Chapter

The Role of Mathematical Modeling in STEM Integration and Education

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Abstract

With the rapidly developing technology, the labor force of the society has changed direction, and in the age of informatics, creative engineering applications have come to the forefront. Accordingly, the education levels of the labor force were also changed. The science, technology, engineering, and mathematics (STEM) education model in most countries aims to teach science, mathematics, technology, and engineering in relation to primary, secondary, high school, and higher education. STEM education, which has an impact in our country in recent years, has an important role in acquiring new skills, supporting creativity, innovation, and entrepreneurship, gaining the ability to transition between professions and adapting to new occupations. Nowadays, technology is expected to have different skills from individuals who will work in different fields with rapid development. Also, different teaching strategies play a major role in STEM integration and training. One of them, mathematical modeling, is the process of analyzing real-life or realistic situation using mathematical methods in the most general sense. The idea that mathematical modeling cycles should be used in STEM education at all levels from primary to tertiary education has gained importance in recent years, since it increases the students' motivation towards the lesson and they learn better by concentrating their attention.

Keywords: mathematical modeling cycle, STEM, real-life problems, metacognition, learning strategies

1. Introduction

Because of the quality of teaching, students find mathematics very abstract and fear mathematics. Thus, there may be difficulties in transferring the information learned in the classroom to the daily life. The fact that the learning environments are teacher-centered and uniform can be one of the reasons why students have difficulty in implementing information in their daily lives. In this sense, the subjects taught in the course should be taught by different practices and activities in a way that is more meaningful and related to daily life. One of these activities, mathematical modeling activities, can be said to show the relationship between real life and its applicability to real life.

Mathematical modeling involves a complex process in which a problem state encountered in real life is formulated mathematically and solved with the help of mathematical models, and the solution is interpreted and evaluated in the real world [1]. In this process, mathematics is used to represent, analyze, predict, or otherwise make sense of real-world situations [2]. In mathematical modeling, the individual tries to create a mathematical model that will solve the problem that he/she encounters in real life or in the future. The model in question includes not only mathematical structures but also estimates, assumptions, and strategies for solution [3, 4]. In other words, the solution plan including the assumptions, estimations, and mathematical tools used to solve the problem is the mathematical model for the problem. In addition to being mathematically correct, the model should be meaningful and adaptable for real life. While solving the problem, the individual should also evaluate the meaning of the solution for the real world. All these processes and all the stages of problem-solving in addition to the individual model are mathematical modeling [5].

Science, technology, engineering, and mathematics (STEM) education with technology age is appeared in the twenty-first century; it plays an important role in shaping cultural and economic development, embracing innovation, caring about creativity and problem-solving [6]. Due to the benefits of STEM education on the development of countries, intensive efforts are being made to reach the desired level between STEM and science education [7, 8].

The United States Bureau of Labor Statistics (2009) stated that 80% of the professions will need technology in 2018, and 8.5 million workforce will be needed in the STEM disciplines. STEM training can help students become problem-solvers and innovative and technologically literate citizens [9]. As the society becomes more dependent on technology, engineering, and mathematics, it is becoming increasingly important that students receive an integrated STEM education.

Due to global developments in the world demanding the thinking skills necessary to create a high workforce in the future, the curriculum for inter-curricular education in schools has been implemented. Initially, the implementation of STEM training was carried out with projects outside the formal classes. However, in the STEM education integrated with the direct curricula like Finland, there are also countries where many disciplines are taught.

STEM training has been implemented in many countries of the world (Korea, Japan, Germany, China, etc.), especially in the United States (USA) and in secondary schools and universities starting from primary schools. As STEM Education Coalition, there are organizations that undertake a roof in STEM education and direct STEM education and develop policy in this context [10].

STEM training, based on the integration of the disciplines of science, technology, engineering, and mathematics, has emerged as a result of the efforts of integrating separated parts in the real-world context [11], because, only by eliminating and integrating the boundaries between disciplines can the complex problems encountered in real life be understood and overcome. [12]. With STEM-oriented activities, the aim is to solve the real-life problems with the applications of technology and engineering disciplines by using the scientific knowledge which is the product of the basic sciences [12, 13]. For this purpose, it is necessary to remove the boundaries between disciplines [14–16]. In other words, STEM education understanding can be structured in the context of real-life problems by establishing a relationship between disciplines and focusing on a certain discipline.

Nowadays, both the training and applications related to STEM have been widely used in the world. On the one hand, many people now agree on what STEM means, interdisciplinary studies, and the common uses of science, technology, engineering, and mathematics. However, Clark-Wilson and Ahmed [17] emphasized that mathematics was included in an integrated curriculum on how M should be interpreted. Therefore, mathematics educators have said that mathematics in STEM should be used more, not as a servant. Coad [18] emphasizes that the use of mathematics as a data presentation tool with its study may lead to discrediting mathematics. Although mathematics is an inevitable component of STEM activities, it is also emphasized in this study that it is important to evaluate mathematical success and participation.

One of the most important tools for transition to STEM education is mathematical modeling [19]. Model-eliciting activities (MEA) are mathematical modeling applications. Mathematical modeling applications are composed of concepts related to different disciplines by their nature [20]. There is not a single definition of mathematical modeling agreed in the literature [21]. Instead, there are definitions, explanations, or shared assumptions made by individual authors. According to Kaiser [22], mathematical modeling is seen as a creative process to interpret the results and make changes to the model in order to define, control, or optimize the situation in order to make the real-life situation meaningful.

One of the many challenges faced by educators is the ways in which complex solutions to unusual problems can be taught to the student in the context of STEM education. One of the tools for transition to STEM training is the MEA [23].

