Chapter

The Study of Magneto-Convection Heat Transfer in a Partially Open Cavity Based on LBM

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Abstract

A free convection heat transfer in a sinusoidally heated enclosure filled with conducting fluid is presented in this chapter by Lattice Boltzmann Method (LBM). The horizontal walls in the enclosures are insulated and there is an opening part on the right wall. The right non-open parts of the vertical wall of the square cavity are maintained at constant cold temperature and the left wall of the cavity is sinusoidally heated. The cavity is get under a uniform in-plane magnetic field. The main aim of this study is to highlight the effectiveness of the LBM mesoscopic approach to predict the effects of pertinent parameters such as the Hartmann number varying from 0 to 150 where Rayleigh number is fixed at moderate value of 10^5 on flow patterns. This in-house numerical code used in this chapter is ascertained and a good agreement with literature is highlighted. The appropriate validation with previous numerical investigations demonstrated that this attitude is a suitable method and a powerful approach for engineering MHD problems. Findings and results show the alterations of Hartmann number that influence the isotherms and the streamlines widely.

Keywords: sinusoidal thermal boundary condition, MHD, partially open cavity, free convection, LBM

1. Introduction

Convective flow and heat transfer in an open cavity has been studied due to the extensive range of applications in engineering science and technology that consider various combinations of imposed temperature gradients and enclosure sketches [1–10]. Open cavity with a modified linear or sinusoidal thermal boundary condition is encountered in many practical engineering and industrial applications, such as solar energy collection, cooling of electronic devices, material processing, grain storage, flow and heat transfer in solar ponds, high-performance insulation for buildings, dynamics of lakes, reservoirs and cooling ponds, crystal growing, float glass production, metal casting, food processing, galvanizing, metal coating, and so on.

Besides, pertinent useful numerical research works had been conducted to simulate the MHD free convection under nonuniform thermic boundaries where recent attention has been intensively focused on the cases of mixed boundary conditions [11–14]. However, few results have been reported for free convection caused simultaneously by both external magnetic field in partially open enclosures subjected to a sinusoidal temperature variation in the left vertical wall although problems of this type are frequently important, and their study is necessary for understanding the performance of complex magneto-convection flow and heat transfer.

MHD forces generated from the interaction of induced electric currents with an applied external magnetic field can alter the flow of an electrically conducting fluid in the presence of magnetic field. An externally imposed magnetic field is an important tool used to control melting flows that grow bulk crystal in semiconductor's applications. A main purpose of electromagnetic control is to stabilize the flow and suppress oscillatory instabilities, which degrades the resulting crystal. In literature, wide ranges of investigations were investigated by researchers in MHD free convection [15–30]. In such complex geometry, the balance is achieved by inertial, viscous, electromagnetic, and buoyancy forces; finding a numerical efficient tool to predict flow and heat pattern inside MHD cavities is a crucial aim for industrial related engineering applications.

The effect of Ra number on free convection MHD in an open cavity was investigated in [29]. Nonetheless several investigations in MHD free convection inside partially open enclosure with sinusoidally wall have been carried out yet.

The mesoscopic approach called Boltzmann method (LBM) joined the microscopic models and the macroscopic dynamics of a fluid. It can recover the Navier-Stokes equation by using the Chapman-Enskog expansion [39]. LB method is not very demanding in terms of memory requirement. In addition, in terms of computational speed, the algorithm is generally simpler and therefore faster than many traditional CFD schemes. It is easy for parallel computation and for implementation of irregular boundary conditions, which is a sought task in many-needed engineering geometry.

To our best knowledge, no previous study on effects of sinusoidally heated boundary on free convection in a partially open MHD enclosure cavity with the LBM had already been studied so far. The main aim of this chapter is to study the effects of linearly heated wall on flow field and temperature distribution in an open MHD cavity filled.

The main aim of the present study is to demonstrate the use of the Lattice Boltzmann Method (LBM) [31–40] for MHD with a simple and clear statement and also solve MHD free convection as a left sinusoidally heated side mixed with a partially open right wall filled with a conducting fluid. Hartmann number varies in a wide range from 0 to 150. First, the results of LBM are validated with previous numerical investigations. Effects of Rayleigh, Hartmann number, and various positions of the open side on flow field and temperature distribution are considered simultaneously.

The proposed configuration with sinusoidal temperatures on the left side wall of a partially open cavity in the presence of a magnetic field has not been focused. A major objective of the present study is to examine the magneto-convection in this configuration filled with a conducting fluid confined between two horizontal walls, which are thermally insulated. The effect of the open side on fluid flow and heat transfer is studied numerically.

