Chapter

Chaos in Human Brain Phase Transition

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Abstract

From a theoretical equation, modeling the dynamic of the time-dependent coefficients of the first and the second Karhunen-Loeve (KL) expansion of a superconducting quantum interference device (SQUID) signal, chaotic phase transition has been studied in the human brain. Through numerical investigations, the bifurcation diagram and the dynamic of Lyapunov exponent have been plotted. These diagrams reveal that throughout the variation of the control parameter here the frequency of the acoustic stimulus, the brain bifurcates from chaotic states to periodic or to quasiperiodic one. Also a chaotic phase portrait of the KL modes and its corresponding Poincaré section have been plotted. The origin of chaos in the human brain could be due to the self-organizing processes of nonequilibrium phase transition occurring in the electrochemical physiological phenomena of the complex nerve cells and neural assembly. Besides, the occurrence of chaos in the absence of stimuli has been remarked and thought to be due to the fact that an intrinsic brain could be chaotic. Moreover it has been found that the range of frequency for which the brain is forced to behave periodically could be harmful to the thinking process.

Keywords: human brain, Karhunen-Loeve coefficients, phase transition, self-organizing phenomena, chaos

1. Introduction

Nonlinearity is one of the most ubiquitous phenomena in nature and in our society. A system is said to be nonlinear if its output is not proportional to its input. The study of nonlinear system can be divided into six domains: fractal, pattern formation, soliton, complex systems, cellular automata, and chaos [1]. Many researches have been carried out in those six domains by scientists through the study of the dynamics of numerous systems. Generally, those systems exhibit spontaneous orders and patterns of organization governed by the self-organizing phenomena. Through the variation of an order parameter, a system can move from one state to another: this is the bifurcation phenomena, which is extremely linked to the phenomena of phase transition in some systems.

Among all the nonlinear phenomena, chaos is one of the most attractive and studied. The word "chaos" was first used in science by Li and York, who are mathematicians [2]. The Greek word chaos was originally a verb meaning to gape open and was used to refer the primeval emptiness of the universe before a thing comes into being (Encyclopedia Britannica, Vol. 5, p. 276; [3]). More simply, it means:

- Utter disorder and confusion
- The unformed original state of the universe

For scientists, chaos is known as a nonperiodic oscillatory state, which stems from the nonlinear nature of a given system [4]. It has been observed in condensed matter physics, turbulent fluid dynamics, the Belousov-Zhabotinsky reaction in chemistry, charge density wave in low dimensional conductors, and nonlinear carrier transport in semiconductors [5]. Chaos has also been reported in economics, in social sciences, and in biological sciences.

Chaos was widely studied in human physiological systems and particularly in the human brain. The brain is the most complex and fascinating organ of the human body [6]. The human brain is constituted of many elements: neurons, neural stem cells, blood vessels, and glial cells. The neurons are about 100 billions, and for each neuron, there are more than 10^4 connections to others [7]. These neuronal and non-neuronal cells are located in the cerebral cortex, outer surface of the cerebral hemispheres or the cerebrum. The brainstem lies under the cerebrum. The cerebellum is situated behind the brainstem and under the cerebrum.

The study of chaos in the brain started in the 1980s [8] when scientists observed that when rabbits inhale an odorant, their electroencephalograms (EEGs) display oscillations in the high-frequency range of 20–80 Hz. Bressler and Freeman have named that behavior "gamma" in analogy to the high end of the X-ray spectrum [9]. The carrier wave of that odor information has exhibited an aperiodic behavior leading to the conclusion that activity of the olfactory bulb is chaotic. Therefore, chaos has been investigated in human brain elements, ranging from subcellular to cellular levels [10]. Chaos has also been studied in single neurons, in coupled neurons, in axonal membranes, and in synapses. Additionally, from the model of Hodgkin and Huxley [11, 12], chaotic dynamics of some neuron's ion conduction has been recorded.

