Chapter

# Boundary Layer Theory: New Analytical Approximations with Error and Lambert Functions for Flat Plate without/with Suction

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## Abstract

In this work, we investigated the problem of the boundary layer suction on a flat plate with null incidence and without pressure gradient. There is an analytical resolution using the Bianchini approximate integral method. This approximation has been achieved by Lambert or Error functions for boundary layer profiles with uniform suction, even in the case without suction. Based on these new laws, we brought out analytical expressions of several boundary layer features. This gives a necessary data to suction effect modeling for boundary layer control. To recommend our theoretical results, we numerically studied the boundary layer suction on a porous flat plate equipped with trailing edge flap deflected to 40°. We showed that this flap moves the stagnation point on the upper surface, resulting to avoid the formation of the laminar bulb of separation. Good agreement was obtained between the new analytical laws, the numerical results (CFD Fluent), and the literature results.


**Keywords:** boundary layer suction, analytical approximation, Error function, Lambert function, trailing edge flap

## 1. Introduction

The advantage of the parietal suction is to delay the transition of the boundary layer to turbulence, reduce drag, and increase lift (avoid stalling) [1]. A method used in aviation is the multi-perforation of the walls. This type of boundary layer control is at the project stage on the wings of airplanes, such as the F-16XL and the A320, and starts to emerge in other industries. Equally, this technic is of interest in different engineering branches as the extraction of geothermal energy, nuclear reactor or electronics cooling system, filtration process, lubrication of ceramic machine parts, etc.

The theoretical and numerical study of laminar boundary layer over a flat plate without and/or with uniform suction is well introduced in the literature [2–8]. The modeling of the effect of this control technics can require analytical expressions of characteristic parameters of flow. Equally, theoretical investigations help understand the underlying physics of boundary layer suction and help to predict, with a minimum waste of time, its effect.

For the homogeneous suction, Schlichting and Bussmann [9] assumed that the longitudinal velocity gradient is null. Based on this hypothesis, he found an asymptotic solution which is expressed by the exponential function, but this profile is not valid in the region near the leading edge. To improve this solution, Preston [10] considered a family of parameters of velocity profiles having the Blasius profile and the asymptotic profile as a limit form. The solution obtained is more accurate by comparing it with the exact numerical solutions of Iglisch [11]. Palekar and Sarma [2] who applied the Bianchini approximate integral method [12] determined an analytical solution of the boundary layer profile expressed by the Error function in the case of nonuniform suction. This law was not compared with the real profile of Blasius. In the case of the uniform suction, the solution was obtained in an asymptotic form.

Kay [13] took velocity measurements out of a blower up a flat plate with uniform suction. The vertical velocity distribution was described by the exponential function. Aydin and Kaya [4] have considered finite difference approximations to resolve the boundary layer equations. Fang et al. [5] studied a similarity equation of the momentum boundary layer for a moving flat plate in a stationary fluid with mass suction at the wall surface. They provided a new solution branch for the Blasius equation. Recently, researchers have studied convergent and closed analytical solution of the Blasius Equation [14–16]. Wedin et al. [17, 18] have studied the effect of plate permeability on nonlinear stability of the asymptotic suction boundary layer. Zheng et al. [8] have proposed a solution of the Blasius equation expressed by two power series. They showed that the method for finding the closed analytical solution of Blasius equation was used in the regulation of the boundary layer injection and slip velocity.

The study of the flow over a flat plate requires a suitable geometry to avoid the transition to turbulence for low Reynolds numbers. Roach and Brierley [19] studied the flow over a flat plate with cylindrical leading edge of 2 mm diameter, tapered to 5° from the upper surface, to avoid instabilities and separation. Palikaras et al. [20] experimentally and numerically studied the effect of the semicircular leading edge on the transition laminar-turbulent from a flow on a flat plate without pressure gradient. They showed that this transition occurs in the presence of a pressure gradient in the region of the leading edge, resulting in the formation of the laminar bulb of separation. Several configurations were designed to avoid this phenomenon [21–26]. To avoid the influence of this disturbance, Walsh et al. [23] have designed and realized preliminary measurements from a new flat plate facility for aerodynamic research. This flat plate was consisted of a leading edge radius of 1 mm with a 5° chamfer in the intrados and adjustable positive or negative trailing edge flap deflections. The plate was made wof aluminum with 10 mm thick, 1 m long, and 0.29 m wide. Patten et al. [26] studied the design effectiveness of a new flat plate with trailing edge flap. They showed that the stagnation point anchored on the upper surface and the measurements along the flat plate were compared favorably to the Blasius profile.

This paper presents the steps to develop new laws of boundary layer profiles on a horizontal flat plate with uniform suction even in the impermeable case. In the first section, we analytically resolved the governing equations by using the integral method of Bianchini and by inserting particular solutions as Error and Lambert functions. Next, we numerically studied, by using CFD Fluent, the effects of the geometrical parameters of the flat plate (leading edge, trailing edge flap angle) on the boundary layer flow. Finally, the analytical solutions of boundary layer equations were validated with the present numerical results and the literature results in all cases with and without suctions.

