Chapter

Scientific Knowledge of Spanish Military Engineers in the Seventeenth Century

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Abstract

The catenary arches were used in Spanish Art Nouveau architecture by Antoni Gaudí (1852–1926). The theory of the chain, in the shape of a hanging collar, was proposed by Robert Hooke (1676) and used by Christopher Wren in Saint Paul's dome (1675). British school modern mechanic theory was introduced in Spain by Spanish Bourbonic military engineers and also by the Catholic Scottish and Irish families during the eighteenth century. The assessment of some drawings of gunpowder warehouses, found in the collection of *Mapas planos y Dibujos* (MPD) of the General Archive of Simancas (*Archivo General de Simancas*, AGS) (AGS 2014), has revealed the use of the chain theory in Miguel Marín's projects for Barcelona (1731) and Tortosa (1733) and Juan de la Feriére ones in A Coruña (1736). A built evidence has also been found: the Carlón wine cellars in Benicarló, built by the O'Connors family from Ireland (1757). The analysis of these examples proved the theory of the chain arrival to Spain during the first half of the eighteenth century.

Keywords: catenary, gunpouder warehouse, military engineering, geometry, Robert Hooke

1. Introduction: the catenary arch in Spain in the nineteenth and twentieth century

Catenary arches are one of the main features of Art Nouveau architecture in Spain. Their shape is based on the modern theory of masonry arches. This theory was developed during the nineteenth century and claimed the work of Antoni Gaudí (1852–1926) as its main exponent [1]. Architect Cèsar Martinell i Brunet (1888–1973) built the so-called wine cathedrals (1918–1924) which were built by the Commonwealth of Catalonia (1907–1925).

These buildings in the nineteenth-century style named as Noucentisme may be regarded as the last ig cluster of constructions featuring Catalan masonry [2]. The catenary arches designed and built by Cèsar Martinell belong to the Catalan modernist architecture Antoni Gaudí i Cornet (1852–1926). Thus, in a hanging chain, any inward pulling force is matched by an equal outward pushing force. Martinell knew Gaudi's work and inherited his techniques. This is evidenced by his many writings on Gaudí [3], whom he met during a visit to the Sagrada Familia church in 1915, when Martinell was about to complete his studies at the Barcelona School of

Architecture. From then on, Martinell joined an exclusive group of disciples who learned a way of doing architecture aside from university teachings [4].

For Martinell, Gaudí was a much more interesting lesson of life and architecture than most of the teachings given at university. Gaudí's words became architectural when the very statement of the scientific truth and the procedures he himself invented were able to explain problems and geometric concepts which remained unclear in the classrooms of the school of architecture. Antoni Gaudí's theory of structures relies on the strength of geometry, in particular on the strength of the parabolic and catenary shapes. The construction technique used by Cèsar Martinell stems from the methods applied by Gaudí to calculate the geometric shapes of vaults and arches [5] (**Figure 1**). Other architects have written about this view; see, for instance, the lecture entitled *La fábrica de ladrillo en la construcción catalana* (1900), by Josep Domènech i Estapà (1858–1917). In this lecture it is claimed that the parabolic and catenary shapes are the lines of equilibrium in a system of evenly distributed loads, where the parabolic shape relates to the horizontal projection and the catenary shape relates to the arch length [6].

Otherwise, these concepts were introduced in the formation of architects through the Escuela Especial de Arquitectura de Madrid (1844). The work *Traite Theorique et Pratique de L'art de bâtir* (1802–1817) by Jean-Baptiste Rondelet (1742–1829) exposes the methodology to lay out catenary arches by means of the theory of the chain and another complicated procedure [7]. In addition, a treatise by John Millington (1779–1868) was also used in architecture schools. It was translated as *Elementos de Arquitectura* (1848) and contained Hooke's theory and the layout of the catenary [8]. Juan Torras i Guardiola (1827–1910) developed the scientific basis for the calculation of these structures in the Barcelona School of Architecture (1875) [9].

1.1 The curve of equilibrium

The theory of the chain, in the shape of a hanging collar, was proposed by Robert Hooke (1635–1703) at the end of his treatise *A description of Helioscopes, and Some Other Instruments* (1676). Hooke presented a solution that would be revealed as "Ut pendet continuun flexile, sic stabit contiguum rigidum inversum" [10]. The

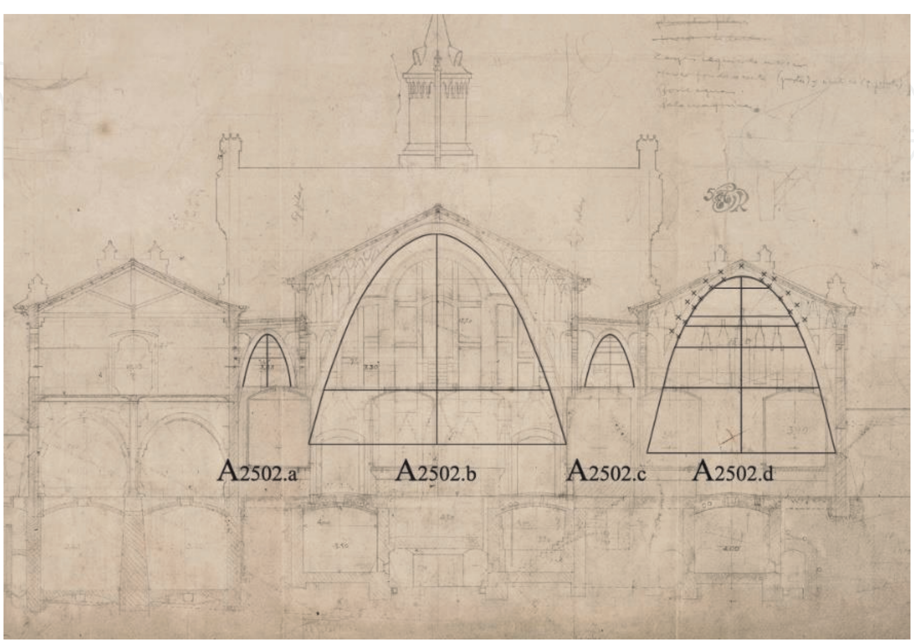

Figure 1.
Cesar Martinell sketch of transversal section El Pinell de Brai [COAC H101I-6-Reg 2502].

awareness about the shape of the catenary was applied by Christopher Wren (1632–1723) in the dome of San Pablo (1675), with the collaboration of Robert Hooke in the design [11].

Simon Stevin (1548–1620), in *De Beghinselen der Weeghconst* (1586), had previously proven the law of equilibrium of a body on an inclined plane. There we can see a hanging cable which has the shape of a catenary [12]. Despite the evidence provided by the figure, there was not any mathematical approach to catenaries. That is why Jakob Bernoulli (1654–1705), in *Acta Eruditorum* (1690), issued a challenge to the mathematical community to solve this problem [13]. The solution was published in *Acta Eruditorum* (1691) by Johann Bernoulli (1667–1748) with the title "Solutio problematis funicaularii" [14] and also by Chistiaan Huygens (1629–1695) with the title "Dynastae Zulichemii, solutio problematis funicularii" [15].

The mathematic equation of the catenary would be formulated some years later by David Gregory (1659–1708) and published in the *Philosophical Transactions of the Royal Society* (1697). Gregory affirms that the catenary is the real shape of an arch, because if these can sustain themselves, it is because a catenary can be drawn in its section [16]. James Stirling (1692–1770), in the *Lineae Tertii Ordinis Neutonianae* (1717), compiled the ideas of the English school building a catenary with hanging spheres, to simulate the behavior of a constructive element [17]. This solution inspired the analysis by Giovanni Poleni (1683–1761) in the *Memorie Istoriche della Gran Cupola del Tempio Vaticano* (1748) [18], who developed a methodology similar to Stirling's, to understand the breaking of the vault of the San Pietro Basilica [19].

In Spain, the development and application of this theory take place in the context of the Mathematics Academy of Barcelona (1720). The work of Bernard Forest de Bélidor (1698–1761) is the main reference of the curve of equilibrium theory. In *La science des ingénieurs dans la conduite des travaux de fortification et architecture civile* (1729), Book II, Chap. III, Prop. V, Bélidor sets out the curve that must be given to a vault, so all its parts weigh the same and stand in equilibrium [20], and as a result its curve will have the shape of a catenary. And so he determines, for military constructions, up to five types of different vaults: rounded, tiers-point pointed, elliptical drawn as a segmental arch, the flat ones, and the derived forms of the catenary [21]. In addition, the work *De la poussée des voûtes*, (1729), by Pierre Couplet (+1743), mentions the *chaînette*, the hanging chain, as the best of all shapes for the construction of vaults. He also says that to build this vault, every part of the hanging rope has to be loaded with the proportional weight of the construction, so the resultant curve will be the one to be used [22].

2. The libraries of the military engineers of the eighteenth century

On 13 January 1710, King Philip V appointed Jorge Prosper Verboom (1665–1744) as engineer in chief. Verboom was a disciple of Sebastián Fernández de Medrano. Together with Alejandro de Retz (c. 1660–c. 1732), chief engineer of the Catalan region, Verboom was the link with the former academy in Brussels. Mateo Calabro was appointed head of the Mathematics Academy of Barcelona (1720–1738), and he had profound disagreements with Jorge Prosper Verboom with regard to the training program. Therefore, in 1738 Verboom offered the head position of the academy to Pedro de Lucuze y Ponce (1692–1779), who held that position until his death. Among other duties, the academy had to build a collection of scientific works which would be used as reference texts for military training. This bibliographic interest led Vicente García de la Huerta (1734–1787) to publish *Bibliotheca Militar Española* (1760), a collection of the most important military engineering treatises which had been written between the sixteenth and the eighteenth centuries [23].

Some of the most relevant texts available for military engineers in the libraries are *L'architecture des voûtes* (1643) by François Derand (1588–1644) (**Figure 2**), in the library of Jorge Prosper Verboom (1665–1744) [24], and the *Treatise on*

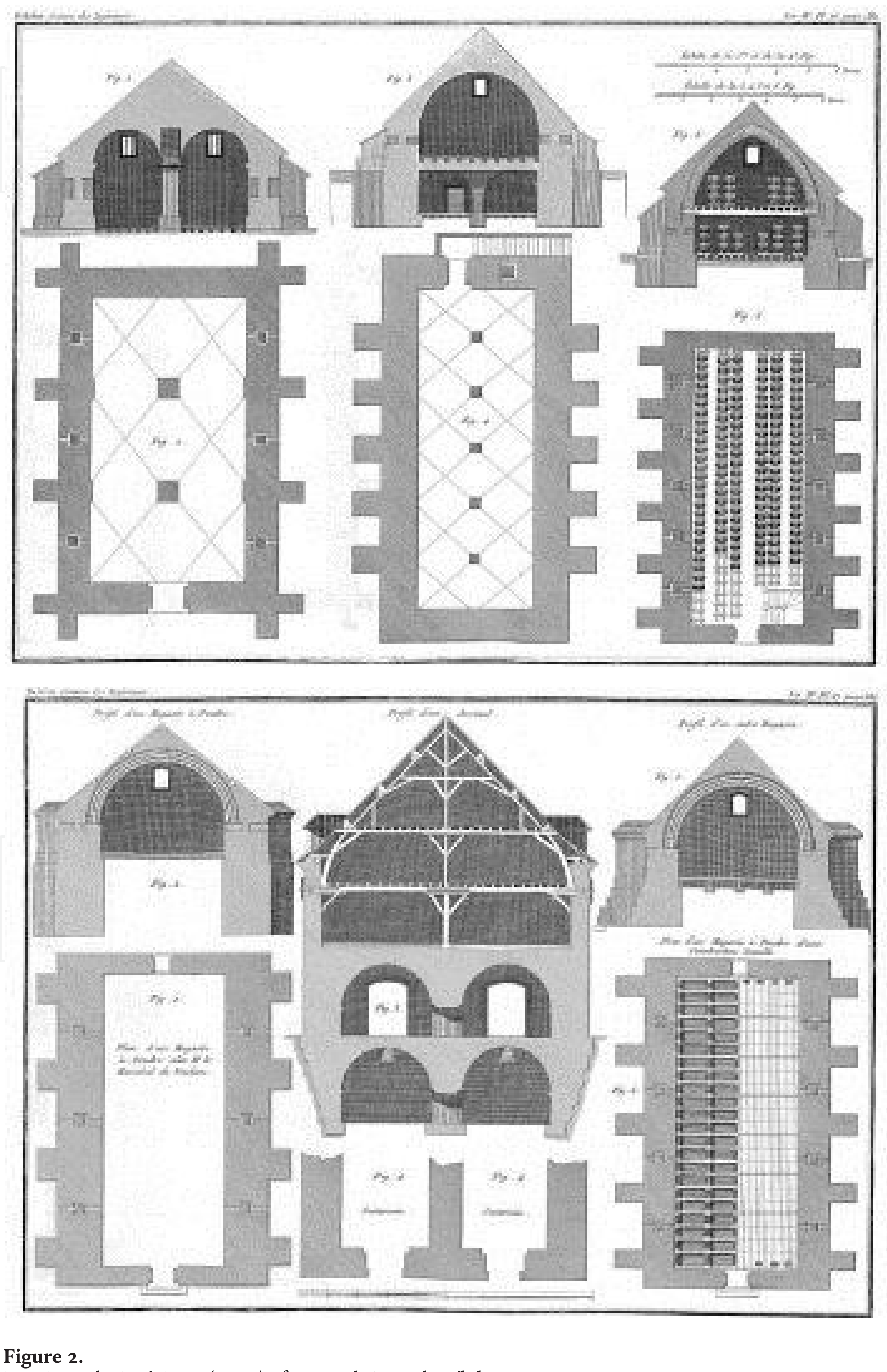

Figure 2.
La science des ingénieurs (1729) of Bernard Forest de Bélidor.