The study of chaos in the human brain has an importance which could no longer be demonstrated. It has been reported that chaos plays an important role in cortical hormone secretion and suppression [13]. Moreover, chaotic behavior stemming from self-organization processes in the human brain could explain "randomness" in neural synchronization related to cognitive functions and consciousness and also in mental disorganization related to psychopathological phenomena such as schizophrenia [14]. In addition, it has been argued that epilepsy is an example of chaos in the human brain [15]. Besides, some researchers think about the dreaming brain as a brain in which some self-organizing processes occur and exhibit chaos-like stochastic properties that are highly sensitive to internal influences [16].

In this paper, we deal with a synergetic view of the human brain. In this view the brain acts by means of self-organization processes through which nonequilibrium phase transitions occur. By means of 37 superconducting quantum interference devices (SQUID), the values of the magnetic field generated by the intracellular dendritic current of the brain of a subject exposed to acoustic stimuli have been recorded [17]. From that experiment, Fuchs and co-workers studied the human brain phase transition and chaos [18, 19]. The aim of this paper is to theoretically study chaos from equations derived by Jirsa and co-workers [20] based on the works of Kelso and co-workers. To reach such a goal, we outline our work as follows: Section 2 deals with the presentation of the equation model and its linear stability, Section 3 describes the numerical studies of chaos, and the paper ends with concluding remarks.

2. Presentation of the model and its linear stability

2.1 Origin of the model

Here we will describe how the equation modeling the brain phase transition was derived. This equation is based on the experimental study carried out by Kelso and co-workers [17]. In that study, a set of 37 SQUID collect the values of the magnetic field generated by the intracellular dendritic current of the brain of a subject exposed to periodic acoustic stimuli. This noninvasive brain exploration was rendered possible because the skull and the scalp are permeable to the magnetic field generated inside the brain. The stimuli served as control parameter, and the subject was invited to press a button in two successive tones. The spatiotemporal behavior of the brain signals was known after the application of a Karhunen-Loeve expansion to the magnetic field collected by the SQUID array. Based on the observations of results of this experiment, Jirsa and co-workers have introduced a mathematical model which mimics the brain behavior [20]. That model was made of two nonlinearly coupled oscillators describing the coefficients of the first KL modes that are driven by the acoustic stimuli. That model provided a mathematical description of the switching from the first KL mode that oscillated at the stimuli frequency to the second KL mode that oscillated at twice the stimuli frequency [20].

The following mathematical model is a set of differential equations [20]:

$$\ddot{x} + \left(\gamma_1 + A_1 x^2 + B_1 y^2\right)\dot{x} + \omega_{01}^2 (1 + \epsilon_1 \sin(2\Omega t))x + C_1 y^2 \sin(\Omega t) = 0, \tag{1}$$

$$\ddot{y} + \left(\gamma_2 + A_2 y^2 + B_2 x^2\right)\dot{y} + \omega_{02}^2 (1 + \epsilon_2 \sin(2\Omega t))y + D\dot{x} = 0. \tag{2}$$

where the parameters x and y represent the time-dependent coefficients of the first and the second Karhunen-Loeve expansions of SQUID signals, respectively. Also, the coefficients γ_i, A_i, B_i, ω_{0i}, $(i = 1, 2)$, C_1, and D are adjustable but then fixable parameters [20]. The control parameter of the system here is Ω which is the frequency of the acoustic signal used to stimulate the subject. Here we neglect the random processes.

2.2 Study of the linear stability of the model

In others to study easily Eqs. (1) and (2), we set

$$\dot{x} = u, \tag{3}$$

$$\dot{y} = v, \tag{4}$$

$$\theta = \Omega t. \tag{5}$$

This leads to

$$\dot{x} = u, \tag{6}$$

$$\dot{u} + \left(\gamma_1 + A_1 x^2 + B_1 y^2\right)u + \omega_{01}^2 (1 + \epsilon_1 \sin(2\Omega t))x + C_1 y^2 \sin(\theta) = 0, \tag{7}$$

