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Chapter

Probabilistic-Entropic Concept of Sustainable Development of the Example of Territories

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Abstract

Nowadays the problem of sustainable development became one of the topical issues. However, many authors point to conceptual complexity, which is that the concept "sustainable development" includes two terms "development" and "sustainability." At the same time, each of these terms is treated not unambiguously. It leads to the emergence of different interpretations of sustainable development in relation to specific systems. A new concept of sustainable development of systems is proposed. According to this concept, the sustainable development of a complex multidimensional system will be understood as the dynamics consisting in the presence of a trend of balanced change in the entropies of randomness and self-organization while maintaining an acceptable risk level in multidimensional systems. The proposed concept is approved on practical examples. Dynamics of vector entropy and multidimensional risk of Yekaterinburg and Sverdlovsk regions in 1992–2017 is given.

Keywords: sustainable development, differential entropy, risk, multidimensional random variable, monitoring, vector, randomness, self-organization

1. Introduction

The term "sustainable development" originally was used in 1972 at the United Nations Conference on the Human Environment in Stockholm. In 1987 in a report entitled as "Our common future" of the World Commission on Environment and Development (WCED) where Norwegian Prime Minister Gro Harlem Brundtland was a chair, the term definition of "sustainable development" has been formulated: "development that meets the needs of the present without compromising the ability of future generations to meet their own needs" [1]. The "sustainable development" interpretation is very common and does not show a particular way to move into practice [2]. One can point out the main conceptual complexity, which is that the concept of "sustainable development" includes two terms "sustainability" and "development." Moreover, each of these terms is interpreted in different points.

These terms are multidirectional. Really, maximizing efficiency usually increases risks, reducing the stability of the functioning of a system. Rather, excessive stability leads to an increase in the costs of its maintenance, reducing the efficiency of functioning of a system.

Thirdly, formalization of sustainable development is a complicated complexity of the studied systems and the phenomena. Currently, there is no unambiguous, accurate interpretation of a concept of the complex system. However, there are characteristic signs, such as multidimensionality, multiconnectivity, a multiloop, multileveled (hierarchy), the composite and multipurpose nature of construction, and also indeterminacy and stochasticity of behavior. We will give below the definition which, in our opinion, most adequately characterizes the concept of the complex system.

Definition 1. A complex system is called a system in the model of which there is not enough information to effectively manage this system [3].

This fact leads to different understandings of "sustainable development" in relation to particular systems [4–11]. For example, in [12], there are more than 50 different interpretations of the "sustainable development" concept.

The implementation of sustainable development implies that certain monitoring of the studied system or phenomenon should be carried out. Monitoring is understood as a system of constant overseeing by the current of any phenomenon for the establishment of its compliance to the initial assumptions or desirable result. This phenomenon can occur in any sphere—in social relations, in nature, in the financial and economic sphere, etc. Within monitoring, there is assessment, control of the system, and the formation of management recommendations (management of its state) depending on the impact of particular factors.

Therefore, the formulation of the formalized concept of monitoring sustainable development, which could be concretized for particular cases, seems to be an urgent problem. One of the possible ways is to use a systems approach [13]. Let us make an attempt to formulate monitoring of the sustainable development concept of complex systems with the example of territories.

2. Problem statement

The systems approach involves the representation of the system S in the form of interrelated elements (infrastructures, key indicators, etc.). Territorial systems are complex ecological and socioeconomic systems, consisting of a large number of interacting elements. They are characterized by a huge variety and complexity of factors, elements of infrastructures, and relations between them [14]. The features of territorial systems can be attributed to [15] multidimensionality, the interconnectedness of components, stochastic nature of the behavior, multicriteriality, and diversity behavior of the elements.

Taking these features into account, the system S can be represented as a random vector $\mathbf{Y} = (Y_1, Y_2, ..., Y_m)$. Each component Y_i of this vector is a one-dimensional random variable characterizing the functioning of the corresponding element of the system (**Figure 1**).

The sustainable development concept has to reflect two components: "sustainability" and "development." Therefore, for its formulation, it is necessary to solve the following three tasks:

1. The development of an integral indicator which characterizes the effective functioning of the system.

2. Ensuring the sustainable functioning of the system.

3. The formation of a criterion which characterizes a sustainable development in the point of solving the first two tasks.
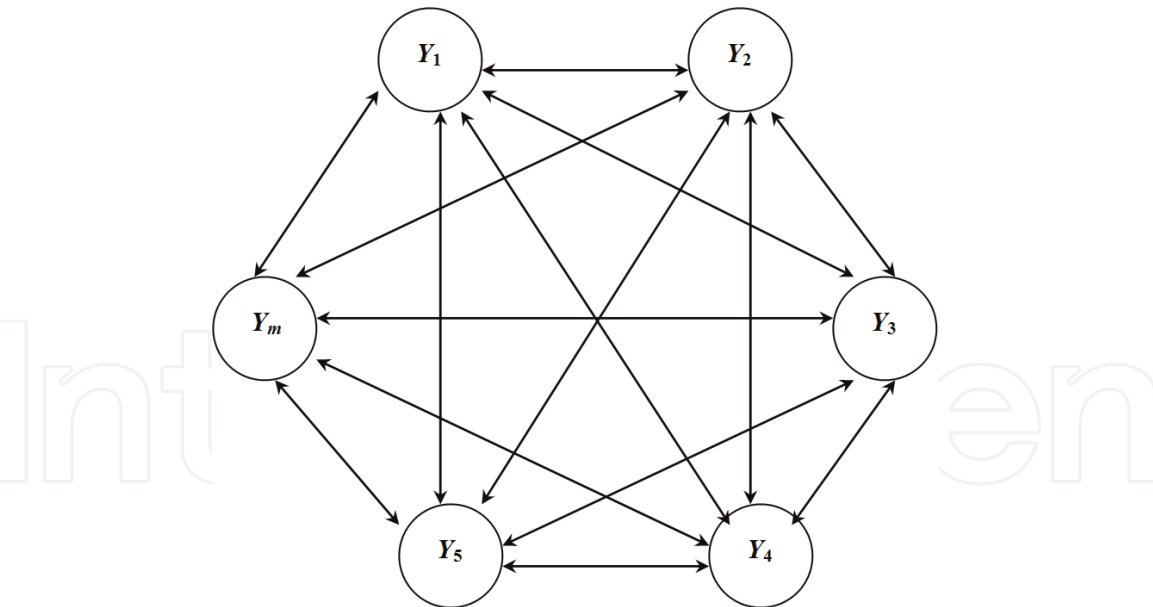
Figure 1.
A model of the system as a structure model.