MEAs, which are integrated into curricula for students to solve complex and difficult real-life problems, force students to build models and encourage them to test their established models, and their theoretical structure is known as a kind of open-ended problem-solving activities based on mathematical modeling perspective [24]. In school mathematics, MEAs have the potential to allow students to use mathematics in a flexible, creative, and powerful way in the STEM field because MEAs support the development of mathematical literacy [25], productive trends in mathematics [26], and a deep and integrated understanding of mathematical content and practices [27]. In MEAs, students clearly document their thought processes, consider their limitations, and use science and mathematics knowledge in the solution of the problem [28, 29]. MEAs offer students the opportunity to work on complex real-life problems involving model development. A framework for quality STEM integration curriculum is linked to the structure of MEAs [30]. According to the framework, the curricula (a) will serve a meaningful purpose and an engaging context, (b) enable students to develop problem-solving skills and engineering designs, (c) allow students to have the opportunity to redesign and learn if they fail, (e) support student-centered pedagogy, facilitator, and cooperative learning, including teacher, and (f) are designed to promote communication skills and teamwork [31].

2. Real-life problems, mathematical modeling cycles, and STEM

One of the first schemes presented as an approach to mathematical modeling is Blum [32]. The mathematical modeling cycle here consists of the real situation and the real world, the mathematical model, and the results in two parallel sections. In the loop, problem-solving is often perceived as a guide for the real situation.

According to Lesh and Doerr [3], it is the basic elements that must be included in a mathematical modeling cycle. There are three basic elements in mathematical modeling (**Figure 1**). According to them, a real-world problem must be started in mathematical modeling. The students generally act in the framework of mathematics and logic with ideas that involve mathematical assumptions and approaches. Then, the mathematics used should be accurate and also in a logical way (**Figure 2**).

The mathematical modeling cycle commonly used in literature is developed by Blum and Leiß [33]. Similar to other models, a distinction is made between the real world and mathematics in this model. A prerequisite for this model is that students

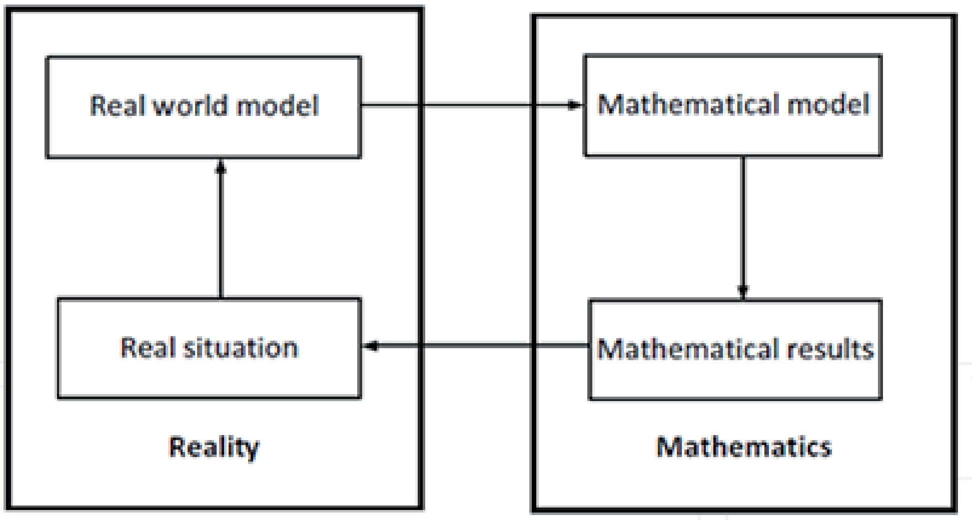

Figure 1.
Mathematical modeling cycle [32].

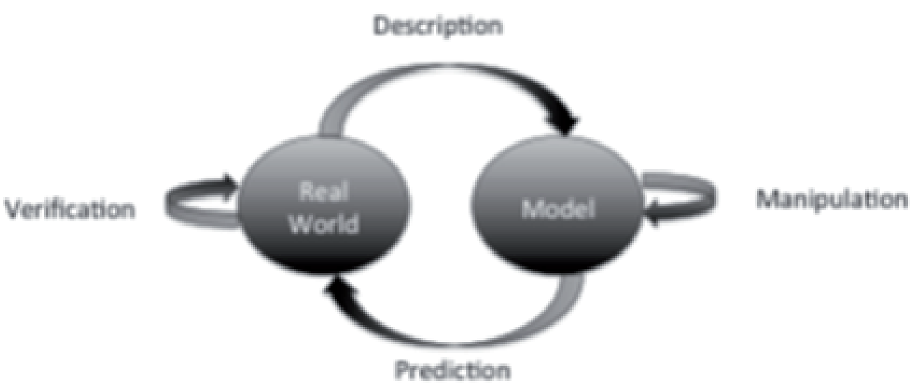

Figure 2.
Lesh and Doerr's modeling cycle.

should understand the mathematical problem and ensure that the model is developed in the real context. Although not mentioned here, it is important to keep in mind that the modeling process is in a repetitive natural loop (**Figure 3**).

Another important element is the existence of basic questions arising from the real-world problem in the mathematical cycle. These key questions can help to solve and study a mathematical modeling activity. Key questions are also very important for solving the problem. A key question can be shown as a real-life example of how long a person may be by spreading from the footprint and the length of the step [3]. Another feature of a key issue is that it allows people to focus on the issue. It can also bring people closer to their jobs or problems. The cycle of Perrenet and Zwaneveld is similar to that of Lesh and Doerr, but it has some differences. We observe that they provide more details and they emphasize three basic elements of mathematical modeling. In the modeling they describe, being outspoken and written communication are of paramount importance.

For example, students can conduct a mathematical modeling study and elaborate their solutions. Students also need to think through the modeling process so that they can clearly explain how well they understand the subject after a certain mathematical use. Thus, this mathematical cycle is repeated in a natural way. The revised solution is required during each cycle. This allows students to progress in different ways throughout the modeling cycle before developing an adequate solution. For the realization of such a process, they argue that the mathematical modeling activities of Perrenet and Zwaneveld must be open-ended (**Figure 4**).