## 2. Theoretical study

In this section, we have presented a new analytical approximation of boundary layer profiles of flow on the upper side of flat plate without and with uniform suction. Based on the approximate integral method of Bianchini and using Lambert and/or Error functions, we have achieved this solution.

### 2.1 Mathematical formulation

We consider a horizontal flat plate placed in incompressible, two-dimensional, steady, and laminar flow with free-stream velocity $U$. The $x$-coordinate and $y$-coordinate are measured from the leading edge and normal to the flat plate, respectively. In the case of permeable flat, the suction velocity $v_p$ is oriented to $y$-negative (**Figure 1**).

Prandtl equations in the boundary layer are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

The boundary conditions are:

$$u(0) = 0; v(0) = v_p \tag{3}$$

$$u(\infty) = U_\infty; v(\infty) = 0 \tag{4}$$

The integration of Eqs. (1) and (2) from 0 to ∞ with the conditions Eqs. (3) and (4) gives the following integral equation:

$$U_\infty^2 \frac{d}{dx}\int_0^\infty \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy - U_\infty v_p = \upsilon\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{5}$$

The basic assumption of the integral method of Bianchini is to pose for the profile speed the following form:

$$\frac{u}{U_\infty} = erf\left(\frac{y}{h(x)}\right) \tag{6}$$

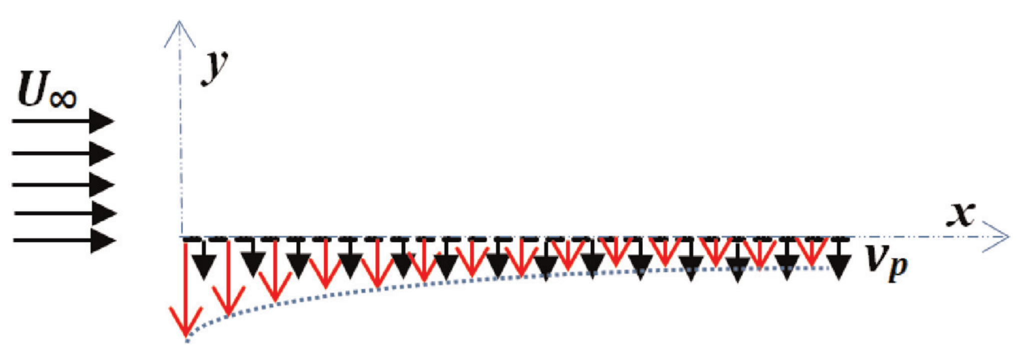


**Figure 1.**
*The problem schematic.*

where $erf\left(\frac{y}{h(x)}\right)$ is the Error function and $h(x)$ is the unknown scale function to be determined. The choice of the Error function has the advantage of a good approximation of the exact solution of Blasius which will be proven below.

The insertion of Eq. (6) in Eq. (5), with the conditions Eqs. (3) and (4), gives a differential equation of $h(x)$, with a boundary condition as below:

$$\begin{cases} U_\infty \alpha_2 \dfrac{dh(x)}{dx} - v_p = \dfrac{v}{h(x)}\alpha_3 \\ h(0) = 0 \end{cases} \tag{7}$$

where

$$\alpha_2 = \int_0^\infty erf(z)(1 - erf(z))dz = \frac{\sqrt{2}-1}{\sqrt{\pi}}, \tag{8}$$

$$\alpha_3 = \left(\frac{derf}{dz}\right)_{z=0} = \frac{2}{\sqrt{\pi}}. \tag{9}$$

### 2.2 Analytical solutions

The analytical resolution of the differential Eq. (7) depends inevitably on the boundary conditions, in particular, the value of suction velocity $v_p$, since we consider two cases: an impermeable flat plate when $v_p = 0$ and a porous flat with uniform suction when $v_p = -v_0 \neq 0$.

#### *2.2.1 Case without suction ($v_p = 0$)*

When replacing in Eq. (7) the value of $v_p = 0$, the differential equation of $h(x)$ is written as

$$U_\infty \alpha_2 \frac{dh(x)}{dx} = \frac{v}{h(x)}\alpha_3 \tag{10}$$

The resolution of Eq. (10) with the boundary condition in Eq. (7) leads to the solution:

$$h(x) = 2\sqrt{1+\sqrt{2}}\sqrt{\frac{vx}{U_\infty}} \tag{11}$$

So, we obtain the profile of the boundary layer velocity of the flow on the impermeable flat plate:

$$\frac{u}{U_\infty} = erf(0,32\eta) \tag{12}$$

These results enable us to determine the various characteristics of the boundary layer as the boundary layer thickness, the displacement thickness, the momentum thickness, and the friction coefficient:

$$\frac{\delta}{\mathrm{x}} = \frac{5,66}{\sqrt{\mathrm{Re_x}}}\frac{\delta}{x} = \frac{5,66}{\sqrt{\mathrm{Re}_x}};\ \frac{\delta_1}{\delta} = 0,31;\ \frac{\delta_2}{\delta} = 0,128;\ \frac{1}{2}Cf = \frac{0,36}{\sqrt{\mathrm{Re}_x}} \tag{13}$$

