Chapter

Selection of Food Items for Diet Problem Using a Multi-objective Approach under Uncertainty

Saman Hassanzadeh Amin, Samantha Mulligan-Gow and Guoqing Zhang

Abstract

It is a problem that concerns us all: what should we eat on a day-to-day basis to meet our health goals? Scientists have been utilizing mathematical programming to answer this question. Through the use of operations research techniques, it is possible to find a list of foods that, in a certain quantity, can provide all nutrient recommendations in a day. In this research, a multi-objective programming model is provided to determine the selected food items for a diet problem. Two solution approaches are developed to solve this problem including weighted-sums and ε-constraint methods. Two sources of uncertainty have been considered in the model. To handle these sources, a scenario-based approach is utilized. The application of this model is shown using a case study in Canada. Using the proposed model and the solution approaches, the best food items can be selected and purchased to minimize the total cost and maximize health.

Keywords: food optimization, diet, nutrition, multi-objective programming, uncertainty

1. Introduction

It is common knowledge that diet affects general health in extraordinary ways. What is less clear is what specific diet results in the best health. Some diets restrict the quantity of carbohydrates or fats, others require particular percentages of the three macronutrients (carbohydrates, fats, and protein), some depend solely on liquids, and the list continues [1–5]. There are unlimited amounts of unique diets being used today by people all over the world, especially since countless health trends have become the new normal. One thing that can be agreed upon is the recommended dietary allowances (RDA), given by the federal government of Canada, which presents the quantity of vitamins and nutrients needed to meet requirements. So it is important to determine the combination of the seemingly infinite food items that reaches nutrient goals in the most efficient manner.

The diet problem was first introduced by Stigler [6] as a way to determine the minimum cost of feeding an adult for a year. Many models have since explored diet optimization with the objectives of reaching recommended nutrient levels while keeping the diets similar to actual intakes, decreasing environmental impact, or satisfying taste. There are numerous models that can be used to create unique diets based on the main target criteria. These models include linear and nonlinear, multi-objective, goal-oriented, integer, and mixed-integer programming. Each yields specific results due to the mathematical basis.

2. Literature review

In this section, some related papers are discussed, classified on the subject of the model's goal. Most research of the diet problem is centered on at least one of the following targets: cost of diet, similarity of diet, environmental sustainability of diet, prevention of health implications by diet, and taste/satisfaction of diet.

2.1 Cost of diet

The first diet problem [6] focused on minimizing the cost of food. The author showed that to feed one man for a year can cost as little as $39.93 (1939 prices). Certainly, lowering the grocery bill is a desire for all and this type of objective is quite common to this day. One paper discusses whether it is too expensive to follow a healthy diet, comparing 2012 and 1980 costs [7]. Stigler's problem was reinvented with updated costs and nutritional information in 1999 [8] and taste of diet was included in the cost minimization by Smith [9].

In recent years, further papers focused on more specific problem statements and how they are affected by expense. In Mozambique, the affordability of a nutritious meal plan was studied and fortified foods were assessed with the hopes of creating more economic value [10]. Specific diets have also been studied to determine whether they can be accomplished in a cost-effective manner [11]. To attempt a solution of high-cost food, James David Ward studied urban agriculture and how it could reduce grocery expenses [12].

2.2 Sustainability of diet

Considering that livestock production is a major contributor to greenhouse gas emissions, worldwide [13], the question of nutritional sustainability has been asked and answered. Multi-objective linear programming was used to formulate three unique diets that minimized cost, environmental indicators (H_2O use, amount of land to regenerate the resources, and CO_2 emissions), and the integration of the two [13]. Another study was completed in which the optimal diet was to be as similar to the general, observed diet [14]. This chapter noted that a sustainable diet reduces greenhouse gas emissions by 27% [14]. Mathematical programming was also used to study which food sources contributed least to environmental footprints such as land use, carbon and nitrogen footprints [15]. Barre et al. [16] found diets by using – on the reduction percentage of environmental impacts being at least 30%. It was concluded that all diets required a decrease in meat consumption to meet the sustainability factors [16]. The cost of feeding cattle was found to increase (by as much as 48.5%) in a hypothetical scenario where there was either a tax on greenhouse gas emissions or a constraint on methane emissions applied [17].