$$\dot{y} = v, \tag{8}$$

$$\dot{v} + \left(\gamma_2 + A_2 y^2 + B_2 x^2\right)v + \omega_{02}^2 (1 + \epsilon_2 \sin(2\theta))y + Du = 0 \tag{9}$$

$$\dot{\theta} = \Omega. \tag{10}$$

For $\Omega = 0$, Eqs. (6)–(10) possesses a circle of fixed points X(0,0,0,0, θ) with θ in R mod 2π. This circle of fixed points becomes a limit cycle of Eqs. (1) and (2). The corresponding Jacobian matrix for $\Omega = 0$ is then given as

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega_{01}^2 & -\gamma_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -D_2 & -\omega_{02}^2 & -\gamma_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{11}$$

The associate Eigenvalues are

$$\begin{bmatrix} 0 \\ -1/2\gamma_2 + 1/2\sqrt{\gamma_2^2 - 4\omega_{02}^2} \\ -1/2\gamma_2 - 1/2\sqrt{\gamma_2^2 - 4\omega_{02}^2} \\ -1/2 + 1/2\sqrt{\gamma_1^2 - 4\omega_{01}^2} \\ -1/2 - 1/2\sqrt{\gamma_1^2 - 4\omega_{01}^2} \end{bmatrix}. \tag{12}$$

When $\Omega \neq 0$, the corresponding Jacobian matrix is then given as

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega_{01}^2(1+\varepsilon_1 \sin(2\Omega)) & -\gamma_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -D_2 & -\omega_{02}^2(1+\varepsilon_2 \sin(2\Omega)) & -\gamma_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{13}$$

The associate Eigenvalues are

$$\begin{bmatrix} 0 \\ -1/2\gamma_2 + 1/2\sqrt{\gamma_2^2 - 4\omega_{02}^2 - 4\omega_{02}^2\varepsilon_2 \sin(2\Omega)} \\ -1/2\gamma_2 - 1/2\sqrt{\gamma_2^2 - 4\omega_{02}^2 - 4\omega_{02}^2\varepsilon_2 \sin(2\Omega)} \\ -1/2 + 1/2\sqrt{\gamma_1^2 - 4\omega_{01}^2 - 4\omega_{01}^2\varepsilon_1 \sin(2\Omega)} \\ -1/2 - 1/2\sqrt{\gamma_1^2 - 4\omega_{01}^2 4\omega_{01}^2\varepsilon_1 \sin(2\Omega)} \end{bmatrix}. \tag{14}$$

It is clear that depending on the values of the parameters γ_i and $\omega_i (i = 1, 2)$, the fixed points could either be stable or unstable.

3. Chaos: numerical simulation and discussions

For numerical investigations of chaos, we consider a subject with the following constant parameters: $\gamma_1 = 0.1$, $\gamma_2 = 0.1$, $A_1 = 3$, $A_2 = 9.8$, $B_1 = 2.2$, $\epsilon_1 = 17.09$, $\epsilon_2 = 2.99$, $\omega_1 = 2\pi$, $\omega_2 = 2\pi$, $D_2 = 10$, $B_2 = 2.2$ and C_1.

The numerical simulations are carried out via Runge-Kutta algorithm. The dynamic of the maximum of the Lyapunov exponent, (i.e., the greatest Lyapunov exponent among the five related to the degree of freedom of the system), is given by **Figure 1**. It has been plotted by following the algorithm of Wolf et al. [4] which is an appropriate tool for numerical calculation of Lyapunov exponents.

It has been obtained through the variation of the order parameter Ω for the values of 0–100.

The bifurcation diagram associated with this system is depicted at **Figure 2** for the same range of parameter values as for the Lyapunov exponent. This diagram has been obtained by means of Runge-Kutta algorithm with a finish value of time of 90, a transient of 30, and a time step of 1.

