Chapter

Towards a Forward-Thinking College Calculus Program

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Abstract

Calculus is perceived as serving many roles in college STEM students' education, including as a way to 'weed out' students who should not be in the major to teaching fundamental concepts. No matter its purpose, it is clear that college calculus is viewed as a critical course in university STEM education. It is also clear that in the US and other countries, STEM education is disproportionately serving men and white and Asian students. In this chapter, I discuss how calculus has come to occupy this position over time and the current state of college calculus drawing on two national studies in the United States. I then define a forward thinking-calculus program as one designed to support a diverse population of students to thrive, provide an example of a program aligned with this approach, and discuss key features to consider in designing a calculus program for the modern age.

Keywords: college, calculus, change, equity, STEM education

1. Introduction

In their summary of calculus education research, Rasmussen, Marrongelle, and Borba describe calculus serving the role of "everything from a 'weeding out' course to fundamental preparation to take on applied problems in partner disciplines, preparing students to bring an understanding of rates, concavity, functional relationships, among other topics to bring to bear on multi-disciplinary problems" ([1], p. 507). Calculus is often thought of as the college mathematics course, with the main goal of mathematically preparing students for degrees in STEM, but it is also often seen as beneficial to students in non-STEM degree programs for developing critical thinking and problem solving experience. As a researcher who has spent countless hours sitting in calculus classes across the country, at schools we have identified as having successful or interesting programs, I frequently find these classes stale, uninspiring, and certainly not supporting critical thinking or creative problem solving. As a mathematician who was inspired to study math because of calculus, I find this troubling, but as an educator I find it immoral. In this chapter, I will explore how calculus has come to occupy its current status as the gateway mathematics class for STEM students, discuss the current state of calculus drawing on my research teams' studies of college calculus for the past decade, and offer an example of a forward-thinking calculus program.

2. How calculus established itself in STEM education

2.1 Brief history of calculus

The invention of calculus is traditionally given shared credit to Isaac Newton and Gottfried Wilhelm Leibniz, who each independently developed the theories around infinitesimal calculus in the late seventeenth century. Bressoud points out that their contributions to calculus lie in connecting theories related to differentiation and integration, rather than in developing the theories of the individual ideas [2]. Newton, primarily a physicist, was motivated to pursue calculus in order to provide a scientific description of motion and magnitude. When documenting his ideas related to calculus, he did so primarily for himself, using a mixture of notations that made sense to him. Leibniz, described as a polymath or someone with a wide array of knowledge akin to a renaissance man (specifically, his interests included metaphysics, law, economics, politics, logic, and mathematics), was motivated to pursue calculus in order to provide a metaphysical explanation of change. He purposefully developed a clear and consistent notation system to document his work that we still essentially use today. While these two men came to be interested in calculus for very different reasons (one from more applied motivations and one coming from more pure mathematical interests), today they continue to share the honor of being credited with this field.

Although Newton and Leibniz are credited with the invention (or discovery, depending on your scientific philosophy), many mathematicians came before them, to develop the ideas they built on, and after them to refine their ideas [3]. Even as recently as 2014, researchers are challenging the credit given to Newton and Leibniz by finding evidence that other mathematicians developed the formal ideas of calculus long before the 1670's. For example, a group of mathematicians in Southern India from the Kerala School developed and published work on many fundamental ideas of infinitesimal calculus 300 years prior to Leibniz and Newton [4]. With this note aside, college calculus today is a direct descendent of the work of Newton and Leibniz, using Leibniz's notation, and so I consider these as the birthplace of the college calculus we still see today.

