Chapter

Parameterization Methods and Autoregressive Model

Boukari Nassim

Abstract

The first phase for the treatment of random signals is the feature extraction; in this we can find several methods for that. In this chapter, we presented the autoregressive (AR) method, some methods of univariate and multivariate measures, and examples of their applications.

Keywords: feature extraction, autoregressive model, univariate measures, multivariate measures, applications

1. Introduction

In machine learning, pattern recognition and image processing, as well as feature extraction, start from an initial set of measured data and construct derived values (features) to be informative and nonredundant, facilitating subsequent learning and generalization steps. Feature extraction is related to the reduction of dimensionality.

When the input data of an algorithm is too large to process and is suspected to be redundant, it can be transformed into a small set (also called a feature vector).

In the following section, we will review the measures commonly used for the classification of electroencephalogram (EEG) signals, some of which have been used to predict seizures. The majority of these techniques use some type of serial analysis method. In order to detect off-line seizures, time series analysis of an EEG is in one of two groups: univariate or multivariate mathematical measures.

Concerning our work, we will quote methods used for the parameterization of EEG signals as well as for the detection of seizures [1].

2. Multivariate measures and autoregressive model

To parameterize our signals, we use different methods for that; like in our experiments, we have the autoregressive model, with the data of EEG signals founded in Bonn University. The work is realized by the logical of math work laboratory, and in the final, we obtained great results of different orders of autoregressive parameter like an important example in the treatment.

3. Univariate measures

The analysis of univariate time series consists of a single observation of sequential recordings on equal time increments, for example, univariate time series: the price of a company's stock, daily fluctuations, humidity levels, and records of the single-channel EEG. Time is also an implicit variable in the time series. Electroencephalogram analysis using univariate measurements contains a single recording site; univariate linear measurements characterize the EEG time series in terms of amplitude and phase information.

In this section, we describe the univaried linear methods most often used to predict epileptic seizures; common to each method is the requirement of stationarity of the time series, and this implies that the statistical parameters (mean and standard deviation) of the process do not change over time, which is probably not true for seizures.

3.1 Fourier transform in the short term

The short-term Fourier transform (STFT), transforming Fourier (TFCT), or Fourier transform with a sliding window is a transformation used to determine the frequency [2, 3], sinusoidal, and the phase of a local section of a signal. Its square module gives the spectrogram.

We then obtain a family of coefficients or represent the frequency and locate the analysis. As for the Fourier analysis, the knowledge of all the real values will completely and uniquely determine the signal.

The short-term Fourier transform, of a function $x(t) \in (R)$, is defined by means of a window. Thereafter, we drag this window used to locate in time the analysis:

$$W_x(\lambda, b) = \int_{-\infty}^{+\infty} x(t)\overline{\omega}(t-b)e^{-2i\pi\lambda t}dt \tag{1}$$

We then obtain a family of coefficients or represent the frequency and locate the analysis. As for the Fourier analysis, the knowledge for all the real values will completely and uniquely determine the signal. The local discrete Fourier transform is defined by

$$STFT\{x[n]\} = x[m, \omega] = \sum_{-\infty}^{\infty} x[n]\omega[n-m]e^{-j\omega n} \tag{2}$$

where ω is the windowing function.

3.2 Wavelet transforms

3.2.1 Continuous wavelet transforms

In mathematics, a continuous wavelet transform (CWT) is used to divide a continuous function into wavelets [4, 5]. In contrast to the transforming Fourier, the continuous wavelet transform has the possibility of constructing a time-frequency representation of a signal that offers a time and frequency.

The four-time transform in continuous time of the function has the scale of $a > 0$, and the transaction value is expressed by the following integral:

$$X_\omega(a, b) = \frac{1}{|a|^{1/2}} \int_{-\infty}^{\infty} x(t)\overline{\psi}\left(\frac{t-b}{a}\right)dt \tag{3}$$

where $\psi(t)$ is a continuous function in the time domain and the frequency domain called the mother wavelet and the highlight represents the operation of the complex conjugate.

3.2.2 Detective wavelet transforms

A wavelet is a function at the base of wavelet decomposition, which is similar to the short-term Fourier transform used in signal processing [6, 7]. It corresponds to the intuitive idea of a function corresponding to a small oscillation, hence its name.

We can adapt the wavelet transform in the case where we are in a discrete set. This technique is particularly used in digital data compression with or without loss.

