destroyed)—energy is merely recycled—hence the First Law of Thermodynamics is not violated [6]. No decrease in entropy occurs— $\Delta S = W/T_H > 0$ for any finite T_H —hence the Second Law of Thermodynamics is not violated [6]: as per Eqs. (20) and (21), ΔS is diminished but still remains positive. To re-emphasize, it is the *diminution* of $\Delta S = W/T$ at higher T : $W/T_H < W/T_C$ (notwithstanding that ΔS still remains positive: $W/T_H > 0$) that yields increased efficiency via HTR, within the restriction of frictional dissipation of W being unavoidable.

Consider the following thought experiment. If an automobile travels at constant speed, the work output of its engine is immediately and continually frictionally dissipated, but the work was done and the efficiency was W/Q_H , not zero. In the operation of automobiles at constant speed, W is immediately and continually frictionally dissipated to ambient (the cold reservoir), and hence the consequent entropy increase is $\Delta S = W/T_C$. (Of course, there are in practice other entropy increases accompanying the operation of automobiles, e.g., owing to irreversible heat flows engendered by finite temperature differences.) But if W could instead be frictionally dissipated into the cylinders of an automobile's engine during power strokes (of course this is impracticable), the entropy increase would be diminished from $\Delta S = W/T_C$ to $\Delta S = W/T_H$, and the required heat input would be reduced from Q_H to $Q_H - W = Q_C$. Hence via HTR the efficiency would be increased from W/Q_H to $W/(Q_H - W) = W/Q_C$, and thus also the required fuel consumption would be decreased by the ratio Q_C/Q_H . Even though this is obviously impracticable, given that $\Delta S = W/T_H > 0$ the Second Law allows it, so we can at least do it as a thought experiment.

If a heat engine's work output is frictionally dissipated into its hot reservoir, the net heat input required from the hot reservoir is reduced from Q_H to $Q_H - W$,

and hence via HTR the engine's efficiency is increased from W/Q_H to $W/(Q_H - W) = W/Q_C$, which can indeed exceed the Carnot limit—even though the efficiency W/Q_H of the *initial* production of work must be within the Carnot limit. If the temperature T_C of the cold reservoir is only a small fraction of the the temperature T_H of the hot reservoir, W/Q_H can be almost as large as unity or equivalently W can be almost as large as Q_H , and hence $W/(Q_H - W) = W/Q_C$ can *greatly* exceed the Carnot limit.

We note that the temperature of the cosmic background radiation is only 2.7 K, while the most refractory materials remain solid at temperatures slightly exceeding 2700 K. This provides a temperature ratio of $R_T \equiv T_C/T_H \approx 10^{-3}$. Could even smaller values of $R_T \equiv T_C/T_H$ be possible, at least in principle? Perhaps, maybe, if frictional dissipation of work into heat might somehow be possible into a gaseous hot reservoir at temperatures exceeding the melting point or even the critical temperature (the maximum boiling point at any pressure) of even the most refractory material.

While in this chapter we do not challenge the Second Law, we do challenge an *overstatement* of the Second Law that is sometimes made: that energy can do work only once. This *overstatement* is *false*. Energy can indeed do work more than once, in principle up to an infinite number of times, and even in practice many more times than merely once, before its ability to do work is totally dissipated. Consider these three examples: (i) Energy can do work in an infinite number of times in perfect (reversible) regenerative braking of an electrically-powered motor vehicle, with the motor operating backward as a generator during braking. Even with real-world less-than-perfect (less than completely reversible) regenerative braking, energy can do work many more times than merely once before its ability to do work is totally dissipated. (ii) Energy can do work in an infinite number of times in perfect (reversible) HTR (in the limit $R_T \rightarrow 0$). Even with real-world less-than-perfect (less than completely reversible) HTR (finite but small $R_T > 0$), energy can do work many more times than merely once before its ability to do work is totally dissipated. (iii) Energy can do work in an infinite number of times in perfect (reversible) thermal recharge of intermediate heat reservoirs—not to be confused with HTR discussed in this present chapter—see Section VI of Ref. [35] and the improved treatment in another chapter [61] in this book. Even with real-world less-than-perfect (less than completely reversible) thermal recharge of intermediate heat reservoirs, energy can do work more times than merely once before its ability to do work is totally dissipated.