3. Vector entropy model for the effective functioning of systems

Consider the problem of developing an integral indicator that characterizes the effective functioning of the system.

Multicriteriality of complex systems functioning, including territorial, and the diversity of their elements functioning, makes the development of universal formal indicators difficult which characterizes the effectiveness of systems as a whole.

It is known that entropy is a fundamental property in any systems with probabilistic behavior [16]. The concept of entropy is flexible and allows interpretation in terms of the branch of science, where it is applied. Therefore, entropy modeling is one of the promising lines of research of complex stochastic systems [17–20].

However, the frequent use of entropy for modeling of open systems, in contrast to thermodynamics, is insufficiently formalized and has generally qualitative and private character; there are no rather simple and adequate mathematical models that allow associating entropy with the actual characteristics of conditions of multidimensional systems. Common in these works is the use of Shannon's information entropy [21]. But, as it is noted in [15], the information entropy allows developing adequate entropy models only for particular problems.

However, in the same work [21], Shannon heuristically offered a formal analog of a concept of information entropy for the m-dimensional continuous random vector of Y with a probability density:

$$H(\mathbf{Y}) = - \int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{+\infty} p_{\mathbf{Y}}(y_1, y_2, \ldots, y_m) \ln p_{\mathbf{Y}}(\mathbf{y}) dy_1 dy_2 \ldots dy_m. \qquad (1)$$

This value Kolmogorov together with Gelfand and Yaglom was called subsequently differential entropy [22].

The differential entropy, being the functional given on the set of the probability density of a random vector of Y, represents a number. Therefore it cannot be an adequate mathematical model of a multidimensional system. However, the practical use of entropy (1) is complicated by the need to know the distribution law of a multidimensional random value of Y.

In [23] it was offered to use a differential entropy (further, an entropy) for modeling multidimensional stochastic systems. It is proven [23] that entropy in Eq. (1) can be represented as a sum of two components:

$$H(\mathbf{Y}) = H(\mathbf{Y})_V + H(\mathbf{Y})_R, \tag{2}$$

$$H(Y)_V = \sum_{i=1}^{m} H(Y_i) = \sum_{i=1}^{m} \ln \sigma_{Y_i} + \sum_{i=1}^{m} \kappa_i \text{—randomness entropy,}$$

$$H(Y)_R = \frac{1}{2} \sum_{k=2}^{m} \ln \left(1 - R^2_{Y_k/Y_1 Y_2 \dots Y_{k-1}}\right) \text{—self-organization entropy,}$$

where $\sigma^2_{Y_i}$—dispersion, $\kappa_i = H(Y_i/\sigma_{Y_i})$—entropy indicator shows a type of random value distribution law Y_i, $i = 1,2,\dots,m$; $R^2_{Y_k/Y_1 Y_2 \dots Y_{k-1}}$ - coefficient of determination of regression dependencies of random vector $\mathbf{Y}$, $k = 2,3,\dots, m$.

In particular, for multidimensional normally distributed random variable $\mathbf{Y}$.

$$H(\mathbf{Y})_V = \sum_{i=1}^{m} \ln \sigma_{Y_i} + m \ln \sqrt{2\pi e}, H(Y)_R = \frac{1}{2} \ln (|R|), \tag{3}$$

where $\mathbf{R}$ is the correlation matrix of random vector $\mathbf{Y}$.

The formula (2) does not always explain the behavior of the system. The addition of the component $H(\mathbf{Y})_V$ and $H(\mathbf{Y})_R$ in terms of systems analysis is incorrect since they characterize various regularities of the complex systems: $H(\mathbf{Y})_V$ is additivity, and $H(\mathbf{Y})_R$ is an integrity of the system.

The practical use of the relation (2) showed that there are situations when systems with different functional states have approximately the same general entropies of $H(\mathbf{Y})$, but the corresponding values of entropies of randomness $H(\mathbf{Y})_V$ and self-organization $H(\mathbf{Y})_R$ have significant differences. It schematically looks as follows. There are two the same systems $\mathbf{Y}^{(1)}$ and $\mathbf{Y}^{(2)}$ with different states. At the same time, $H(\mathbf{Y}^{(1)}) = 0$, $H(\mathbf{Y}^{(1)})_V = 1$, $H(\mathbf{Y}^{(1)})_R = -1$ and $H(\mathbf{Y}^{(2)}) = 0$, $H(\mathbf{Y}^{(2)})_V = 10$, $H(\mathbf{Y}^{(2)})_R = -10$.

Complex systems, including territorial ones, are open, and their entropy can both increase and decrease. Moreover, the directions of change in the entropies of randomness $H(\mathbf{Y})_V$ and self-organization $H(\mathbf{Y})_R$ of systems may be different. To build adequate models and investigate multidimensional stochastic systems, differential entropy should be considered not in scalar, but in vector form as two components—the entropies of randomness and self-organization as [15]:

$$h(\mathbf{Y}) = (h_V; \; h_R) = \left(H(\mathbf{Y})_V; \; H(\mathbf{Y})_R\right). \tag{4}$$

In specific situations, the direction and values of the entropy vector Eq. (4) should be set on the basis of the features of the studied system. In other words, complex systems should have a balance between the entropies of randomness and self-organization.

The complex systems are open. Influence of entropy on the evolution of open systems was investigated by many scientists. In their publications, it is noted that the change of open systems either leads to degradation or it is self-organization process as a result of which more complex structures appear. Prigogine [24] in 1955 formulated an extended version of the second law of thermodynamics. According to this law, the total change of entropy dS of an open system must be represented in the form of two parts. The reason of the first of them serves internal processes

which are irreversible and by all means are followed by the transition of a part of the energy of ordered processes (kinetic energy of a moving body, energy of electric current, etc.) into the energy of the disordered processes and eventually in warmth. The second part is caused by the exchange of energy and substance between a system and a surrounding medium:

$$dS = dS_{in} + dS_{out}, \qquad (5)$$

where dS is the total change in entropy of an open system, dS_{in} is the change in entropy during the processes occurring in a system, and dS_{out} is the change of entropy during the processes of exchange with the environment.