*2.2.2 Case with uniform suction*

We considered the case of a flow on a permeable flat plate with uniform suction $v_p = -v_0 \neq 0$. To simplify the resolution of the differential Eq. (7), we imposed the particular solution:

$$h(x) = A_1 + A_2 W(g(x)) \tag{14}$$

where $A_1$ and $A_2$ are constant parameters, $W$ is the Lambert function, and $g(x)$ is a function of $x$ to determine.

By inserting Eq. (14) in the differential Eq. (7), we supplied the parameters of the scaling function:

$$A_1 = A_2 = \frac{b_1}{\frac{v_p}{U_\infty}}; g(x) = -\frac{1}{2v} \exp\left(-\frac{b_2}{\left(\frac{U_\infty}{v_p}\right)^2} x + b_3\right) \tag{15}$$

where

$$b_1 = -\frac{\alpha_3 v}{U_\infty} = -\frac{2v}{\sqrt{\pi} U_\infty}, b_2 = \frac{(1+\sqrt{2})\pi U_\infty}{2v}, \text{ and } b_3 = \ln(2v) - 1 \tag{16}$$

Thus, the profile of the boundary layer velocity of the flow on the permeable flat plate with uniform suction is

$$\frac{u}{U_\infty} = erf\left(y / \frac{b_1}{\frac{v_p}{U_\infty}}\left[1 + W\left(-\frac{1}{2v} \exp\left(-\frac{b_2}{\left(\frac{U_\infty}{v_p}\right)^2} x + b_3\right)\right)\right]\right) \tag{17}$$

These results enable us to determine the various characteristics of the boundary layer as the boundary layer thickness, the displacement thickness, the momentum thickness, and the parietal friction coefficient:

$$\delta(x) = 1,82\, h(x); \delta_1(x) = 0,564 h(x); \delta_2(x) = 0,231 h(x); Cf(x)\mathrm{Re}_x = \frac{4}{\sqrt{\pi}} \frac{x}{h(x)} \tag{18}$$

We can rewrite this friction coefficient in the universal form of law recommended by Iglisch (1949) [11].

$$\frac{Cf(t)}{2\frac{v_p}{U_\infty}} = f\left(t = -\frac{v_p}{U_\infty}\sqrt{\mathrm{Re}_x}\right) \tag{19}$$

Thus

$$\frac{Cf(t)}{2\frac{v_p}{U_\infty}} = -\frac{1}{1 + W\left(-\frac{1}{2v} \exp\left(-\frac{(1+\sqrt{2})\pi}{2} t^2 + b_3\right)\right)} \tag{20}$$

## 3. Validations of the new boundary layer theories

In order to validate the new laws of boundary layer without and with suction, we studied the 2D, laminar, and incompressible flow around the flat plate by means of CFD using the software package Fluent.

### 3.1 CFD study of the boundary layer

Theoretically, the boundary layer equations were studied with hypothesis of zero pressure gradient flow. In order to validate and compare the numerical results with the new boundary layer laws, it is essential to avoid separation and instability of the flow. A critical part of the experiment that must be addressed is the leading edge of the flat plate. The boundary layer development is considerably influenced by the stagnation point location. A flat plate equipped with adaptable trailing edge flap ensures that the boundary layer is developed smoothly and a negligible stream-wise pressure gradient is achievable. So, the laminar separation bulb, which is one of the phenomena at the origin of the transition to turbulence, is avoided [23, 24, 26].

For this reason, we used a flat plate with a semicircular leading edge and provided with trailing edge flap deflected to an angle $\beta$ = 40°. Its length and thickness are $L$ = 0.9 m and $e$ = 0.01 m, respectively, the flap chord l = 0.1 m. The leading edge is chamfered at an angle of 5° relative to the lower surface, and its diameter is $d$ = 0.002 m (see **Figure 2**). This is similar to the flat plate designed by Patten et al. [26] for boundary layer research.

A judicious choice of the suction system is required to control the laminar flow [1, 27–29]. The suction area is placed in 0.1 m from the leading edge and extends to 0.7 m. It consists of holes of the same diameter $d_{suc}$ = 0.2 mm and equidistant from 1 mm (**Figure 2**). This corresponds to a longitudinal dimensionless spacing $d_p/d_{suc}$ = 5 (where $d_p$ is the spacing between two successive holes), respecting the mechanical strength of the plate [30].