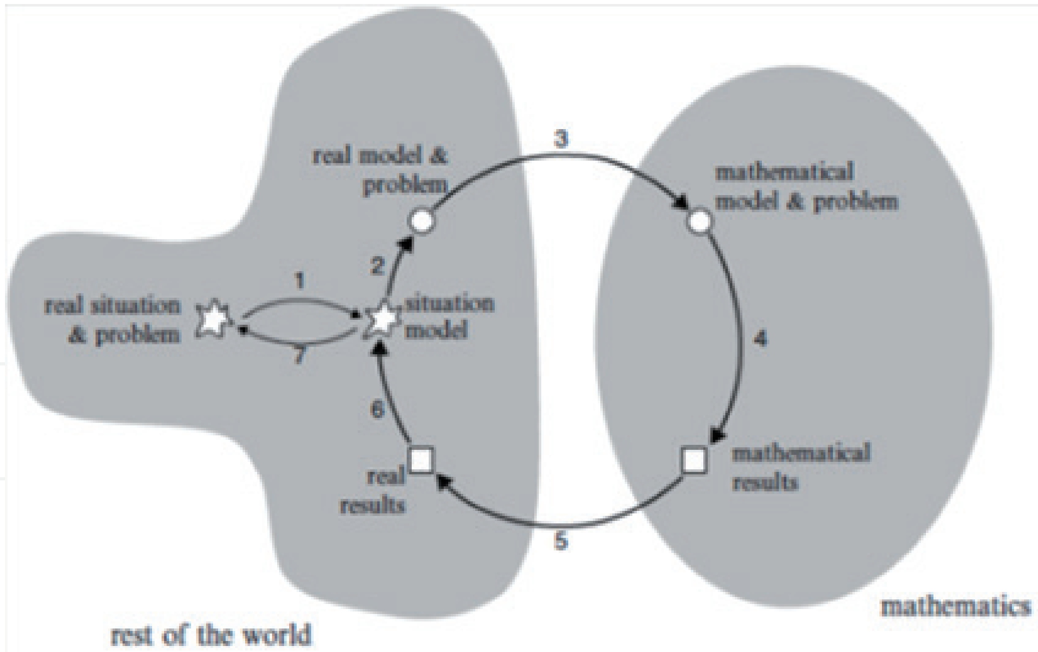

Figure 3.
Blum and Leiß [33] modeling cycle.

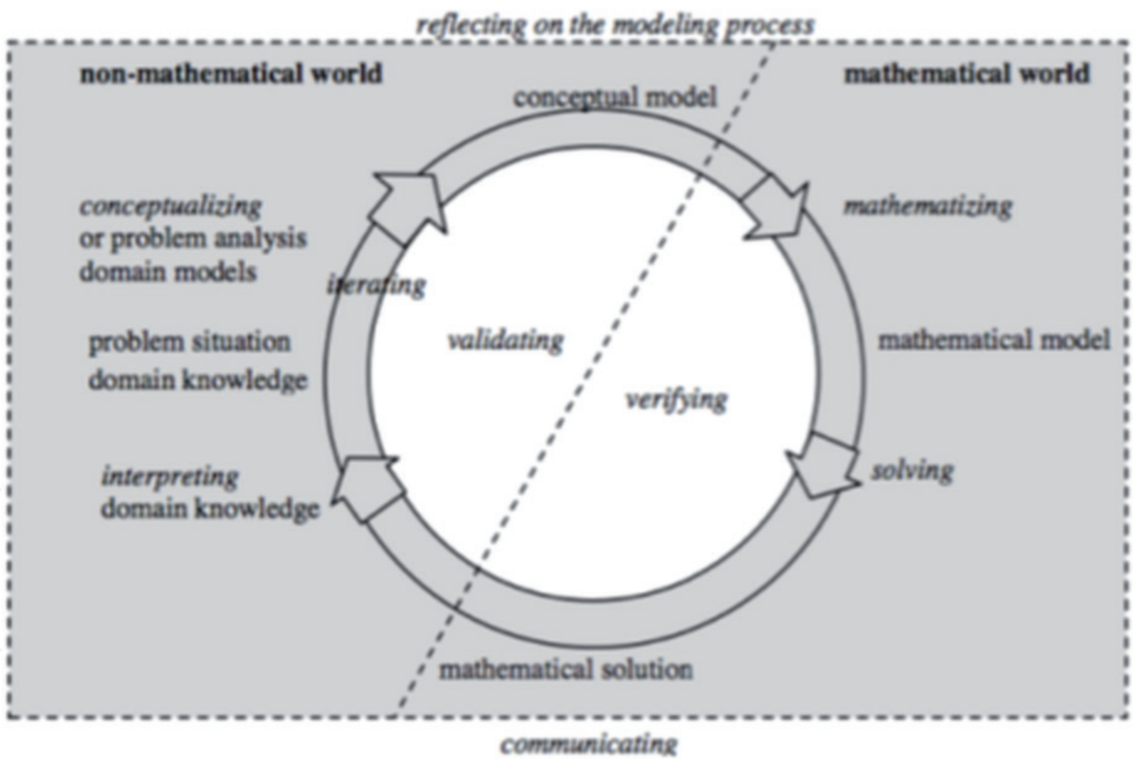

Figure 4.
Perrenet and Zwaneveld's modeling cycle.

Stohlmann and Albarracin [34] stated that there should be seven items in a mathematical modeling. The first one is that the problem should start with a real-world problem. Second, key questions should be addressed. The third one of these basic elements is the logical thinking of the solution of the problem with mathematical assumptions and approximations. Fourth, the mathematics used must be related to the real situation. The fifth of these elements plays an important role in written communication. The sixth, which is the mathematical modeling process, is an iterative process with open-ended problems. The seventh and last item is the reflections in the mathematical modeling used. However, the most widely used curricula are the modeling activities based on the models and modeling perspective.