2.2 Evolution of calculus education

So how did calculus come to hold the place as the integral (pun intended) component of so many students' college educations? To answer this question, I draw significantly from Alan Tucker's "History of the undergraduate program in mathematics in the United States" [5]. In the 1700s and 1800s, mathematics was studied as one of the main topics (along with Latin, Greek, and Hebrew) in college following the English college model. The goal of mathematics in such an education was as a "classical training of the mind instead of the language of science and engineering it is today" ([5], p. 689). The students attending these colleges were mostly male and mostly from the upper-class. Although Newton's and Leibniz's work developing calculus into a more systematized and valued field occurred in the late 1600s, calculus wasn't taught widely in college until the late 1800s. It was during this time period that colleges shifted from delivering a classical curriculum to a more practical curriculum. This is largely due to the fact that land-grant public universities were established in 1862 by the Morrill Act, and calculus became more standard for technically-oriented students.

By the early 1900s, most colleges allowed students to choose the courses for their study, which led to an increase in college enrollment and a decrease in mathematics study. It was during this time that mathematics was no longer viewed as part of a classic education, and instead a tool useful for engineers and scientists (as it continues to be seen today). During this time, calculus became an elective in US college preparatory high-schools and a mandatory subject for college preparatory high schools in Europe.

Also around this time, the presentation of calculus changed from following the order of topics in which the theory was developed (first integration, then differentiation, then series, and lastly limits), to the order most beneficial for rigorously proving theorems of calculus (first limits, then differentiation, then limits, and lastly series) [3]. Little has changed since this time. The undergraduate mathematics curriculum for mathematics majors has become more and more solidified, aided by guidance from the Mathematical Association of America and their Committee on the Undergraduate Program in Mathematics (CUPM) guide. Tucker notes that in the more recent history, though the curriculum has not changed much, "the greatest area of change and concern in the past 40 years has been the articulation between high school and college mathematics." By the early 1980s, "mathematics faculty were dealing with large numbers of students in freshman courses who showed limited knowledge of needed algebra skills" ([5], p. 702). He attributes these changes to states increasing their high school mathematics requirements and watering down the material to meet these higher expectations, as well as the rise of high stakes testing. Near this time, AP Calculus became an increasingly expected course in high school. During the late 1990s (during which time I was in high school and also the earliest data recorded online of AP participation by subject), approximately 100,000 high school students took the AP Calculus AB exam [6]. From my experience, AP Calculus AB was viewed as a course that only students extremely interested in pursuing mathematics or physics as their college major would take. By 2018 that number had tripled [7]. As part of a large, national study on college calculus conducted in 2010, my research team identified that two-thirds of all students in a college calculus class had already taken a course in high school called calculus (many of these being AP Calculus AB or BC), and half of the students we surveyed believed they needed to take a calculus course in high school to be successful in college [8].

Over the past nearly 50 years, there has been tremendous attention paid to reforming college calculus, which has resulted in more attention to problem solving in applied contexts, an increased focus on supporting students' development of conceptual understanding rather than only procedural fluency, and often more active learning techniques employed in the classroom, including student-centered instruction and more technology use [1, 9, 10]. These changes have certainly resulted in more variation in the college calculus instruction across the country, with some programs very much still rooted in these reforms and others holding on to a pre-reform calculus model. That said, the basic content being taught in all of these programs is still essentially a course on Newton & Leibniz's ideas, taught in the order best suited for proving calculus, regardless of the presentation, the students being taught, the pedagogy, or the contexts for the word problems.

2.3 Current state of college calculus education

For the past decade, I have been a part of a large research team studying college calculus. This research team has been led by David Bressoud, run under the auspices of the Mathematical Association of America, and funded by the National Science Foundation. Our research has come from two projects, the first begun in 2009 and focused on mainstream college differential calculus programs (typically called Calculus I) in all institution types, called *Characteristics of College Calculus (CSPCC);* the second begun in 2014 and focused on precalculus, differential and integral calculus programs at Masters and PhD-granting institutions, called *Progress through Calculus (PtC).* Our work has been generally focused on identifying aspects of college calculus programs that are more successful or innovative than comparative institutions, and supporting more mathematics departments to improve their programs based on these findings. For our purposes, success in college calculus is primarily marked by a large percentage of the students who plan to complete both differential and integral calculus (typically called Calculus I and Calculus II; requirements of most STEM-degrees) reporting that their confidence, interest, and enjoyment of mathematics did not decrease after the first course in calculus, and that these students primarily planned to continue studying calculus (and thus continue studying STEM) after taking the first course[1].