The quality of mesh has a great importance on the results of a numerical resolution. For this, we choose a structured and very tight mesh in the near-wall and in the area of the leading edge (Y+ = 1). This computational domain is made up of 53.970 cells in the case of the impermeable plate and 100.000 cells in the case of the

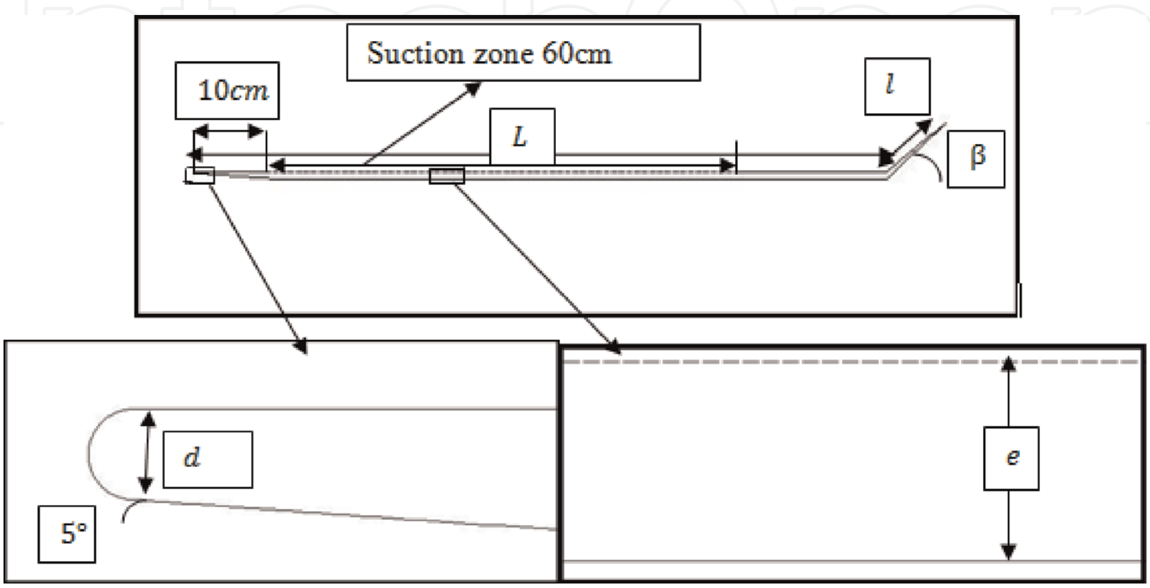


**Figure 2.**
*Design characteristics of the flat plate.*

permeable plate, because of the mesh refinement above the suction zone to account for the velocity gradient. The Navier-Stokes equations for 2D, laminar, and incompressible flow were resolved by using the finite volume method (FVM). We used the algorithm "SIMPLE" for the pressure-velocity coupling.

In the case without suction, we studied the flow with free-steam velocity $U = 5$ m/s around the flat plate to compare their boundary layers with Blasius profiles. **Figure 3** shows the effect of the trailing edge flap angle on the position of the stagnation point. For $\beta = 40°$, the stagnation point is displaced to the upper surface of plate resulting in the reduction of the separation flow at the leading edge compared to the case of $\beta = 0°$. This result is compared to those obtained in the literature [26].

In **Figure 4**, we compared the Blasius profiles with the results from the CFD of the flow boundary layer on the upper side of the impermeable flat plate for $\beta = 40°$. It is shown that the boundary layer of the flat plate at different positions favorably follows the Blasius profile. Thus, the shapes of the leading edge and the deflected trailing edge have an effect to neglect the pressure gradient in the flow of the upper side of the plate which greatly influences the formation of the boundary layer. In the continuation of this work, we select the case of the trailing edge deflected to 40°.

## 3.2 Validation and discussion

Many solutions were found based on the Prandtl equations such as Blasius and Schlichting [9] profiles. The differential equations of Blasius have no solutions for