2. Governing equations and mathematical formulation

2.1 Problem statement

The considered geometries of the present problem are shown in **Figure 1**. They display a two-dimensional closed, open or partially open east side cavity with the height of H. A constant, linear, or sinusoidal temperature is imposed along the left vertical wall. Then opening side boundaries are correlated with temperature

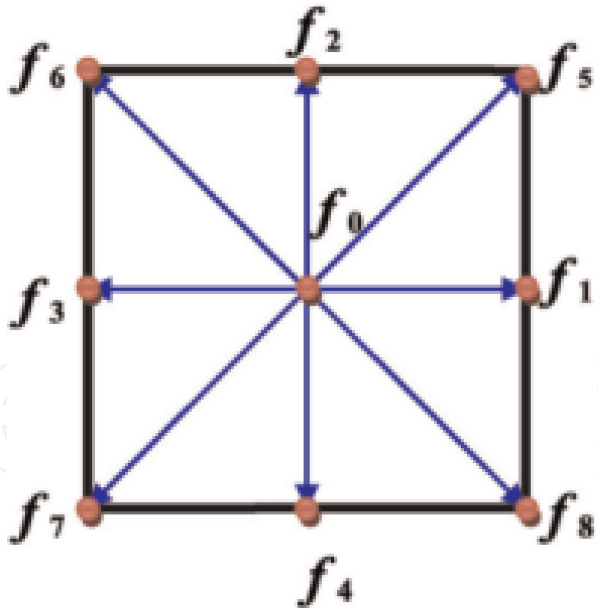


Figure 1.
The standard D_2Q_9 LBM lattice.

conducting fluid at the cold temperature (Tc). The north and south horizontal boundaries are adiabatic. We consider a horizontal uniform magnetic field applied to a two-dimensional Newtonian, laminar, and incompressible conducting fluid. The radiation effects, the viscous dissipation, and Joule heating are neglected in the present study. The thermophysical properties of the conducting fluid are constant, and the density variation in the liquid gallium is approximated by the standard Boussinesq (**Figure 2**).

2.2 Governing equations

Governing equations for MHD free convection are written in terms of the macroscopic variable depending on position x,y as:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equations

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+F_x \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+F_y \tag{3}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where $\nu=\mu/\rho$ is the kinematic viscosity and F_x and F_y are the body forces at horizontal and vertical directions, respectively, and they are defined as follows [30]:

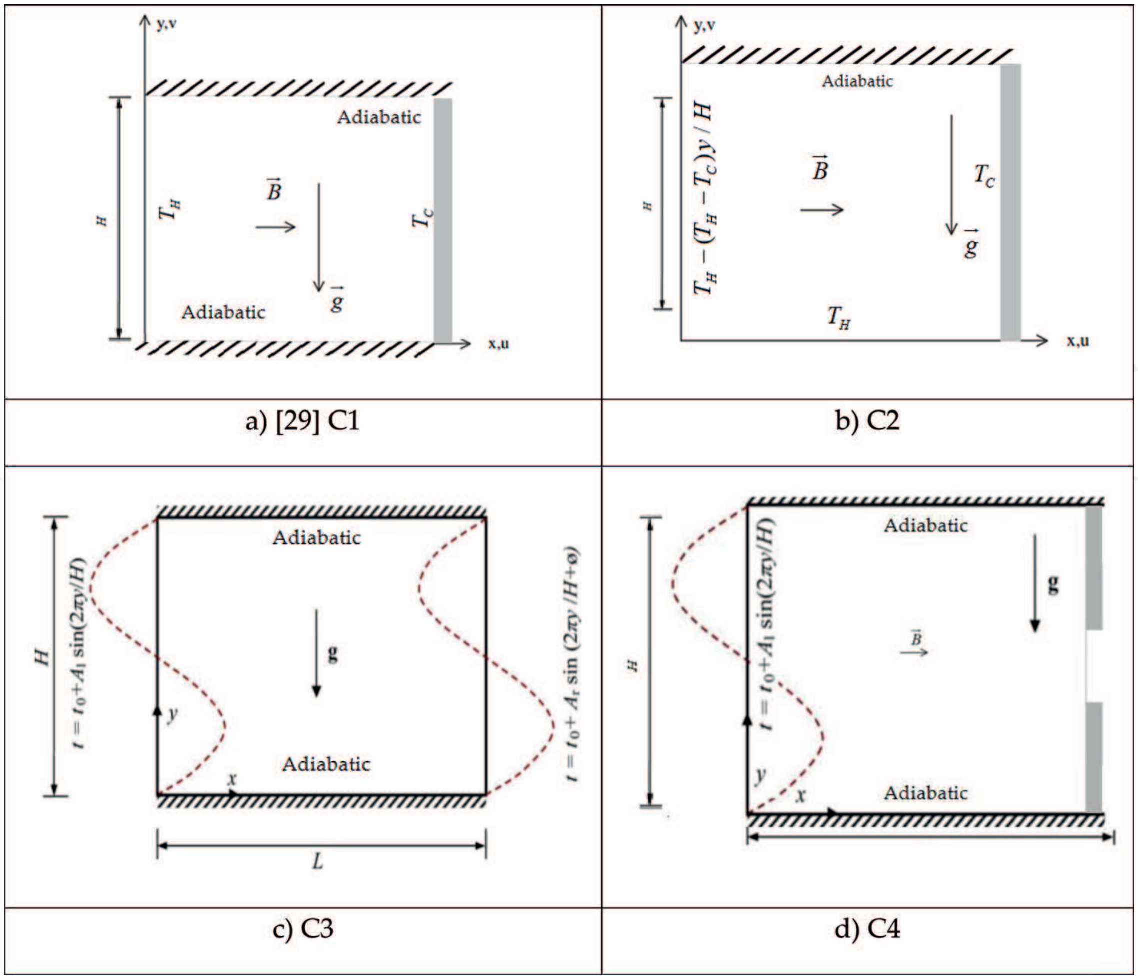


Figure 2.
Used configurations C1–4 with different boundary conditions.