Stereotomy by Abraham Bosse (1604–1676), in addition to *La pratique du trait à preuve de M. des Argues Lyonnois pour la coupe des pierres en Architecture* (1643), in the library of the Barcelona Academy [25].

Another reference work for engineers is *Compendio Mathematico* (1707–1715) by Tomás Vicente Tosca (1651–1723). Treatise XVI in volume V, which is divided into six parts, deals with military architecture (1712). It is available in the academy's library, in Verboom's library (+1744), in Cermeño's library (+ 1790), and in Hermosilla's library (+1776). [26] This treatise deals with arch and vault dimensioning, as well as their collapse mechanisms. It claims that the perfect shape of an arch is a mixed one, made up by the intrados of a round arch and the extrados of a pointed arch. Furthermore, Vicente Tosca improves the three-centered arch or basket-handle arch (*arco apaynelado* or *arco carpanel*), since in Book II, Prop. III, he establishes for the first time the geometrical construction of ovals, which are defined by the length of their two main axes [27].

Yet another work of reference in the libraries of the military engineers was *Traité des Ponts* (1716) by Hubert Gautier (1660–1737) in the libraries of Aylmer (+1788), Aedo Espinosa (+1787), Roncali (+1794), and Cermeño (+1790) [28].The second edition (issued in 1723) includes the famous statement "Ut pondera libra, sic aedificia architectura," referring to the difference in thrust between a round arch (which tilts the balance) and a pointed arch [29]. This edition also includes an additional dissertation: *Augmenté d'une Dissertation sur les Culées, Piles, Voussoirs, et Poussées des Ponts*. The 1728 edition includes a revised and enlarged version of *Dissertation sur l'Epaisseur des Culées des Ponts, sur la Largeur des Piles, sur la Portée des Voussoirs, sur l'Erfort & la Pesanteur des Arches à differens surbaissemens*. [30].

The *Oeuvres de Monsieur Maroitte* (1717), by Edme Mariotte (1620–1684), was also used for teaching purposes. The most interesting part is Volume 2, which includes *Traité du mouvement des eaux et des autres corps fluides, divisé en V parties*; this work is in Verboom's library (+1744) and in Cermeño's library (+1790) [31], previously published by de La Hire (1686).

However, the main references for the Spanish military engineers were definetly the works of Bernard Forest of Bélidor (1698–1761): the *Nouveau cours de Mathématique* (1725), *La science des ingénieurs* (1729), and the first (1737) and second (1739) volumes of his *Architecture hydraulique* (1739), in the Library of Verboom (+1744), Espinosa (+1787), Burgo (+1788), Cermeño (+1790), and Juan Miguel de Roncali (+1794). In *Nouveau cours de Mathématique* (1725), Bernard Forest de Bélidor discusses a practical application of masonry mechanics to the construction of gunpowder magazines [32]. Bélidor calculates the abutment for a barrel vault and for a third-point arch. He includes a table summarizing the size of the pieds droits depending on their curvature and their location, specifying as well if they are supporting the basement floor or the roof. In *La science des ingénieurs dans la conduite des travaux de fortification et architecture civile* (1729) (**Figure 2**), Book II, Chap. III. Prop V, Bélidor establishes the curvature that a vault should have so that all its parts have the same weight and are well balanced (the result is a curve with the shape of a catenary). Bélidor differentiates five vault topologies in military constructions: barrel vaults, third-point vaults, surbased vaults with an elliptical profile, flat vaults, and vaults with a shape that results from the chain [33].

2.1 Gunpowder warehouses in the military architecture treatises

Military architecture treatises of José Cassani (1673–1750), developed at the end of the seventeenth and eighteenth centuries, make reference to the construction of these warehouses, especially if they have an element of high resistance, such as having been made bombproof or having been constructed underground (whether in manmade excavations or in caves) [34].

The principal work of reference is *Maniere de fortifier selon la methode de Monsieur de Vauban*, of Sébastien Le Prestre Vauban (1633–1707), edited by the abbot Du Fay in 1681. The morphology of the gunpowder warehouses with a double enclosure is defined in that treatise, together with the design of its roofing [35].

In the Spanish treatise *El Ingeniero Primera Parte, de la Moderna Architectura Militar* (1687), by Sebastián Fernández de Medrano (1646–1705), that question is

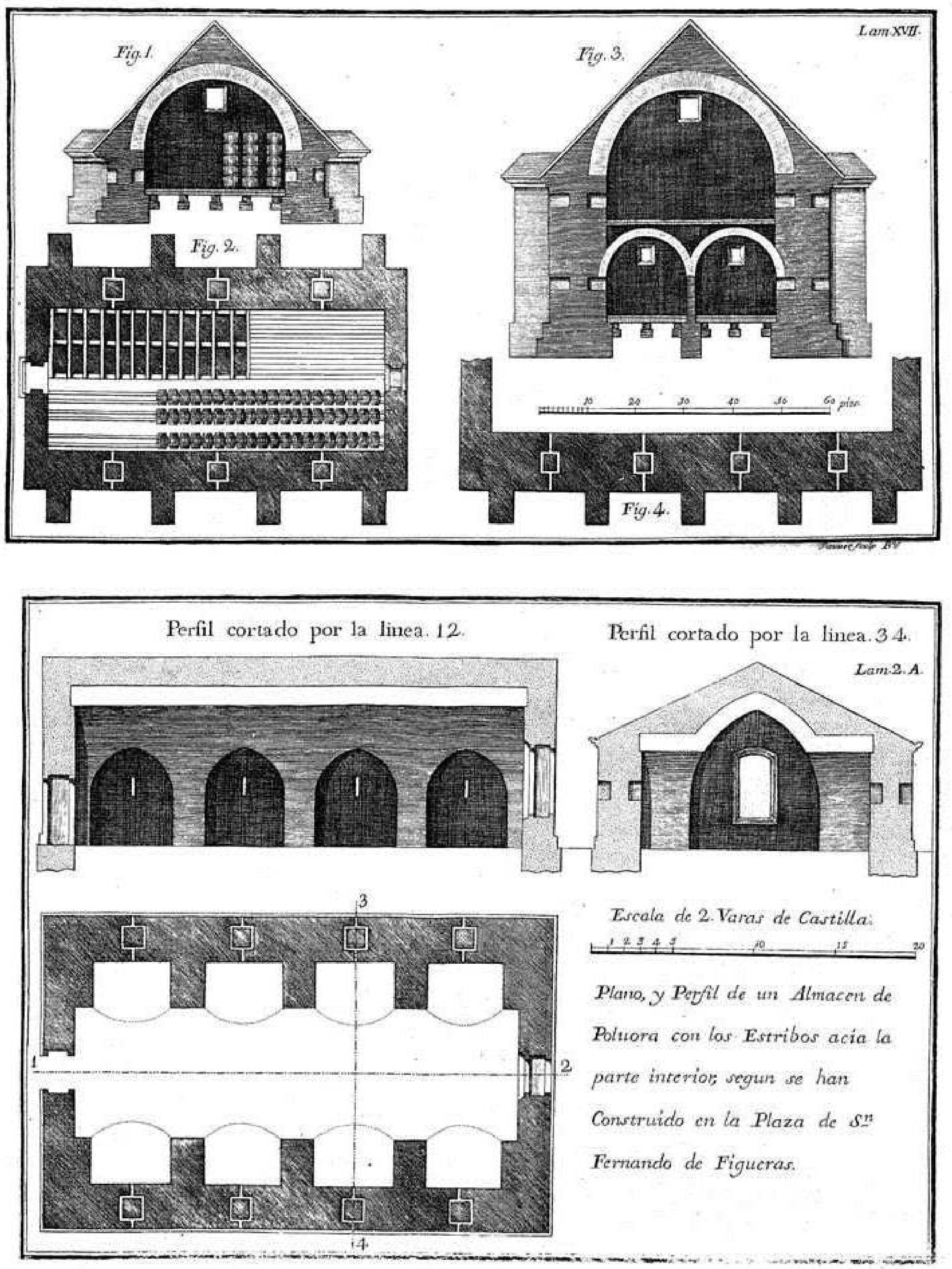

Figure 3.
Tratado de fortificación, ó Arte de construir los edificios militares, y civiles (1769), John Müller.

solved with a masonry vault of 4 feet (0.324 m) width of Spanish "vara" [36]. Subsequently, in the *El Architecto Perfecto en el Arte Militar* (1700), the thickness is increased from 12 to even 14 feet [37]. In the *Escuela de Palas ò sea Curso Mathematico* (1693), attributed to José Chafrion (1653–1698) in the *Libro de Arte Militar*, XI Treatise [38], constructions similar to the Vauban's constructions exist. This vault is solid and is built with lime masonry, which differs from Sebastián Fernández's vault.

In *Nouveau cours de Mathématique* (1725), Bernard Forest de Bélidor considered a practical application of masonry mechanics for the construction of gunpowder warehouses. He determined the abutment for a canon vault and for a tiers-point pointed arch. In a table, he synthesized the dimension of the pieds droits, in ratio with its curve and localization [32]. The importance of these strategic elements obligates Bélidor to dedicate the entirety of Chapter IX of the Book IV of *La science des ingénieurs dans la conduite des travaux de fortification et architecture civile* (1729) (**Figure 2**) to the construction of gunpowder warehouses. In Book II, Chap. III, Proposition V, Bélidor presented that the curve must be given to a vault in order not only to keep up its weight throughout but also to keep itself in equilibrium [33]. As a result, its curve would have the shape of a catenary. Thus, for military constructions, Bélidor determined up to five different types of vaults: semicircular, third-tip pointed, elliptical (drawn as a segmental arch), plane, and (the derived forms of the) catenary.

The work of Bélidor is translated into English by John Müller (1699–1784) and published under the title *A Treatise Containing the Elementary Part of Fortification, Regular and Irregular. For the use of the Royal Academy of Artillery at Woolwic*h (1755) in Part. III, Sect. XIX, of this translation is entitled *Of Powder-Magazines* [39]. This text is also translated into Spanish by the Mathematics Academy Professor Miguel Sánchez Taramas in Barcelona (1769) under the title *Tratado de fortificación, ó Arte de construir los edificios militares, y civiles* (1729) (**Figure 3**) for the use of the pupils of the mathematics school. For the construction of gunpowder warehouses, their references are Vauban (1681) and Bélidor texts [40].

Élémens de Fortification (1739) by Guillaume Le Blond (1704–1781) is another influential text. With regard to the construction of gunpowder magazines, Le Blond raises Problem II: *Tracer de rempart et le parapet* [41]. Lastly, we would like to highlight *Principios de Fortificación* (1772), a treatise by Pedro de Lucuze (1692–1779) for the academy. With regard to the construction model, in Chapter XIX (entitled *Edificios Principales*), he defines the main characteristics that a building must have *robustness*, *convenience*, and *symmetry*. He draws a distinction between two types of structures according to their robustness: a simple model and a bombproof model. Bombproof structures need a sufficiently thick vault built with stone or brick or a cushioning system consisting of a wooden framework covered with earth [42].

3. Morphologies of gunpowder magazine projects (1715–1798)

Gunpowder magazine projects made by the Spanish military engineers of the eighteenth century are based on previous military architecture treatises. Therefore, most of these ancillary constructions are built bombproof by shielding the roof. A distinction is made between two types of designs: vaults and wooden structures. The latter are protected by elastic components capable of cushioning the impact of a pyroballistic weapon. From a morphological point of view, gunpowder magazines can be classified depending on their protecting enclosures. From a formal point of view, a distinction is made between the following three morphologies:

Gunpowder magazines having a simple construction body, with the outer wall directly exposed to hostile fire. This is the case of the gunpowder magazines built in Zaragoza (1729) [MPD, 39, 041] (**Figure 4**), Cádiz (1749) [MPD, 56, 029], San Sebastián (1738) [MPD, 27, 092], or Peñíscola (1739) [MPD, 18, 262].