Overall, based on these measures, we did not see great evidence of success in college calculus across the country. Among the students surveyed, we saw significant decreases in confidence, enjoyment, and interest in continuing to study mathematics [8], and we found that nearly 18% switched out of the calculus sequence after taking differential calculus [12]. The main reasons for switching out of the calculus sequence given by students were changing their majors and no longer needing to finish the sequence, not having the time and effort to put into calculus to do well, and having a negative experience in differential calculus. Women students switched out at significantly higher rates than men, and disproportionately credited a lack of confidence in their mathematical abilities as the reason why [12].

From the 213 schools that participated in the CSPCC survey, we identified 18 schools that showed promise, including community colleges, Bachelor's-granting, Master's-granting, and PhD-granting schools. We conducted case studies at these sites, and based off of these case-studies have identified a number of components of calculus programs potentially related to student success. A collection of these findings can be found on our project website, www.maa.org/cspcc. For this paper, I focus on the findings that have had the most direct impact on the follow up study, PtC. From the five doctoral-granting departments we visited, we identified seven features that were common and that we believed were related to their success [13]. These features are: a coordinated calculus program, collection and use of local data to inform changes to the calculus program, rich and engaging curriculum, support of active learning, teaching preparation of the graduate students involved in the program, tutoring centers and other supports available for students, and adaptive placement systems into the calculus program. Since publishing those findings, we have seen a number of calculus programs across the country use these findings to guide improvements to their own programs, showing the impact that such studies can have on shifting the national landscape of calculus education.

I am confident these features provide concrete aspects of calculus programs that departments can focus their improvement efforts on, and that these are likely to lead to some improvements. However, I have recently argued [14] that it is also likely that focusing on these aspects alone can lead to programs making improvements that better serve the populations of students already being supported through calculus programs. In **Table 1**, I provide demographic data of the students earning Bachelor's degrees in any major, and specifically in STEM, from each school near the time of our data collection in 2010 from the five universities visited.

Table 1 highlights the low population of students of color at the institutions visited, and the lack of STEM degrees earned by women of all ethno racial backgrounds and students of color of both sexes. The percentage of students who switched out of the calculus sequence at these institutions varied drastically by institution, from as low as 2% at one technical institutions to 30% at one large, public. However, the trends of women students switching at higher percentages than men and low enrollment by students of color are common across each of these sites. A deficit-oriented interpretation of this data would argue that these differences in interest, success, and persistence by different student populations are due to internal deficiencies of some populations of students, playing into common

[1] The surveys used can be found at [11].

	PTI[2]	LPU1	LPU2	PTU	LPrU
Total	620 (542)	6473 (1822)	5323 (2004)	1073 (816)	6864 (1350)
Woman	26.1 (23.7)	51.2 (29.9)	52.5 (43)	21.7 (15.2)	50.9 (23.7)
White, non-Hispanic/Latinx	79 (80.1)	67.4 (62.7)	31 (26)	87.2 (88.4)	87.3 (87.2)
Hispanic/Latinx	3.4 (3)	4.6 (2.5)	10.9 (8.1)	1.6 (1.5)	3.2 (2.1)
African American and Black	1.8 (1.7)	5.7 (3.8)	1.6 (1.0)	1.5 (1.2)	0.5 (0.4)
Asian and Native Hawaiian/ other pacific Islander	6.9 (6.8)	12.3 (17.0)	43.2 (52.2)	1.1 (1.2)	3.3 (3.8)
Percent American Indian/ Alaska Native	0.5 (0.4)	0.9 (0.5)	0.5 (0.5)	0.7 (0.5)	0.8 (1.0)

[1]*IPEDS data retrieved from: https://nces.ed.gov/ipeds/datacenter/Data.aspx*
[2]*University pseudonyms follow those given in [8]: LPU1 and LPU2, large public universities; LPrU, large private university; PTU, public technical university; PTI, private technical institute.*

Table 1.
Percentage of bachelor's degrees earned in 2009 (percentage of bachelor's degrees earned in STEM fields in 2009 in parenthesis).[1]

stereotypes of women and students of color not being as good at or as interested in STEM as white, Asian, and male students [15].