The analysis of these two diagrams reveals the fact that depending on parameter range, the system is subject to various behaviors. For example, for $\Omega \epsilon [0, 0.1]$ the system behaves chaotically, while for $\Omega \epsilon [0.1, 0.12]$ it behaves periodically. Throughout the variation of the control parameter, the system bifurcates from chaotic states to periodic or to quasiperiodic ones. This means that the brain

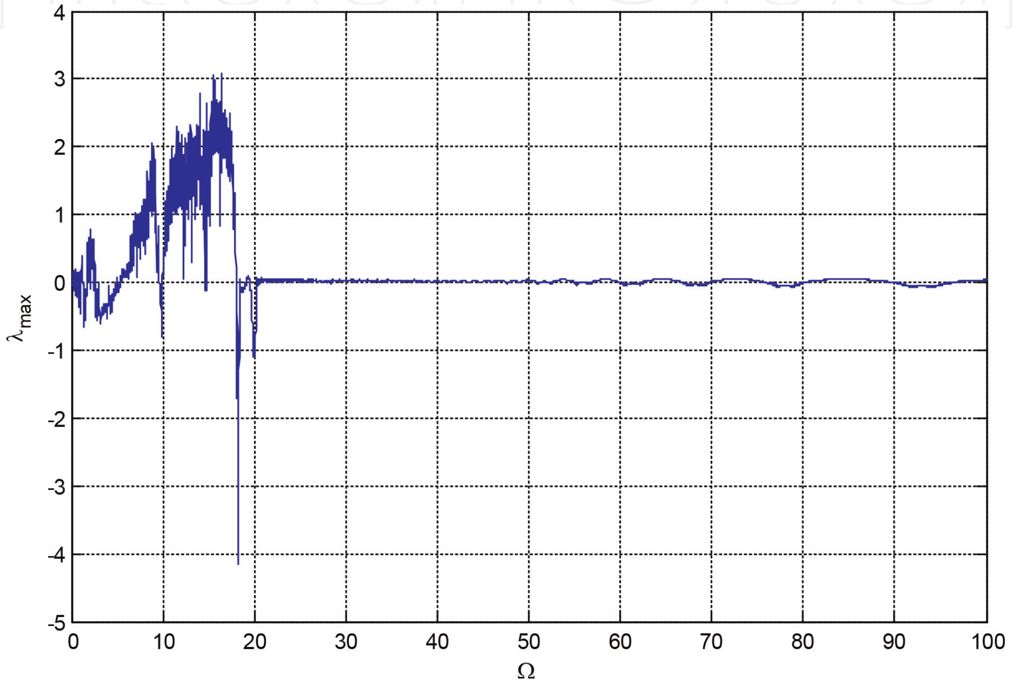

Figure 1.
Dynamics of the maximum of the Lyapunov exponents obtained through the variation of the stimuli frequency Ω. It has been plotted by using the following initial conditions (0.1 0.6 0.1 0.8 0.2) and parameters of the text.