For example, let us assume that there is an overflow after a heavy rainfall in a water-filled dam. Thus, in the case of an increase in the water level in the dam, the walls that protect the dam may break and the nearby city may cause a flood. This situation can be overcome by keeping the dam covers open at a certain time after each filling. As the precipitation continues, this situation will be constantly renewed. The important factors in this case are the water level in the dam, the amount of water discharged when the dam covers are open and the time. If necessary, a mathematical formula can be developed by considering the precipitation status for this problem. Even then, different variables or parameters can be found. The formulas that need to be considered here must be adjusted. For example, the amount, size, and time of the caps are important.

Some changes can be made in the model recursively. For such examples and similar representations, the example shown in the figure can be used as a mathematical model (**Figure 5**) [35, 36].

Güder and Gürbüz [38] aimed to improve the ability of interdisciplinary relations in the fields of mathematics, science, and technology in the field of "Energy-Saving Problem" for seventh grade students. In this problem, the concepts of power, motor power, power units (watt-kilowatt), and their transformation into each other are taught. In line with the purpose of the study, they tried to reveal the development of participants from a different perspective in a conceptually enriched environment in line with the multilayered teaching experiment [24, 39]. The multi-tier teaching experiment is designed to help students understand the modeling activities of teachers and teacher trainers in order to develop models for describing and explaining mathematical structures. The models are the teaching experiments consisting of three stages [24, 39, 40].

In this study, in the first 4 weeks of the study, thoughtful and supportive modeling problems are included. As a second step, the "Energy-Saving Problem" together with the Science teacher was developed by researchers. Finally, in the third stage, the researchers made observations and inferences during the application of the

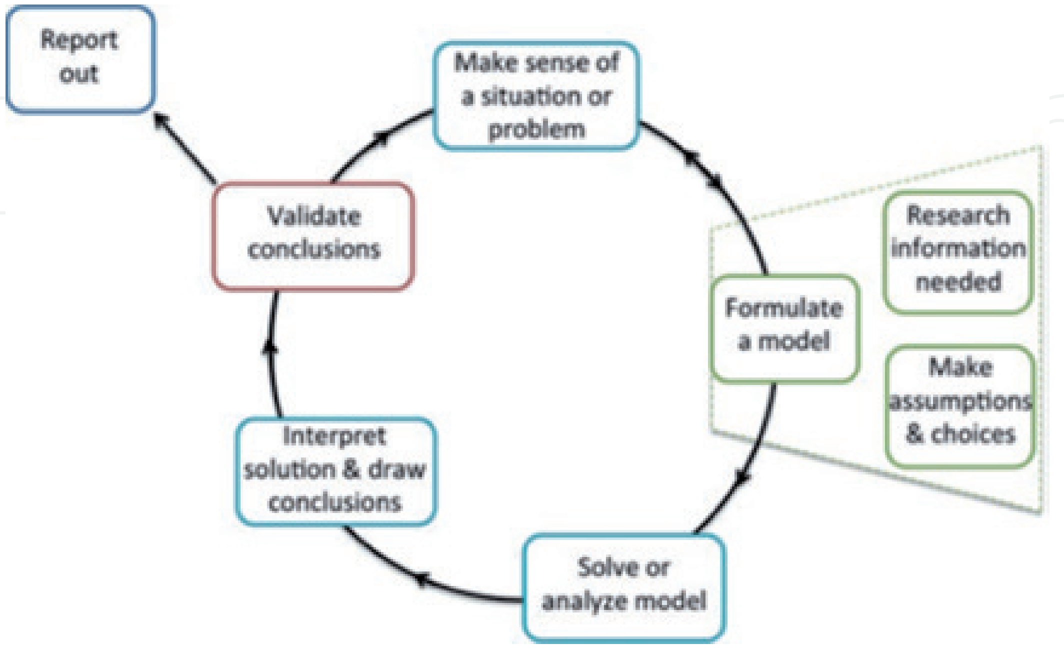

Figure 5.
A representation of the mathematical modeling process [37].

problem and reported the participants' progress in this matter. The general framework used in this study is as follows (**Figure 6**):

Real-life problems are discussed in mathematical modeling activities. Real-life situations are complex and cover many areas. Therefore, mathematical modeling is suitable for the different disciplines, and it is seen as an effective tool that can be used in STEM education [19, 41]. This type of activity, which is defined as interdisciplinary mathematical modeling (IMM), includes an understanding of different disciplines. In the understanding of IMM and in the solution of the problems of real-life situation, one or several disciplines are used together with mathematics [19]. In their study IMM is dealt with in mathematics and science. Therefore, IMM activities represent the activities associated with mathematics and science.

As mentioned above, the mathematical modeling process is a cyclical process consisting of several steps. Similarly, the IMM process is a cyclical and cascading process. However, unlike mathematical modeling, the inclusion of more than one area of IMM activities leads to a differentiation in the modeling process. Doğan et al. also defined a framework of the interdisciplinary mathematical modeling process for the mathematics and science disciplines in their studies. The IMM process begins in the real world, and first of all the individual needs to understand the real-life problem. The first step, which is expressed as an understanding of the problem, enters the STEM world.

A conceptual framework proposed by Daniels [42] was included in a study conducted on theories and assumptions developed before STEM training. This framework, which is designed as the theoretical framework of STEM integration, has been shown as three Venn diagram using mathematical modeling applications. While the first circle represents the elements of metacognitive theory (metacognitive knowledge, processes, skills, and strategies), the second circle includes social development theory (social mediated interaction—promoting communication). Lastly, the third circle consists of the teaching elements which are considered as basic for education.