Gunpowder magazines having an outer protection enclosure and a simple enclosure for storing the gunpowder. This is the most common type of magazine in the military treatises. Examples that stand out are the gunpowder magazines in Cardona (1718) [MPD, 19, 028] (**Figure 5**), San Sebastián (1722) [MPD, 28, 034], Ceuta (1724) [MPD, 39, 083], Málaga (1724) [MPD, 59, 046], and Gerona (1738) [MPD, 01, 018].

Lastly, gunpowder magazines having an outer protection enclosure and a two-element central construction body (where the inner wall is at ground level and the main magazine is above ground level). This is the case of the gunpowder magazines built in Tortosa (1721) [MPD, 64, 019], Málaga (1721) [MPD, 59, 044] (**Figure 6**), Barcelona (1726) [MPD, 10, 060], Cádiz (1728) [MPD, 08, 236], or Zaragoza (1729) [MPD, 28, 010].

3.1 Wood beam structure supported by load-bearing walls

These gunpowder magazines are built on load-bearing walls which are parallel to the vault's longitudinal axis, with a perpendicular framework of wooden beams. Where the width is small (until about three toises), the project is built with a single span. Examples that stand out are the gunpowder magazines in Tortosa (1721) [MPD, 64, 019] (**Figure 7**), Málaga (1721) [MPD, 59, 044], Barcelona (1726) [MPD, 10, 060], Cádiz (1728) [MPD, 08, 236], or Zaragoza (1729) [MPD, 28, 010]. Where the width is greater, there are two structural spans. The latter projects are divided into three types: firstly, projects where the central body has load-bearing walls and the two vaulted spans are connected by small doors, such as in Cádiz (1728) [MPD, 08, 237] and Cartagena (1745) [MPD, 18, 258] or Alicante (1750)

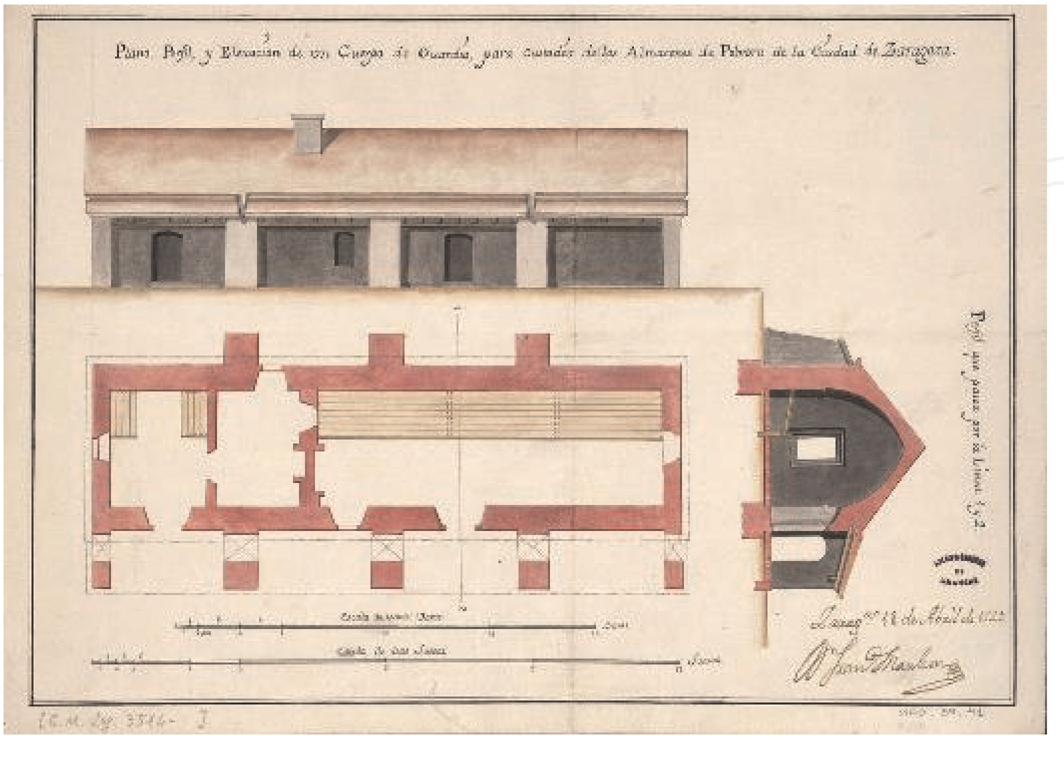

Figure 4.
Zaragoza (1729) [MPD, 39, 041].

[MPD, 65, 088 and MPD, 65, 092]; secondly, projects where the two spans are separated by pillars and the roof is supported by wooden main beams, such as the gunpowder magazines in Hondarribia (1733) [MPD, 65, 044], Cartagena (1745) [MPD, 18, 257], and Tortosa (1798); and thirdly, projects where the roof is

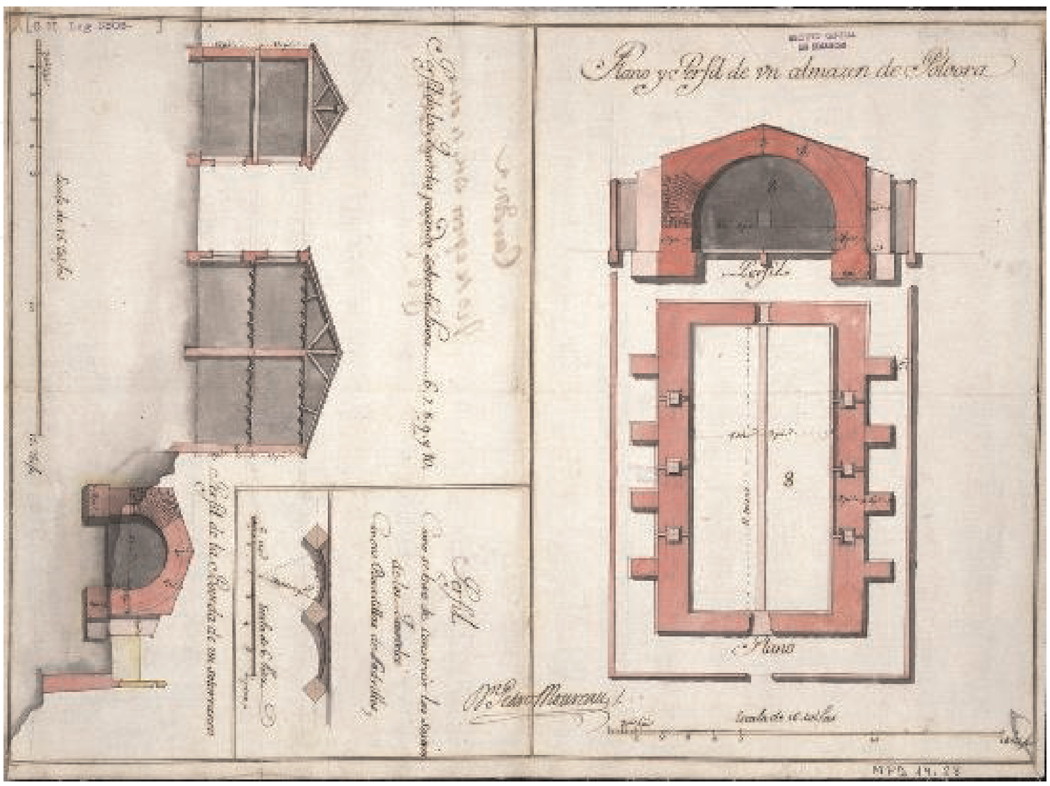

Figure 5.
Cardona (1718) [MPD, 19, 028].

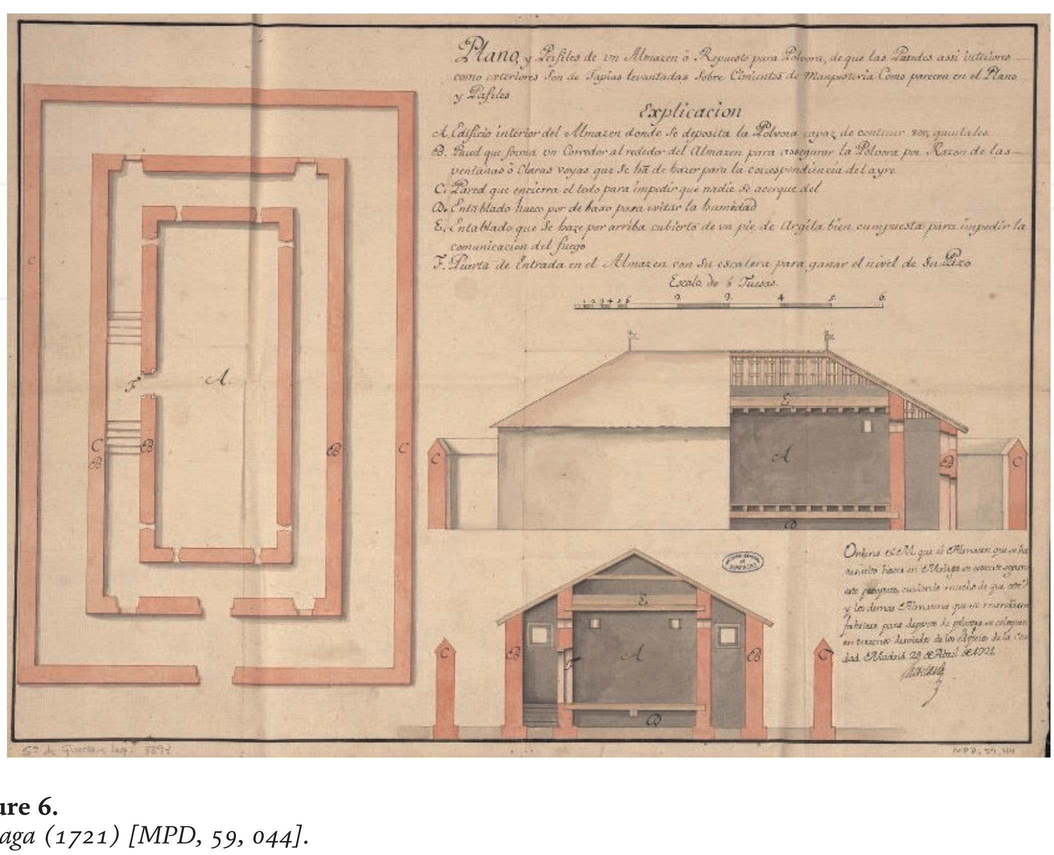

Figure 6.
Málaga (1721) [MPD, 59, 044].

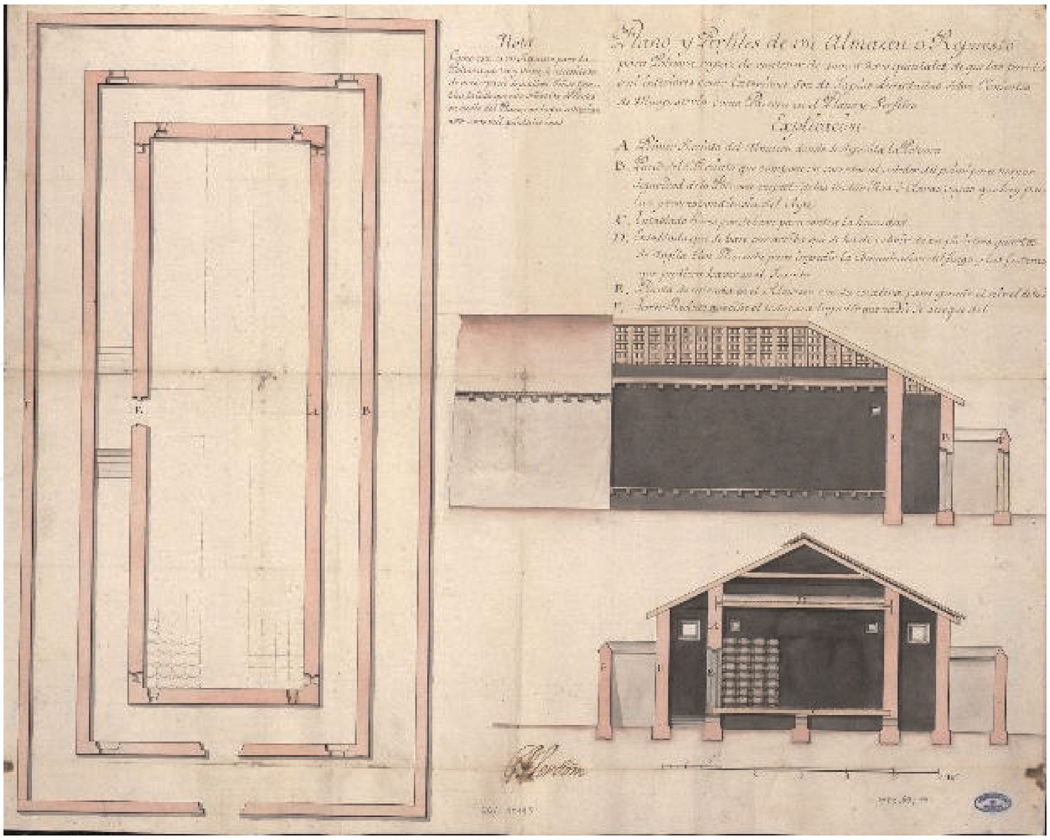

Figure 7.
Tortosa (1721) [MPD, 64, 019].