An anti-deficit interpretation rejects this assumption and rather assumes (1) that women and students of color can thrive in STEM, (2) that the disproportionate enrollment of white and Asian students, and disproportionate persistence of men indicate a failure of the system and not of the students, and (3) we can learn how to better support women and students of color to thrive in STEM by studying the women and students of color who are already thriving in STEM [16].

The enrollment and persistence data indicates that the five schools we visited, and that we based our "features of successful calculus programs" (which have come to shape improvements to calculus programs across the country) were based on programs serving a predominantly white and Asian, male student population. That is, the demographics of the students in these calculus programs were predominantly white and Asian men and women students, and the students persisting through the sequence were disproportionately men of all races and ethnicities. Knowing this information and coming from an anti-deficit perspective, I argue that the seven features can only offer possible improvements for calculus programs when considered in conjunction with diversity, equity, and inclusion practices. Diversity practices refer to actions done within the calculus program and mathematics department that attract and retain a diverse population of students. Equity practices refer to actions that (1) acknowledge the multiple ways in which some people face barriers (both visible and invisible) to their success, and (2) work to dismantle these barriers [17, 18]. Inclusion practices refer to actions that support the full participation of a diverse student population within the classroom community and within the broader departmental and institutional communities. By focusing on the original seven characteristics alone, departments may foster inequities by further supporting the populations of students who are already successful in calculus. Instead, departments should explicitly implement diversity, equity, and inclusion practices while also improving their programs through focus on the seven characteristics.

As a follow-up project to CSPCC, the PtC project has identified 12 research-oriented mathematics departments implementing a combination of the seven features in the Precalculus and calculus programs. For the PtC project, we used IPEDS data to very purposefully consider the demographics of the students enrolled at the schools, and the demographics of the students graduating with STEM degrees, when selecting the 12 institutions involved in our study. This resulted in a number of institutions serving a more racially and ethnically diverse student population, and with a few of those institutions implementing programs specifically designed to support women and/or students of color and/or first-generation students to be successful in STEM. This work is ongoing, and we are in the process of learning more about these programs so that we can share more about them with others schools. One disappointing finding in our recent work has been the general lack of programs geared to increasing the diversity in STEM among research-oriented math departments across the country; while there were programs at the university and college level developed to foster diversity, equity, and inclusion in STEM, there were very few programs at the department level [19].

3. An example of a college calculus program affords an anti-deficit perspective

Through the PtC work, I hoped to find a mathematics department where the calculus program was thoughtfully crafted to best support today's college students – a more diverse population of students, that includes more students of color, and more first-generation and low-income students than before [20]. We did not find a program that had an explicit focus on supporting a diverse population of students to thrive in mathematics, but we did see a calculus program developed to support every student in their construction of mathematical meanings in calculus. This program was developed based on research rooted in radical constructivism, and not with an explicit attention to equity. However, I believe this program affords an anti-deficit approach to mathematics by viewing every student's mathematical understanding as valuable and part of the construction of richer mathematical meanings. This calculus program illustrates that by sincerely valuing every student's mathematical understandings, and leveraging research to support each student's rich construction of mathematical meaning, a diverse population of college calculus students can mathematically thrive.