If a good STEM integration is to be made, elements of the metacognitive [43] and Vygotsky's social development [44, 45] should also be included. As shown in

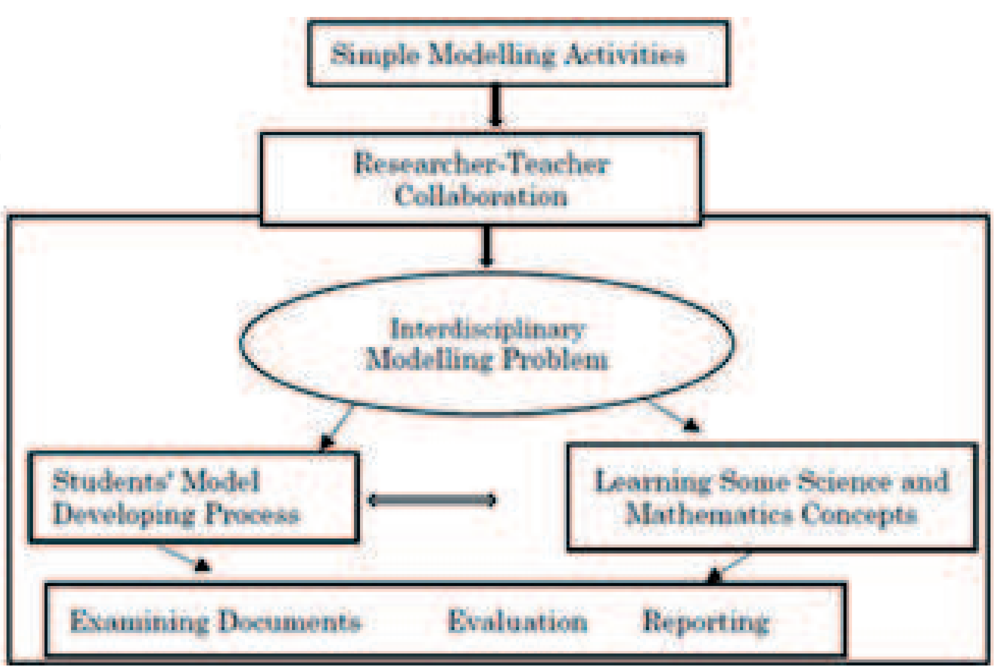

Figure 6.
Theoretical framework of the study.

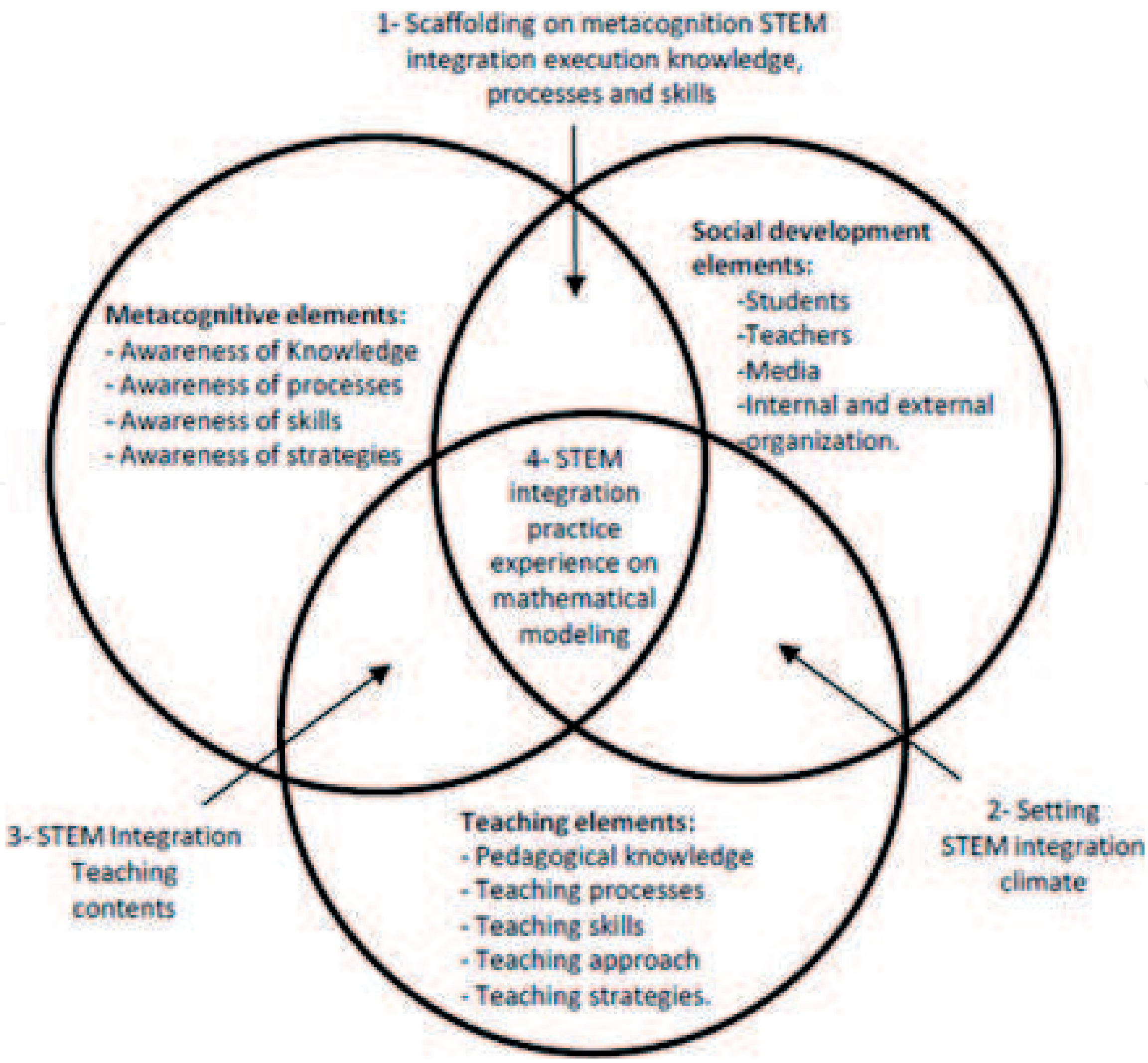

Figure 7.
The theoretical framework on metacognition of STEM integration from mathematical modeling perspectives.