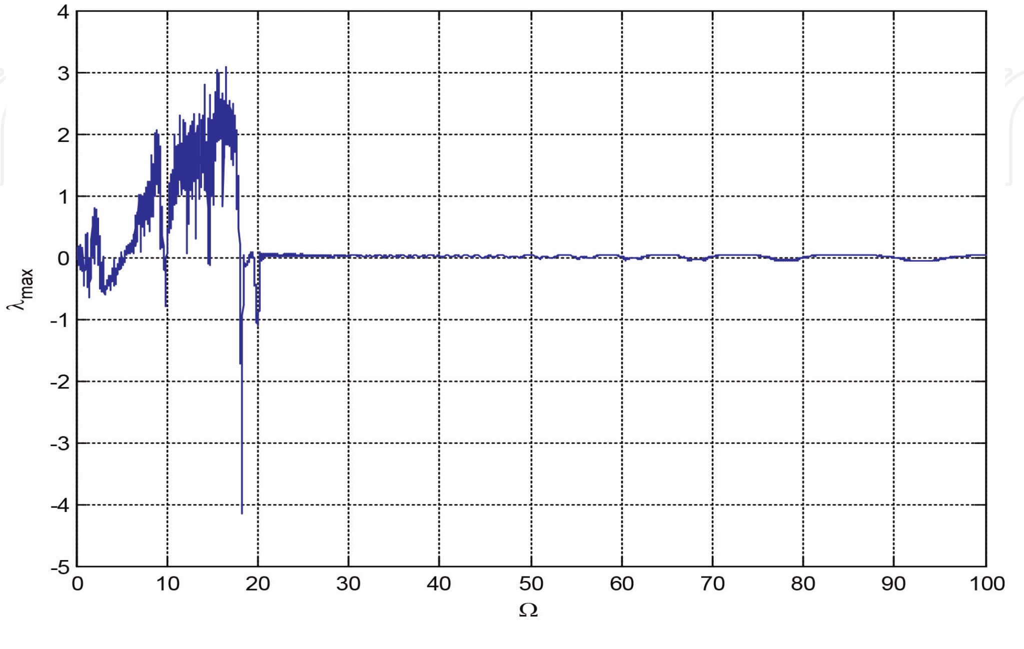

Figure 2.
Bifurcation diagram corresponding to the maximum of the first KL coefficient (i.e., x). It has been obtained for the same range of parameters as for the Lyapunov exponent.

experiences periodic states, quasiperiodic states, intermittencies, and chaos. The fact that chaos occurs even for $\Omega = 0$ could explain the chaotic behavior of an intrinsic human brain, the dreaming brain, for example [15].

In addition to the above diagrams, we present the 2D phase portrait (x, y) of the KL given by **Figure 3** for $\Omega = 15$ with the parameters of **Figure 2**.

The topological mixed nature of this phase portrait reveals its chaotic nature. **Figure 4** represents the Poincaré section corresponding to the 2D phase portrait of **Figure 3**. It has been obtained by cutting the 3D phase space (x, u, y) at u = 5.

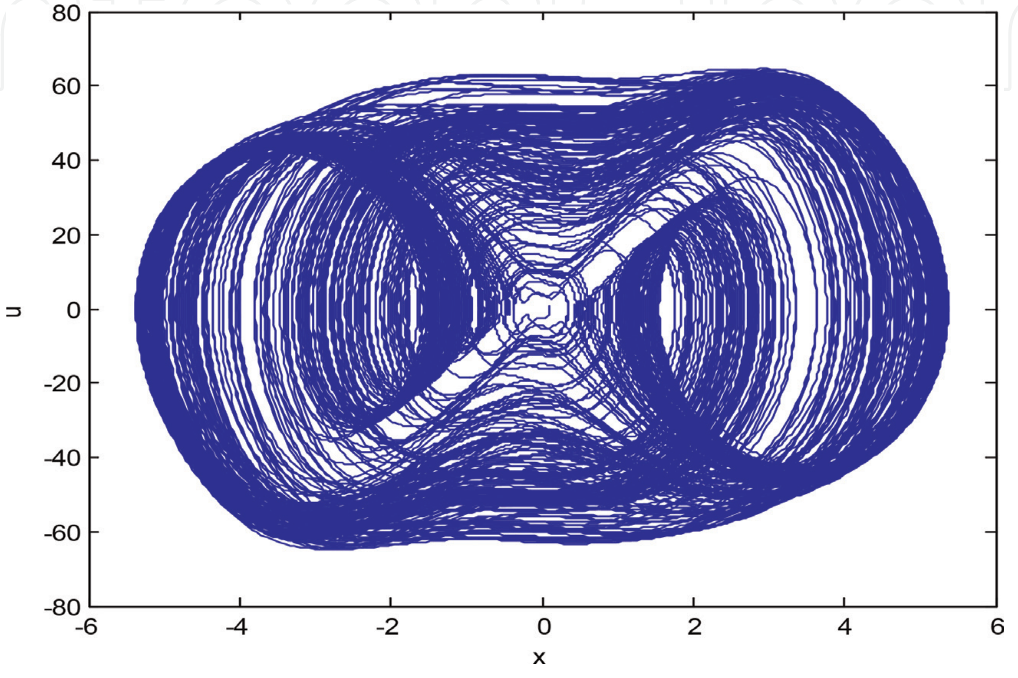

Figure 3.
2D phase portrait (x, y) of the KL coefficients obtained for Ω = 15.