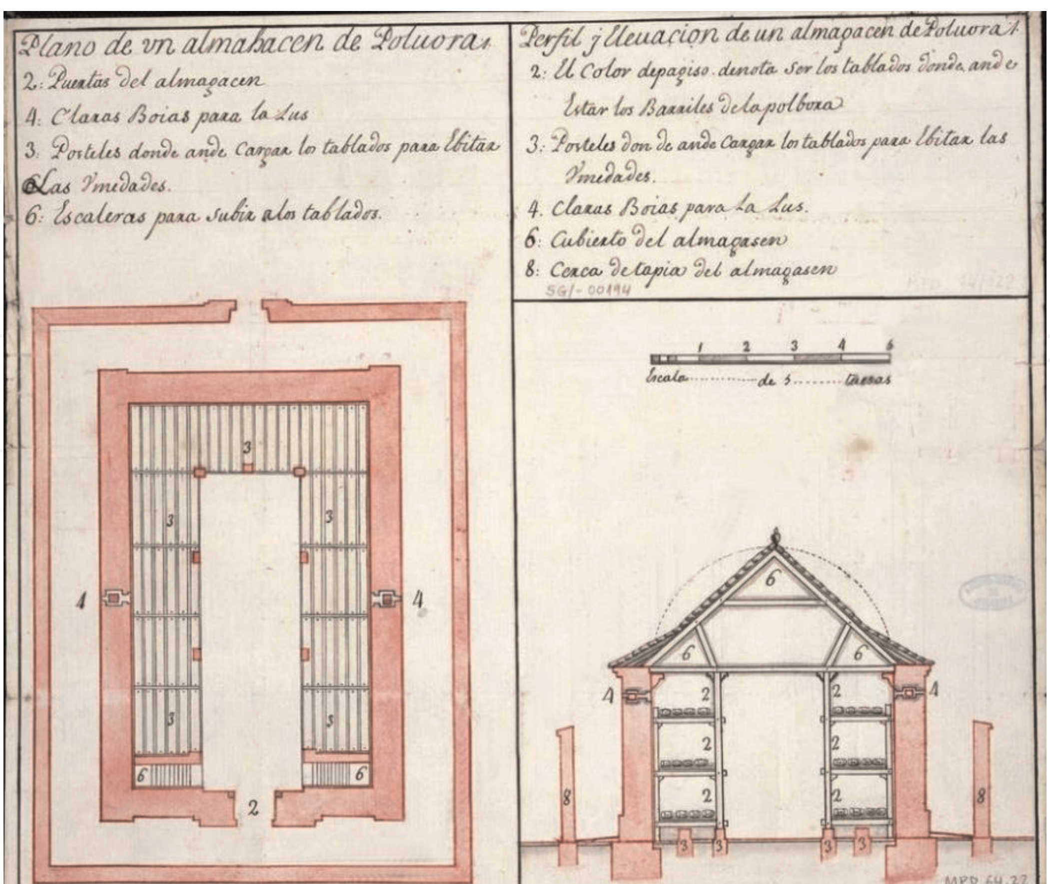

Figure 8.
Málaga (1721) [MPD, 64, 022].

supported by masonry arches, such as in the gunpowder magazines of Ceuta (1735) [MPD, 07, 179] or Lleida (1739) [MPD, 07, 001].

3.2 Wood structure supported by diaphragm arches

Military architecture treatises require the outer walls of the gunpowder magazines to be reinforced with buttresses. Thus, buttresses were used very often as the abutment of diaphragm arches. These types of design emerged later, and structures thus generated may have a single span. This is the case of the gunpowder magazines in Benimàmet (1751) [MPD, 06, 169] (**Figure 4**), Valencia (1756) [MPD, 07, 028] (where the arch abutment is built outwards), and A Coruña (1774) [MPD, 28, 027] (where the abutment is concealed in the interior space, as Müller set out in his treatise (1769)). Other larger powder magazines feature two parallel vaulted spans, for instance, the one in Barcelona (1761) [MPD, 20, 031], with a central pillar between each pair of arches.

3.3 Wood trusses

These are gunpowder magazines having a pitched roof and a timber framing. In some instances a joggle-truss is used, i.e., in the projects for Cádiz (1718) [MPD, 64, 020] and Gerona (1755) [MPD, 10, 073]. In other instances a collar-beam truss is used, i.e., in Barcelona (1731) [MPD, 18, 100 and MPD, 18, 101]. But mainly these magazines are built using the Spanish double-framed roof. This is the case of the gunpowder magazines in Zaragoza (1729) [MPD, 28, 009], Zamora (1734) [MPD, 65, 042], El Ferrol (1772) [MPD, 04, 089], or Cartagena (1795) [MPD, 46, 051 and MPD, 46, 052]. Sometimes they are Spanish double roofs with small variations affecting the inclined tie beams, i.e., in Pamplona (1723) [MPD, 64, 023]

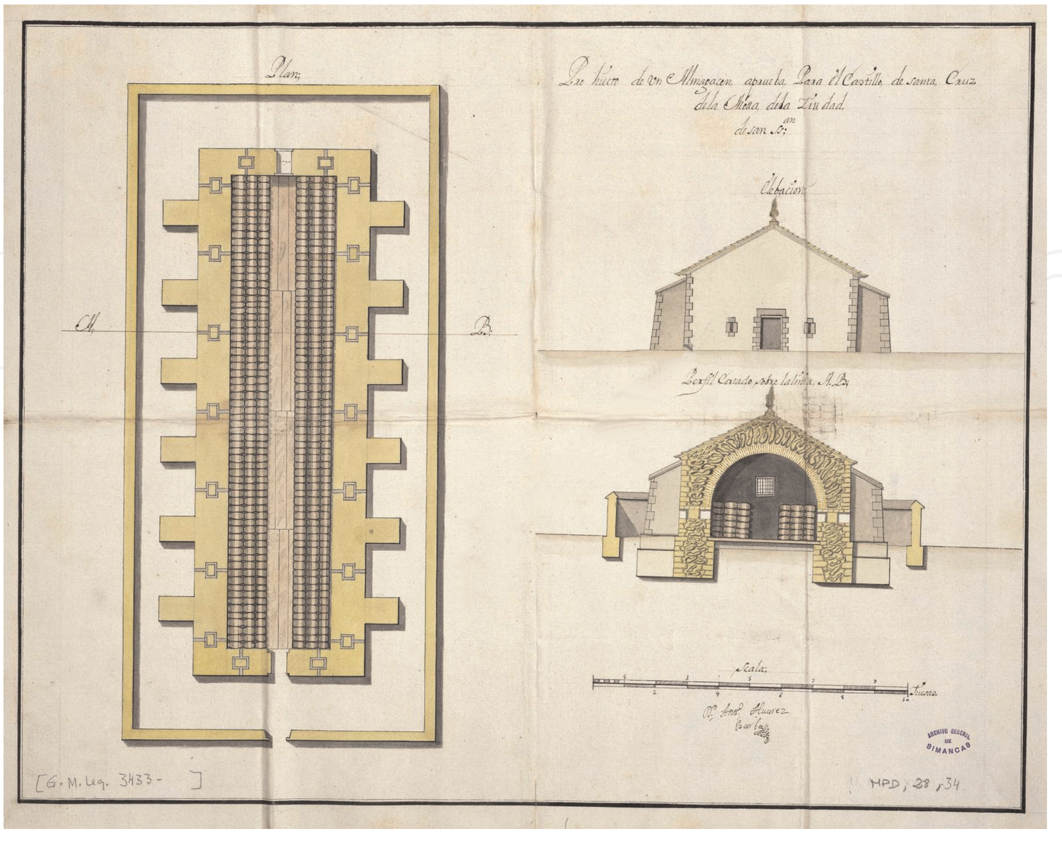

Figure 9.
San Sebastián (1722) [MPD, 28, 034].

and El Ferrol (1738) [MPD, 47, 094]. Other projects feature scissors trusses, i.e., in Monzón (1740) [MPD, 54, 049], Palma (1748) [MPD, 65, 047], and Valencia (1754) [MPD, 06, 170]. The project for Barcelona (1796) [MPD, 46, 035] features a double-framed roof but with two horizontal rafters as tie beams. In other cases, such as the project for Málaga (1721) [MPD, 64, 022] (**Figure 8**), there is a mansard truss with two horizontal rafters (this structure is more similar to the models used in the upper body of military barracks).

3.4 Barrel vaults

Masonry barrel vaults are the most commonly used type of bombproof vaults. The gunpowder magazines are the direct result of military architecture treatises. Because it is such a common construction technique, most of the engineers do not even mention the material used for the vault of the magazine. This is what happened in the projects for Ceuta (1724) [MPD, 39, 083], Longone (1725) [MPD, 12, 222], A Coruña (1738) [MPD, 17, 058], San Sebastián (1738) [MPD, 27, 092], Peñíscola (1739) [MPD, 18, 262 and MPD, 18, 263], Játiva (1748) [MPD, 54, 012], Palma (1748) [MPD, 65, 048], San Sebastián (1750) [MPD, 27, 093], Viveiro (1778) [MPD, 19, 241], and Santa Cruz de Tenerife (1792) [MPD, 05, 033]. With regard to the projects which do specify the material used for the construction of the barrel vault, a distinction must be made between those which use stone for the first layer of the vault and those which use ceramic bricks.

The main projects which use stone masonry include those for Barcelona (1715) [MPD, 18, 097] and Dénia (1748) [MPD, 65, 085]. There is a variant which uses an ashlar arch with wooden beams on top, such as the project for Santa Cruz de Tenerife (1758) [MPD, 18, 050] (this variant was already used by Fernández de Medrano in the previous century). In other cases, ashlar masonry is substituted by

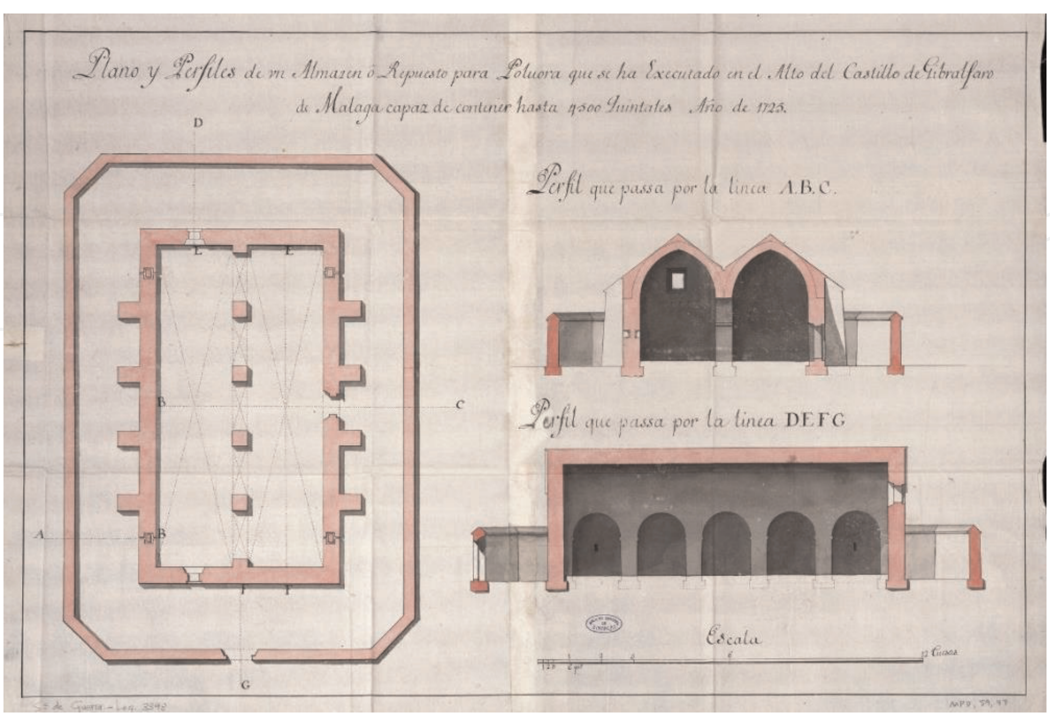

Figure 10.
Gibralfaro Castle in Málaga (1724).

brick masonry with a lime and pebble covering, as in the case of San Sebastián (1722) [MPD, 28, 034] (**Figure 9**).

