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# The General Kinematic Pair of a Cam Mechanism

*Jiří Ondrášek*

## Abstract

At present, there are still increasing demands on the performance parameters of machinery equipment as well as cam mechanisms that belong to it. For this reason, the operating speeds and hence inertial effects of moving bodies, which limit the utilizable working frequency of machines, are increasing. These facts are the cause of higher wear and a decrease of the overall lifetime and reliability of machines. The force ratios in the general kinematic pair created by contact between the cam and the follower cause the contact stress. The generated stresses are transient and have a pulse shape. Fatigue damage of the cam working surface or the follower working surface may occur after exceeding a certain limit value of these stresses during the cam mechanisms running. This damage is in the form of cavities (pitting), which develop from cracks on the working surface. The chapter aim is to outline the issues of the dynamic stress of a general kinematic pair of a cam mechanism. One of the possible methods of the complex solution of the stress of the general kinematic pair is to use the possibilities of the finite element method in combination with the knowledge and conclusions of the contact mechanics.

**Keywords:** cam mechanism, cam, follower, general kinematic pair, contact stress

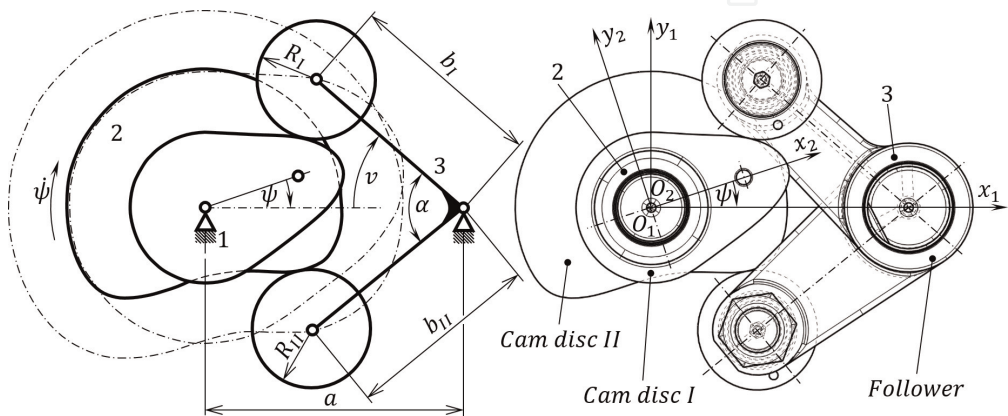
## 1. Introduction

Cam mechanisms are one of the basic objects in the design of production machines and equipment, whose characteristic feature is a high degree of automation and optimization of production and work processes. These mechanical systems are characterized by the transmission of large load possibility at high speed and positional accuracy of the working member of the relevant machinery. Their application is mainly connected with the so-called hard automation, which is characterized by unchangeable or difficult to change operations of the given technical equipment. Their widespread use is known in manufacturing and handling machines of the manufacturing industry, with their dynamic effects and properties greatly affecting the overall behavior, operation, and efficiency of such machinery. At present, increasing demands are placed on the performance parameters of such machinery. Therefore, the operating speeds and thus the inertial effects of the moving bodies are increased, thereby reducing the usable operating frequency of the machines. These facts cause greater wear and reduce overall machine lifetime and reliability and must be taken into account when designing and developing

them. The development of computer technology, numerical mathematics, and informatics enables the use of analytical and numerical methods in the design, development, and construction of cam mechanisms.

A general cam mechanism is typically referred to as a three-link mechanism with a single degree of freedom, which consists of two moving members mounted on a fixed frame (1). The moving members are the cam (2) and the follower (3) (see an example of a cam mechanism in **Figure 1**). In the case of the general cam mechanism, the general kinematic pair is formed of the contact of the working surfaces of the cam and the follower. The contact strain of the surface of the contact areas of the said type of kinematic pair, and in the vicinity of this surface, has a periodic course. As a result of the contact strain action, fatigue damage can occur on the contact surfaces. One of the criteria for such damage may be a value of the largest compressive principal stress in the contact area and in its vicinity. This is subsequently brought into the relationship with the strength limit of the respective material. The lifetime of the cam mechanisms is closely related to the choice of materials and their physical and mechanical properties from which the individual components are made, the method of material processing, and the technology of manufacturing of the individual parts or the way and the intensity of loading. It is connected with the way, intensity, and conditions of loading too. This issue connects the knowledge of theoretical, applied, and contact mechanics, tribology, material engineering, and structural analysis, though this theoretical knowledge must be supported by results from experimental identification.

In technical practice, multi-body cam systems are often encountered in addition to the basic three-body cam systems, which may also contain transforming linkages and mechanisms with constant gear ratio. Such mechanical systems are termed cam hinge or also combined cam mechanisms. These mechanical systems with a single degree of freedom are characterized by a uniform motion of the driving member, which generally does not need to be a cam, in most of their applications. The shape of the working surface of the cam realizes the working motion of the working member of the system, which is not necessarily a follower. In many cases, the kinetostatic analysis method is sufficient for the basic determination of the time course of the dynamic behavior of these mechanical systems. The kinetostatic solution determines the driving force effects, reactions in joints, and the force effects transmitted by the cam mechanism and linkages. For the solution, the knowledge is necessary of the kinematic quantities and geometrical mass parameters of all members of the mechanism as well as action force effects acting on



**Figure 1.**  
*A conjugate cam mechanism with an oscillating roller follower.*

individual bodies. The results of kinetostatic analysis thus become the basic data for determining the distribution of contact stress in the contact area of the cam and follower.