2.3 Disease prevention of diet

It is commonly known that what you eat affects your health. Studies have been conducted to explain how diets can reduce your risk of certain health risks and concerns. Observed diets and those recommended by the World Cancer Fund and the American Institute of Cancer Research were compared by Masset et al. [18].

Furthermore, obesity and its relation to dietary intake has been a frequent topic of interest. Silva et al. [19] presented the possibility of a diet that constricts amount of calories by increasing the quantity of proteins eaten in a day, which will then create the opportunity for weight loss.

2.4 Similarity of diet

In many papers, one of the objectives (usually secondary) is to minimize the difference between the proposed diet and the current, observed diet of a group of people being studied [16, 20–23]. This is done for many reasons: to ensure palatability, ease of acceptance, and culturally appropriate solutions. This focus is often the backbone of the programming calculation as it guarantees a certain level of logic and reality.

In **Table 1**, related papers are organized by which mathematical programming approach was utilized. Linear programming (LP) is used for optimizing (maximizing or minimizing) a linear function of many variables [24], while nonlinear programming (NLP) does the same when there is(are) one or more nonlinear functions in the problem [25, 26]. For computing problems with multiple objectives, goal programming (GP) is often used [27]. This technique is popular in recent diet studies due to its potential ability to achieve a more realistic food balance [28]. Linear programming has been seen in **Table 1** as the most commonly used technique for diet problems, including a paper done to disavow goal programming [29].

Publication	LP	NLP	GP	MOP	Un	Final Diet	Nutrients	Model Only
[9]	✓					✓		
[32]	✓					✓		
[33]	✓					✓		
[34]				✓		✓		
[8]	✓					✓	✓	
[20]								
[21]	✓							
[10]	✓		✓			✓	✓	
[11]	✓					✓	✓	
[18]	✓							✓
[35]	✓					✓	✓	
[22]				✓	✓			✓
[10]	✓					✓		
[17]	✓							
[15]	✓		✓				✓	
[7]	✓		✓					✓
[13]			✓					
[23]	✓					✓		
[12]	✓							
[13]	✓			✓				
[14]	✓					✓		✓
[19]				✓		✓	✓	
[16]	✓	✓						
Our paper	✓			✓	✓	✓		

Goal Programming (GP); Linear Programming (LP); Multi-Objective Programming (MOP); Non-Linear Programming (NLP); Uncertainty (Un).

Table 1.
Review of some related papers and their operations research techniques [32–34].

Multi-objective programming (MOP) is used when there are multiple, competing objectives that result in more than one optimal solution [30]. With uncertain environments, fuzzy set theory (FST) and some specific techniques can be applied so that qualitative statements can be described numerically without losing precision [31].

Included in **Table 1** is the format of the results in each respective paper. Some papers explicitly create day-to-day diets, including exact foods and their quantities. We call them "Final Diet." Other papers note the nutrients that their proposed model offers if created into a diet. These are called "Nutrients" in **Table 1**. Other papers only present the model that is used to create a diet without stating which foods should be chosen. In these papers, no specific food intake is specified, rather only the math is presented.

2.5 Research gaps

There are some gaps in the literature of diet problems. There are a few papers in the literature that have considered multiple objectives in diet problems. Among them, most publications have focused on two objectives. Therefore, the research gap can be filled by considering more than two objectives. The other point is related to the availability of data. In recent years, companies have been forces to provide nutritional information for the packages and products. Therefore, a lot of useful information is available that is new and valuable in this field. Case studies can be conducted based on the available information. The other gap in the literature is about the uncertainty of the parameters in diet problems. Most of the papers in this area have ignored uncertain parameters and their effects on the results.

2.6 Research contributions

The main research contributions of this paper are defined in this section.

- A novel optimization model is provided to determine personal food selection.
- Multiple objectives are considered in the mathematical model. To our knowledge, these objectives have not been considered simultaneously in the other papers in the literature.
- The mathematical formulation is solved by two solution approaches including weighted-sums and ε-constraint approaches. As a result, the efficient solutions are obtained.
- Uncertainty in the parameters is considered using an effective scenario-based solution approach. Different combinations of the scenarios are analyzed in this paper.
- The application of the model is shown using real data and a case study in Canada.