As for the projects which use ceramic bricks, a distinction must be made between four different types: firstly, those which feature a single-layer ceramic vault, like the ones for Zamora (1738) [MPD, 13, 113] or Ciudad Rodrigo (1739) [MPD, 12, 154]; secondly, those which feature a bottom layer of ceramic bricks, an intermediate layer of stone, and a top layer of lime and pebble, like the gunpowder magazine in Cardona (1718) [MPD, 18, 028]; thirdly, vaults designed with a double layer of ceramic bricks and a top fill of lime and pebble, like in the projects for Pamplona (1718) [MPD, 31, 032; MPD, 31, 033; MPD, 31, 034; MPD, 31, 035; MPD, 31, 036]; and lastly, projects which feature three layers of ceramic bricks, for instance, in Longone, Italy (1728) [MPD, 12, 221], or even four layers, like in Badajoz (1749) [MPD, 65, 045].

3.5 Pointed vaults

Pointed vault structures were described by Bélidor (1729), and Müller (1769) said they are less resistant to bomb impacts than barrel vaults. They belong to the Gothic building tradition, and they need a smaller abutment than barrel vaults, even though magazine walls in military treatises are dimensioned depending on the impact of pyroballistic weapons (those employing gunpowder) and not on the basis of masonry mechanic criteria. This is the case of the gunpowder magazines in Zaragoza (1729) [MPD, 39, 041] and San Fernando de Cádiz (1749) [MPD, 56, 029], which have only one enclosure. It is the same construction type as the projects for Gerona (1738) [MPD, 01, 018] and San Sebastián (1749) [MPD, 27, 094], but these two magazines are protected by an encircling wall, and therefore they have two enclosures. The gunpowder magazine projects for Gibralfaro Castle in Málaga (1724) [MPD, 59, 046 and MPD, 59, 047] (**Figure 10**) and Ceuta (1737) [MPD, 07, 180] feature two parallel vaulted vans which are separated by square pillars supporting round arches, thus forming the central valley of the roof.

4. The curve of equilibrium and the Spanish military engineers

In the construction of gunpowder warehouses, barrel and pointed vaults are generally used, although there are some examples with elliptical vaults, such as that one built in 1694 by Hércules Torelli in Pamplona. This construction was remodelled by Francisco Larrando de Mauleón (1718) [MPD, 31,031] (**Figure 11**) [43]. Mauleón was professor at the Mathematics School of Barcelona and Zaragoza and authored the *Estoque de la Guerra y Arte Militar*, published in Barcelona (1699). The viceroy had ordered the repair of the fortifications and the gunpowder warehouses to make them bombproof. The elliptical vault was replaced by a barrel vault to make it less visible and vulnerable to enemy artillery. It was reinforced and reduced in height according to the concept introduced in the military treatises by General Ambroise (d. 1587) in *Le Timon du Capitaine* (1587) [44].

The simple vault of the warehouse of Montjuïc mountain in Barcelona (1731) [MPD, 07, 057], a project attributed to Miguel Marín (**Figure 12**), is not generated through an arch of circumference. The geometric study reveals that the vault has a length of 16 feet in toise, a rise of 11.5 feet, a width of 3 feet, and a buttress of 7 feet. A geometric element having the shape of a catenary can be drawn running through

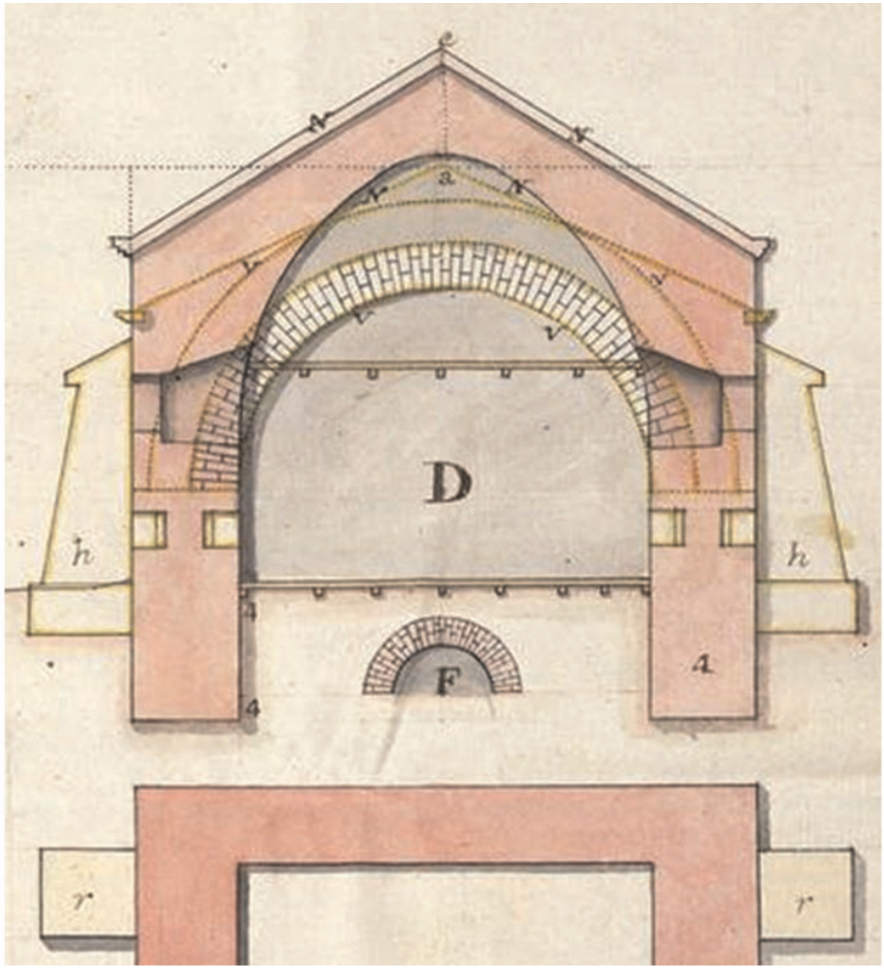

Figure 11.
Warehouse of Pamplona, Francisco Larrando de Mauleón (1718) [MPD, 31,031].

the springing points and the key of the vault (**Figure 13**). The shape of this element, which has the same span and rise as the vault, is very similar to the shape drawn in the project.

Other projects, such as the project from Miguel Marín for Tortosa (1733) [MPD, 13, 035] (**Figure 12**), have a span of 21 feet in toise, a rise of 14 feet, a width of 3.5 feet, and a buttress of 7 feet (**Figure 13**). Another similar project is the simple warehouse layout by Juan de la Feriére y Valentín in A Coruña (1736) [MPD, 17, 057] (**Figure 12**), which contains a span of 22 feet in toise, a rise of 14 feet, a width of 3 feet, and a buttress of 7 feet (**Figure 13**).

The design of the pointed vault is initially compared with the catenary, as obtained with a chain over a reproduction of the plans on a larger scale (**Figure 14**). Thus, the arch described by the chain is very similar but not coincidental to the profile of the vault because there are small deviations near the springline of the vault. This deviation is because it is not possible to lay out the catenary with traditional drawing tools, such as rulers and compasses.

The assessment of the original section drawing of the warehouse reveals three compass marks. One point is made over the vertical axis of symmetry of the figure, while the other two are made over the perpendicular axis, slightly below the springline of the vault. An oval was drawn on each project using these compass marks, and the obtained curves were coincident with the curves of the projects. Thus, to draw the projects of the warehouses, both Miguel Marín and Juan de Feriére y Valentín used the geometrical solution of an oval. Therefore, the curves drawn in the three projects are oval, but the major axes of the oval are higher than

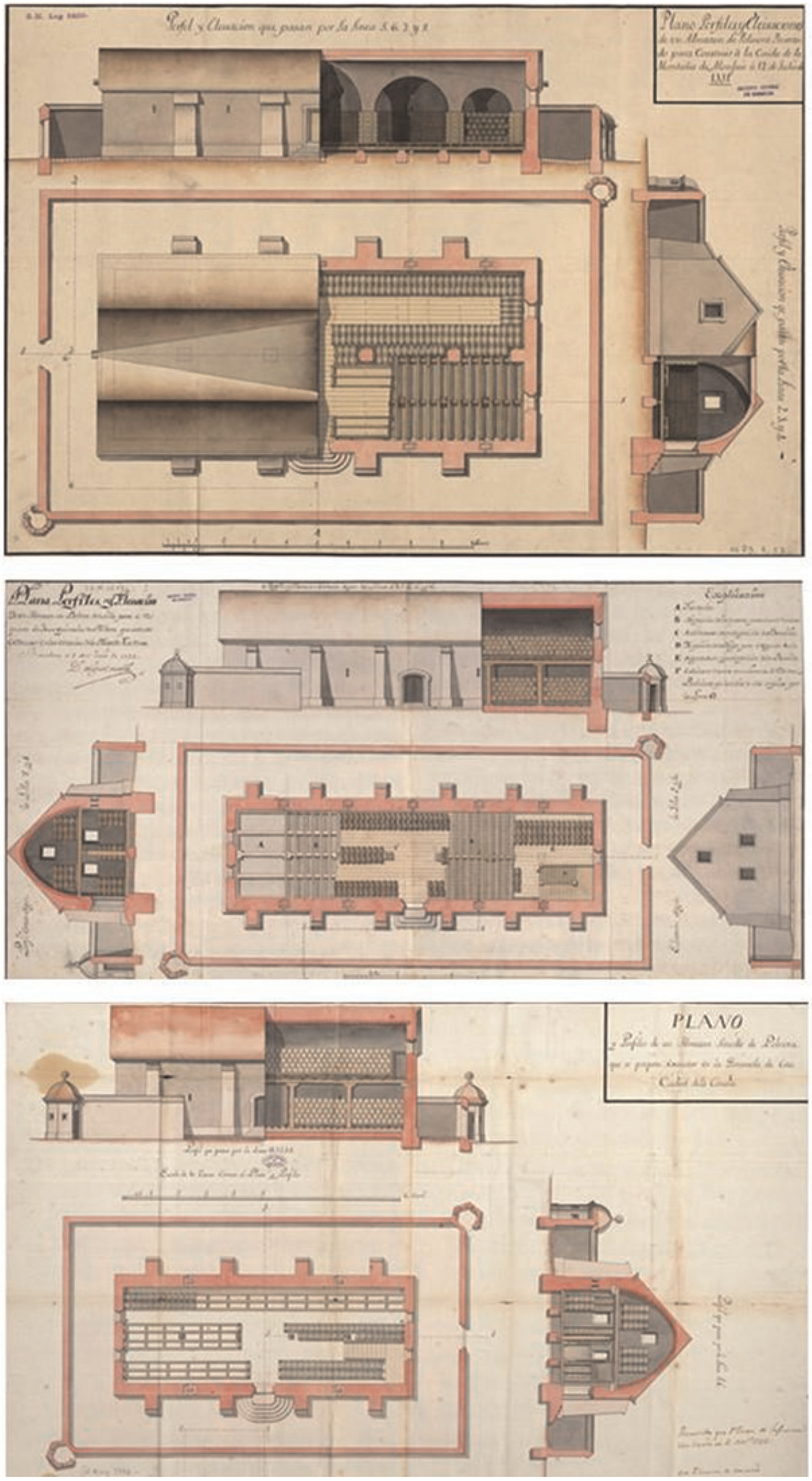

Figure 12.
Gunpowder warehouses: Marín (1731), Marín (1733), and Feriére y Valentín (1736).

the springline of the arch. As a consequence, the curves are not tangential at the springing. So, the military engineers drawn the curve of the vault as an arch *apaynelado, carpanel* of Tosca (1712), or *anse de panier* of Bélidor (1729) (**Figure 15**).