We thus define the discrete wavelet transform:

$$g[t] = \sum_{m \in z} \sum_{n \in z} \left(x, \psi_{m,n}\right).\psi_{m,n}[t] \tag{4}$$

3.3 Nonlinear measurements

For nonlinear measurements, we cite the correlation dimension [8], which makes it possible to distinguish the signals from the deterministic time series; its value decreases before the crisis 9, as well as the dynamic similarity index [9], which follows the spatiotemporal modifications of brain dynamics before the crisis. A crisis.

In addition, there are other methods such as the Lyapunov exponent that is used to identify changes before seizure in the EEG basis for epilepsy surgery [10]. In addition, we must not forget the methods of entropy (SampEn) [11], which characterize by definition the degree of disorganization or unpredictability of the information content of a system, as well as the approximate entropy method (ApEN) [12], a technique used to quantify the amount of regularity and unpredictability of fluctuations over temporal data. ApEN was first developed to analyze medical data, such as heart rate, and then in finance, psychology, etc.

Indeed, to obtain entropy results of good quality, it is necessary to have many data, as well as the reliability of the system and the absence of undesirable phenomena.

4. Bivariate and multivariate measures

Electroencephalogram (EEG) has been used frequently to study synchronization in the brain [13]. EEG is also the technique of choice for various computer interface design applications because of its low cost and easy-to-use architecture.

Electroencephalogram signals can be conceptualized as numerical values (voltages) in time and space (collected electrodes). The techniques available in signal processing and dynamic systems have a long history of applications to the EEG. In this section, we will mention the bivariate and multivariate methods most used in the world of electroencephalography and epilepsy.

4.1 Bivariate measures

For this type of index, the key point is to measure the amount of synchronism between two time series. Here, we review a number of bivariate measures that are often used to compute interdependence between two time series:

a. **Cross-correlation:** Cross-correlation is a simple synchronization measure that is frequently used for linear interdependence capture in the time domain.

The Pearson product moment correlation coefficient between and is calculated by

$$Corr_{y1,y2} = \frac{\sum_{t=1}^{L}\left(Y_1^t - \frac{1}{L}\sum_{t=1}^{L}Y_1^t\right)\left(Y_2^t - \frac{1}{L}\sum_{t=1}^{L}Y_2^t\right)}{\sqrt{\sum_{t=2}^{L}\left(Y_1^t - \frac{1}{L}\sum_{t=1}^{L}Y_1^t\right)^2}\sqrt{\sum_{t=2}^{L}\left(Y_2^t - \frac{1}{L}\sum_{t=1}^{L}Y_2^t\right)^2}} \quad (5)$$

b. **Cross-consistency:** Correlation is a measure in the time domain. In order to obtain the dependency in a certain frequency band, the data must be filtered in this band.

The cross-coherence of two signals and at the frequency f is obtained as

$$Coh_{y1,y2}(f) = \left|P_{y1y2}(f)\right|^2 / \left(\left|P_{y1y1}(f)\right|\left|P_{y2y2}(f)\right|\right) \quad (6)$$

where is the spectral density of cross power at frequency f. Coherence at a certain frequency band can be obtained by averaging above the values in this range.

4.2 Multivariate measures

One approach for calculating synchronization within multivaried time series is to consider all series as components of a single interdependent system and to use multivariate measures; the measures discussed here have been introduced into the literature and successfully applied in the literature. The EEG signal.

4.2.1 Omega complexity

A multivariable time series can be thought of as a representation of. The complexity of omega evaluates the dimensionality of these trajectories in the state space based on analysis of the main components.

Let C = Cij be the matrix covariance, in which the input Cij is the cross-correlation between the time series and calculated from Eq. (2). We calculate the complexity of omega by

$$\Omega = \exp\left(-\sum_{i=1}^{N}\frac{\lambda_i}{N}\log\frac{\lambda_i}{N}\right) \quad (7)$$

The complexity of omega varies between 1 (maximum synchronization) and N (minimum synchronization, i.e., desynchronization). In order to scale the above measurement between a value close to 0 (for minimum synchronism) and 1 (for maximum synchrony), we can calculate omega by

$$Omega = \frac{1}{\Omega} \quad (8)$$

5. Autoregressive method

5.1 Principle

The (parametric) model-based methods are based on the modeling of the data sequence x(n) as an output of a linear system characterized by a rational structure [14]. The AR method is the most frequently used parametric method.

In the AR method, the data can be modeled as the output of a discrete causal filter, all poles, whose input is a white noise. Indeed, the autoregressive (AR) model describes each EEG signal sample as a linear combination of previous samples.