However, the question of the practical application of this theory for research of real systems has not been disclosed. Let us express the change of the total entropy of $\Delta H\ (Y)$ through the changes of entropies of randomness and self-organization:

$$\Delta H(\mathbf{Y}) = \Delta H(\mathbf{Y})_V + \ \Delta H(\mathbf{Y})_R \qquad (6)$$

Let us try to give an interpretation of Eq. (5) according to Eq. (6). First, it is apparent that $dS \cong \Delta H(Y)$. The sign of the conditional equality "$\cong$" is used in view of the fact that in [24], change of a thermodynamic entropy of dS was considered.

Let us consider the influence on the entropy of processes of exchange with the environment. From the environment multidimensional open system takes or gives energy, which can be treated as a change of mean square deviations σ_{Y_i}. Besides the appearance of new properties, states can also occur from the outside, from the environment. Therefore, the change of distribution type, and therefore entropy indicators, is also due to the process of exchange of the system with the environment, that is, it is possible to consider that the change of entropy during processes of exchange with the environment represents a change of randomness entropy:

$$dS_{out} \cong \Delta H(Y)_V. \qquad (7)$$

System elements in the process of functioning can strengthen or weaken the interaction between them due to the increase or decrease of the narrowness of correlation communication. Therefore, the change of entropy during the processes happening in a system is a change of self-organization entropy:

$$dS_{in} \cong \Delta H(Y)_R. \qquad (8)$$

On the basis of Eqs. (7) and (8), it is possible to make the following hypothesis.

Hypothesis 1. The total change of entropy of an open system consists of the sum of two items. The first item characterizes the impact of the interaction of a system with the external environment and represents a change of randomness entropy. The second item characterizes the processes occurring within a system and represents a change of self-organization entropy.

Example 1. Entropy analysis of Yekaterinburg (regional center of Sverdlovsk region) development dynamics in 1992–2017.

The effective functioning of the megalopolis as a complex system according to the vector entropy model Eq. (4) consists in the simultaneous growth of diversity, opportunities for all elements of this system, and the presence of a close interrelation between these elements. This is manifested in the fact that with the development of a megalopolis, its randomness entropy should gradually increase, and the self-organization entropy will decrease.

The analysis will be performed according to the official data from the Russian Federal State Statistics Service (Rosstat) [24]. Of the many basic socioeconomic indicators of cities, we will form a system of signs that characterize all the main aspects of the city's infrastructure [15]:

1. Natural increase, decrease (−) per 1000 population.

2. The share of the working population in organizations, %.

3. Average monthly nominal accrued wages (in 2017 prices), thousand rubles.

4. The share of retirees registered with the social security authorities, %.

5. The total area of residential premises per one urban resident (at the end of the year), m².

6. Number of pupils in preschool educational organizations, thousand people.

7. The number of doctors per 1000 population, people.

8. The number of registered crimes per thousand people.

9. The volume of work performed under construction contracts (in 2017 prices), ths. rub. for one person.

10. Retail trade turnover (in 2017 prices), ths. rub. for 1 person.

11. Investments in fixed assets (in 2017 prices), ths. rub. for one person.

When calculating the entropy, the estimates were performed for periods of 13 years. This period turned out to be optimal, on the one hand from the statistical smoothing point of view and on the other hand because it takes into account the dynamics of entropy change. Entropy was estimated in the vector form Eq. (4). Accounting for inflation was carried out by recalculation in 2017 prices based on consumer price indices; the different populations of cities were taken into account by the transition to relative indicators per inhabitant. Since the sample was quite small, the deviations of the empirical distributions of the considered features from the normal distribution are practically impossible to establish. Therefore, when calculating the entropies of randomness and self-organization, we use Eq. (3).

Figure 2 shows the graphs of changes in the entropies of randomness and self-organization in Yekaterinburg. **Figure 3** shows the entropy dynamics.

Analysis of graphs in **Figures 2** and **3** allows us to make the following conclusions:

1. The period of stabilization of operation (until 2008). The randomness entropy h_V increases, and the self-organization entropy h_R decreases.