The geometric layout of these vaults, based on ovals, was well known by the eighteenth-century military engineers. They began from the essential feature that oval vaults are tangential to the springline of these building elements. When

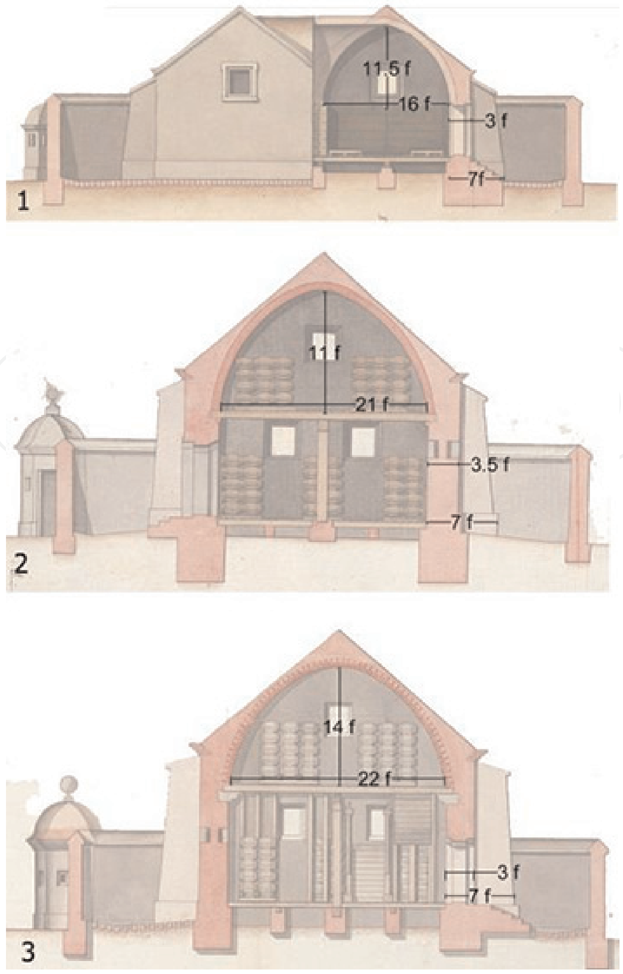

Figure 13.
Metrology of the projects for the gunpowder magazines.

the springline is higher than the axis, a non-tangential curve is obtained, which is a feature of the catenary definitions given by Frézier (1738). Concurrently, Bélidor (1729) specifies the method to lay out the true shape of the catenary vault. By knowing the rise and the span of the vault, the architectonic shape is determined with a hanging chain. Thus, a scale model can be built and can easily be taken to the construction site. By contrast, the layout of the catenary in military engineers' projects is more complex, because it needs the use of an approximation of the catenary through the geometrical shape of a lowered oval.

The ovals are derived from the centres of the circumferences (the compass center points on the paper). They are referred to (O1) for Barcelona (1731) [MPD, 07, 057], (O2) for Tortosa (1733) [MPD, 13, 035], and (O3) for A Coruña (1736) [MPD, 17, 057]. They are consistent with three different types of ovals, and they all share the common feature that the origin of the vertical tangent to the minor axis of the oval is located 1 foot below the impost line (**Figure 16**).

The main ovals' geometric data are shown in **Table 1**, where:

- e1 describes the clear span.
- e2 describes the rise.
- a1 describes the distance between centres of the minor axis.

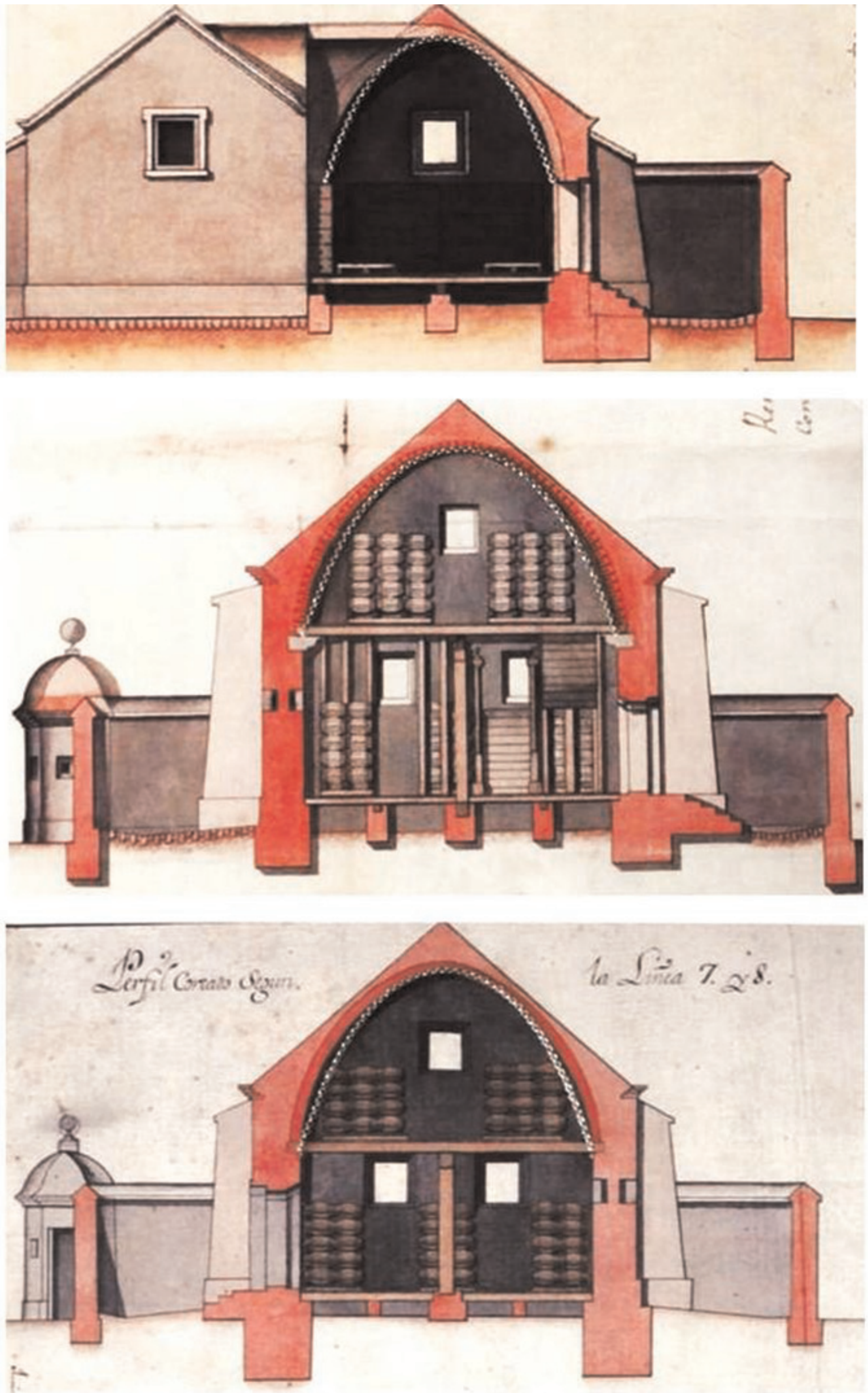

Figure 14.
Chaînette method applied to the projects for the gunpowder magazines.

- a2 describes the distance between the centre of the minor arc and the minor axis.
- d1 describes the ratio between the length of the semimajor axis and the vertical distance from the semiminor axis to the springline (feet).
- d2 describes the ratio between the length of the minor axis and the distance from the center of the major arc to the point of tangency between the major arc and the minor axis.

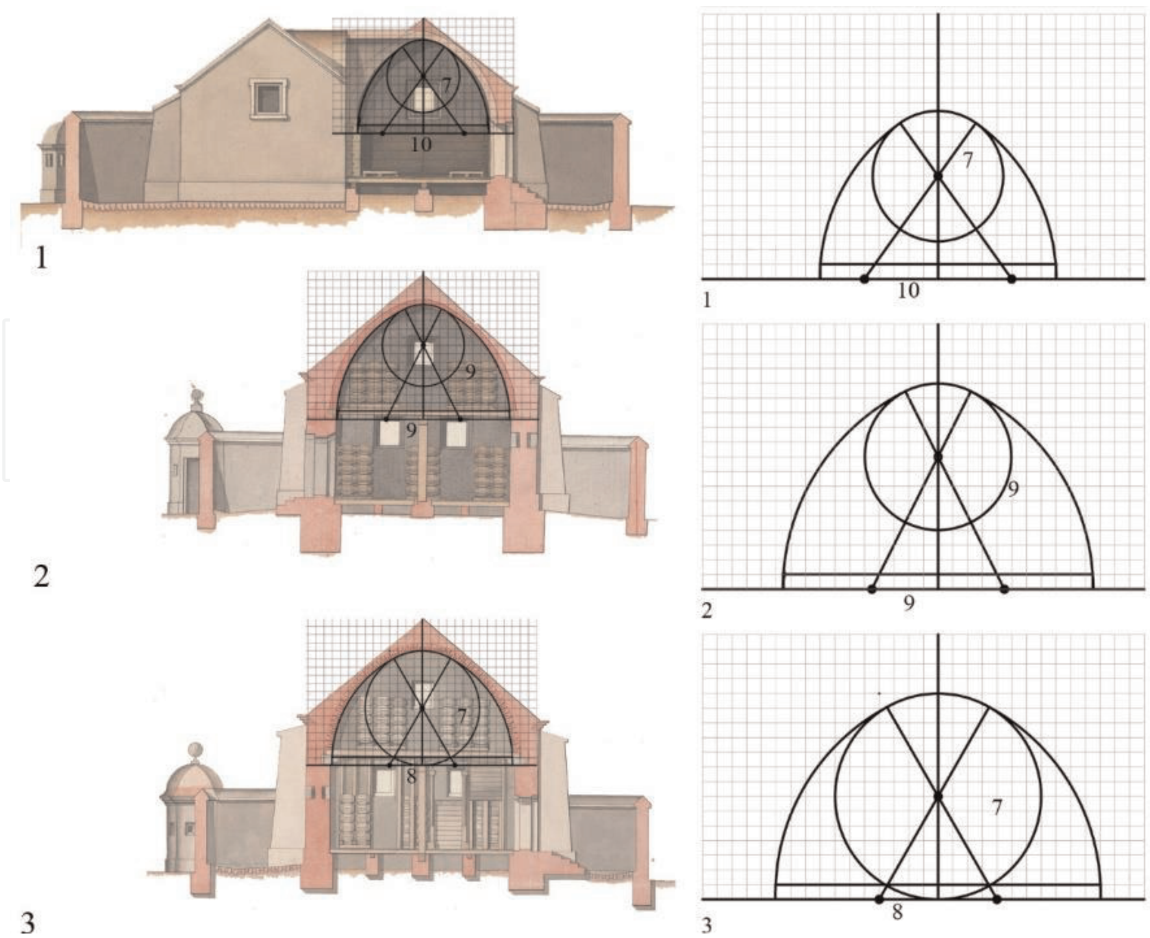

Figure 15.
Oval method applied to the projects for the gunpowder magazines.

- d3 describes the ratio between the length of the semimajor axis and the radius of the oval's minor arc.

- p1 describes the ratio between the semimajor axis and the minor axis.

- p2 describes the ratio between the center-to-center distance on the minor axis and the distance from the center of the semimajor axis to the minor axis.

The ovals used in the layout of the gunpowder magazines are thus used as a reference for purposes of comparison with the cellar's layout. The layout of [O1, O2, O3] is based on a ratio between d3 and e2 of [0.39:0.36:0.50].

In addition, the layout of each oval is compared with a catenary that has the same rise and span, which is drawn using InnerSoft software. According to the results, the inner surface defined between the corresponding geometric shape and the springline is different (1.33 m^2 vs. 0.98 m^2). Furthermore, the ratio between the maximum distance between geometric shapes and the arch's span ranges from 2.14 to 3.44%. From these data, we can conclude that the approximation made by the engineers by drawing ovals in the three projects considered is sufficiently precise for the drawing scale used, between E: 1:90 and E: 1:70. Finally, the curves are compared with an elipse with the same rise and span. The obtained shape is clearly not coincident with the curves of the projects.

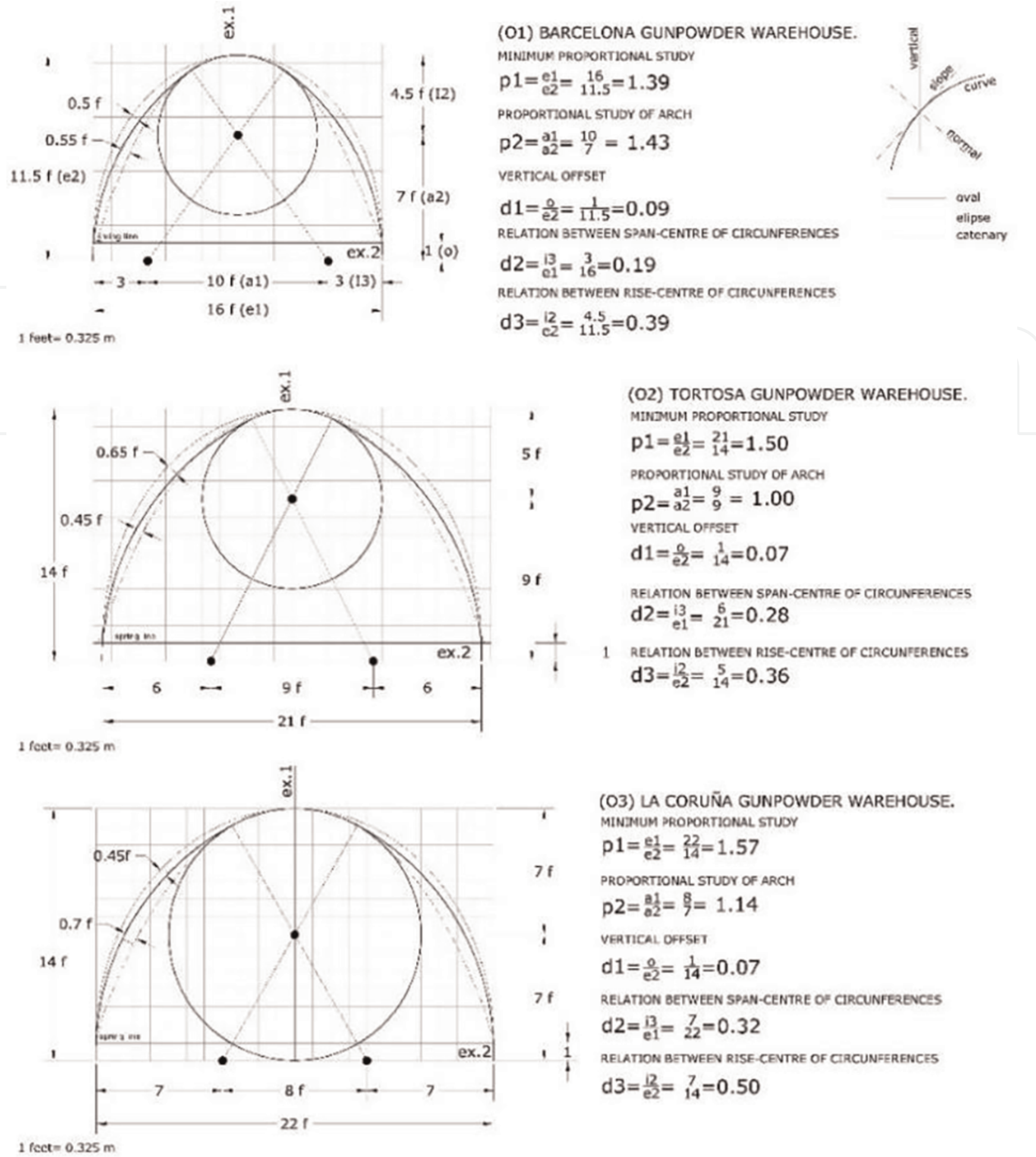

Figure 16.
Geometrical analysis of the vaults of the projects for the gunpowder magazines.