## 2. Cam mechanisms

This section gives only basic information on how to solve the tasks of combined cam mechanisms. Fundamental terms and methods of analysis for the investigation of this type of the mechanical systems are specified. For example, the results of the kinematic analysis and synthesis are further used in kinetostatic or dynamic analysis tasks of these mechanical systems. The main goal of these analyses is to determine the courses of reactions in kinematic pairs, force loading of the cam and the follower, force effects acting on individual members, driving effects needed in the systems' movement, etc. Knowledge of the acting forces in the cam mechanism general kinematic pair is significant to determine the contact stress distribution in the contact of the cam and the follower.

Detailed knowledge of this issue may be found in [1, 2].

### 2.1 General cam mechanism

By definition, a general cam mechanism refers to a three-link mechanical system with a single degree of freedom that contains at least one cam linked with other members by means of at least one general kinematic pair. In this case, the general kinematic pair is formed by contacting the cam and the follower, whose movement is translation, rotation, or general. The cam mechanisms can implement a required working motion within a very precisely prescribed path with the use of a small number of bodies housed inside a relatively small space. The cam is the driving (or also input) member; in terms of shape, it is possible to define the basic cam types: radial, axial (cylindrical), and globoid. The follower is termed as the driven (or output or also working) member of a cam mechanism, which carries out the desired motion. The translating follower motion is defined as a translational or a general. The rotating follower, which performs a rotational motion, is usually called the lever. In order to reduce the passive resistance, the follower is often equipped with a roller in technical practice, whereby pure rolling in the interaction between the cam profile and the follower occurs, as is shown in **Figure 1**. This arrangement does not influence on the required follower motion.

One of the main conditions for proper operation of the cam mechanism is to maintain permanent contact of the follower with the cam during the action. This constraint of a general kinematic pair is achieved by a load or a redundant kinematic constraint. In the first case, the given contact is held using preloaded returnable compression springs, gravity forces, or inbuilt hydraulic or pneumatic elements. The disadvantage of this arrangement is the increased force loading and wear of the cam mechanism, which is caused by the preload required for the permanent contact between the follower and the cam. In the second case, contact by the redundant constraint is ensured by adding an extra linkage. For example, a grooved cam can realize such an arrangement. This embodiment is simple, but its disadvantage is the change in the rotation direction of the roller in the cam groove during the relative movement between the roller follower and the cam. This phenomenon is caused by a change in the sense of the transmitted normal reaction between the roller and the cam groove, because the pole of relative motion changes during the cam mechanism operation. As a result, the working surfaces of the groove are more worn in the points of the change in the roller rotation. Dual cam and roller follower systems are

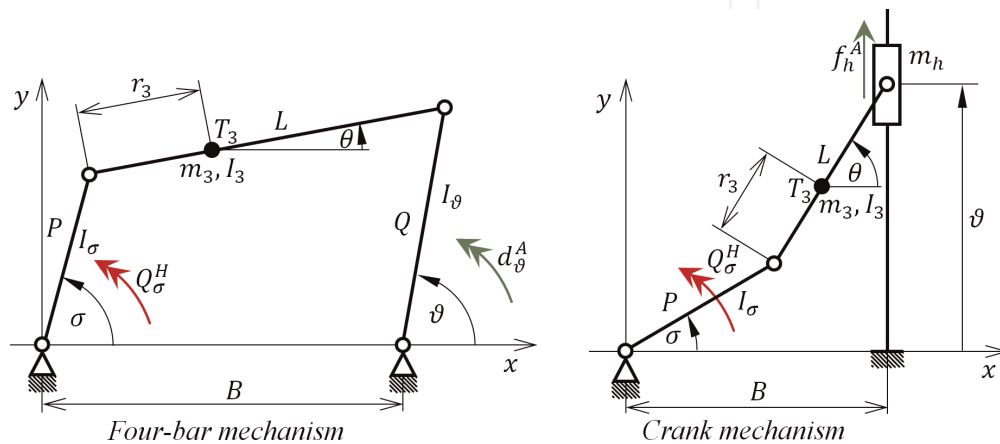
a more preferred design of the cam mechanism with the redundant kinematic constraint, although this solution is more expensive and complicated to manufacture. The conjugate, complementary, or double-disc cam is one including dual radial discs, each in contact with at least two driven followers coupled by a rigid or a kinematic linkage. The mobility of this mechanism is ensured by a special dimension arrangement, where the actions of both working surfaces of the dual cam must correspond exactly to each other. A schematic representation of a cam mechanism with radial conjugate cams and an oscillating dual roller follower is shown in **Figure 1**. The given constraint consists of another radial cam  $II$  and a roller follower  $II$ .