$$F_x = R\left(v\sin\gamma\cos\gamma - u\sin^2\gamma\right) \tag{5}$$

$$F_y = R\left(u\sin\gamma\cos\gamma - v\cos^2\gamma\right) + \rho g\beta(T - T_m) \tag{6}$$

where the Ha number is defined as:

$$Ha = HB_x\sqrt{\sigma/\mu} \tag{7}$$

The LBM method [31–40] with standard, two-dimensional, nine velocities (D2Q9) for flow and temperature is used in this chapter (**Figure 1a**); for completeness, only a brief discussion is given in the following paragraphs. The Bhatnagar-Gross-Krook (BGK) approximation Lattice Boltzmann equation with external forces F_{tot} can be written as:

$$f_\alpha(\mathrm{r} + \mathrm{e}_\alpha\Delta t, t + \Delta t) = f_\alpha(\mathrm{r}, t) - \frac{\Delta t}{\tau_f}\left[f_\alpha(\mathrm{r}, t) - f_\alpha^{eq}(\mathrm{r}, t)\right] + \Delta t\,\mathrm{F}_{tot} \tag{8}$$

where $f_\alpha(\mathrm{r}, t)$ is the particle distribution defined for the finite set of the discrete particle velocity vectors e_α. r and t are the coordinates of Eulerian node and time.

where τ_f is the relaxation time and $f_\alpha^{eq}(\mathrm{r}, t)$ is the local equilibrium distribution function.

The equilibrium distribution can be formulated as:

$$f_\alpha^{eq} = w_\alpha\rho\left(1 + \frac{\mathrm{e}_\alpha.\mathrm{u}}{c_s^2} + \frac{(\mathrm{e}_\alpha.\mathrm{u})^2}{2c_s^4} - \frac{\mathrm{u}.\mathrm{u}}{2c_s^2}\right) \tag{9}$$

where u and ρ are the macroscopic velocity and density, respectively, and w_α are the values of the weighting constant factors for e_α that must be assigned as $w_0 = 4/9$ (rest-particle) for $|e_0| = 0$, $w_{1-4} = 1/9$ for $|e_{1-4}| = 1$, and $w_{5-9} = 1/36$ for $|e_{5-9}| = \sqrt{2}$.

where $c = \Delta x/\Delta t$, Δx and Δt are the lattice space and the lattice time step size, respectively, which are set to unity.

For scalar function (temperature), another distribution is defined:

$$g_\alpha(\mathrm{r} + \mathrm{e}_\alpha \Delta t, t + \Delta t) = g_\alpha(\mathrm{r}, t) - \frac{\Delta t}{\tau_g}\left[g_\varepsilon(\mathrm{r}, t) - g_\alpha{}^{eq}(\mathrm{r}, t)\right] \tag{10}$$

The equilibrium distribution function can be written as:

$$g_\alpha^{eq} = w_\alpha T\left(1 + \frac{\mathrm{e}_\alpha.\mathrm{u}}{c_s^2} + \frac{(\mathrm{e}_\alpha.\mathrm{u})^2}{2c_s^4} - \frac{\mathrm{u.u}}{2c_s^2}\right) \tag{11}$$

For momentum equation, we have:

$$\tau_f = 0.5 + \left(3\nu/c^2\Delta t\right) \tag{12}$$

where ν is the kinematic viscosity and for the scalar:

$$\tau_g = 0.5 + \left(3\alpha/c^2\Delta t\right) \tag{13}$$

where α is the diffusion coefficient (thermal diffusion coefficient) and Δt is the lattice time step. The buoyancy force term is added as an extra source term to Eq. (1) as:

$$F_\alpha = 3w_\alpha g_y \beta \Delta T \tag{14}$$

where g_y, β, and ΔT are gravitational acceleration, thermal expansion coefficient, and temperature difference, respectively.

In the LBM, the total force is

$$F = F_x + F_y \tag{15}$$

$$F_x = 3w_k\rho\left[R(v \sin\gamma \cos\gamma) - u \sin^2\gamma)\right] \tag{16}$$

$$F_y = 3w_k\rho\left[g_y\beta(T - T_m) + R(u \sin\gamma \cos\gamma) - v \cos^2\gamma\right] \tag{17}$$

where $R = \mu Ha^2$ and γ is the direction of the magnetic field.

After completing streaming and collision processes where the Boussinesq approximation is considered for free convection, the macroscopic fluid quantities, namely, the macro density, velocity (obtained through moment summations in the velocity space), and temperature are computed.

$$\rho(\mathrm{r}, t) = \sum_k f_k(\mathrm{r}, t) \tag{18}$$

$$\mathrm{u}(\mathrm{r}, t) = \sum_k \mathrm{e}_k f_k(\mathrm{r}, t)/\rho(\mathrm{r}, t) \tag{19}$$

$$T = \sum_k g_k(\mathrm{r}, t) \tag{20}$$

3. Results and discussion

In all cases, both the computations for flow and temperature fields are based on the D2Q9 LBM approach.