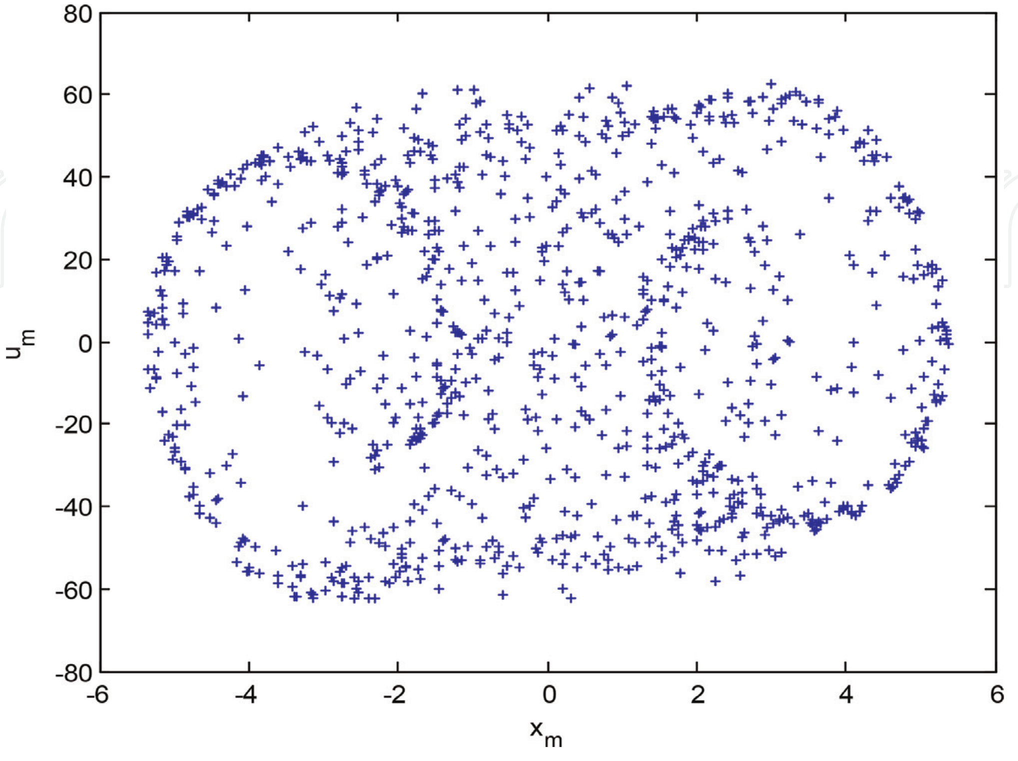

Figure 4.
2D Poincare section of the previous phase portrait obtained by cutting the 3D phase space (x, u, y) at u = 5.