3. Problem statement

As discussed previously, the optimization of diets is a continuously important problem since we all eat every day. What is more, the food costs money and affects our health and well-being. Some diseases related to obesity (e.g., cardiovascular and diabetes) have significant impacts in Canada. Some factors of diets such as sugars, sodium, and fat play important roles on health of people. A study done by Hajizadeh et al. [35] found that body mass index, an indicator of obesity, is negatively related to household income (and fruit and vegetable consumption). Families across Canada suffer from food poverty: the inability to purchase healthy, nutritious food for their loved ones [36]. There are people who have to make the difficult decision to either pay rent, or buy groceries. These people should be able to know that what money they put toward food is being used in the most efficient way possible. On the other hand, if food can be used to combat major health concerns within our population, this information should be taken advantage of. Obesity is an extensive issue in all regions of Canada and the major contributor is nutrition [35].

The government of Canada provides health and nutrition information online. The federal government has also created legislation that ensures all food items show a nutrition facts table on the packaging. This information covers facts on recommended macronutrients and micronutrients. Carbohydrates give the body energy and are separated into three categories: starch, fiber, and sugars [36]. Fat is a macronutrient that also provides energy to the body as well as helps digest essential vitamins. Fats are categorized into trans, saturated, and unsaturated but only trans and saturated are needed on nutrition facts tables as they are the fats that increase blood cholesterol level [36].

Cholesterol is a type of fat that is produced by the body but can also be consumed through food. High levels of cholesterol can increase risk of developing heart disease. Only animal-based foods contain cholesterol. Protein is the third macronutrient which helps tissues and muscles build recover from strain and as well as provide energy [36]. Sodium is a nutrient that is prevalent in our society due to our use of salt to preserve food, which raises blood pressure, increasing the risk of stroke, heart and kidney diseases. Calcium is a mineral found in our bones but can also be eaten in order to strengthen our bones and help our muscles work [36]. Another mineral, iron, helps produce red blood cells and helps carry oxygen through the body. Some important vitamins the government emphasizes are vitamins A and C [36]. Vitamin A maintains healthy skin and normal bone growth. Vitamin C facilitates the absorption of iron, is an antioxidant, and helps heal wounds [36].

Another resource from the Government of Canada is the recommended number of food guide servings per day [37]. They have created a table that presents the number of servings needed of each food group for all ages and genders of the Canadian population. The four food groups are: vegetables and fruit (VG), grain products (GP), milk and alternatives (MA), and meat and alternatives (ME). The ages of population categories are split into 2–3, 4–8, 9–12, 14–18, 19–50, and 51 and over [37].

In this problem, selected food items should be determined (to be purchased) according to some constraints and goals. There are four goals in this diet problem based on the available information for foods in Canada. They allow the cost of the food to be minimized while decreasing the trans/saturated fats and sugar, and maximizing the amount of fiber. This combination of objectives aims to limit nutrients that are harmful to the human body, as noted above. The diet will be based on the consumption of the chosen foods for 1-month period. Since nutrition guidelines vary based on age, the chosen population group for this study is 51 and older. This group was chosen due to the aging population of Canada.

Figure 1 shows an example of nutrition facts table. In addition, examples of four food groups (vegetables and fruit (VF), grain products (GP), milk and alternatives (MA), meat and alternatives (EA)) are illustrated in **Figure 2**.

Nutrition Facts
Per 9 crackers (23 g)

Amount	% Daily Value
Calories 90	
Fat 4.5 g	**7** %
Saturated 2.5 g + Trans 0 g	**13** %
Cholesterol 0 mg	
Sodium 280 mg	**12** %
Carbohydrate 12 g	**4** %
Fibre 1 g	**4** %
Sugars 0 g	
Protein 3 g	

Vitamin A	0 %	Vitamin C	0 %
Calcium	2 %	Iron	8 %

Figure 1.
An example of nutrition facts table [38].

Figure 2.
Examples of four food groups (vegetables and fruit (VF), grain products (GP), milk and alternatives (MA), meat and alternatives (EA)).

4. Optimization model

In this section, a multi-objective programming formulation is proposed to determine the numbers of the foods that should be consumed. The definitions of sets, parameters, and decision variables are provided in this section.

Sets

i = Type of food (1, 2, ..., I).
h = Food group (1, 2, ..., H).

t = Period (1, 2, ..., T).
j = Vitamin (1, 2, ..., J).