5. The curve of equilibrium and the civil constructions of the eighteenth century

After the Bourbon dynasty's ascendancy to the Spanish throne (1700), Catholic diplomatic and military families of Irish and Scottish origin emigrated under royal protection, preserving their status. O'Connor family was installed in Benicarló in the eighteenth century, and they associated with the McDonnells in the wine export business.

The Bourbon dynasty, which established itself in Spain in 1700 with King Philip V (1683–1746), created the Army Corps of Engineers by the Royal Decree of 17 April 1711. Several Irish families moved to eastern Spain in the mid-seventeenth century. Patrick White Limerick, a trader in agricultural products and wine, and the

O'Connor family settled in Benicarló in 1749. Gaspar White and the O'Gorman family were based in Alicante, whereas the Lilikells, the Tuppers, and Henry O'Shea lived in Valencia. Against this backdrop, the O'Connor family built a Carlón wine cellar in Benicarló in 1757 [45] (**Figure 17**).

The wine cellar's construction, using diaphragm arches, is very similar to the gunpowder magazine projects built by the Army Corps of Engineers in a neighbouring geographical area. These include the project by Carlos Beranger [MPD, 06, 169] for Benimàmet (1751) and the one by Juan Bautista French (1756) for Peñíscola [MPD, 07, 208]. In these projects, as opposed to the O'Connor cellar, the arch abutment is on the outside of the building. Nonetheless, there is a subsequent project by Antonio López Sopeña [MPD, 28, 027] for A Coruña (1774), in which he uses diaphragm arches with inside abutments similars to those used in the Benicarló's wine cellar.

The O'Connors family Benicarló's building was built in 1757. The geometric study of the cellar arches is based on the topographical survey conducted with a laser scanner. The Carlón wine cellar has a rectangular floor plan; its inside measures are 12.42 m in width and 43.01 m in length. In the nave, there are eight diaphragm arches, each having a single two-piece offset-jointed ring and an average depth of 0.60 m. The arches are made of solid ceramic bricks (measuring $0.37 \times 0.18 \times 0.04$ m), and they rest on a limestone base that was brought from Santa Magdalena de Polpís. The top of this base determines the springline of the ceramic arch. The arch's abutment and the outside walls are made of ordinary uneven masonry. The formal characteristics of the arches are different: arch a_1 has a clear span of $e1_{a1} = 9.65$ m and a rise of $e2_{a1} = 5.82$ m, whereas the other seven arches can be grouped together. Their span is within a range of $e1_{a(2–8)} = [9.76–9.69$ m$]$, similar to arch a_1, but their rise significantly differs from the first arch, within a range of $e2_{a(2–8)} = [5.46–5.45$ m$]$. All of the arches share a special feature: they do not have a vertical tangent on the stone base. The angle of incidence (α) of these arches with respect to both vertical sides, left (α_a)and right (α_b), has the following values: $\alpha_{a.a1} = 13.94°$ and $\alpha_{b.a1} = 8.97°$ in arch a_1 and $\alpha_{a.a(2–8)} = [4.49°–1.80°]$ and $\alpha_{b.a(2–8)} = [7.38°–2.58°]$ in arches $a_{(2–8)}$ (**Figure 8**). By statistically analysing the

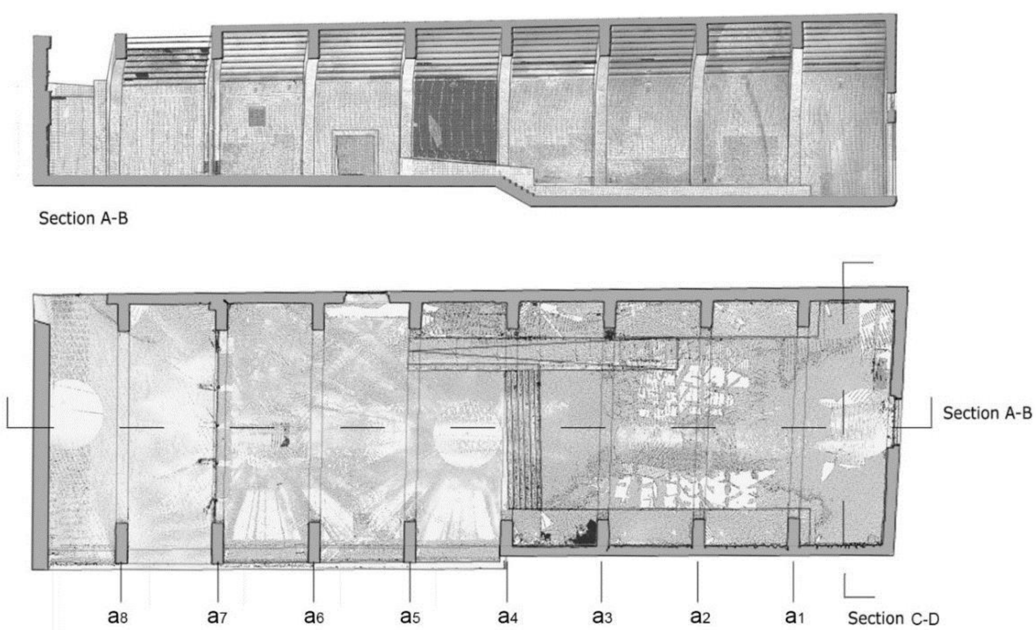

Figure 17.
Floor plan and cross section of the O'Connor cellar in Benicarló (1757).

parameters, for arches $a_{(2–8)}$ the average span calculated is $e1_{a(2–8)}$ = 9.72 m, and the average rise is $e2_{a(2–8)}$ = 5.46 m (**Figure 18**).

It seems that the measurement units used for the construction of the cellar were the toise (194.90 cm) and the toise foot (32.48 cm). Arches $a_{(1–8)}$ have an average span of 29.92 toise feet (9.71 m), with an error of 0.02 m for 30 feet (5 toises, 9.74 m). The rise of arch a_1 is 17.92 feet (5.82 m), i.e., there is an error of 0.02 m in 18 feet (5.84 m), which are 3 toises (**Figure 19**). Arches $a_{(2–8)}$ have a rise of 16.80 toise feet (5.46 m), i.e., there is an error of 0.06 m in 17 feet (5.52 m). The arches are 0.60 m in width (1 + 10/12 feet). Regarding the outside measurements, the nave is 41.50 feet (13.48 m) wide and 92.36 feet (30 m) long, and the arches' abutments are structures measuring 5 + 9/12 feet. The inside length of the cellar is 43.01 m, i.e., 132 + 5/12 toise feet. The enclosure wall on the façade is 2 feet thick; thus, the span-to-arch ratio is 5.75/30 feet (**Figure 20**).

A metrological analysis of the arches in Benicarló's cellar reveals that the eight arches show the same metric relations, i.e., a 5 toise span and a 3 toise rise. The dimension of the catenary arch a_1 are 30 × 18 feet (exactly 5 × 3 toises). The dimensions of the elliptical or oval-shaped arches $a_{(2–8)}$ are 30 × 17 feet. If we follow the hypothesis that the minor axis (either the ellipse minor axis or the oval minor axis) is 1 foot below the impost, then the geometric relation of arches $a_{(2–8)}$ is also 30 × 18 toise feet.

A statistical analysis is now performed on each of the eight arches $a_{(1–8)}$ to determine the difference between the shape of the arches built and the shapes of reference: ellipse (E), catenary (C), and ovals [O1, O2, O3]. The following values are calculated for 29 points on each cellar arch:

a. The average and maximum deviation of these 29 points

b. The angle of incidence on the springline

The mean deviation has a spread of only 0.03 m, which is approximately 0.31% of the arch's span, making it difficult to conclude whether it is a catenary or an oval. The determining feature is that arch a_1 has an angle of incidence on the springline

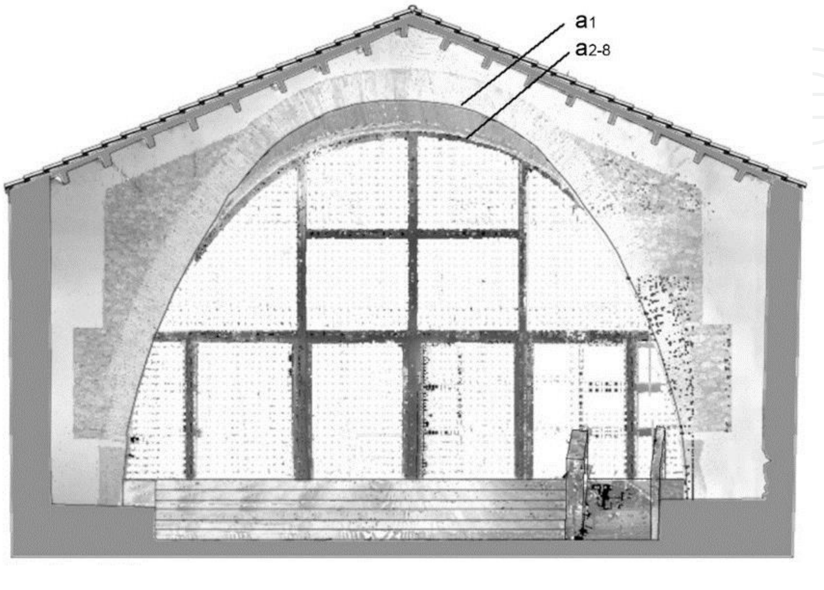

Figure 18.
Transversal section of the O'Connor cellar in Benicarló (1757).

[$\alpha_{a.a1}$ = 13.94°, $\alpha_{b.a1}$ = 8.97°]. Because the catenary's angle of incidence is 19.38°, the geometric shape that most closely resembles the arch is the catenary.