Estimation of the spectral density of a random process is a problem that can be solved by parametric modeling. This modeling makes it possible to represent all the spectral information by a small number of parameters (**Figure 1**).

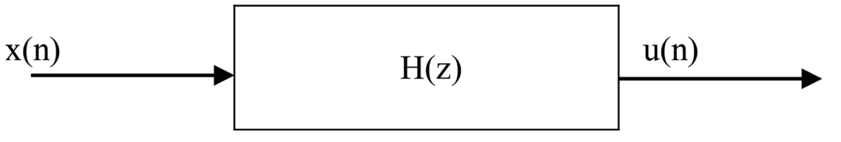

Figure 1.
A parametric model of the autoregressive (AR) type.

where x(n) is the modeled process, H(z) is the all-pole transfer function of the model, and u(n) is a variance white noise. Such a model is governed by the following difference equation:

$$X(n) = -\sum_{i=1}^{p} a_i x(n-i) + u(n) \tag{9}$$

La densité spectrale du processus AR, x(n) s'exprime sous la forme:

$$S_x(Z) = \frac{\varphi^2}{\left|1 + \sum_{i=1}^{p} a_i z^{-i}\right|^2} \tag{10}$$

5.2 Order determination

The best choice of the order of the AR model is often unknown a priori, so it is necessarily practical to postulate several orders and choose the order that seems most appropriate [15]. For this purpose, different criteria based on the prediction error are evaluated to indicate which order of the model to choose.

Two methods have been proposed by AKAIKE, for the choice of the optimal order. The first method, known under the name of FPE criterion (final prediction error or final error prediction), estimates the order of the model which minimizes

$$FPE(k) = \frac{N+K}{N-K} {\varphi^2}_k \tag{11}$$

where ${\varphi^2}_k$ represents the estimated noise variance (or power of the prediction error) and N represents the number of samples of the analyzed signal x(n).

The FPE criterion is an estimator of the power of the prediction error adapted to the estimation of the prediction coefficients directly from the data. The second method, by far the most used, is called AIC or Akaike information criterion. This criterion minimizes.

$$AIC(k) = N\log\left(\varphi^2{}_k\right) + 2k \tag{12}$$

This applies even to the determination of the order of models other than the AR model, for example, MA or ARMA models.

5.3 Coefficient determination

5.3.1 Yule-Walker

The principle consists in determining the link between the unknown AR parameters and the autocorrelation function assumed to be known [16]:

$$C_{xx}(K) = E\{x(n+k)x^*(n)\} \tag{13}$$

Expression of Yule-Walker can be written in the following matrix form:

$$\begin{bmatrix} C_{xx}(0)\;\; C_{xx}(-1)....C_{xx}(-p) \\ C_{xx}(1)\;\; C_{xx}(0)...C_{xx}(-(p-1) \\ .... \\ C_{xx}(p)\;\; C_{xx}(p-1)\;\; C_{xx}(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ ... \\ a_p \end{bmatrix} = \begin{bmatrix} \varphi^2 \\ 0 \\ ... \\ 0 \end{bmatrix} \tag{14}$$

5.3.2 Algorithm of Levinson-Durbin

The application of the Levinson-Durbin algorithm to solve the Yule-Walker equation system requires operations only, while the most advanced classical numerical analysis algorithm (e.g., Gauss) requires operations.

The AR parameters are calculated recursively:

$$\left(a_{11}, \varphi_1^2\right), \left(a_{21}, a_{22}, \varphi_2^2\right), ..., \left(a_{p1}, a_{p2}, ...a_{pp}, \varphi_p^2\right) \tag{15}$$

5.3.3 Algorithm of Burg

The parameters obtained by the Levinson-Durbin algorithm may be greater than unity, resulting in instability of the AR filter.

In order to eliminate this major inconvenience, Burg proposes a computational algorithm based on the minimization of the sum of the progressive mean squared error and the retrograde mean squared error (the taking of the mean is done, under the assumptions stationarity and ergodism, using a simple summation on a set of samples):

$$\xi_p = \sum_{n=p}^{N-1} \left|e_p^p(n)\right|^2 + \sum_{n=p}^{N-1} \left|e_p^\tau(n)\right|^2 \tag{16}$$

5.4 Applications

5.4.1 Transmission of the speech signal

The large flow of information passing through the channels suggests the adoption of coding techniques that reduce the size of the storage memory and the computational volume; in general, it is the delta modulation that is used for this [17].