Parameters

a_{ih} = Size of food i in food group h.
A_{iht} = Cost of food i in food group h and period t.
B_t = Minimum total calories in period t.
C_{ih} = Calories of food i (in each unit) in food group h.
D_t = Maximum total calories in period t.
E_{jt} = Minimum total vitamin j in period t.
F_{jih} = Vitamin j in food i (in each unit) and food group h.
G_{jt} = Maximum total vitamin j in period t.
K_{ih} = Saturated and trans fats of food i (in each unit) in food group h.
L_{ih} = Sugar of food i (in each unit) in food group h.
M_{ih} = Fiber of food i (in each unit) in food group h.
O_t = Minimum total cholesterol in period t.
P_{ih} = Cholesterol of food i (in each unit) in food group h.
Q_t = Maximum total cholesterol in period t.
R_t = Minimum total sodium in period t.
S_{ih} = Sodium of food i (in each unit) in food group h.
V_t = Maximum total sodium in period t.
X_t = Minimum total protein in period t.
Y_{ih} = Protein of food i (in each unit) in food group h.
Z_t = Maximum total protein in period t.
β_t = Minimum total calcium in period t.
λ_{ih} = Calcium of food i (in each unit) in food group h.
α_t = Maximum total calcium in period t.
ρ_t = Minimum total iron in period t.
γ_{ih} = Iron of food i (in each unit) in food group h.
σ_t = Maximum total iron in period t.
ψ_{ht} = Amount of food guide servings per month in food group h in period t.

Decision Variables

N_{iht} = Number of food i in food group h and period t.

$$\text{Min } z_1 = \sum_t \sum_h \sum_i A_{iht} N_{iht} \tag{1}$$

$$\text{Min } z_2 = \sum_t \sum_h \sum_i a_{ih} K_{ih} N_{iht} \tag{2}$$

$$\text{Min } z_3 = \sum_t \sum_h \sum_i a_{ih} L_{ih} N_{iht} \tag{3}$$

$$\text{Max } z_4 = \sum_t \sum_h \sum_i a_{ih} M_{ih} N_{iht} \tag{4}$$

$s.t.$

$$B_t \le \sum_h \sum_i a_{ih} C_{ih} N_{iht} \le D_t \qquad \forall t \tag{5}$$

$$E_{jt} \le \sum_h \sum_i a_{ih} F_{jih} N_{iht} \le G_{jt} \qquad \forall j, t \tag{6}$$

$$O_t \leq \sum_h \sum_i a_{ih} P_{ih} N_{iht} \leq Q_t \qquad \forall t \tag{7}$$

$$R_t \leq \sum_h \sum_i a_{ih} S_{ih} N_{iht} \leq V_t \qquad \forall t \tag{8}$$

$$X_t \leq \sum_h \sum_i a_{ih} Y_{ih} N_{iht} \leq Z_t \qquad \forall t \tag{9}$$

$$\beta_t \leq \sum_h \sum_i a_{ih} \lambda_{ih} N_{iht} \leq \alpha_t \qquad \forall t \tag{10}$$

$$\rho_t \leq \sum_h \sum_i a_{ih} \gamma_{ih} N_{iht} \leq \sigma_t \qquad \forall t \tag{11}$$

$$\sum_i a_{ih} N_{iht} = \psi_{ht} \qquad \forall h, t \tag{12}$$

$$N_{iht} \geq 0 \qquad \forall i, h, t \tag{13}$$

The total cost of the foods is minimized in the first objective function. The second objective minimizes saturated and trans fats in the foods. In addition, the third objective minimizes the sugar of the foods. Besides, the fourth objective function maximizes the fiber of the foods.

Constraint (5) is related to the minimum and the maximum required calories in the foods. Constraint (6) is about the vitamins in the foods. In addition, constraints (7)–(11) are about the minimum and the maximum values of cholesterol, sodium, protein, calcium, and iron in the diet, respectively. Constraint (12) considers the recommended amount of food guide servings. Finally, the last constraint ensures that the variables are nonnegative.

5. Solution approach

In this section, a solution approach counting weighted-sums method and ε-constraint method is described. The main goal is to convert the multi-objective model to a single objective one.