We have validated our in-house Fortran computer code for the free convection in a square open cavity with insulated horizontal walls filled with air with a uniform right (west) vertical temperature by reference [29]. The obtained numerical results are compared with the numerical ones reported in [29]. It can be seen from **Figure 3** that there is a good agreement for the distribution of streamlines and isotherms among the present solution and literature and this for Pr = 0.71, Ha = 0, and Ra = 10^5. The used configuration is C1 (**Figure 2**).

In **Figure 4**, the considered matter is MHD free convection in an open cavity with linearly heated west wall (C2), which is filled with liquid gallium (Pr = 0.025) and Ha = 150. As shown in **Figure 4**, an increase in Rayleigh number makes the thermal boundary layer to become narrower for the constant parameters Pr = 0.025 and Ha = 150.

Figure 5 displays steady-state contour maps for the isotherms and the streamline contours at Pr = 0.025 for a high Rayleigh number Ra = 10^6 and high Hartmann

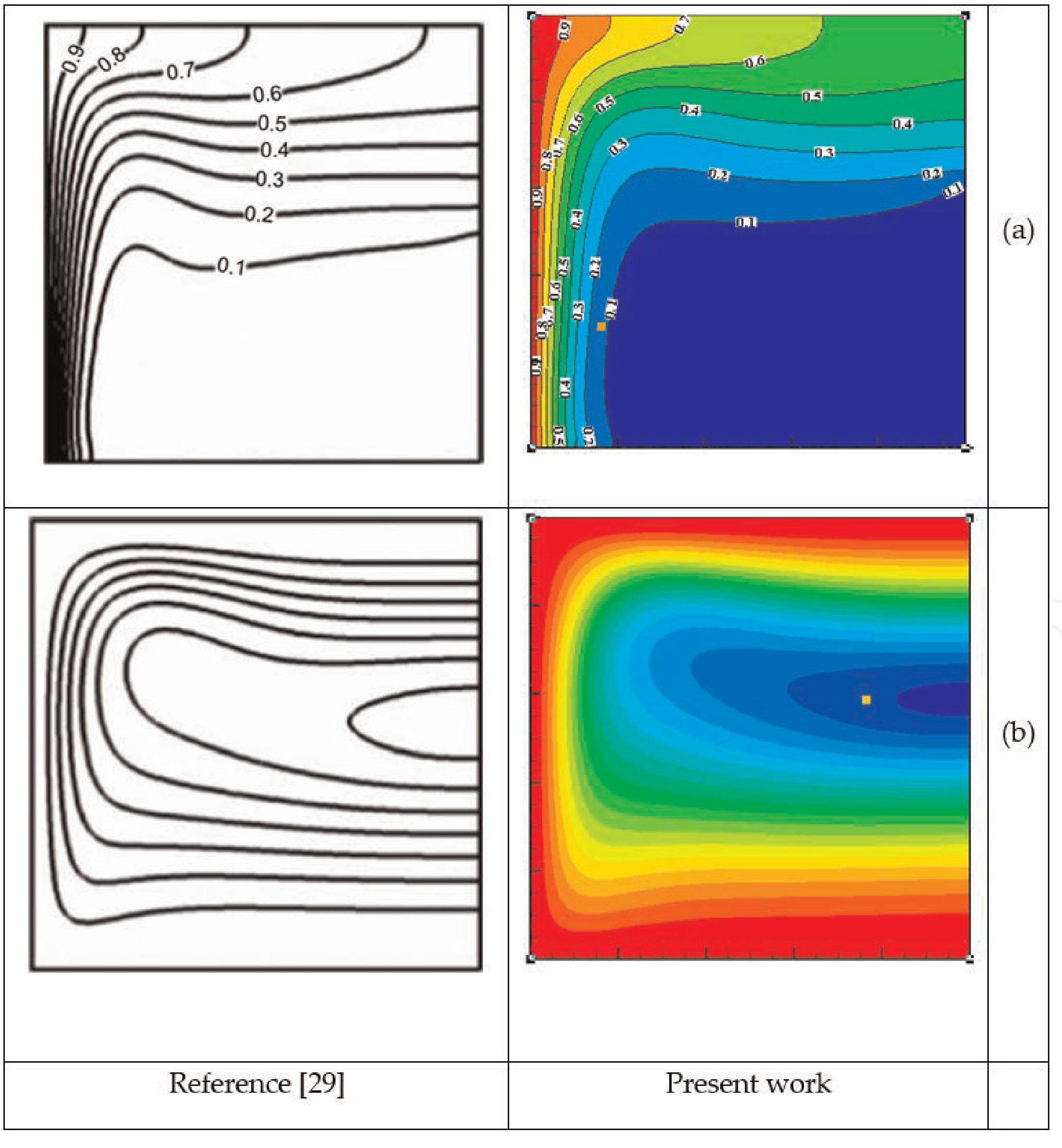


Figure 3.
Comparison of the steady state isotherms (a) and streamlines (b) at Pr = 0.71 for Ha = 0 and Ra = 10^5 between [29] and the present work.