Figure 7, STEM integration can be facilitated if the instructors implement these selected theories. These two theories have been proposed based on the following principles for implementation.

When Piaget [45], who explained the theory of cognitive development, explained only the characteristics of the cognitive development age stages, he mentioned the best level of learning and the importance of age for thinking development. Thus, Flavell's [47] theory (metacognitive knowledge, metacognitive experience, and metacognitive strategies) can explain the students' thinking, strategies, and actions to solve mathematical modeling problems [46, 48]. The problem-solving model of Polya may not be sufficient for a STEM practitioner [48]. This is because the problem is defined here in only three ways.

In particular, supporting mathematical and quantitative processes in science, mathematics, and engineering, and thus increasing mathematical reasoning, is the main objective. Technology provides tools to perform quantitative calculations more efficiently or to produce alternative visualization tools for experimental outputs. All modeling processes share the standard features shown in **Figure 8**. It has been demonstrated that there is a capability of researching modeling techniques, mathematical reasoning to model engineering design, and the ability to make scientific inquiry and then produce a structure. Mathematical modeling is of particular importance because it is important to produce appropriate tools to predict how quantification methods, new designs, and new situations will behave [49].

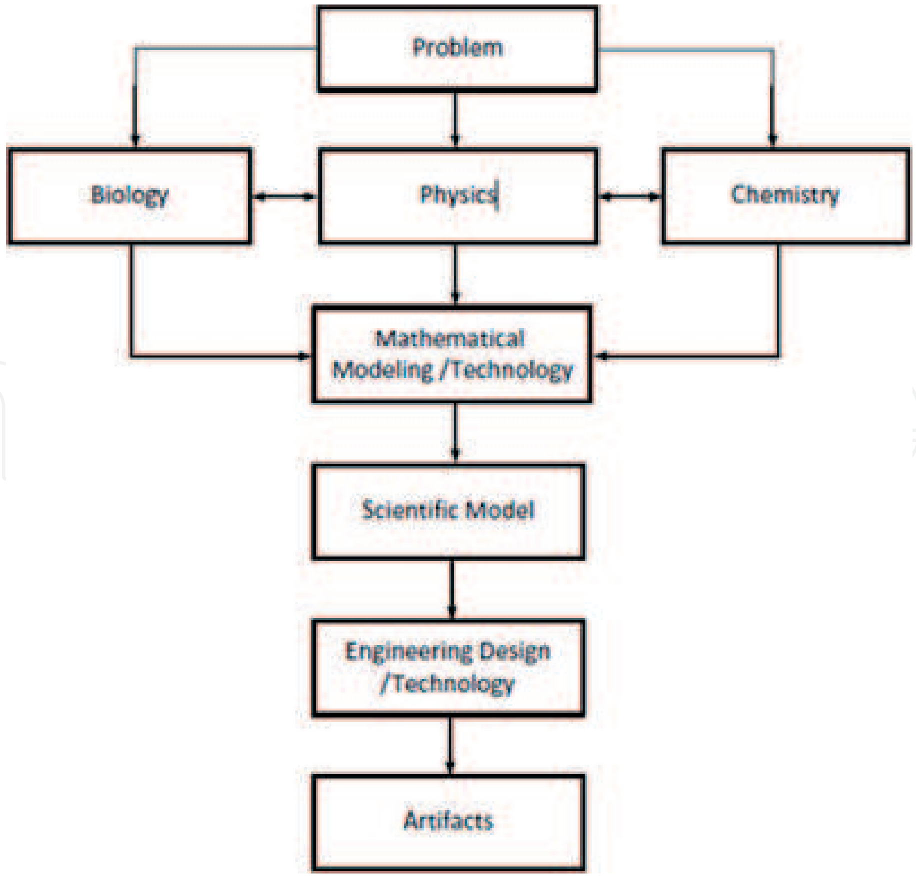

Figure 8.
Phases of STEM projects.

3. Discussion

Niss [50] stated that only theoretical mathematics knowledge is not always sufficient to solve real-life problems. In this case, the importance of mathematical modeling for the transfer of mathematical knowledge emerges. However, because the rapidly developing technologies and science are keys to solving real-life problems in different disciplines, STEM activities have been spread to schools. Thus, STEM integration has been added to the training program of many countries. In the educational programs of some countries, the presence of STEM activities combined with mathematical modeling also stands out [51].

Similarly, when questioning which stages of the mathematical modeling process improve students' problem-solving skills [49], the question of which stages of the "mathematical modeling together with STEM activities" should students use problem-solving skills raises too.

In the early 2000s, three and four stage cycles were used in mathematical modeling to solve real-life problems. However, over time, due to the need seen, these mathematical modeling cycles have been further elaborated by adding some steps. An example of this is Stohlmann and Albarracin [34] mathematical modeling cycle. Using science, technology, education, and mathematics together with mathematical modeling to solve real-life problems will facilitate to solve these problems [48]. Kertil and Gurel [52] and Sokolowski [49] supported this idea and were among the thinkers of STEM and mathematical modeling together. In the researches, teachers stated that this kind of instruction encourages students, focuses their attention on the subject and they learn the lesson better by leaving a positive effect on them [31, 53, 54]. An example of this is the STEM project conducted in conjunction with mathematical modeling to investigate the impact of student competences on sustainability in a university classroom [55]. In recent years, it has seen that teachers have been given courses for STEM training based on model-eliciting activities within STEM integration [30, 56].

4. Conclusion

When the studies are examined, more and more detailed explanations have been made about the cognitive activities in the modeling process. It is seen that technological developments are taken into consideration in the conceptualization of the mathematical modeling process. Considering the modeling used in the mathematical modeling process, the emergence of different frameworks and approaches reveals the complex structure of the process. For this reason, it is seen that the studies related to the mathematical modeling process, taking into account the different effects of technology, are combined with STEM, and this leads to the emergence of richer cognitive and metacognitive processes.