5.1 Weighted-sums method

In this technique, a weight is assigned to each objective function. Then, the objective functions are combined to build a single objective function [39–45]. Suppose that the weight of objective function w is W_w. Thus, W_1, W_2, W_3, and W_4 should be determined in this problem. The summation of the weights is one. The weights represent the importance of the objectives for the decision-makers. The proposed optimization model is converted to the following optimization formulation using the weighted-sums method.

$$Miz \;\; z_5 = W_1 z_1 + W_2 z_2 + W_3 z_3 - W_4 z_4 \tag{14}$$

$$s.t.$$

$$\sum_w W_w = 1 \tag{15}$$

Constraints (1)–(15).

5.2 ε-constraint method

In ε-constraint technique, the most prominent objective among others is chosen as the primary objective function. Other objective functions are considered as constraints of the optimization model [46–50]. The first objective function is the most important one in this model. Therefore, it is selected as the main objective function. Three constraints are added to the mathematical model [constraints (17)–(19)]. It is noticeable that the signs of the inequalities are related to the types of the objective functions (minimization or maximization).

$$Miz \;\; z_6 = z_1 \tag{16}$$

$$s.t.$$

$$z_2 \leq \varepsilon_2 \tag{17}$$

$$z_3 \leq \varepsilon_3 \tag{18}$$

$$z_4 \geq \varepsilon_4 \tag{19}$$

Constrains (5)–(13)

6. Results of the case study

Four types of foods are considered in four food groups including vegetables and fruit, grain products, milk and alternatives, and meat and alternatives. The recommended number and amount of food guided servings in a month are provided in **Table 2**. This table is based on the information in Food-guide-basics, 2018. We focus on 51+ year-old females in this case. The last column of the table shows 50% of the required amount of food. It is supposed that the other 50% nutrition is supplied by other sources. Two periods (months) are considered in this case study. Two types of vitamins including vitamin A and vitamin C are taken into account because information about them is provided in nutrition facts tables of the products in Canada. Mentioning the values of other vitamins in the tables is optional for Canadian food producers. The other data of the case are provided in Appendix A.

In this research, the General Algebraic Modeling System (GAMS) software is employed to write the codes and find the solutions. First, different weights are devoted to the objective functions and the problem is solved. Each solution of the multi-objective model is called efficient solution. Efficient solutions cannot be improved without scarifying other objective functions [46, 51–56]. The results have been collected in **Table 3**. As it can be seen, the weights are assigned between 0 and 1. The efficient solutions are presented to the decision-makers. The second part of

Food group	Examples of one food guide serving	Number of food servings	Amount of food servings	50% of the amount
Vegetables and fruit (VF)	125 mL (½ cup) fresh, frozen or canned vegetable or fruit or 100% juice	210	26,250 ml	13,125 ml
Grain products (GP)	1 slice (35 g) bread or ½ bagel (45 g)	180	6,300 g	3,150 g
Milk and alternatives (MA)	250 mL (1 cup) milk or fortified soy beverage	90	22,500 mg	11,250 mg
Meat and alternatives (EA)	75 g (2 ½ oz.)/125 mL (½ cup) cooked fish, shellfish, poultry or lean meat	60	4,500 g	2,250 g

Table 2.
Recommended number and amount of food guided servings in a month for 51+ year-old females.

a) Weighted-sums method				
(w_1, w_2, w_3, w_4)	z_1	z_2	z_3	z_4
(0.7, 0.1, 0.1, 0.1)	147.197	372.000	3,957.000	588.000
(0.25, 0.25, 0.25, 0.25)	288.572	228.000	3,642.000	588.000
(0.1, 0.2, 0.5, 0.2)	297.812	181.059	3,464.118	148.235
(0.9, 0.025, 0.025, 0.05)	106.997	372.000	4,587.000	588.000
b) ε-constraint method				
$(\varepsilon_2, \varepsilon_3, \varepsilon_4)$	z_1	z_2	z_3	z_4
(1,820, 34,900, 586)	98.436	835.500	4,587.000	588.000

Table 3.
The efficient solutions.

Table 3 includes the results of ε-constraint method. The main objective function is about the cost objective. Based on the information in **Table 3**, more efficient solutions have been obtained in weighted-sums method. Consequently, this method is selected to solve the mathematical model.