6. Reply to criticisms of HTR

The concept of HTR (without being dubbed HTR) was criticized by Makarieva, Gorshkov, Li, and Nobre [58] and by Bejan [59], as being in conflict with the First and Second Laws of Thermodynamics, especially with the Second Law (see especially Sections 4 and 5 of Ref. [58] and Section 4 of Ref. [59]). These criticisms are addressed directly in Ref. [60]. They are also addressed in works concerning (a) HTR in hurricanes [27–37, 62] and (b) the *experimental verification* of HTR in increasing efficiency of thermoelectric generators [38]. [Concerning (a) immediately above, on the one hand, the importance of HTR (dubbed as “dissipative heating”) has been confirmed in a study of Hurricane Andrew (1992) [36], and, as one might expect, “dissipative heating appears to be a more important process in intense hurricanes, such as Andrew, than weak ones” [36]. But, on the other hand, more recently it has been contended [37] that, while HTR exists in hurricanes, it is of lesser importance than previously supposed [27–36].]

Perhaps the simplest and most straightforward reply to these criticisms [58, 59] is that provided by Spanner (see Ref. [6], pp. 11–12, 60–65, and 263–265, especially pp. 263–265): Friction resulting from dissipation of work can in principle generate arbitrarily high temperature T_H without violating the Second Law of Thermodynamics: The entropy increase resulting from frictional dissipation of work W at temperature T_H , namely, $\Delta S = W/T_H$, decreases monotonically with increasing T_H but is positive for any finite T_H —and the Second Law requires only that $\Delta S \geq 0$ [6]. A heat engine operating between this high temperature T_H and a low (cold-reservoir) temperature T_C arbitrarily close to absolute zero (0 K) can in principle recover essentially all of the frictional dissipation as work [6]—and the recycling of energy from work to heat via frictional dissipation and then back to work via the heat engine can in principle then be repeated essentially indefinitely [6]. No energy is created (or destroyed)—energy is merely recycled—hence the First Law of Thermodynamics is not violated [6]. No decrease in entropy occurs— $\Delta S = W/T_H > 0$ for any finite T_H —hence the Second Law of Thermodynamics is not violated [6].

As has been previously emphasized [35], it is only recycling of a heat engine's waste heat Q_C into its hot reservoir at T_H instead of rejection thereof into its cold reservoir at T_C —not recycling of heat generated by frictional dissipation of its work output W back into its hot reservoir at T_H —that would violate the Second Law of Thermodynamics. Recharging W to the hot reservoir does not violate the Second Law, because the entropy change $\Delta S = W/T_H$ is positive—albeit less strongly positive than $\Delta S = W/T_C$ that obtains if W is frictionally dissipated into the cold reservoir. Only recharging Q_C to the hot reservoir would violate the Second Law, because the entropy change $\Delta S = Q_C/T_H - Q_C/T_C$ would be negative. And recycling of a heat engine's waste heat Q_C into its hot reservoir at T_H instead of its rejection into its cold reservoir at T_C has never been claimed [27–38, 60, 62].

There is one caveat: the entropy increase $\Delta S = W/T_H > 0$ owing to frictional dissipation of W at T_H could in principle be employed to pay for pumping a heat engine's waste heat Q_C from T_C to T_H , but no capability to do work would be gained by this procedure. For, even if this procedure could be executed perfectly (reversibly), e.g., via a perfect (reversible) heat pump, we would have [applying Eqs. (1) and (5)]

$$\begin{aligned} \Delta S_{\text{total}} &= \frac{W}{T_H} - \frac{Q_C}{T_C} + \frac{Q_C}{T_H} = \frac{W}{T_H} - Q_C \left(\frac{1}{T_C} - \frac{1}{T_H} \right) = \frac{W}{T_H} - Q_C \frac{T_H - T_C}{T_C T_H} = 0 \\ \Rightarrow W &= Q_C \frac{T_H - T_C}{T_C} = Q_C \left(\frac{T_H}{T_C} - 1 \right) = Q_H \frac{T_C}{T_H} \left(\frac{T_H}{T_C} - 1 \right) \\ &= Q_H \left(1 - \frac{T_C}{T_H} \right) = Q_H (1 - R_T) = Q_H \epsilon_{\text{Carnot, std}}. \end{aligned} \quad (22)$$

What Eq. (22) brings to light is that the operation of the heat pump, even if perfect (reversible), results merely in the recovery of W . But W is recoverable more simply by avoiding this unnecessary procedure, as per Section 5 and the first three paragraphs of this Section 6.