2. The global financial crisis of 2008–2009. Short-term sharp change in the direction of vector entropy to the opposite.

3. The period of economic recovery after the financial crisis (2009–2011), followed by a decrease in the growth rate of gross domestic product
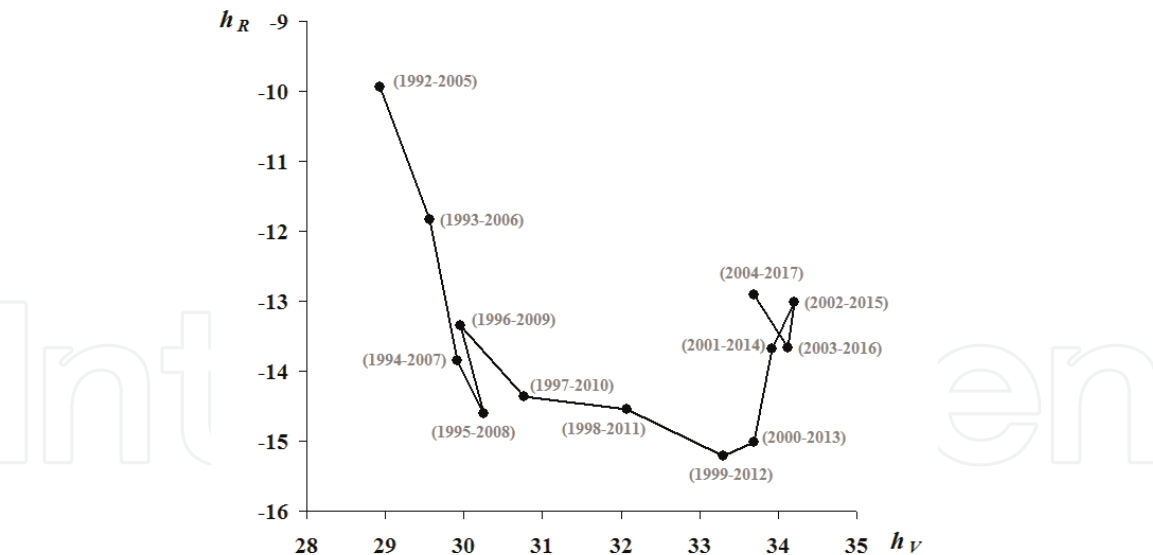
Figure 2.
The change in self-organization and randomness entropies in Yekaterinburg.
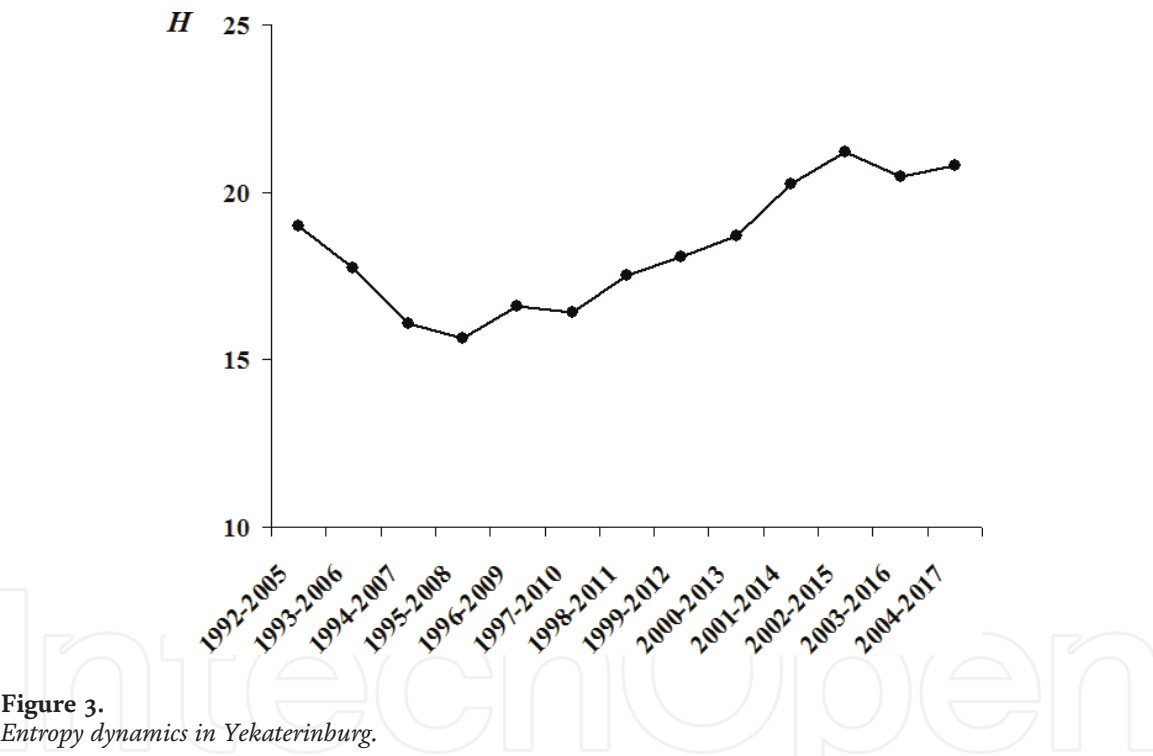
Figure 3.
Entropy dynamics in Yekaterinburg.

(2012–2013). The direction of vector entropy gradually changes (at first h_V increases, and h_R decreases, but gradually this trend fades).

4. Announcement of sanctions (2014). A steep increase in h_R.

5. Functioning in terms of sanctions (since 2014). There is a practical lack of vector entropy dynamics, with the value of self-organization entropy h_R fixed at the level of 2006–2007, and randomness entropy h_V at the level of 2013.

6. The total entropy $H(\mathbf{Y})$ in the period under consideration has changed slightly.

Example 2. The modeling of a system that characterizes the safety of the production.

Seventeen coal mining enterprises were investigated [23]. On the basis of two generalized factors (Y_1 is the factor characterizing the organization of safe production; Y_2 is the factor reflecting the professionalism of the staff), all enterprises were divided into two groups: (1) enterprises with a low level of injury; (2) enterprises with a high level of injury. For the first and second groups of mines, respectively, we have.

$$\left(h_V^{(1)}; h_R^{(1)}\right) = (2, 42;\ -0, 31), H\left(Y^{(1)}\right) = 2, 11,$$

$$\left(h_V^{(2)}; h_R^{(2)}\right) = (3, 74;\ -0, 70), H\left(Y^{(2)}\right) = 3, 04.$$

In this case, the direction of the entropy change vector will differ from Example 1: at the enterprises with a high level of injury, the randomness entropy needs to be reduced, and the self-organization entropy needs to be increased.

For example, this can be accomplished as follows: to bring the state of the second group of mines to the state of the first group, it is necessary to reduce the dispersion of the factor characterizing the organization of safe production and reduce interrelation with the factor reflecting the professionalism of the staff. This means a more specific and accurate organization of production. The organization of safe production should not depend on the degree of professionalism and competence of staff strongly.

Example 3. We investigate the possibilities of entropy modeling on the example of the population analysis in terms of prevention of chronic noninfectious diseases (CNID) by biological risk factors [23]. For carrying out the analysis of change of population entropy depending on the health status, two equal age groups were formed: 18–26 years and 27–35 years. Four risk factors were identified: "total cholesterol," "systolic blood pressure," "body mass index," and "glucose level." The results of the analysis are given in **Table 1**.

As the health of the population deteriorates, the total population entropy and the randomness entropy increase. This can be explained by the fact that the additional damaging influence of CNID, in general, is added to the pathological influence of risk factors on a human body separately and on all population.