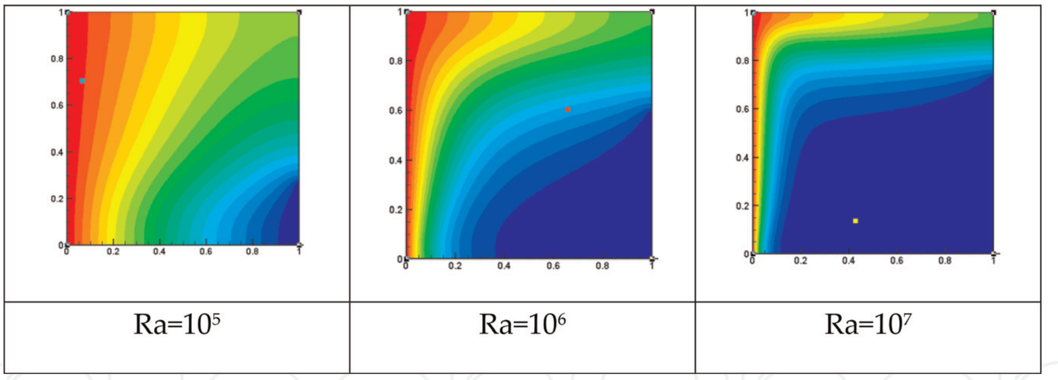


Figure 4.
Steady-state isotherms (b) at Pr = 0.025 and Ha = 150 for different Rayleigh numbers.

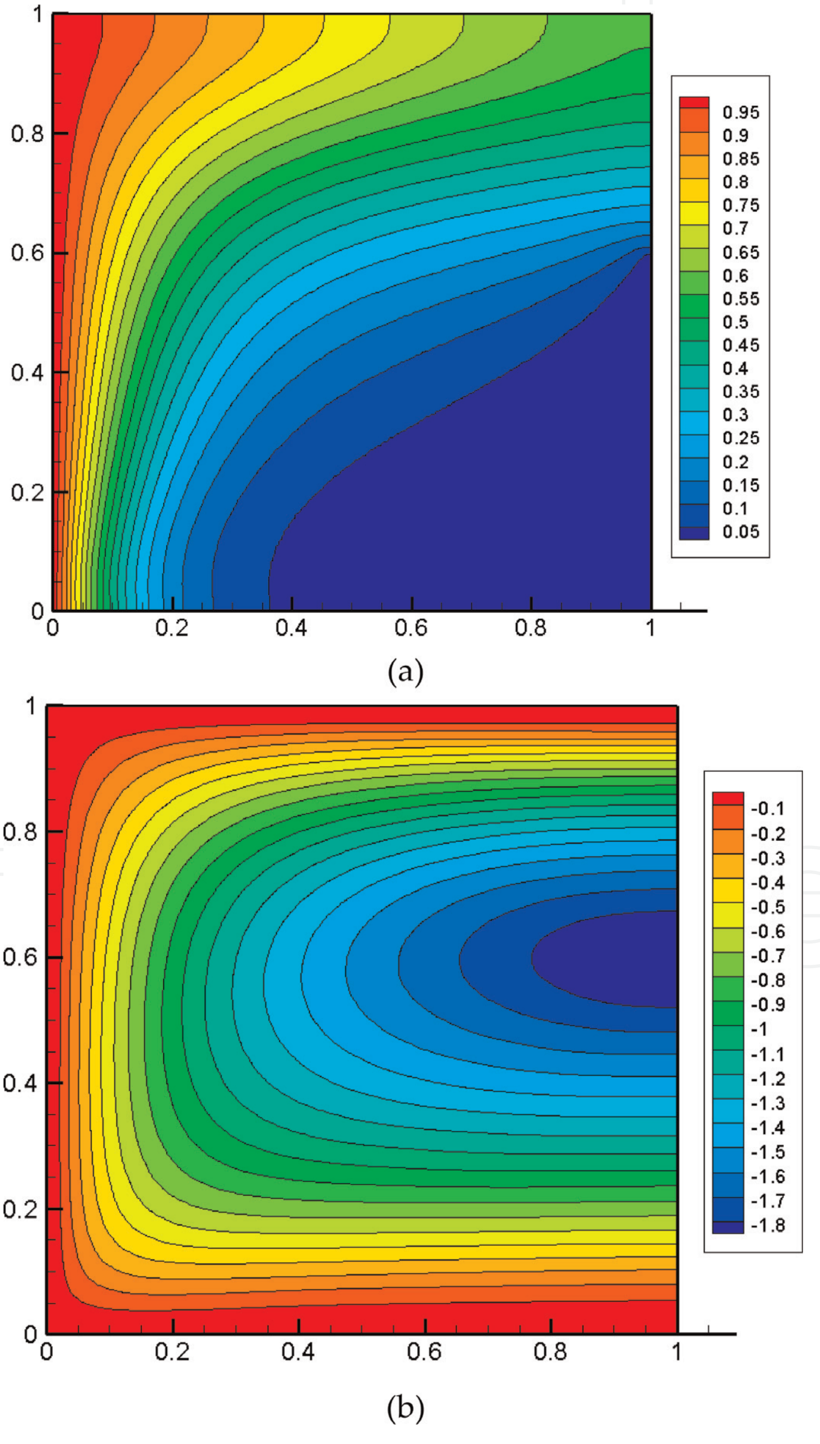
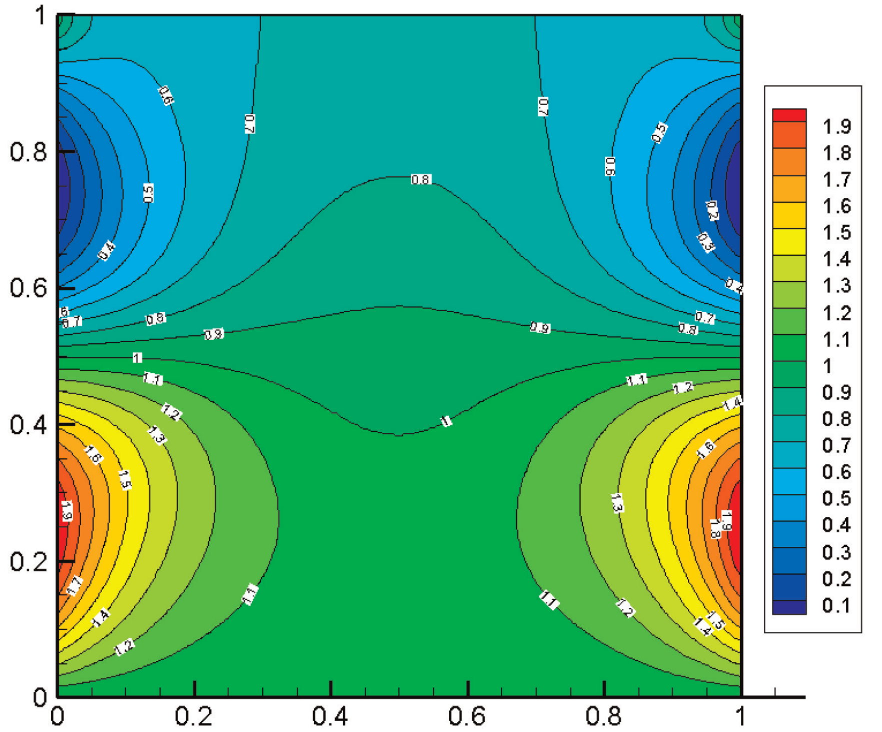