As a result of the importance of STEM activities in solving mathematical modeling and real-life problems in different disciplines, STEM activities continue to be integrated into schools. While many countries have added STEM to their education programs, some of them have been combining mathematical modeling with practices. Even teacher trainings on this subject are continuing.

As a result, it can be said that the teaching done by using mathematical modeling together with STEM increases the students' motivation toward the lesson; they learn better by concentrating their attention on the subject, leaving a positive effect on them; and the students' success and attitudes toward the lesson increase. Solving real-life problems in the future through STEM and mathematical modeling will continue to play an important role in providing innovative and creative problem-solving perspectives in the cultural and economic development of the countries.

Conflict of interest

The author declares no conflict of interest.

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References

[1] Berry J, Houston K. Mathematical Modelling. Bistrol: J. W. Arrowsmith Ltd; 1995

[2] Bliss K, Libertini J. What is mathematical modeling. In: Guidelines for Assessment and Instruction in Mathematical Modeling education (GAIMME). SIAM Society for Industrial and Applied Mathematics; 2016. pp. 7-21

[3] Lesh R, Doerr HM. Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In: Lesh R, Doerr HM, editors. Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching. Mahwah, NJ: Lawrence Erlbaum; 2003. pp. 3-33

[4] Tezer M, Cumhur M. Mathematics through the 5E instructional model and mathematical modelling: The geometrical objects. Eurasia Journal of Mathematics, Science and Technology Education. 2017;**13**(8):4789-4804

[5] Çavuş EZ. Matematiksel Modelleme Etkinliklerine Dayalı Öğrenim Sürecinin Alan Ölçme Konusu Bağlamında İncelenmesi [tezi]. Adıyaman: Adıyaman Üniversitesi; 2018

[6] Cooper R, Heaverlo C. Problem solving and creativity and design: What influence do they have on girls' interest in STEM subject areas? American Journal of Engineering Education. 2013;**4**(1):27-38

[7] David GH, Sharon KS. Conference Prologue. In David GH, Sharon KS, editors; Proceedings of the Conference on K-12 Outreach from University Science Departments; Raleigh, NC: The Science House, North Carolina State University; 2006. p. 67

[8] Tseng K, Chang C, Lou S, Chen W. Attitudes towards science, technology, engineering and mathematics (STEM) in a project-based learning (PBL) environment. International Journal of Technology and Design Education. 2013;**23**(1):87-102

[9] National Academy of Sciences. STEM Integration in K-12 Education: Status, Prospects, and an Agenda for Research. Washington, DC: National Academies Press; 2014

[10] Akgündüz D, Aydeniz M, Çakmakçı G, Çavaş B, Çorlu MS, Öner T, et al. STEM eğitimi Türkiye raporu. İstanbul: Scala Basım; 2015

[11] Moore TJ, Stohlmann MS, Wang HH, Tank KM, Glancy AW, Roehrig GH. Implementation and integration of engineering in K-12 STEM education. In: Engineering in Pre-College Settings: Synthesizing Research, Policy, and Practices. West Lafayette: Purdue University Press; 2014. pp. 35-60

[12] Bybee RW. Advancing STEM education: A 2020 vision. Technology and Engineering Teacher. 2010;**70**(1):30-35

[13] Morrison J. TIES STEM education monograph series, attributes of STEM education. Teaching Institute for Essential Science [Internet]. 2006. Available from: https://www.partnersforpubliced.org [Accessed: 26 May 2019]

[14] Chiu A, Price AC, Ovrahim E. Supporting Elementary and Middle School STEM Education. In: NARST 2015 Annual Conference; Chicago

[15] Dugger W. Evolution of STEM in the United States. In: Technology Education Research Conference; Queensland; 2010

[16] Meng CC, Idris N, Eu LK. Secondary students' perceptions of assessments

in science, technology, engineering, and mathematics (STEM). Eurasia Journal of Mathematics, Science and Technology Education. 2014;**10**(3):219-227

[17] Clark-Wilson A, Ahmed A. Optimising the M in STEM: Researching Collaborative CPD Models that Address the 'M' in the STEM Agenda. Chichester, UK: University of Chichester

[18] Coad L. The M in STEM: What is it really? Australian Mathematics Teacher. 2016;**72**(2):3-6

[19] Doğan MF, Şahin S, Çavuş Erdem Z, Gürbüz R. Investigation of teachers' awareness of interdisciplinary mathematical modeling problem. In: International Conference on Mathematics and Mathematics Education (ICMME-2018), Ordu University; 27-29 June 2018; Ordu

[20] Stohlmann M, Moore TJ, Roehrig GH. Considerations for teaching integrated STEM education. Journal of Pre-College Engineering Education Research (J-PEER). 2012;**2**(1):4

[21] Perrenet J, Zwaneveld B. The many faces of the mathematical modeling cycle. Journal of Mathematical Modelling and Application. 2012;**1**(6):3-21

[22] Kaiser G. The teaching and learning of mathematical modeling. In: Cai J, editor. Compendium for Research in Mathematics Education. National Council of Teachers of Mathematics. Reston, VA; 2017

[23] Zawojewski JS. Teaching and learning about mathematical modeling. In: Hirsch C, McDuffie AR, editors. Annual Perspectives in Mathematics Education: Mathematical Modeling and Modeling Mathematics. Reston, VA: NCTM; 2016. pp. 51-52

[24] Lesh R, Hoover M, Hole B, Kelly A, Post T. Principles for developing thought-revealing activities for students and teachers. In: Kelly A, Lesh R, editors. Handbook of Research Design in Mathematics and Science Education. Mahwah, NJ: Lawrence Erlbaum Associates; 2000. pp. 591-645

[25] Steen LA, Turner R, Burkhardt H. Developing mathematical literacy. In: Modelling and Applications in Mathematics Education. Boston, MA: Springer; 2007. pp. 285-294

[26] Lesh R, Yoon C. What is distinctive in (our views about) models & modelling perspectives on mathematics problem solving, learning, and teaching? In: Modelling and Applications in Mathematics Education. Boston, MA: Springer; 2007. pp. 161-170