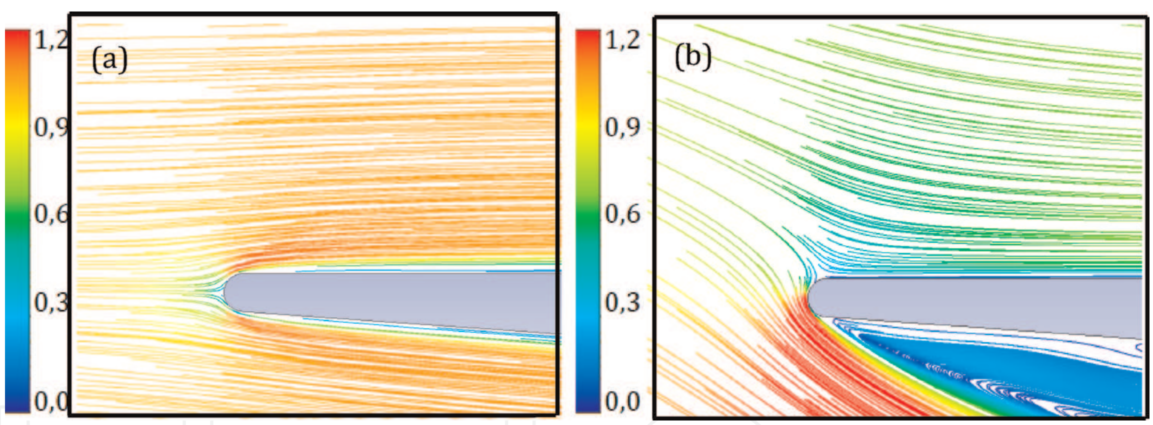


**Figure 3.**
*Streamline velocity colored by dimensionless velocity magnitude $((u^2 + v^2)^{0,5}/U)$: (a) trailing edge flap angle $\beta = 0°$; (b) trailing edge flap angle $\beta = 40°$.*

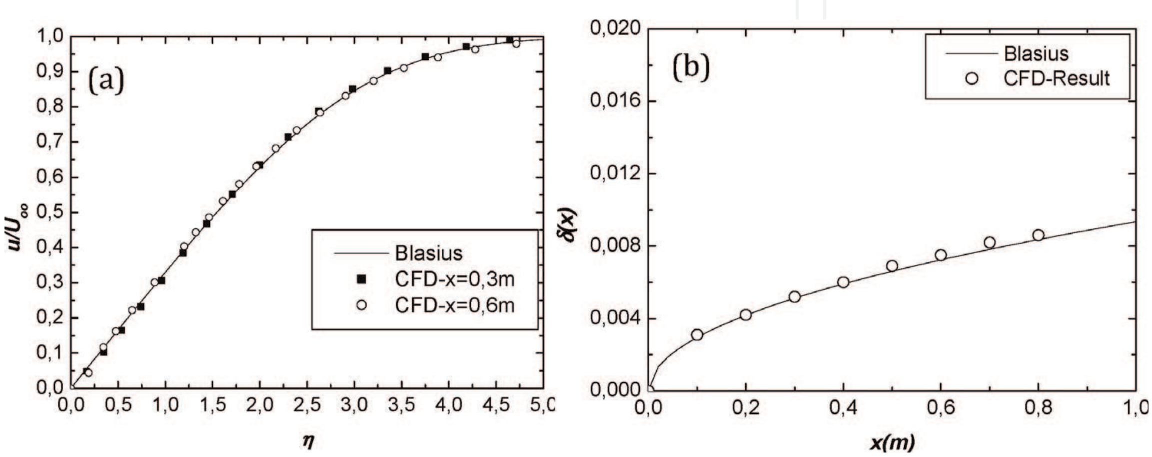


**Figure 4.**
*Comparison between Blasius and CFD profiles for impermeable flat plate for $\beta = 40°$.*

the case of uniform suction. Schlichting [9] neglected the dependency between the boundary layer velocity and the $x$-coordinate; this hypothesis is not acceptable in the region near the leading edge. In the case with uniform suction, Palekar and Sarma [2] found two solutions for boundary layer profiles: the first one nears to the leading edge and the second one far the leading edge. The advantage of the new solutions is that the Error function defines well the boundary layer near and far the leading edge in the cases with and without uniform suction, as well as, in the case of nonuniform suction.

**Figure 5** compares the boundary layer profile defined in Eq. (12), with the CFD, and the Blasius profiles for the impermeable flat plate. This result confirms the choice of the profile shape $(erf(y/h(x)))$. This shows that the Error function has the advantage of a good approximation of the boundary layer theory. We compared in **Table 1** the new characteristic parameters for the boundary layer with those from the literature. The values found by this approximation are quite comparable to Blasius and generally better than the other approximations.

Concerning the case of uniform suction ($v_p = -0.0118$ m/s), we compare the profile defined by Eq. (17) with the Palekar profiles for small and gross $x$. Our analytical solution verifies well both cases at once (**Figure 6**). As shown in **Figure** 7, the relation Eq. (20) well verified the universal law of friction found by Iglisch (1949) [11]. The comparison between the analytical profiles Eq. (17) with the numerical results (CFD) shows a little difference. This is caused by the nature of the parietal suction for each case, where in the theoretical study, the boundary condition at the wall is defined as a continuous suction along the plate; however, in the CFD study, the suction is discrete.