One of the efficient solutions in **Table 3** has been obtained by considering equal weights (w_1 = 0.25, w_2 = 0.25, w_3 = 0.25, and w_4 = 0.25). The values of the decision variables related to this solution are as follows: $N_{1.3.1}$ = 11.250, $N_{1.3.2}$ = 11.250, $N_{2.2.1}$ = 4.667, $N_{2.2.2}$ = 4.667, $N_{3.4.1}$ = 3.775, $N_{3.4.2}$ = 3.775, $N_{4.1.1}$ = 7.500, and $N_{4.1.2}$ = 7.500. In other words, Food 1 in Group 3 in Periods 1 and 2, Food 2 in Group 2 in Periods 1 and 2, Food 3 in Group 4 in Periods 1 and 2, and Food 4 in Group 1 in Periods 1 and 2 should be purchased to have optimal solution.

7. The optimization model under uncertainty

In reality, several parameters are uncertain. In this section, the effects of uncertainty in two parameters including cost of foods and amount of food guide servings per month are examined in the mathematical model. These two parameters are very important factors of food items. Suppose that u represents a scenario among U scenarios. The decision variables (nonnegative variables in this case) are written based on each scenario [39]. A_{ihtu} is defined as cost of food i in food group h and period t in scenario u. It is noticeable that the costs of foods in different stores are usually different. Furthermore, Ψ_{htu} represents the amount of food guide servings per month in food group h in period t in Scenario u. m_u is introduced as the probability related to Scenario u. The new optimization model under uncertainty is written as follows:

$$\text{Min } z_1 = \sum_u \sum_t \sum_h \sum_i m_u A_{ihtu} N_{ihtu} \tag{20}$$

$$\text{Min } z_2 = \sum_u \sum_t \sum_h \sum_i m_u a_{ih} K_{ih} N_{ihtu} \tag{21}$$

$$\text{Min } z_3 = \sum_u \sum_t \sum_h \sum_i m_u a_{ih} L_{ih} N_{ihtu} \tag{22}$$

$$\text{Max } z_4 = \sum_u \sum_t \sum_h \sum_i m_u a_{ih} M_{ih} N_{ihtu} \tag{23}$$

s.t.

$$B_t \le \sum_h \sum_i a_{ih} C_{ih} N_{ihtu} \le D_t \qquad \forall t, u \tag{24}$$

$$E_{jt} \le \sum_h \sum_i a_{ih} F_{jih} N_{ihtu} \le G_{jt} \qquad \forall j, t, u \tag{25}$$

$$O_t \le \sum_h \sum_i a_{ih} P_{ih} N_{ihtu} \le Q_t \qquad \forall t, u \tag{26}$$

$$R_t \le \sum_h \sum_i a_{ih} S_{ih} N_{ihtu} \le V_t \qquad \forall t, u \tag{27}$$

$$X_t \le \sum_h \sum_i a_{ih} Y_{ih} N_{ihtu} \le Z_t \qquad \forall t, u \tag{28}$$

$$\beta_t \le \sum_h \sum_i a_{ih} \lambda_{ih} N_{ihtu} \le \alpha_t \qquad \forall t, u \tag{29}$$

$$\rho_t \le \sum_h \sum_i a_{ih} \gamma_{ih} N_{ihtu} \le \sigma_t \qquad \forall t, u \tag{30}$$

$$\sum_i a_{ih} N_{ihtu} = \psi_{htu} \qquad \forall h, t, u \tag{31}$$

$$N_{ihtu} \ge 0 \qquad \forall i, h, t, u \tag{32}$$

It is supposed that the values of the two sources of uncertainty can increase, decrease, or remain same. Therefore, three situations exist for each source of uncertainty. The combination of the two sources of uncertainty produces nine

Scenario	Cost of food	Amount of food servings	Probability
1	$1.05A_{iht}$	$1.05\ \Psi_{ht}$	0.04
2	$1.05A_{iht}$	Ψ_{ht}	0.16
3	$1.05A_{iht}$	$0.95\ \Psi_{ht}$	0.04
4	A_{iht}	$1.05\ \Psi_{ht}$	0.16
5	A_{iht}	Ψ_{ht}	0.2
6	A_{iht}	$0.95\ \Psi_{ht}$	0.16
7	$0.95A_{iht}$	$1.05\ \Psi_{ht}$	0.04
8	$0.95A_{iht}$	Ψ_{ht}	0.16
9	$0.95A_{iht}$	$0.95\ \Psi_{ht}$	0.04