Another coding technique is to model the speech signal and to represent it by a set of parameters. This type of coding based on linear prediction (AR modeling) makes it possible to carry out a large data compression that can reach a factor of 50 and offers great interest because of its ease of implementation.

This method is based on a very simplified modeling of the mechanism of speech production which consists in applying periodic vibrations or a source of white noise at the input of an AR filter according to whether the sounds to be produced are voiced or unvoiced [18].

5.4.2 Detection of gear defects by vibration analysis

Gear reducers are widely used in mechanics (automotive industry, aeronautics, etc.), highly stressed and complex to dimension, and to achieve.

Some defects appear in these machines, which are due to shocks resulting in local non-stationarity on the vibratory signals.

So, it should be detected early, for which several techniques are in use such as traditional techniques based on Fourier analysis, but the most common techniques use an autoregressive (AR) model.

Indeed, this model gives a good approximation of the estimated spectrum with a relatively small number of samples.

6. Conclusion

In this chapter, we discussed the parameterization methods and their approach to the art; in fact, we approached the univariate, bivariate, and multivariate measures and the method used in this chapter, the principle, algorithms, applications, etc.

Finally, the method is very efficient and very interesting, so it is valid in several areas such as the EEG signal, so this method of parameterization is part of the subject of our work.

Author details

Boukari Nassim
Skikda University, Annaba, Algeria

*Address all correspondence to: boukarinassim@yahoo.fr

References

[1] Agarwal R, Gotman J, Flanagan D, Rosenblatt B. Automatic EEG analysis during long-term monitoring in the ICU. Electroencephalography and Clinical Neurophysiology. 1998;**107**(1): 44-58

[2] Adeli H, Zhou Z, Dadmehr N. Analysis of EEG records in an epileptic patient using wavelet transform. Journal of Neuroscience Methods. 2003;**123**(1): 69-87

[3] Hazarika N, Chen JZ, Tsoi AC, Sergejew A. Classification of EEG signals using the wavelet transform. Signal Processing. 1997;**59**(1):61-72

[4] Radke RJ, Andra S, Al-Kofahi O, Roysam B. Image change-detection-algorithms: A systematic survey. IEEE Transactions on Image Processing. 2005;**14**(3):294-307

[5] Harrison MA, Frei MG, Osorio I. Accumulated energy revisited. Clinical Neurophysiology. 2005;**116**:527-531

[6] Iasemidis LD, Pardalos P, Sackellares JC, Shiau DS. Quadratic binary programming and dynamical system approach to determine the predictability of epileptic seizures. Journal of Combinatorial Optimization. 2001;**5**: 9-26

[7] Iasemidis LD, Shiau DS, Chaivalitwongse W, et al. Adaptive epileptic seizure prediction system. IEEE Transactions on Biomedical Engineering. 2003;**50**:616-627

[8] Ferraty F, Vieu P. Dimension fractale et estimation de la régression dans des espaces vectoriels semi-normés. Comptes Rendus de l'Académie des Sciences-Series I-Mathematics. 2000; **330**(2):139-142

[9] Litt B, Esteller R, Echauz J, et al. Epileptic seizures may begin hours in advance of clinical onset: A report of five patients. Neuron. 2001;**30**:51-64

[10] Ansari-Asl K. Mesure de couplage statistique entre signaux EEG: application à l'évaluation quantitative des relations fonctionnelles entre structures cérébrales en épilepsie. Diss. Université Rennes; 2005. p. 1

[11] Güler NF, Übeyli ED, Güler I. Recurrent neural networks employing Lyapunov exponents for EEG signals classification. Expert Systems with Applications. 2005;**29**(3):506-514

[12] Amoud H. Detection d'une evolution du risque de chute chez les personnes agees. Diss. Troyes; 2006

[13] Mazeran C. Sur la structure mathématique et l'approximation numérique de l'hydrodynamique lagrangienne bidimensionnelle. Diss. Bordeaux. 2007. p. 1

[14] Periodic EEG. Clinical significance of periodic EEG patterns. Archives of Neurology. 1980;**37**:15-20

[15] Abdelajal il Ouahabi. Techniques Avancées de traitement du signal et Applications. 1993

[16] Drouiche K. Signaux CETIM, analyse spectrale. Telecom-Paris, departement signal; 1991

[17] Nigam VP, Graupe D. A neural-network-based detection of epilepsy. Neurological Research. 2004;**26**(1): 55-60

[18] Boite R. Traitement de la Parole. PPUR presses polytechniques; 2000