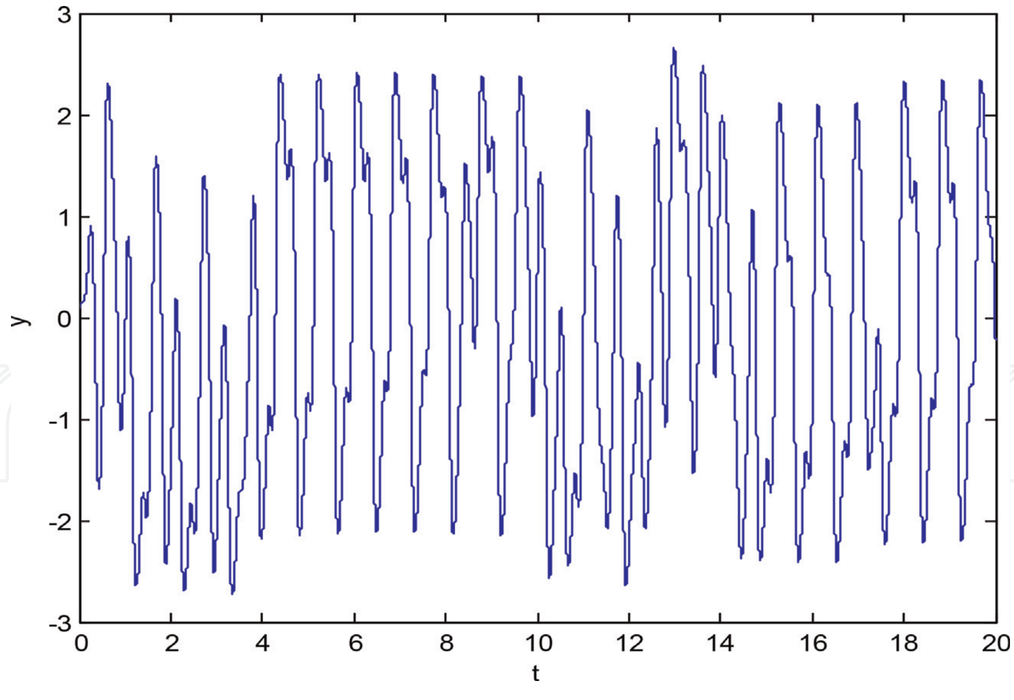

Figure 5.
Chaotic wave form of the second KL coefficients obtained for Ω = 15.

Its non-ordered nature confirms that the previous phase portrait is really chaotic.

The origin of chaos in the human brain could be due to the self-organizing process of nonequilibrium phase transition occurring in the electrochemical physiological phenomena of the complex nerve cells and neural assembly.

The chaotic wave form of the second KL coefficients obtained for $\Omega = 15$ is represented by **Figure 5**. The wave does not conserve its form throughout the time meaning that there is a loss of memory while the system evolves.

4. Conclusion

In this study, we dealt with chaos in the human brain. By means of numerical investigations through an equation modeling a human brain phase transition, some interesting results have been found. The linear stability of the equation model has been done: the system possesses one fixed point which could either be stable or unstable depending on the values of some parameters. Additionally, the bifurcation diagram and the dynamic of the Lyapunov exponent have been plotted. The examination of those diagrams reveals the fact that the brain can undergo periodic phase transition, quasiperiodic phase transition, and chaotic phase transition. Chaos may be due to a nonequilibrium phase transition, happening even when there are no stimuli, meaning that an intrinsic brain could be a chaotic one. The Lyapunov exponent and the unrepeatable waveform of the second KL coefficients underline the loss of memory by the system revealing that nonequilibrium phase transition of the brain is not reversible. Therefore the electrochemical physiological phenomenon of the brain is a dissipative one.

There are two opinions about human brain chaos. Chaos was firstly discovered at a neural level and has led researchers to a thing about chaos as a source of some diseases: schizophrenia, insomnia, and epilepsy [21]. The studies of chaos on the macroscopic level by other researchers reveal the fact that chaos is related to thinking and only the brain of falling sickness is periodic [22]. Also chaos helps the brain to quickly recognized previously learned patterns and behaviors [23]. From these above ideals, the importance of chaos in the brain is no longer to be demonstrated.

In this study, we have realized that there are some frequencies for which the brain behaves periodically and others for which its behavior is chaotic. This means that some acoustic stimulus at some frequencies could alter the brain functioning. For example, for >20, the brain is forced to behave periodically and could have some harmful feedback on the thinking process.

Also, this study can help for the development of neuromorphic and artificial neural network architectures since such technologies are based on human brain functioning. The understanding of chaos in human brain electromagnetic activity could help for deeper understanding of the role of chaos in the brain. This could be the object of future work.

Conflict of interest

The authors declare there is no conflict of interest.

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