Table 4.
Nine scenarios in the diet problem.

	u	1	2	3	4	5	6	7	8	9
i	*h.t*									
1	3.1	11.812	11.250	10.687	11.812	11.250	10.687	11.812	11.250	10.687
1	3.2	11.812	11.250	10.687	11.812	11.250	10.687	11.812	11.250	10.687
2	2.1	4.899	4.667	4.433	4.899	4.667	4.433	4.899	4.667	4.433
2	2.2	4.899	4.667	4.433	4.899	4.667	4.433	4.899	4.667	4.433
3	4.1	3.963	3.775	3.763	3.963	3.775	3.763	3.775	3.775	3.763
3	4.2	3.963	3.775	3.763	3.963	3.775	3.763	3.775	3.775	3.763
4	1.1	7.875	7.500	7.125	7.875	7.500	7.125	7.875	7.500	7.125
4	1.2	7.875	7.500	7.125	7.875	7.500	7.125	7.875	7.500	7.125

Table 5.
The values of the decision variables (N_{ihtu}) *under uncertainty.*

scenarios. Based on the historical data, 5% change in the values of each source of uncertainty is examined. The basic scenario is Scenario 5. A summary of different scenarios in this problem is provided in **Table 4**.

The new model under uncertainty is solved by GAMS, and the values of the decision variables are calculated. There are 365 equations and 4,149 nonzero elements. **Table 5** includes the results. For instance, $N_{1.3.1.1} = 11.812$. The results of Scenario 5 are the numbers that were calculated in the deterministic multi-objective model in the previous section. The maximum deviations are observed in scenarios 1, 3, 7, and 9.

8. Conclusions

Diet problem has been formulated in the form of optimization models in the literature. The main goal of the models is to minimize the total cost of the foods. In this chapter, a unique optimization model has been developed based on a case study in Canada. Four proposed objectives consist of minimizing the total cost, saturated and trans fats, and sugar; and maximizing the fiber of the foods. The data of this problem have been gathered based on the information in the official website of the government of Canada. The recommended number of food guide servings and the nutrition information are available in that website. In addition, nutrition facts tables are good sources of the core nutrients in the foods. They are mandatory for most of the foods in Canada. The proposed multi-objective model has been solved by two approaches containing weighted-sums and ε-constraint solution approaches. Then, the efficient solutions have been provided in two tables.

The effects of uncertainty in two parameters of the mathematical model have been investigated by a scenario-based solution approach. To this aim, nine scenarios for two sources of uncertainty (cost of foods and amount of food guide servings per month) have been investigated. Furthermore, the results have been analyzed. The proposed multi-objective model under uncertainty can be applied in real cases, and determine the food items accurately.

There are several opportunities to extend this research. We focused on a case in Canada. The proposed mathematical model can be extended based on the other cases in other countries such as European countries. Another future opportunity for research is related to the uncertainty in the problem. We concentrated on two sources of uncertainty. It is interesting to investigate the impacts of more sources of uncertainty at the same time. For the case of four uncertain sources, 3*3*3*3 = 81 scenarios should be considered. Therefore, computational time is an important factor for several sources of uncertainty.

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Appendix A

Based on the information in [57], the maximum total calories in 1 month (D_t) is estimated as 2100 * 30 = 63,000 (for females over 51). The minimum is 0.

Based on some studies, no more than 300 * 30 = 9000 milligrams (mg) of cholesterol for each month is recommended [58].

Based on the information in [36], the maximum total sodium in 1 month (V_t) is estimated as 2300 * 30 = 69,000 mg (for people over 51). The minimum is considered 0.

The maximum amount of vitamin A for 1 month is considered 1000 * 30 = 30,000 RE (retinol equivalents). In addition, the maximum value of vitamin

a_{ih}	1 (ml)	2 (g)	3 (ml)	4 (g)
1	1,750	675	1,000	1,224
2	1,750	675	1,000	750
3	1,750	675	1,000	596
4	1,750	400	1,000	537

Table A1.
Sizes of the foods.