Conversely, the self-organization entropy as the deterioration of the health status of the population decreases which corresponds to the strengthening of the narrowness of the interrelations between subsystems. This can be explained by the fact that the development of diseases in the organism happens in many respects and it is interdependent. On the other hand, at the development of diseases, some

Age (years)	Health status	Randomness entropy $H(Y)_V$	Self-organization entropy $H(Y)_R$	Total entropy $H(Y)$
18–26	Healthy	5.500	−0.514	4.986
	Apparently healthy	7.131	−0.578	6.553
	Patient	7.847	−0.696	7.151
27–35	Healthy	5.731	−0.299	5.432
	Apparently healthy	8.376	−0.542	7.834
	Patient	8.720	−0.781	7.939

Table 1.
Entropy levels in different groups of people.

subsystems can adapt to others, compensating shortcomings their functioning (substitution effect).

4. Multidimensional risk model of complex systems

Consider the task of ensuring the system sustainable functioning. Here you first need to concretize the "sustainability" concept. Typically the stability of the system functioning is interpreted in terms of its safety. Security issues are resolved with the help of risk analysis [25]. Some authors note that growth rates of damage considerably exceed growth rates of the economy [26, 27]. This can be explained with a constant increase of risk in the conditions of a scientific and technical revolution and the forced development of a technosphere [28]. Therefore, we will assume that the functioning stability of the territorial system is intimately connected with risk; the lower the risk level, the more stable the system state. Thus, the diagnosis of system sustainability can be made on the basis of monitoring its risk. This requires adequate risk models.

Let S be some multidimensional stochastic system. Let us consider an adequate representation of this system as a random vector $\mathbf{X} = (X_1, X_2, ..., X_l)$ with a certain probability density $p_{\mathbf{X}}(\mathbf{x})$. The development of a complex system and an increase in the efficiency of its functioning are an inevitable cause of increasing risks. Therefore, it is necessary to assess the risks of such systems. Consider the risk model of multidimensional stochastic systems proposed in [29].

Instead of the generally conventional selection of concrete dangerous situations, we will define the geometric area D of adverse outcomes. Formally this area can look arbitrarily depending on a specific objective.

The concept of dangerous states as larger and improbable deviations of a conception of dangerous conditions as large and improbable deviations of random variables from some best provision $\mathbf{\Theta}$ is mostly distributed. In this case, D represents an external area of an m-axis ellipsoid.

Setting the function of consequences from dangerous situations (risk function) in the form of $g(x)$, we will receive a model for the quantitative assessment of risk [30]:

$$r(X) = \int\int ... \int_{R^m} g(x) p_X(x) dx. \tag{9}$$

If in Eq. (9) to accept $g(x) = 1 \ \forall x \in D$ and $g(x) = 0 \ \forall x \notin D$, that $r(X) = P(X \in D)$, that is, the risk is estimated as a probability of an unfavorable outcome.

If at an early stage of system analysis is difficult to describe enough precisely the $g(\mathbf{x})$ function, then Eq. (2) becomes an assessment of $P(D)$ and is a convenient initial approximation of risk model.

To define a function $g(\mathbf{x})$ requires a quantitative assessment of consequences for the studied system depending on values of risk factors. It demands to carry out separate research. Let us note that values of the function $g(\mathbf{x})$ are given in the nominal units. But they are usually quite simply interpreted in the respective subject area. The essence of the function $g(\mathbf{x})$ is as follows. It accepts the least nonnegative (e.g., zero) value in a point of $\mathbf{\Theta}$ or in its neighborhood of $U(\mathbf{\Theta})$. Further in each direction during removal from $U(\mathbf{\Theta})$ the $g(\mathbf{x})$ function has to increase monotonously. For scaling on each risk factor, we will set some limit values, at which consequences become dangerous (or irreversible). Let us set values $g(\mathbf{x})$ at each

such point equal to some value, for example 1. For convenience, it is desirable to impose a number of restrictions on the function $g(\mathbf{x})$: convexity, continuity, etc.

In [30] the variant of the task of the $g(\mathbf{x})$ function in the form of a paraboloid is given. By way of illustration in **Figure 4**, the example of the risk function for a case $m = 2$ is shown. The ellipse describing the area $\overline{D}$ of admissible values of risk factors and lying on the Ox_1x_2 ($r = 0$) plane is shown by black color. The paraboloid above the plane represents possible values of risk $r(X)$. White points on the plane are values of risk factors; to them there correspond points on paraboloid surface which set risk values; the image of the border of an ellipse $\overline{D}$ is shown in the form of the black line. All corresponding couples of points (values of risk factors and risk values) are connected among themselves by vertical dashed lines.

In the problems of risk monitoring, along with risk assessment, $r(X)$ on all risk factors of $X_1, X_2 ..., X_m$ of the multidimensional system is expedient to estimate the contribution of each factor to total risk. We introduce a random vector $X_k^- = (X_1, ..., X_{k-1}, X_{k+1}, ..., X_m)$. Then the absolute change of risk of the multidimensional system due to the addition of factor X_k is equal:

$$\Delta r(X_k) = r(X) - r(X_k^-). \tag{10}$$

Dividing $\Delta r(X_k)$ of the risk $r(X_k^-)$, we will receive the relative change of risk of the multidimensional system by the addition of factor X_k:

$$\delta r(X_k) = \Delta r(X_k)/r(X_k^-). \tag{11}$$

Let us note that along with a contribution to the common risk of one factor, Eqs. (10) and (11) allow us to estimate influence and groups of factors.

Monitoring risk on the basis of the model in Eqs. (9)–(11) consists of serial estimation in time of the actual values of $r(\mathbf{X})$, $\Delta r(X_k)$, $\delta r(X_k)$, $j = 1, 2, ..., m$, and also dynamics of their change.