Conversely, the statistical analysis of the remaining seven arches $a_{(2–8)}$ shows that the geometric shape that they most resemble is the ellipse, with an average deviation ranging between 0.001 and 0.015 m. The range for oval-shaped arches is 0.006–0.186 m (*arco apaynelado* or *arco carpanel* according to Tosca, i.e., three-centred arch or basket-handle arch; or *anse de panier* according to Bélidor), so the

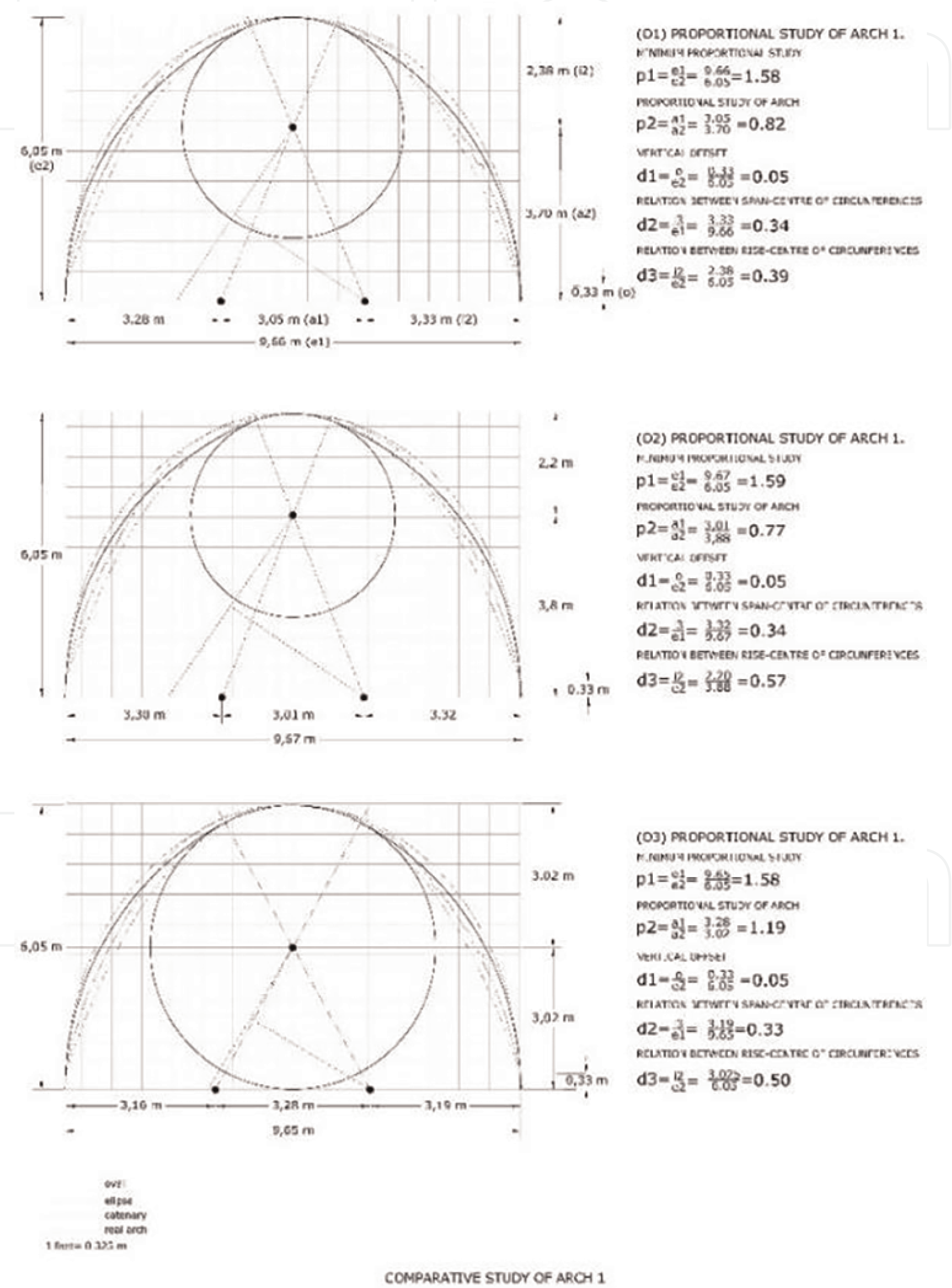

Figure 19.
Geometrical analysis of the arch 1 in the O'Connor cellar (1757).

curves used by the Bourbon engineers to layout the projects for the gunpowder magazines are very similar to the cellar's arches. Nevertheless, the angle of incidence on the springline tends not to have a vertical tangent, which is a fundamental feature of both the ellipse and the oval in the arches considered here. In the springline of these arches, the angle of incidence ranges between $\alpha_{a\ .a5}$ = 1.54° and $\alpha_{b\ .a5}$ = 7.38°.

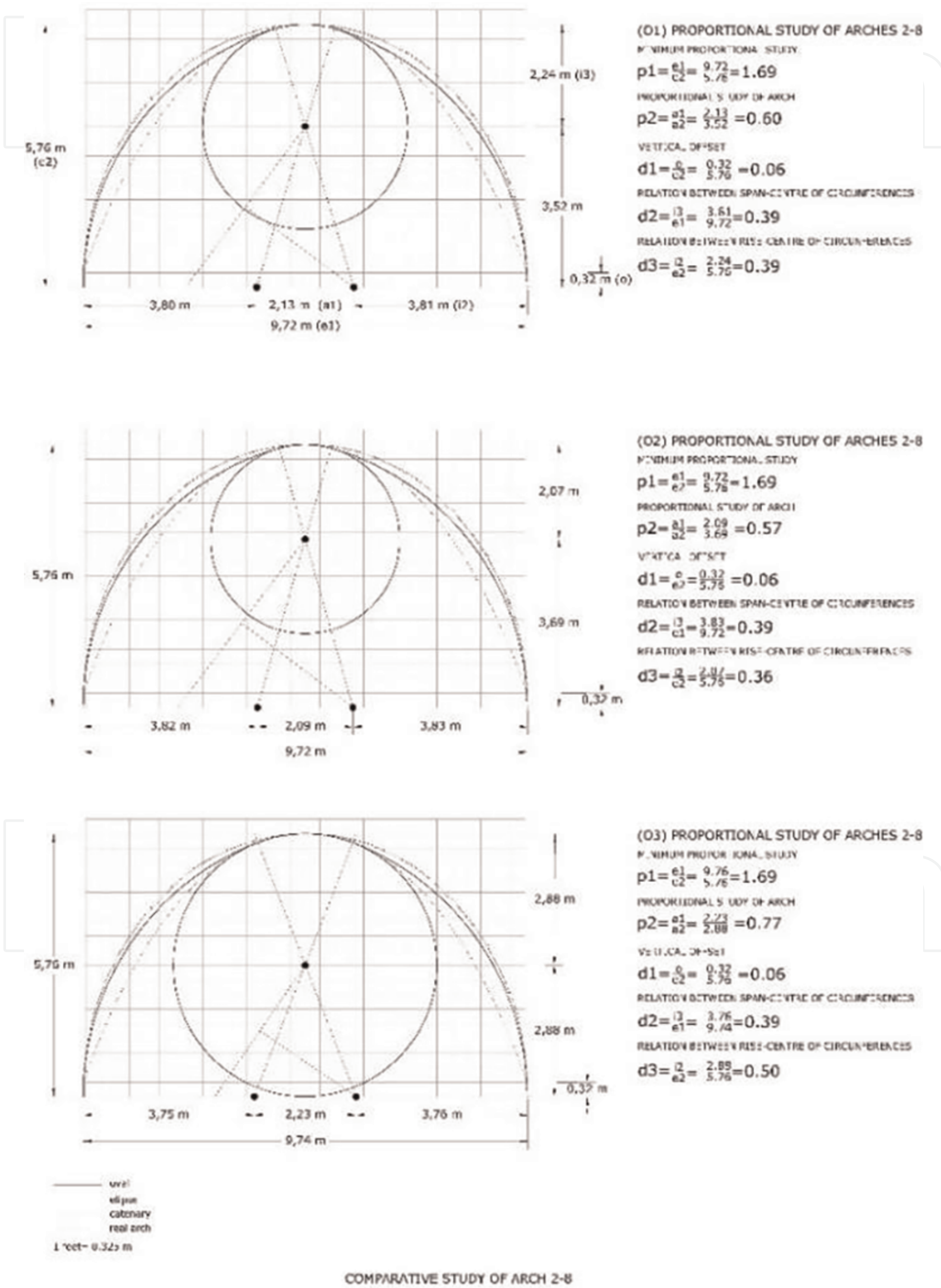

Figure 20.
Geometrical analysis of the arches 2–8 in the O'Connor cellar (1757).

Thus, arch a_1 resembles a catenary arch, whereas the other seven arches $a_{2–8}$ tend to be ellipses. These seven arches do not have a vertical tangent on the springline because their horizontal axis has been moved 1 foot below the arch's springline. As defined by Frézier (1738), the shape of the catenary has the following essential property: the vertical line which is tangent to the curve at the springline does not form a right angle with the horizontal plane. Therefore, geometrically, the catenary can be understood as any curve that does not have a vertical tangent at its springline. This is what happens in the springline of St. Paul's dome in London [11], which was designed by Christopher Wren in collaboration with Robert Hooke [46]. Otherwise, it should be noted that from a mechanical perspective, catenary arches are an optimal solution to build masonry arches, since the material has very low tensile strength.

Finally, from the construction point of view, the catenary shape can be approximated using other geometric forms such as ovals or ellipses, under the condition that there is not a vertical tangent at the springline. The catenary shape forms a barycentric axis, which minimizes the tensions on a linear element that is subject to only vertical loads. In the arch, the inverted catenary shape prevents the appearance of stresses other than compression stresses.

Thus, there are two hypotheses regarding the construction of the wine cellar. The first one is that the construction work was started from the inside toward the façade; thus, arches $a_{(2–8)}$ were constructed before the catenary arch a_1. The second hypothesis is that the construction work began with arch a_1. According to the second hypothesis, there is also a difference between both types of arches: on the first brick courses from the springline of arch a_1 (the first 9 courses on 1 side and the

Figure 21.
Springing of arches no. 1 and no. 2 in the O'Connor cellar.

first 17 courses on the other side), the ring is 0.36 m wide. On the remaining seven arches, the ring is 0.60 m wide (just like the arch's depth). It is clear that less ceramic material is necessary for the construction of arch a_1 than for the other seven elliptical arches $a_{(2–8)}$ (**Figure 21**).

6. Conclusion. The origin of the catenary arch in Spain

The assessment of some drawings of gunpowder warehouses, found in the collection of *Mapas planos y Dibujos* (MPD) of the General Archive of Simancas (*Archivo General de Simancas*, AGS) (AGS 2014), has revealed the use of the chain theory in Miguel Marín's projects for Barcelona (1731) and Tortosa (1733) and Juan de la Feriére ones in A Coruña (1736). A built evidence has also been found: the Carlón wine cellars in Benicarló, built by the O'Connors family from Ireland (1757). The analysis of these examples proved the theory of the chain arrival to Spain during the first half of the eighteenth century. However, 50 years before Antoni Gaudí, Catholic families emigrating from Scotland and Ireland already initiated some of the catenary's form mathematical theory in some practical uses, a theory that begun to be taught at the Mathematics Academy of Barcelona in 1720.

This paper addresses the introduction of the concept of the catenary arch in Spain before the nineteenth century. After an exhaustive review of the theoretical framework, some cases are assessed. The aim of the research is to find out if the mechanical concept of the chain was used by the Spanish military engineers and by the exiled English engineers, who built several wine cellars in Spain. Thus, we intend to determine whether there is any geometrical relationship between the layout of several gunpowder magazines made by Spanish military engineers in the 1730s and the construction of a civil building—the Carlón wine cellar in Benicarló (1757)—in which catenary arches may have been used.

The assessments of the gunpowder warehouses by Miguel Marín for Barcelona (1731) and Tortosa (1733) and by Juan de la Feriére y Valentín in A Coruña (1736) are only a mere 4.05% of the projects analysed. However, they prove the intention to lay out the vault as a catenary. These authors knew that in a catenary the tensility in the shape of a hanging chain has the same compression values in the inverted geometrical figure. These engineers had a vast knowledge of the mechanical principles of the modern theory for masonry. From a scientific perspective, catenary vaults are the most interesting because they introduce the principles established by Hooke (1676). Both the arches of gunpowder magazines and the arches $a_{(2–8)}$ of Benicarló were laid out using the geometrical construction of an oval. Otherwise, the location of the horizontal axis of the ovals under the springline reveals the application of one of the characteristics of the catenary. This causes that the vertical line which is tangent to the curve in the springing does not form a right angle with the horizontal, so they used the chain's theory in the layout of the projects.

Formally, if the distance between the axes and the springline of the arch is small, then the angle of incidence has a minimum influence on the thrust and the line of pressure. Otherwise, the location of the axis under the springline reveals the intention to minimize stresses in this point and in the neighbouring areas, even though the final mechanical influence is small.

Although there is no evidence of the construction of the gunpowder warehouses, it is possible to confirm the use of catenary arches in the construction of the Carlón cellars of the O'Connor in Benicarló (1757). There are significant differences between the measures of the arches of the gunpowder magazines (maximum span: 22 feet; maximum rise: 14 feet) and the arches of the Benicarló cellar (span: 30 feet; rise: between 17 and 18 "toise" feet, until the springline). In addition, the span-to-rise ratio of the oval arches in the gunpowder magazine studies is [1.39:1.57], whereas in Benicarló, this ratio is [1.67:1.76]. It can be concluded that arch a1 is a catenary arch, whereas arches $a_{(2–8)}$ tend to be elliptical. Arches $a_{(2–8)}$ show the special feature that their (x) axis is located below the springline; therefore, the tangent of the curve on the springline does not form a right angle with the horizontal. This is a feature of the definition of the catenary.

The theory of the equilibrium curve, followed by most of the British engineers, became known to the Bourbon military engineers through the academy of mathematics in the eighteenth century, and it was used by some immigrants of English origin, such as the O'Connors, a century before the modernist architecture of Antoni Gaudí.

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