A_{iht}	1	2
1.1	4.27	4.27
1.2	2.99	2.99
1.3	2.97	2.97
1.4	2.97	2.97
2.1	1.59	1.59
2.2	2	2
2.3	2.89	2.89
2.4	2.47	2.47
3.1	2.58	2.58
3.2	2.47	2.47
3.3	2	2
3.4	2.58	2.58
4.1	12.24	12.24
4.2	9	9
4.3	12.47	12.47
4.4	7.17	7.17

Table A2.
Costs of the foods ($).

C_{ih}	1	2	3	4
1	110/250	230/85	110/250	270/100
2	130/250	180/75	160/250	220/100
3	130/250	220/79	130/250	100/100
4	120/250	110/43	130/250	160/100

Table A3.
Calories of the foods (in each unit).

F_{jih}	1	2	3	4
1.1	0/250	0/85	100/250	0/100
1.2	0/250	0/75	100/250	0/100
1.3	0/250	0/79	100/250	0/100
1.4	0/250	0/43	100/250	20/100
2.1	1000/250	0/85	0/250	0/100
2.2	1000/250	0/75	0/250	0/100
2.3	1000/250	0/79	0/250	0/100
2.4	1100/250	0/43	0/250	0/100

Table A4.
Vitamins of the foods (in each unit), $j = 1$ (Vitamin A), $j = 2$ (Vitamin C).

K_{ih}	1	2	3	4
1	0/250	0.5/85	1.5/250	10.5/100
2	0/250	1/75	1.5/250	7/100
3	0/250	1/79	3.1/250	0.2/100
4	0/250	0.4/43	3.1/250	3/100

Table A5.
Fats of the foods (in each unit).

L_{ih}	1	2	3	4
1	25/250	1/85	12/250	0/100
2	31/250	3/75	26/250	0/100
3	28/250	2/79	12/250	0/100
4	22/250	2/43	12/250	0/100

Table A6.
Sugar of the foods (in each unit).

M_{ih}	1	2	3	4
1	0/250	2/85	0/250	0/100
2	0/250	7/75	0/250	0/100
3	0/250	2/79	0/250	0/100
4	0/250	1/43	0/250	0/100

Table A7.
Fiber of the foods (in each unit).

P_{ih}	1	2	3	4
1	0/250	0/85	10/250	70/100
2	0/250	0/75	10/250	65/100
3	0/250	0/79	20/250	60/100
4	0/250	0/43	20/250	75/100

Table A8.
Cholesterol of the foods (in each unit).

S_{ih}	1	2	3	4
1	15/250	480/85	120/250	55/100
2	20/250	300/75	170/250	60/100
3	10/250	420/79	120/250	50/100
4	0/250	230/43	120/250	80/100

Table A9.
Sodium of the foods (in each unit).

Y_{ih}	1	2	3	4
1	0.4/250	9/85	9/250	18/100
2	1/250	8/75	9/250	19/100
3	1/250	8/79	9/250	25/100
4	2/250	4/43	9/250	18/100

Table A10.
Protein of the foods (in each unit).

λ_{ih}	1	2	3	4
1	22/250	66/85	33/250	0/100
2	0/250	88/75	33/250	0/100
3	22/250	66/79	33/250	0/100
4	0/250	22/43	33/250	22/100

Table A11.
Calcium of the foods (in each unit).

γ_{ih}	1	2	3	4
1	0/250	2.8/85	0/250	2.1/100
2	0/250	1.4/75	0.28/250	2.1/100
3	0.28/250	2.8/79	0/250	1.12/100
4	0/250	1.4/43	0/250	1.12/100

Table A12.
Iron of the foods (in each unit).

C for 1 month is considered a big number. Furthermore, the maximum total calcium is assumed 1100 * 30 = 33,000 mg for each month. The maximum amount of iron is supposed 14 * 30 = 420 mg for 1 month. These values have been calculated according to the information in Percent-daily-value, 2018. The maximum total protein is considered as a big number because no daily-value has been mentioned for this element in [58, 59]. **Tables A1** to **A12** include other data of the problem.

Author details

Saman Hassanzadeh Amin[1]*, Samantha Mulligan-Gow[1] and Guoqing Zhang[2]

1 Department of Mechanical and Industrial Engineering, Ryerson University, ON, Canada

2 Mechanical, Automotive and Materials Engineering, University of Windsor, ON, Canada

*Address all correspondence to: saman.amin@ryerson.ca

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