After validation of the new analytical laws, we presented the effect of the suction rate on the characteristics of the boundary layer. **Figure 8a, b**, and **c**, shows

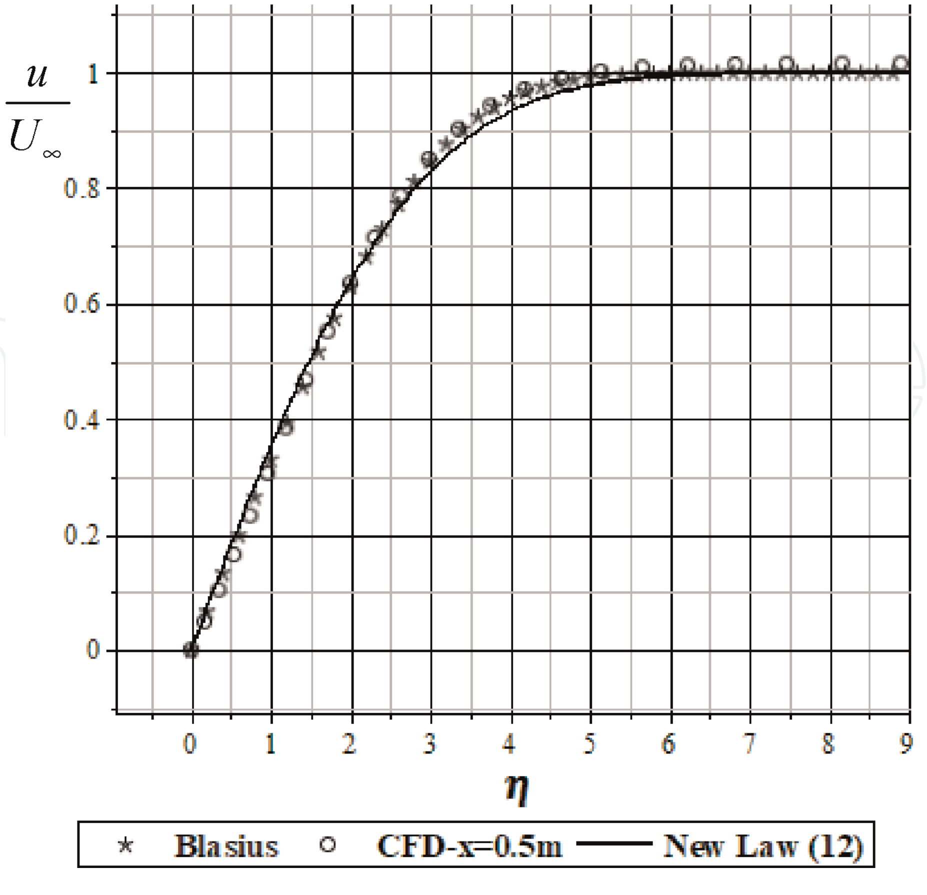


**Figure 5.**
*Comparison of the analytic velocity profile Eq. (12) with the profile of Blasius and the profile obtained by using CFD Fluent for the impermeable flat plate.*

| $\frac{u}{U_\infty}$ | $\delta\sqrt{\frac{Re_x}{x}}$ | $\frac{\delta_1}{\delta}$ | $\frac{\delta_2}{\delta}$ | $\frac{Cf}{2}\sqrt{Re_x}$ |
|---|---|---|---|---|
| $2\eta-\eta^2$ | 5, 4 | 0, 33 | 0, 4 | 0, 36 |
| $\frac{3}{2}\eta-\frac{1}{2}\eta^3$ | 4, 6 | 0, 375 | 0, 139 | 0, 33 |
| $2\eta-2\eta^3+\eta^4$ | 5, 8 | 0, 3 | 0, 121 | 0, 34 |
| *Blasius solution* | 5 | 0, 344 | 0, 132 | 0, 332 |
| $erf(0, 32\eta)$ | 5, 66 | 0, 31 | 0, 128 | 0, 36 |