3.1 Background on DIRACC

Project DIRACC (*Developing and Investigating a Rigorous Approach to Conceptual Calculus)* is an NSF-funded college calculus curriculum developed by Pat Thompson and his colleagues based on years of research on student understandings of calculus (see [21] for a description). This curriculum is self-described as "Newton meets Technology", focusing on developing meaning for infinitesimals (while utilizing animations and interactive apps) rather than emphasizing the notation and formality of Leibniz. This curriculum is shared online for free, and is currently being implemented in at least two large, public, doctoral granting mathematics departments, including one involved in Progress through Calculus.

In this chapter, I will draw on my experience at the one university involved in PtC (referred to as Large State University; LSU), where DIRACC is the curriculum used for all calculus courses for science, computer science, and mathematics majors. The undergraduate population of LSU is approximately 50% white students, 20% Hispanic and Latinx students, 7% Asian, and 5% Black and African American. In the DIRACC calculus courses I observed, I estimated that approximately 30% of students were Black, Latinx, and/or Native (based on appearance). At LSU, there is a separate (and more procedurally oriented) college calculus course for engineering majors. The DIRACC courses are taught by instructors, mathematics education faculty, and doctoral students pursuing degrees in pure and applied mathematics and mathematics education. This course is coordinated by a full-time instructor, and this coordination includes weekly meetings for all instructors, where the topics of discussion during the meetings include understanding the mathematics and student thinking related to the mathematics for the upcoming section. Preliminary results from PtC indicate that students in DIRACC outperform students at comparable universities on a calculus content assessment and maintain positive beliefs towards mathematics more than students at other institutions.

3.2 Shift in curriculum

To best serve the students in our calculus classes, we need to learn what is motivating them to pursue degrees requiring calculus – whether future career goals or general interest in learning – and rethink our calculus curriculum to be in line with these interests. It is well established that in today's economy, STEM jobs pay significantly more, on average, than non-STEM jobs [22]. Given this widespread knowledge, we cannot ignore that one contributing motivation for students to pursue STEM is future job and wage prospects. When sitting in Calculus I classes across the country, it often seems that everyone knows the students are there not to learn deep and interesting mathematics, but to get a grade in the course that allows them to continue pursing whatever STEM degree they are hoping for in order to get a good job. I believe that we are missing a big opportunity in our calculus classes to inspire these STEM-intending students about the magic and beauty of calculus. The great majority of calculus courses I have visited have been "mainstream" courses, meaning to serve all STEM students, although in actuality the great majority of the content is driven by the needs of the engineering students, with occasional word problems being set in other contexts.

In a forward-thinking calculus system, there would be a meaningful connection between the content taught in calculus, the needs of the majors whose students are taking calculus, and the interests and motivations of the students enrolled in our courses. It would be these latter two driving the content, rather than historical precedents. The DIRACC curriculum achieves this by forgoing Leibniz's precise notation in favor of Newton's more intuitive ideas – skipping the formalities of ideas such as limit to spend more time supporting students to understand the ideas of infinitesimals and how this can support meaningful understanding of rate of change functions and accumulation functions. This curriculum was designed explicitly to support students in developing rich mathematical meanings, and is thus inherently responsive to how students think about calculus and what todays' students should be learning in a calculus course. As currently taught, I witnessed this curriculum equitably engaging a racially diverse student population in rich mathematics. This curriculum could go further in the future by engaging the diverse learners as whole people, by situating the mathematical content in contexts that are especially interesting and relevant for them (where these contexts could be identified by talking to students and using local data to identify trends in women and students of color's majors).