7. Conclusion

We provided introductory remarks, an overview, and general considerations in Section 1. A misconception pertaining to the efficiencies of engines (heat engines or

otherwise) was discussed and corrected in Section 2. Then we discussed the work outputs, efficiencies, and entropy productions of Carnot (reversible) and Curzon-Ahlborn (endoreversible) heat engines. In Section 3, we reviewed the standard (without HTR) work outputs, efficiencies, and entropy productions of Carnot (reversible) and Curzon-Ahlborn (endoreversible) heat engines, first without frictional dissipation of heat engines' work outputs and then with frictional dissipation thereof into their cold reservoirs. In Section 4 we considered them with frictional dissipation of heat engines' work outputs into their hot reservoirs (with HTR). (If HTR is employed, we construe the terms "the highest practicable temperature for HTR" and "the hot reservoir" to be synonymous.) We showed that the efficiencies of both Carnot and Curzon-Ahlborn engines can be increased, indeed in some cases greatly increased, via employing HTR. The increases in efficiencies via employing HTR are minimal in the limit $R_T \equiv T_C/T_H \rightarrow 1$ but become arbitrarily large in the limit $R_T \equiv T_C/T_H \rightarrow 0$. Efficiencies via employing HTR can exceed unity and can even approach ∞ .

We provided recapitulation, as well as generalization, in Section 5. We replied to criticisms [58, 59] of HTR in Section 6.

As we have already noted in Section 1, the increases in efficiency attainable via HTR are not practicable if frictional dissipation of work into other than the cold reservoir is not practicable. Thus they are *never* practicable for noncyclic (necessarily one-time, single-use) heat engines: however the work output of a noncyclic (necessarily one-time, single-use) heat engine might be frictionally dissipated, the heat thereby generated *cannot* restore the engine to its initial state. Moreover in many cases the work outputs of noncyclic (necessarily one-time, single-use) heat engines are not frictionally dissipated *at all*, at least not during practicable time scales, for example, a noncyclic rocket heat engine's work output is sequestered essentially permanently as kinetic and gravitational potential energy in the launching of a spacecraft (but typically most of the kinetic energy accelerates the exhaust gases, not the payload). They also are *never* practicable for reverse operation of cyclic heat engines as refrigerators or heat pumps, because for both refrigerators and heat pumps, the *total* energy input (the work W , plus the heat Q_C extracted from a cold reservoir at the expense of W as required by the Second Law of Thermodynamics) *always* is deposited as heat Q_H into a hot reservoir ($Q_H = Q_C + W$): thus there is *never* any *additional* energy to be deposited into the hot reservoir (as there is from frictional dissipation of work done via forward operation of cyclic heat engines). {See Ref. [1], Section 20-3; Ref. [2], Sections 4.3, 4.4, and 4.7 (especially Section 4.7); Ref. [3], Sections 4-4, 4-5, and 4-6 (especially Section 4-6); Ref. [5], Section 5.12 and Problem 5.22; Ref. [7], pp. 233–236 and Problems 1, 2, 4, 6, and 7 of Chapter 8; Ref. [16], Chapter XXI; Ref. [17], Sections 6.7, 6.8, 7.3, and 7.4; and Ref. [54], Sections 5-7-2, 6-2-2, 6-9-2, and 6-9-3, and Chapter 17. [Problem 2 of Chapter 8 in Ref. [7] considers absorption refrigeration, wherein the entire energy output is into an *intermediate*-temperature (most typically ambient-temperature) reservoir, and hence for which HTR is *even more strongly never practicable*.]} They also are not practicable for cyclic heat engines in cases wherein a cyclic heat engine's work output is not frictionally dissipated immediately or on short time scales [16, 17], for example, as gravitational potential energy sequestered for a long time interval in the construction of a building. For a building once erected typically remains standing for a century or longer. Even if, when it is finally torn down, its gravitational potential energy were to be totally frictionally dissipated into a hot reservoir, it is simply impracticable to wait that long. Thus HTR is not practicable in all cases. But in the many cases wherein cyclic heat engines' work outputs are frictionally dissipated immediately or on short time scales [16, 17], practicability obtains: improved conversion—and reconversion—of

frictionally dissipated heat into work, and hence improved cyclic heat-engine performance, can then obtain.

We emphasize yet again that First and Second Laws of Thermodynamics are *not* violated. The First Law is not violated because no new energy is created (or destroyed): super-unity efficiencies via employment of HTR obtain via recycling and reusing the same energy, not via the creation of new energy. The Second Law is not violated because the change in total entropy is positive if HTR is employed and frictional dissipation of work as heat is into the hot reservoir, albeit less strongly positive than if HTR is not employed and frictional dissipation of work as heat is into the cold reservoir. The improved heat-engine performance that HTR provides ultimately obtains from this reduction of entropy increase.

While in this chapter we do not challenge the First or Second Laws of Thermodynamics, we should note that there have been many challenges to the Second Law, especially in recent years [64–69]. By contrast, the First Law has been questioned only in cosmological contexts [70–72] and with respect to fleeting violations thereof associated with the energy-time uncertainty principle [73, 74]. But there are contrasting viewpoints [73, 74] concerning the latter issue.

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Conflicts of interest

The author declares no conflicts of interest.

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