**Table 1.**
*Comparative table of characteristic parameters of boundary layer.*

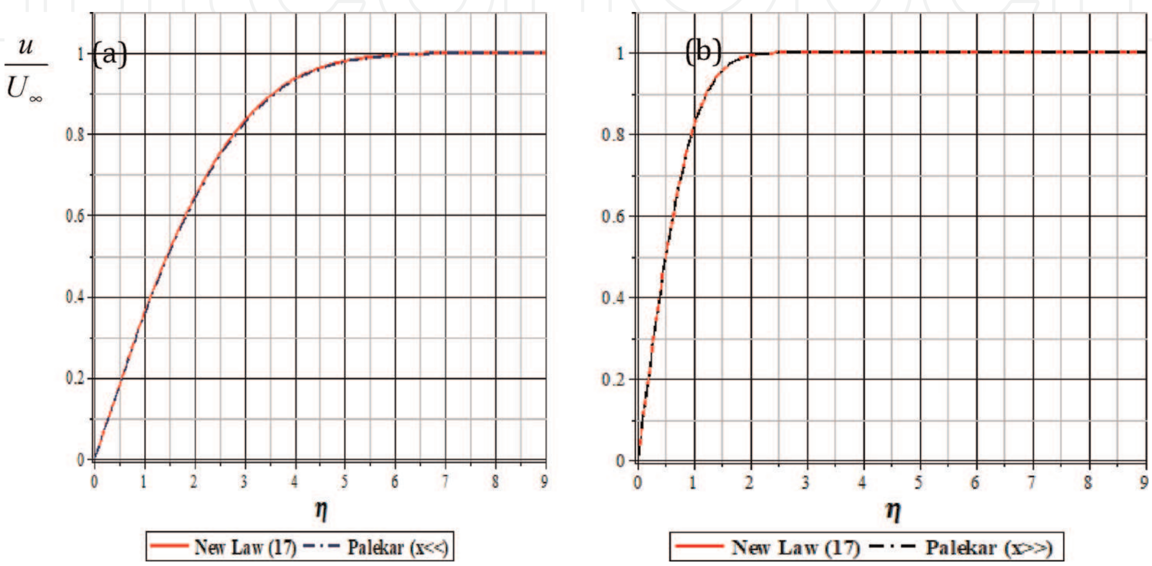


**Figure 6.**
*Validation of the new law in the case of uniform suction with the solutions found by Palekar (1984) [2], for small x (x = 0.001 m $\times=10^{-5}$) and gross x (x = 0.5 m $\times=10^{-5}$).*

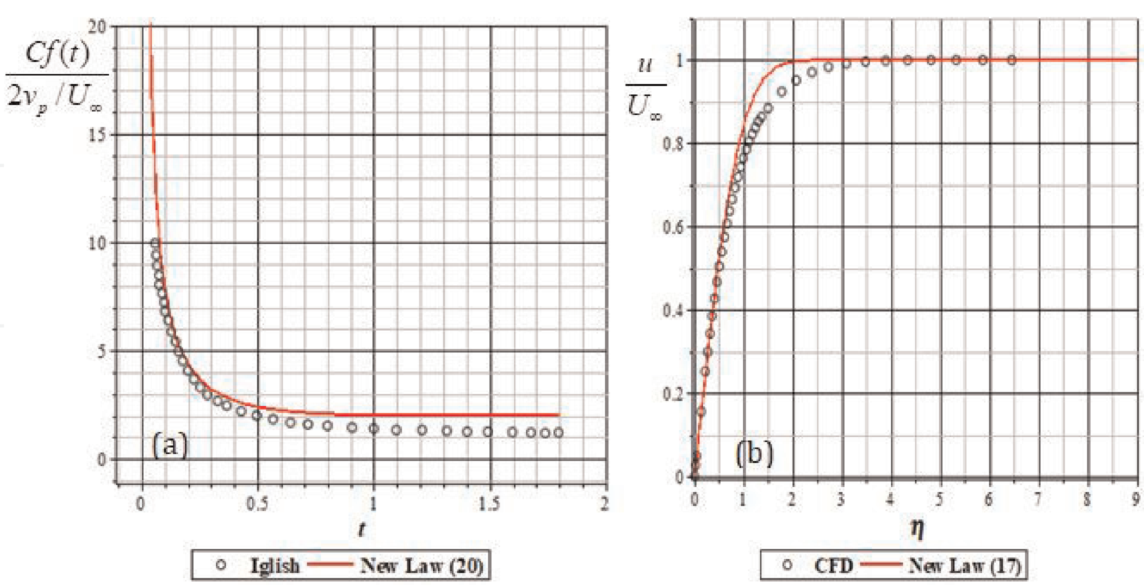


**Figure 7.**
*Representing curves of (a) the universal law of friction $\frac{Cf}{2\frac{v_p}{U_\infty}}$ depending on t; (b) boundary layer profile (x = 0.5).*

the velocity profiles, the parietal friction coefficients, and the boundary layer thicknesses for different suction rates. It is clear that when the suction rate increases, the thickness of the boundary layer decreases. As a result, the boundary