3.3 Shift in pedagogy

Through PtC, I observed three DIRACC calculus courses at LSU, and though the three courses looked different, in each I witnessed a racially diverse group of students equitably engaging in rich mathematics, contributing to constructing mathematical meaning as a class. In one class, the instructor stood in front of a 40-person class, while he randomly selected students to answer questions related to a context problem they worked on. The questions he asked were substantive and open ended, allowing every student to contribute thinking related to the question rather than simply answering correctly or incorrectly. The second class was a 120-person class where the instructor presented a slide presentation wearing a microphone, with three Learning Assistants circulating the room, and students discussing problems in small groups. The third class was a 30-person class where students spent the entire class working in groups of three-four on rich tasks while the instructor floated around the room, visiting with individual groups, and then bringing the class together for a whole-class discussion. The common element of these courses, in addition to the content being taught, was that the instructors authentically cared to understand what their students were thinking related to the mathematics, and that the instructors used this understanding of their students' thinking to connect the mathematics to the students' understanding of the mathematics – what Hackenberg has called exhibiting mathematical caring relations [23]. The DIRACC curriculum and its enactment at LSU illustrate a forward-thinking calculus program by centering the mathematics, and every individual student's construction of the mathematics, as the guiding forces.

4. An example of the process to develop an anti-deficit college mathematics program

As noted, the DIRACC curriculum was not developed with an explicitly focus on equity, though it affords an equitable enactment. Here, I provide an example of a college Precalculus program developed with an explicitly focus on changing the program to better support a diverse population of students. This example comes from a mathematics class designed at Bates College to prepare students for calculus, called *Mathematics Across the Sciences.* Meredith Greer described the development of this course in depth in a recent *PRIMUS* article called "Interdisciplinarity And Inclusivity: Natural Partners in Supporting Students" [24]. I will summarize some key aspects of this course and its development, but encourage interested readers to read the article for more details.

A group of mathematics faculty at Bates College developed this new course mainly informed by (1) input from faculty from every science department on their campus, (2) a multidisciplinary group of faculty focused on diversity and inclusion, and (3) mathematics education research and national conversations. Input from science faculty was gathered primarily based off meetings centered on which concepts they teach draw significantly on mathematics and what mathematical topics they want their students to know better. After meetings with all science departments on campus, trends surfaced which were used to guide the content of the course. One or two faculty members from each department then came together to refine the topics and include examples from their own fields. After the content was decided on, presentations were made to science and mathematics department chairs and faculty. While the interdisciplinary group worked together on the content, another interdisciplinary group of faculty was working together on learning how to support diversity and inclusion on their campus. This group was supported by the college and motivated, in part, by the Association of American Colleges and Universities (AAC&U) Making Excellence Inclusive project (which offers many very useful resources for departments interested in diversity and inclusion). This group primarily leveraged research on student experiences in higher education, especially the experiences of students of color, as well as the resources from the AAC&U Making Excellence Inclusive project. Based off these readings, the work developed pedagogical strategies that could be used across campus. These were then translated to the mathematics course being developed, resulting in a number of new pedagogical strategies. Lastly, the group developing this course also read and brought in recommendations from national mathematics education conversations, including the Mathematics Association of America's Curriculum Foundations Project [25] and the Inquiry Based Learning community [26].

Informal conversations with students were also used to understand more about their program, especially among the faculty group focused on diversity and inclusion, but not as directly as with the science departments. Input from students, primarily from student evaluations but also from informal conversations, was used to make improvements to each future iteration of the course (which in 2018 had been offered three times).

Greer describes how these components came together to inform the development of the course curriculum and the pedagogical approach: "Class time, course topics, and out-of-class assignments are designed to encourage a diverse set of students to succeed in this course as well as when they later proceed to more advanced mathematics and science courses" ([24], p. 2). This quotes perfectly reflects what should be the guiding principle of all college calculus programs, and can be cultivated through shifts in both the curriculum and the pedagogical approach: A forward-thinking calculus program is developed so that calculus courses, including the class-time, course topics, and out-of-class assignments, are designed to support a diverse set of students to succeed in the course as well as in courses building on calculus and in their STEM careers.

5. How such a program relates to the seven features of successful college calculus programs

Based on our site visits to five doctoral-granting mathematics departments with college calculus programs which we identified as more successful than other programs, we identified seven features of college calculus programs that we hypothesize are related to these programs' successes [13]. In [14], I discuss how each of these features can be thought of while implementing diversity, equity, and inclusion practices. Here, I consider how the above articulation of a forward-thinking calculus program would relate to the seven characteristics.