Let us consider the most common case when $\mathbf{X}$ has a joint normal distribution with a probability density:
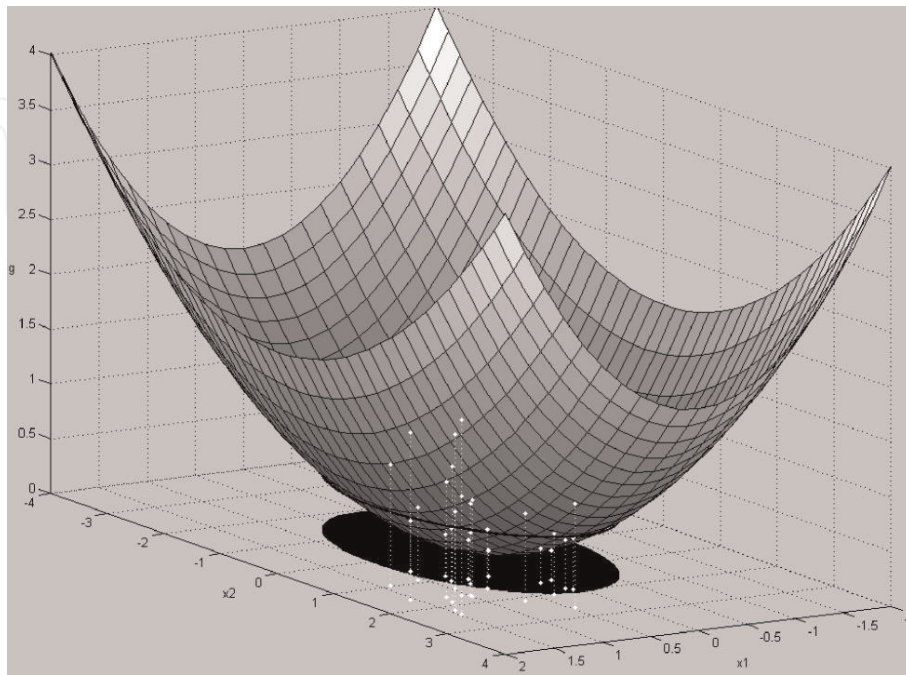
Figure 4.
An example of a two-dimensional risk functions.

$$p_X(x) = \frac{1}{\sqrt{(2\pi)^m |\Sigma|}} \exp\left\{ -\frac{1}{2}(x-a)^T \Sigma^{-1}(x-a) \right\},$$

where $a = (a_1, a_2, \ .., a_m)^T$—a vector of expectations, $\Sigma = \{\sigma_{ij}\}_{m \times m}$—a covariance matrix, $\sigma_{ii} = \sigma_i^2$—dispersion of factor X_i.

The use of a Gaussian random vector is based on the central limit theorem [31]. As approbation on a number of examples has shown, such idealization is not so critical, and if there are any bases to consider that density of probabilities is a component of the vector of **X** having more extended tails, then this can be corrected by setting the $g(\mathbf{x})$ function accordingly.

Example 4. Let us consider a two-dimensional Gaussian random vector which components have a zero average and single dispersion. In **Figure 5** the example of realization of such accidental vector is shown for: (a) $\rho = 0,4$; (b) $\rho = 0,9$, where ρ is a coefficient of correlation between *X1* and *X2*.

From **Figure 5** we see that the probability of large deviations of the random vector from the origin increases with the increase in the closeness of the correlation.

Example 5. Let us estimate the probability of P(D) for a Gaussian random vector of X, with the different narrowness of correlation communication $D_e(X) = 1 - |R_X|^{1/m}$ [32], where $|R_X|$ is a continuant of a complete correlation matrix ($D_e(X) = 0$ is independence of components, and $D_e(X) = 1$ is a rigorous linear relation). Let us consider the following cases: $D_e(X) = 0$, $D_e(X) = 0,5$, $D_e(X) = 1$. The results of the calculation of *P(D)* are given in **Figure 3**. For descriptive reasons we will accept $A_1 = A_2 = ... = A_m = A$, $A_j = b_j/\sigma_j$.

The analysis of schedules in **Figure 6** indicates the following.

The increase in the probability of an unfavorable outcome is influenced by both the increase in the dimension of m and an increase in the narrowness of correlation communication between the components of a random vector of X. Let us note that even the average narrowness of correlation ($D_e(X) = 0,5$) leads to a significant increase in the probability of an unfavorable outcome. The effect increases with the increase in values A_j that correspond to less probable, but more dangerous, adverse outcomes. Therefore, risk modeling should take into account both a multidimensionality factor and narrowness of correlations.

Example 6. Approbation of risk model of a multidimensional stochastic system for monitoring of risk of Sverdlovsk region in 1999–2017.