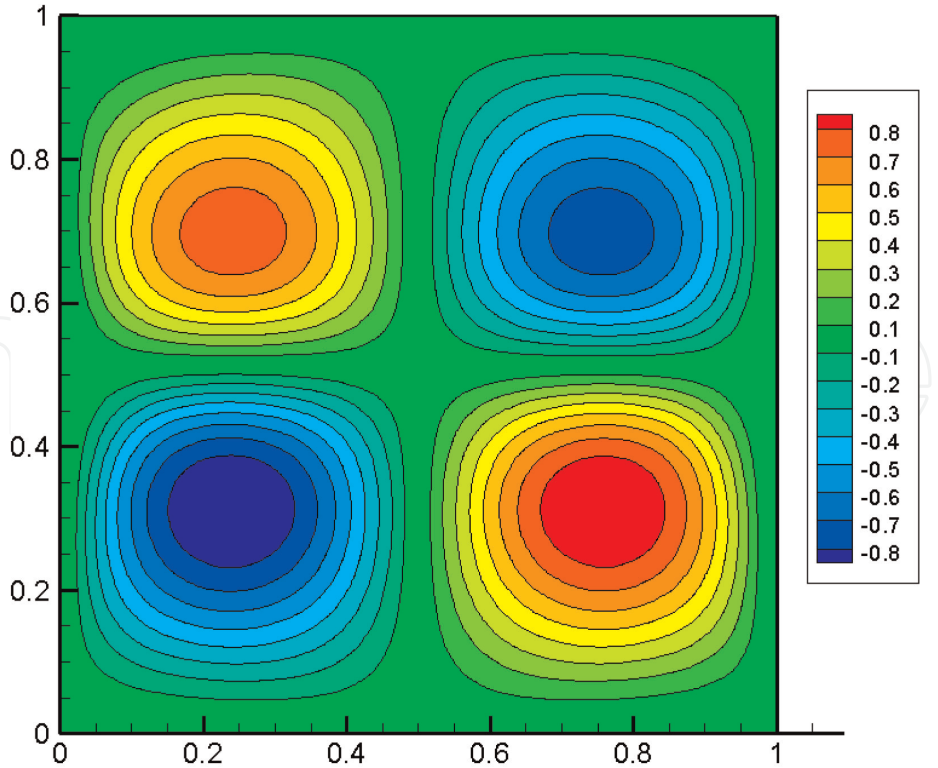
Figure 5.
Steady-state isotherms (a) and isotherms (b) at Pr = 0.025, Ra = 106, and Ha = 150.

number Ha = 150. Configuration C2 is considered for this case. The effect of the open right side wall is depicted in **Figure 5a** via streamline display.