By my definition above, a forward-thinking calculus program is designed so that all components of the course support a diverse population of students to thrive. A diverse student population will include a diversity of mathematical backgrounds and experiences, cultural diversity, language diversity, as well as diversity of genders, ages, races and ethnicities, sexual orientations, and physical and mental abilities. While each of these types of diversity can influence the design of a forward thinking calculus program, here I foreground the role of diversity in mathematical backgrounds and cultural diversity.

A rich and engaging calculus curriculum designed to support a diverse population of students to thrive would acknowledge the needs of the students taking the course, including what additional mathematical preparation they need to thrive in the course and what components of calculus are needed in their future courses and careers. At my own institution, the calculus coordinator is often surprised and disappointed by calculus students' algebraic knowledge – one example is how persistent many students' belief that

$$a^2 + b^2 = (a + b)^2. \tag{1}$$

One way to respond to this realization is to blame the students for not being prepared enough, and to continue assessing their calculus learning by inherently relying on their lacking algebraic understanding, resulting in believing that the students also lack calculus understanding. A different way to respond to this realization is to blame the system responsible for educating these students, and either infuse algebraic lessons in to the calculus lessons or to not rely on students' algebraic skills for them to demonstrate their calculus understanding (for example, by not assigning algebraically messy functions and by delegating algebraic manipulations to technology). A forward-thinking calculus program would additionally learn what majors the students are pursuing and what calculus content students need to thrive in those majors – while STEM is constantly developing and growing, as should the mathematics we teach students to support them in STEM.

A mathematics department engaging in bringing their calculus program into the modern day should use *local data* to inform these changes. The types of local data collected can include quantitative outcome measures (such as grades and persistence) and qualitative measures of experience (such as focus groups with students who have persisted and those who have not). The value in the quantitative data is that it can identify trends and patterns and can be used to examine the prevalence of an observation. One downside is that the experiences of the majority can overshadow the experiences of the minority, and when designing a calculus program to support a diverse population of students to thrive, it is the voices of the minority that become especially important. Qualitative data can complement the quantitative data by illuminating experiences of a smaller number of students. One way to gather such data is to hold a number of focus group interviews with students (as done in the bates College example previously discussed), especially students from demographic groups and with experiences not held by the majority of the population. This could be holding a focus group of students of color in calculus to identify how they are experiencing the calculus program, and specifically how they are experiencing the calculus program as students of color. A similar focus could be taken by speaking to transfer students, first generation students, "non-traditional" students (typically older than traditional students), and students who have not taken calculus before. The quantitative and qualitative data gathered can be used together to inform curricular decisions (what do our students need from this course?), pedagogical decisions (what have students been experiencing in our courses, and what needs to change?), and programmatic decisions (is this calculus program achieving the goals that we want it to?).

Coordination of a calculus program designed to support a diverse population of students to thrive raises questions about what is fair. A primary goal of coordination is to ensure that all students (including those being taught by different instructors) experience a similar course and that their grades reflect this objectively. This need for similarity and objectivity speaks to a desire for the course to be fair for all students, though this inherently assumes that all students are coming in with the same preparation and resources. By acknowledging that this is not the case, the role of coordination becomes not to ensure fairness but to ensure justice for all students. A fair coordination system will seek to ensure that students are graded as objectively as possible and that this grade is only based on their knowledge. A just coordination system will seek to ensure that all students are given an opportunity to communicate what they have learned – which may entail acting in ways that do not seem fair to other students.