Let us execute monitoring of Sverdlovsk region on the dynamics of macroeconomic risk factors, taken as an interval of 9 years. Risk factors and their threshold levels are given in **Table 2**.
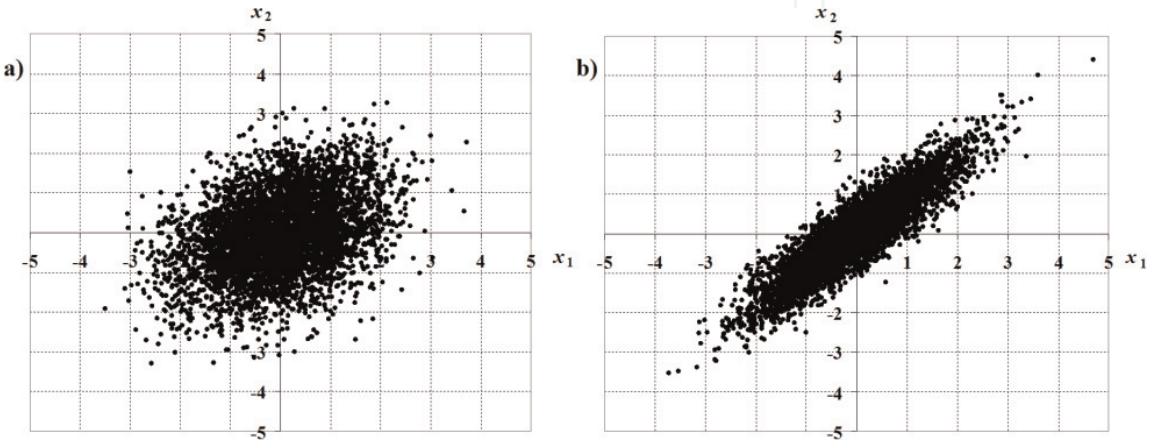
Figure 5.
Realization of a standard normal random vector.
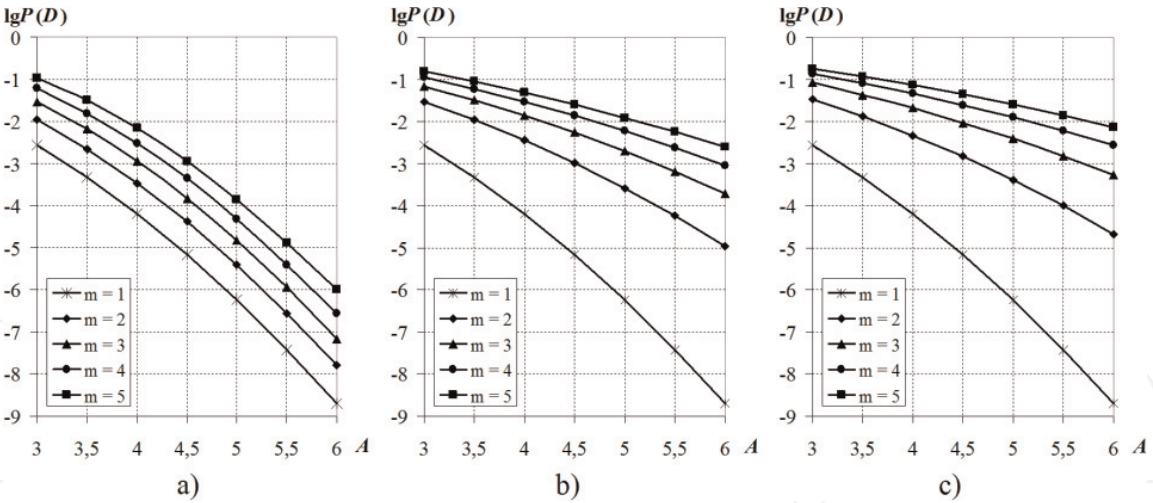
Figure 6.
Dependences of lgP(D) on threshold level A: (a) $D_e(X) = 0$; (b) $D_e(X) = 0.5$; (c) $D_e(X) = 1$. Designations: Row 1 (m = 1), row 2 (m = 2), row 3 (m = 3), row 4 (m = 4), row 5 (m = 5).

Risk factors	Threshold levels K_j
X_1—real income movement, in % to the previous year	79.93
X_2—the ratio of the average size of pension to subsistence minimum of pensioners	0.66
X_3—morbidity on 1000 people of the population	960
X_4—mortality from external causes, number of the dead on 100,000 people of the population	322.1
X_5—wear of fixed assets on the end of the year, %	71.33
X_6—the volume of budget revenues per capita, in the prices of 2017, thousand rubles	21.75
X_7—quantum index of gross regional product, % to the previous year	88.4
X_8—unemployment rate, in %	18

Table 2.
Macroeconomic risk factors of the region.

We consider that random vector **X** has a joint normal distribution.

In **Figures 7** and **8**, results of the calculation of the probability of unfavorable outcome *P(D)* and risk *r*(**X**) for the threshold levels of risk factors K are shown.

Analysis of the results of monitoring of multidimensional risk in the Sverdlovsk region showed the following:

1. During the initial period, the greatest socioeconomic instability (the highest risk values) was observed. Then gradually the dynamics of sustainability in the region have increased (decreased risks).

2. After the sanctions were imposed, the risk began to increase. The lower rate of growth of *r*(**X**) than *P(D)* indicates that the probability of occurrence of very dangerous situations has been growing slightly since 2014.

3. In the Sverdlovsk region until 2010, the main contribution to regional instability was made by factor X_6, then X_2 became such a factor, and since 2015, the main contribution to instability was made by factor X_7.
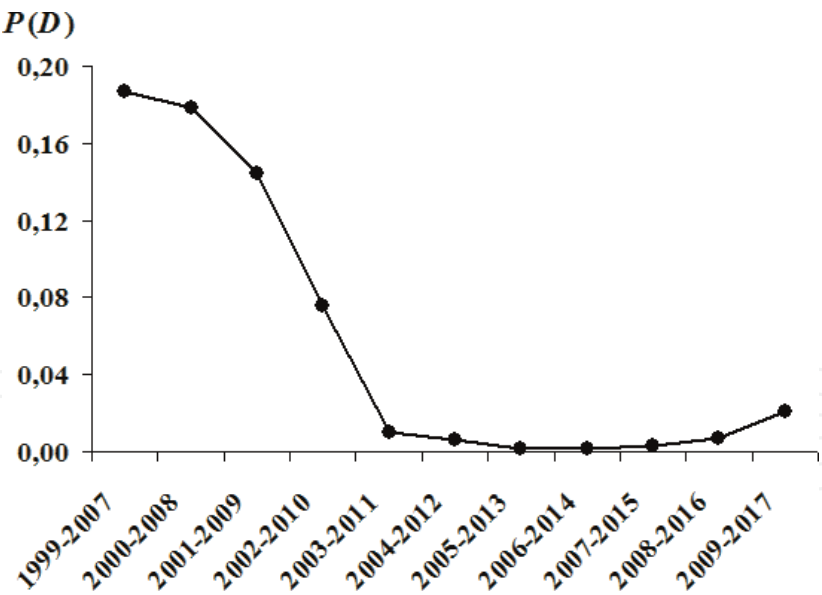
Figure 7.
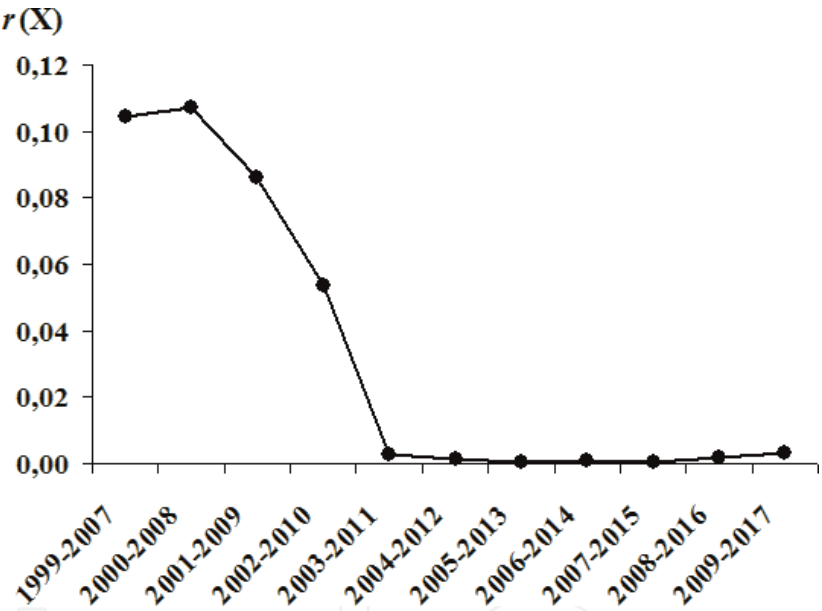*Estimates of P(D) in the Sverdlovsk region.*

Figure 8.
Estimates of r(X) in the Sverdlovsk region.