In this chapter, we aim to test the ability of the LBM to deal with a future complex configuration (C4), which is a MHD partially open cavity with sinusoidal input excitation on the west wall. The horizontal walls are adiabatic. The cavity is



(a)

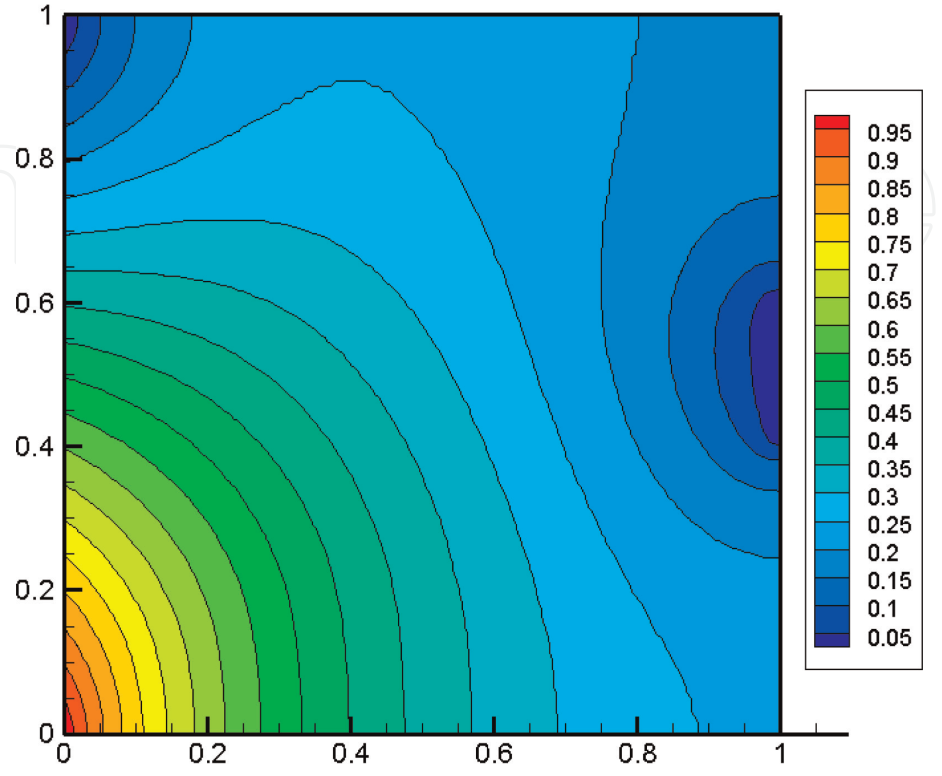


(b)

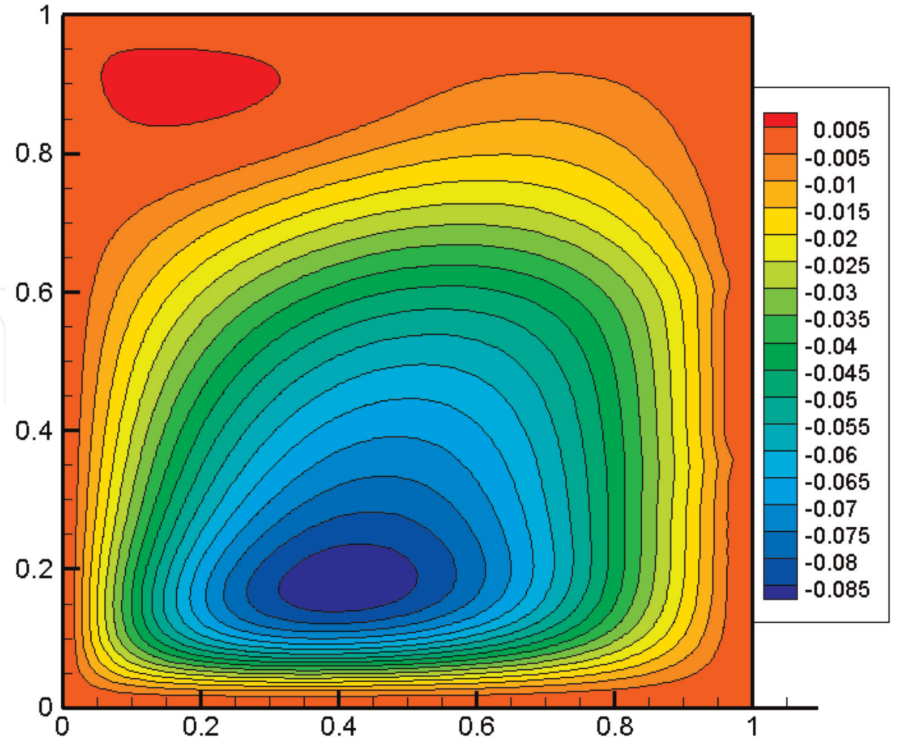
Figure 6.
Steady-state isotherms (a) and streamlines (b) of sinusoidally heated side walls in a partially open MHD cavity for Ha = 150, Pr = 0.025 and Ra = 10^5.

filled with liquid gallium, and the simulation is done for Ha = 50 and a moderate Rayleigh number of Ra = 10^5.