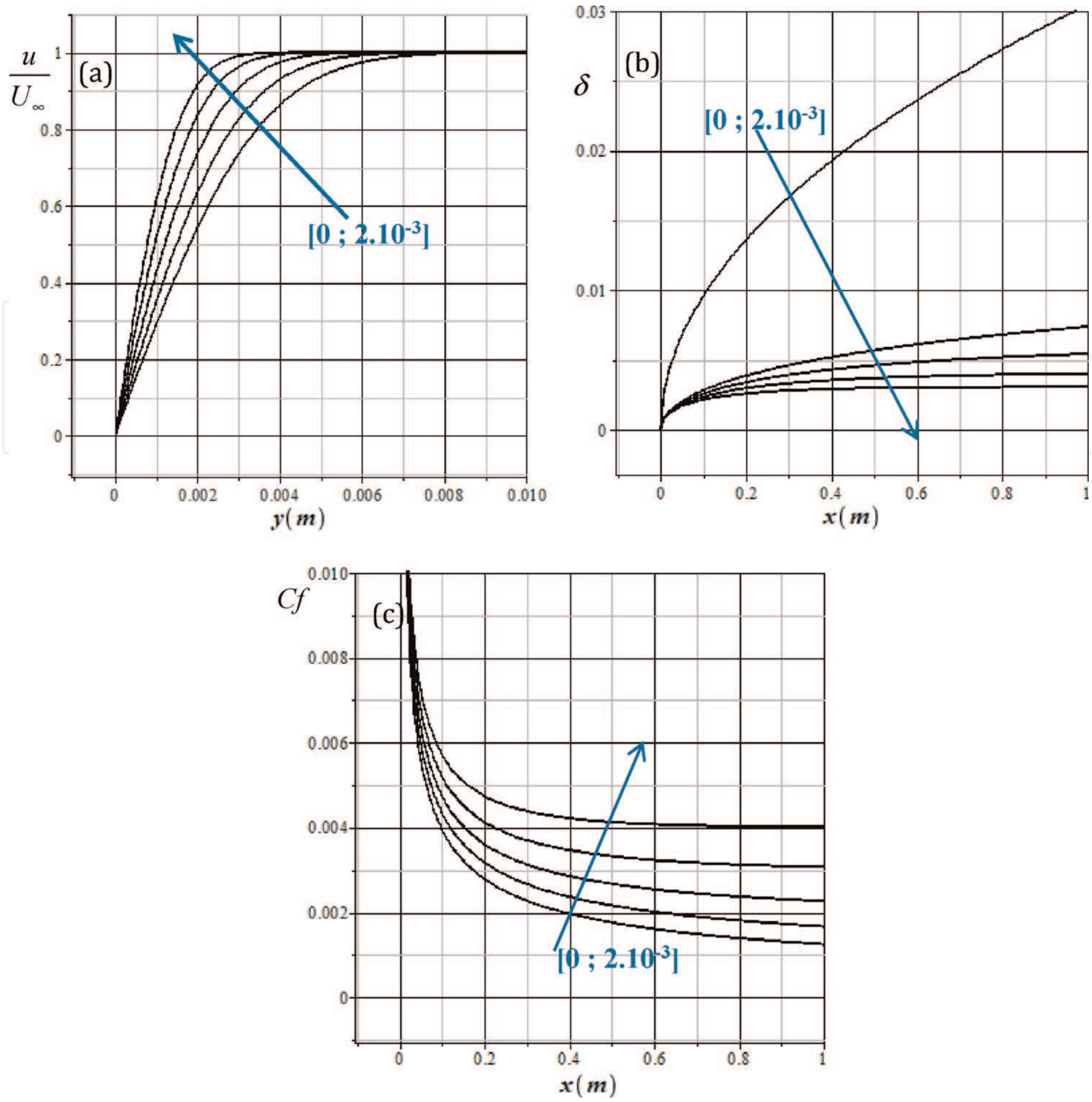


**Figure 8.**
*Parameters of boundary layer for different values of suction rate ($v_p/U = 0; 5.10^{-4}; 10^{-3}; 1,5.10^{-3}; 2.10^{-3}$) for $U = 5$ m/s; (a) velocity profiles; (b) boundary layer thicknesses; (c) parietal friction coefficients.*

layer profile flattens and the skin friction coefficient increases. This increase has no great effect on the total drag because it depends essentially on the form of drag. The contribution of the friction drag is negligible. This result is in accord with the literature.

## 4. Conclusion

This article has an objective to provide the analytical solutions of profile of boundary layer without and with uniform suction and to contribute to a better description of the structure of the flat plate to control the boundary layer. So, we presented the analytical resolution of the boundary layer equations by using the Bianchini integral method. This leads to new theoretical approximations, with the Error and/or Lambert functions. This result allows us to bring out the analytical expressions of several boundary layer features.

The new boundary layer theories were validated with literature results, as well as, with results obtained from numerical simulations using CFD Fluent.

## Nomenclatures

| | |
|---|---|
| Cf | skin friction coefficient |
| $d$d | diameter of the leading edge of the plate (m) |
| $d_{asp}$ $\mathrm{d_{asp}}$ | thickness of the suction holes (m) |
| $d_p d_p$ | distance between two successive holes (m) |
| e | plate thickness (m) |
| L | length of the plate (m) |
| l | length of the trailing edge flap of the plate (m) |
| $\mathrm{Re}_x$ | Reynolds number based on $x$ |
| $u$u | velocity component along $x$ (m/s) |
| $U_\infty$ | velocity inlet (m/s) |
| $v$v | velocity component along $y$ (m/s) |
| $\mathrm{v_p}$ | suction velocity (m/s) |
| $\mathrm{v_0}$ | absolute value of the suction velocity (m/s) |
| x | longitudinal coordinate (m) |
| y | transversal coordinates (m) |
| β | trailing edge flap angle (°) |
| $\nu$ | kinematic viscosity of the fluid (air) ($m^2$/s) |
| $\delta$ | boundary layer thickness (m) |
| $\delta_1$ | displacement thickness (m) |
| $\delta_2$ | momentum thickness (m) |
| $\tau p$ | parietal friction force ($N/m^2$) |
| $\eta = y\sqrt{\frac{U_\infty}{\nu x}}$ | Blasius parameter |

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