5. Formation of the sustainable development concept

Consideration of examples shows that "development" and "sustainability" characterize various aspects of the complex systems operation. And for ensuring sustainable development, they need to be taken into account together.

Hypothesis 2. We will understand dynamics consisting of available trends of the balanced change of a vector entropy while maintaining an acceptable level of risks as sustainable development of the complex system.

For this purpose, we combine the vector entropy model and the risk model of a multidimensional stochastic system. Moreover, it is necessary to consider elements of the system (components of the random vector $\mathbf{Z}$), both as risk factors X_i and as indicators Y_j, characterizing the functioning of the system, that is,

$$\mathbf{Z} = \mathbf{X} \cup \mathbf{Y} = (Z_1, Z_2, ..., Z_n), \ \max(l, m) \leq n \leq l + m.$$

The case $n < l + m$ appears, when $X \cap Y \neq \varnothing$.

Within the framework of the proposed concept, along with the tasks of monitoring complex systems discussed above, it is possible to solve management problems (development of control recommendations).

The idea of vector entropy control is the transfer of the vector $\mathbf{h}(\mathbf{Z})$ from the state $h(Z^0) = (h_V^0; h_R^0)$ in the state $h(Z^*) = (h_V^*; h_R^*)$, which corresponds to the effective functioning of the stochastic system.

For a Gaussian system, the vector entropy control consists in directing the entropy from some initial point $(h_V^0; h_R^0) = (H(Z^0)_V; H(Z^0)_R)$ with the covariance matrix Σ_0 to the final point $(h_V^*; h_R^*)$ with a minimal change of the covariance matrix $\Sigma^0 = \{\sigma_{ij}^0\}$ and the expectations vector a^0 and acceptable risk (**Figure 9**).

The problem of the vector entropy control of the Gaussian system to ensure sustainable development will take the form:

$$
\begin{cases}
G(\Sigma) = \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\sigma_{ij} - \sigma_{ij}^0\right)^2 + \sum_{i=1}^{j}\left(a_i - a_i^0\right)^2 \to \min_{a,\,\Sigma}, \\[2mm]
H(Y)_V = A, \\[1mm]
H(Y)_R = B, \\[1mm]
r(X) \leq r, \\[1mm]
\Sigma \in G_\Sigma, \quad a \in H_a, \\[1mm]
\sigma_{ij}^2 < \sigma_{ii}\sigma_{jj}, \quad \sigma_{ij} = \sigma_{ji}, \quad \sigma_{ii} > 0 \ \forall \ 1 \leq i, j \leq n, \\[1mm]
\Sigma > 0,
\end{cases}
\tag{12}
$$

where $A = h_V^*$, $B = h_R^*$, $\mathbf{a}$—the expectations vector of the components X_i, $i = 1, 2, ..., l$.

The last constraint in Eq. (12) means positive definiteness of the matrix Σ. Note that the performance criterion in Eq. (12) may be different, depending upon the characteristics of a particular system S.
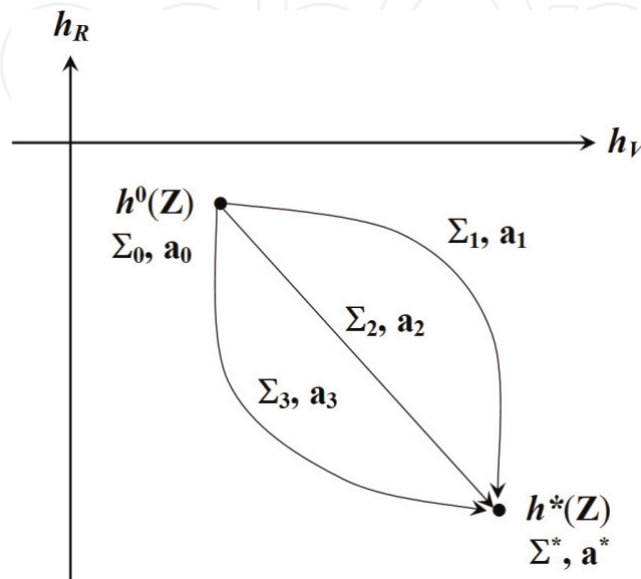
Figure 9.
An illustration of vector entropy management to ensure sustainable development of the system.

6. Discussion of results

Thus, on the basis of the use of two original models—a vector entropy and multidimensional risk—it was succeeded to formalize the new concept of sustainable development of complex systems. Both models are successfully approved on real data.

This concept can be implemented by means of monitoring of the studied system. As observed parameters efficiency factors of the functioning of a system and its risk factors are used. The direction of development is given by an entropy vector, and stability is provided due to an acceptable risk level.

Management recommendations are formed in the form of the solution to an extreme problem Eq. (12). This problem is solved by methods of penalty functions. Currently, the work is at a stage of practical approbation of monitoring of sustainable development of Sverdlovsk region.

7. Conclusion

1. The probability-entropy concept of sustainable development of complex stochastic systems is formulated. It is based on vector entropy and multidimensional risk models.

2. According to the formulated concept, the sustainable development of a complex system will be understood as the dynamics consisting of the tendency of a balanced change in vector entropy while maintaining an acceptable level of risk.

3. The proposed concept of sustainable development has been tried out in application to territorial systems.

Acknowledgements

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