Many researchers were considering a sinusoidal heating wall in their simulation [11–14]. For brevity, we resume their important findings.



(a)



(b)

Figure 7.
Steady-state isotherms (a) and streamlines (b) of sinusoidally heated right side wall partially open MHD cavity for Ha = 150, Pr = 0.025 and Ra = 10^6.

In literature, we find that the heat transfer rate was increased as the amplitude ratio of the sinusoidal excitation increases with Rayleigh and Hartmann numbers.

For a uniform heating wall, the heat transfer rate remains low, which makes that the nonuniform heating of both walls is advised for enhancing and improving heat transfer. It is proven that the heat transfer rate is increased first and then decreased on increasing the phase deviation from 0 to pi [11–14].

In addition, with a phase deviation from 0 to 3pi/4, heat transfer rate is enhanced for all Rayleigh numbers, and the average Nusselt number reaches its highest value at 3pi/4.

A further finding proves that when both walls are in the same temperature distribution in the absence of phase deviation, the heat transfer rate is low for all values of Hartmann and Rayleigh numbers. The right wall is widely influenced by the variation of the amplitude ratio and the phase deviation of the sinusoidal temperature distribution. However, those physical variations have very little effect on the left wall. Besides, an increase in Hartmann number decreases the heat transfer [11–14].

In the present chapter, we seek to deal with a new configuration C4. For this goal, we deal first with the configuration (C3) of **Figure 2**. The sidewalls of the cavity have spatially varying sinusoidal temperature distributions. The horizontal walls are adiabatic. Simulation is established for MHD cavity for Ha = 150, Pr = 0.025, and Ra = 10^6. The heat transfer rate is highlighted within the evolution of isotherms and streamlines inside the cavity in **Figure 6**.

Figure 7 shows the dynamic and thermic behavior of configuration C4. Numerical results in terms of flow and thermic structure show that the flow within the cavity takes place owing to the thermic buoyancy effects caused by the sinusoidally heated right wall. In C3, the flow is characterized by a symmetric multicellular behavior in which the recirculating eddies or cells of relatively high velocity are formed within the enclosure and this in the presence of a high Hartmann Ha = 150 and a high Rayleigh number Ra = 10^5.

In C4, the presence of the partially open sidewall changes the flow and heat transfer. We note the presence of a one dominant cell in the core region of the cavity and a little cell that occurs at upper left side.

As forecasted, because of the partially open side effects, the temperature field was sketched by a noticeable drop in its behavior near the open side of the cavity and these both in flow and heat behaviors. Besides, we highlight that the temperature contour maps lose the sinusoidal behavior as they move to the partially open side.

We infer that a partially open sidewall has the tendency to control efficiently the movement of the fluid in such given configuration (C4).

We subject in the convergence criterion that the relative change in two successive iterates of the solution (temperate and velocities) at each computational point be below a prescribed small value of 10^{-6}.

For all simulations, the criterion convergence is considered to be reached, for velocity and temperature when the following convergence is satisfied:

$$\left|\xi^{\sigma+1}-\xi^{\sigma}\right| \leq 10^{-6} \tag{21}$$

where ξ is velocity or temperature and σ is the iteration number.

4. Conclusions

To the author's knowledge, studies have thus far addressed a mesoscopic approach in an MHD open cavity with sinusoidally heated wall (**Figure 2**). The objective of the present chapter is therefore to predict dynamic and thermic heat transfer in a crucial engineering application. The cavity is investigated at the high Ra number of 10^5, high Ha number (Ha = 50), and Pr number of 0.025. The present chapter extends the study to deal with free convection in MHD open cavity with sinusoidal heated west wall which is filled with liquid gallium with Ha = 50 for Ra = 10^5. Numerical study has been made of free convection in a square enclosure with spatially varying sinusoidal temperature distributions on the vertical left sidewall, whereas the horizontal walls are thermally insulated. The right wall is a partially open one. Lattice Boltzmann method is considered for flow and heat transfer simulation of this problem inside the cavity. After validation, the present in-house Fortran code is extended to deal with the present complex geometry in order to highlight the workability and the ability of LBM to deal with such mixed boundary condition sketch. This investigation demonstrated ability of LBM for simulation of different boundary conditions at various elements affecting the stream in a partially open cavity with sinusoidal heating vertical wall. An analysis of the opening mass flow is highlighted in the dynamic and thermal behavior of the streamlines and the isotherms.

Nomenclature

C	lattice speed
c_i	discrete particle speeds
c_p	specific heat at constant pressure
F	external forces
f^{eq}	equilibrium density distribution functions
g^{eq}	equilibrium internal energy distribution functions
g	gravity
G	buoyancy per unit mass
H	enclosure height
Ma	Mach number
Nu	Nusselt number
Pr	Prandtl number
Ra	Rayleigh number
T	temperature
x,y	Cartesian coordinates
X	horizontal length of the cavity
Y	vertical length of the cavity
Ha	Hartmann number

Greek letters

ω_i	weighted factor indirection i
β	thermal expansion coefficient
τ	relaxation time
ν	kinematic viscosity
Δt	time increment
α	thermal diffusivity

Subscripts

av	average
H	hot
C	cold
b	bottom

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