[27] Lehrer R, Schauble L. Contrasting emerging conceptions of distribution in contexts of error and natural variation. In: Thinking with Data. 2007. pp. 149-176

[28] Chamberlin SA, Moon SM. Model-eliciting Activities: An introduction to gifted education. Journal of Secondary Gifted Education. 2006;**17**:37-47

[29] Shahbari JA, Peled I. Modelling in primary school: Constructing conceptual models and making sense of fractions. International Journal of Science and Mathematics Education. 2017;**15**(2):371-391

[30] Stohlmann MS, Moore TJ, Cramer K. Preservice elementary teachers' mathematical content knowledge from an integrated STEM modelling activity. Journal of Mathematical Modelling and Application. 2013;**1**(8):18-31

[31] Maiorca C, Stohlmann MS. Inspiring students in integrated STEM education through modeling activities. In: Hirsch C, McDuffie AR, editors. Annual

Perspectives in Mathematics Education: Mathematical Modeling and Modeling Mathematics. Reston, VA: NCTM; 2016. pp. 153-161

[32] Blum W. Anwendungsbezüge im Mathematik-unterricht—Trends und Perspektiven. In: Kadunz G et al., editors. Trends und Perspektiven. Schriftenreihe Didaktik der Mathematik. Vol. 23. Wien: Hölder-Pichler-Tempsky; 1996. pp. 15-38

[33] Blum W, Leiß D. How do students and teachers deal with modelling problems. In: Haines C, Galbraith P, Blum W, Khan S, editors. Mathematical Modelling: Education, Engineering and Economics. Chichester, England: Horwood Publishing; 2007. pp. 222-231. DOI: 10.1533/9780857099419.5.221

[34] Stohlmann M, Albarracin L. What is known about elementary grades mathematical modelling. Education Research International. 2016;**2016**:1-9. DOI: 10.1155/2016/5240683

[35] Blum W, Leiss D. “Filling up”—The problem of independence-preserving teacher interventions in lessons with demanding modeling tasks. In: Proceedings of the fourth Congress of the European Society for Research in Mathematics Education; Sant Feliu de Guíxols, Spain: European Research in Mathematics Education IV; 2005. pp. 1623-1633

[36] Yoon C, Dreyfus T, Thomas M. How high is the tramping track? Mathematising and applying in a calculus model-eliciting activity. Mathematics Education Research Journal. 2010;**22**(2):141-157

[37] Anhalt CO, Staats S, Cortez R, Civil M. Mathematical modeling and culturally relevant pedagogy. In: Cognition, Metacognition, and Culture in STEM Education. Cham: Springer; 2018. pp. 307-330

[38] Güder Y, Gürbüz R. Teaching concepts through interdisciplinary modeling problem: Energy conservation problem. İlköğretim Online. 2017;**16**(3):1101-1119

[39] English LD. Reconciling theory, research, and practice: A models and modelling perspective. Educational Studies in Mathematics. 2003;**54**(2-3):225-248

[40] Steffe LP, Thompson PW. Teaching experiment methodology: Underlying principles and essential elements. In: Kelly AE, Lesh RA, editors. Handbook of Research Design in Mathematics and Science Education. Mahwah, NJ: Erlbaum; 2000. pp. 267-307

[41] English LD. STEM: Challenges and opportunities for mathematics education. In: Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics Education; PME; 2015; Vol. 1, pp. 4-18

[42] Daniels H. Vygotsky and Research: Activity Theory and Interventionist Research. New York, NY: Routledge; 2008. DOI: 10.4324/9780203891797

[43] Vygotsky L. Interaction between learning and development. In Cole M, John-Steiner V, Scribner S, Souberman E, editors. Mind in society. Cambridge, MA: Harvard University Press; 1978

[44] Kozulin A, Gindis B, Ageyev VS, Miller SM. Vygotsky’s Educational Theory in Cultural Context. Cambridge: Cambridge University Press; 2003

[45] Piaget J. The Psychology of Intelligence: The Nature of Intelligence. London: Routledge Classics; 1950

[46] Polya G. How to solve it. The Mathematical Gazette. Princeton University Press; 1945

[47] Flavell JH. Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. American Psychologist. 1979;**34**:906-911. DOI: 10.1037/0003-066X.34.10.906

[48] Proust J. Metacognition as cognition about cognition: Attribute view. In: The Philosophy of Metacognition. 1st ed. New York, NY: Oxford University Press; 2013. DOI: 10.1093/acprof:oso/9780199602162.001.0001

[49] Sokolowski A. Scientific Inquiry in Mathematics-Theory and Practice: A STEM Perspective. New York: Springer; 2018

[50] Niss M. Modeling a crucial aspect of students' mathematical modeling. In: Lesh R, Galbraith PL, Haines CR, Hurford A, editors. Modeling Students' Mathematical Modeling Competencies. New York: Springer; 2010. pp. 43-59

[51] Colakoglu MH. Integration of transdisciplinary STEM approach to single discipline-based national education systems. Education Research Highlights in Mathematics, Science and Technology. 2018. p. 94

[52] Kertil M, Gurel C. Mathematical modeling: A bridge to STEM education. International Journal of Education in Mathematics Science and Technology. 2016;**4**(1):44-55

[53] Betanga SA. The effects of mathematical modeling instruction on precalculus students' performance and attitudes toward rational functions. 2018

[54] Koenig K, Schen M, Edwards M, Bao L. Addressing STEM retention through a scientific thought and methods course. Journal of College Science Teaching. 2012;**41**(4):23-29

[55] Suh H, Han S. Promoting sustainability in university classrooms using a STEM project with mathematical modeling. Sustainability. 2019;**11**(11):3080

[56] Baker CK, Galanti TM. Integrating STEM in elementary classrooms using model-eliciting activities: Responsive professional development for mathematics coaches and teachers. International Journal of STEM Education. 2017;**4**(1):10