The acknowledgment that not all students are entering college calculus with the same mathematical experiences, preparation, and resources has a significant affect on the role of *placement* into mathematics. During our site visits to the more successful college calculus programs, we observed placement systems designed to place students into the highest course in which students could be successful. A key component of a placement system that is able to place students in this way is to have multiple options for courses that acknowledge the differences in student experiences. In our more recent work, we have seen examples of a broadened variety of college calculus courses that acknowledge that students come into college calculus with different prior experiences. The majority of these courses focused on supporting students on the lower cusp of placing into calculus (as determined by a placement exam, standardized test scores, or high school grades), such as calculus infused with precalculus and co-calculus (see [27] for details about these course structures), though we did observe courses designed to support students at the higher end of the placement, such as in the accelerated calculus course developed to support students who had already been exposed to calculus in high school. Such course variations enable a placement system to give students the course options in which they can be successful.

Through CSPCC and PtC, we observed growing support of *active learning* in the calculus sequence. While the specific implementations of active learning vary (including partner talk, group work, whole class discussions, and student presentations at the board), the common underlying element is that these classes engage students in mathematical activity during class. To engage students in rich mathematics during class time in a way that supports all students to thrive involves deep attention to and care of the mathematics of the students rather than only of the textbook or the instructor [23]. Another way to say this is that instead of describing the classes as "student-centered" I would describe them as "student-thinking centered." Such classes assume that students make sense of mathematics differently from one another and differently from the textbook or the instructor, and that such differences do not make their meanings incorrect; rather, drawing out multiple mathematical meanings for one problem leads to richer discussion and a richer understanding of the content. Forward-thinking calculus programs value and leverage the diversity of ideas present in a mathematics class composed of a diverse student population by engaging students in rich mathematics, eliciting their meanings of the mathematics, and engaging with the students' meanings of the mathematics.

What I describe here as a forward-thinking calculus program is far different from my own experiences as a college calculus student, and likely far different from the experiences of the majority of novice college calculus instructors (including graduate students, post-doctoral fellows, and new faculty). With this in mind, it becomes even more critical to provide teaching preparation to novice instructors involved in the teaching of calculus. One critical need for such preparation is purely pedagogical – while secondary teachers go through years of pedagogical preparation and apprenticeship, new college instructors are often expected to learn on the job. An additional need, that becomes pronounced when teaching to a more diverse population of students, is to help novice instructors understand that their students are not all like them (and are not all on their way to an advanced degree in mathematics) and to value what these students bring to their class. One professional development experience that can support this is to look at student work in a non-evaluative way; by looking at student work to understand what the students do understand and how they are making sense to come to their solution, rather than evaluating how many points a solution earns, instructors can learn to appreciate the richness of their students' mathematical thinking.

The final component of a forward-thinking calculus program to consider is the *supports that exist outside the classroom* that are designed to support a diverse population of students to thrive. Through the CSPCC and PtC work, we have observed tutoring centers specific to calculus content and shared workspaces in the mathematics department for students to informally gather to work on calculus together. Through the sites we have visited, we have seen much value in these supports, with many students sharing how impactful they were to their learning. We have also seen a number of rich supports for students that reside outside the mathematics department; for example, a mentoring program for students of color in STEM and a tutoring center and gathering space for Native students in STEM. Such programs could be made richer with more of a partnership with the calculus programs. While the mathematics departments' main focus is on supporting students mathematically, there is an opportunity for calculus programs to acknowledge calculus students as multifaceted people, and identify existing supports on campus that the calculus program could integrate into.

6. Conclusion

My intention in writing this chapter has been to blend my observations as a researcher with my desires as a mathematics professor and as a human. Calculus was the course that both enticed me to love mathematics and almost convinced me that mathematics was not for me, or, more honestly, that I was not for mathematics. Articulating and envisioning a calculus program that is explicitly developed to support a diverse population of students to not simply exist or persist in calculus, but to thrive has rejuvenated me to be optimistic about the role that calculus can play in students' STEM education. There are many big questions that remain both unanswered and unasked, and I am excited for a diverse population of students to become inspired to ask and answer these questions by experiencing a forward-thinking calculus program.

Acknowledgements

This material is based upon work supported by the National Science Foundation under grant no. 0910.

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