The shape of a cam contour is determined by the synthesis which is on the basis of the knowledge of the displacement law of the given cam mechanism and its dimensional parameters. The position of the cam relative to the frame of the cam mechanism is determined with an angular variable  $\psi$ , and the position of the follower is indicated with the generalized variable  $v$  (see **Figure 1**).

## 2.2 Combined cam mechanism

As a combined cam mechanism, it is generally called a mechanical system usually with a single degree of freedom which includes at least one general cam mechanism. This system usually also includes sets of various transforming linkages with not only a constant but also a generally variable gear ratio. They are most often complemented by simple linkages with lower kinematic pairs. In practice, lower pairs are generally planar couplings between two movable adjusted neighboring members. The members connected by a kinematic pair with the frame are referred to as the basic members of the transforming linkages. They perform rotational or translational motion. The basic representatives of such mechanisms are four-bar mechanism, crank mechanism, oscillating mechanisms, gears, etc. (see examples in **Figure 2**). The following findings are presented in accordance with the knowledge in publication [1, 3, 4]:

The input of the relevant linkage is an ordered triple of variables  $\sigma_k = (\sigma_k, \dot{\sigma}_k, \ddot{\sigma}_k)$  expressing motion of the input link of a mechanical system. The output is a triple of variables  $\vartheta_k = (\vartheta_k, \dot{\vartheta}_k, \ddot{\vartheta}_k)$ , which represents the motion of the output link of the same system. Index  $k$  denotes the numerical indication of the relevant linkage, and it will be neglected in the next part of the text. The general equation of the linkage may be defined as an implicit function, in which the positional magnitudes  $\sigma, \vartheta$  of basic members are time-related:



**Figure 2.**  
Some common types of transforming linkages.



$$F(\sigma, \vartheta) = 0, \quad \sigma = \sigma(t), \quad \vartheta = \vartheta(t) \quad (1)$$

By differentiating Eq. (1) in time, a relation may be obtained between the velocity and the acceleration:

$$\frac{\partial F}{\partial \sigma} \dot{\sigma} + \frac{\partial F}{\partial \vartheta} \dot{\vartheta} = 0, \quad \frac{\partial F}{\partial \sigma} \ddot{\sigma} + \frac{\partial^2 F}{\partial \sigma^2} \dot{\sigma}^2 + 2 \frac{\partial^2 F}{\partial \sigma \partial \vartheta} \dot{\sigma} \dot{\vartheta} + \frac{\partial F}{\partial \vartheta} \ddot{\vartheta} + \frac{\partial^2 F}{\partial \vartheta^2} \dot{\vartheta}^2 = 0 \quad (2)$$

Differentiation with respect to time is denoted by dots in Eq. (2). The linkage ratio is called a magnitude, which is dependent on the position of the linkage, and it is given by Eq. (3):

$$\frac{d\vartheta}{d\sigma} = -\frac{\partial F}{\partial \sigma} \left( \frac{\partial F}{\partial \vartheta} \right)^{-1}, \quad \frac{d\sigma}{d\vartheta} = -\frac{\partial F}{\partial \vartheta} \left( \frac{\partial F}{\partial \sigma} \right)^{-1} \quad (3)$$

The derivatives of Eq. (3) with respect to positions are given by Eq. (4):

$$\begin{aligned} \frac{d^2 \vartheta}{d\sigma^2} &= - \left[ \frac{\partial^2 F}{\partial \sigma^2} + 2 \frac{\partial^2 F}{\partial \sigma \partial \vartheta} \cdot \frac{d\vartheta}{d\sigma} + \frac{\partial^2 F}{\partial \vartheta^2} \left( \frac{d\vartheta}{d\sigma} \right)^2 \right] \left( \frac{\partial F}{\partial \vartheta} \right)^{-1}, \\ \frac{d^2 \sigma}{d\vartheta^2} &= - \left[ \frac{\partial^2 F}{\partial \vartheta^2} + 2 \frac{\partial^2 F}{\partial \sigma \partial \vartheta} \cdot \frac{d\sigma}{d\vartheta} + \frac{\partial^2 F}{\partial \sigma^2} \left( \frac{d\sigma}{d\vartheta} \right)^2 \right] \left( \frac{\partial F}{\partial \sigma} \right)^{-1} \end{aligned} \quad (4)$$

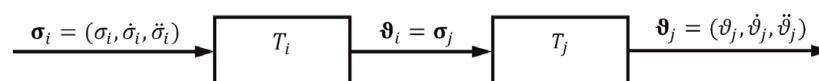
Introducing Eqs. (3) and (4) into Eq. (2) gives:

$$\dot{\vartheta} = \frac{d\vartheta}{d\sigma} \dot{\sigma}, \quad \ddot{\vartheta} = \frac{d\vartheta}{d\sigma} \ddot{\sigma} + \frac{d^2 \vartheta}{d\sigma^2} \dot{\sigma}^2, \quad \dot{\sigma} = \frac{d\sigma}{d\vartheta} \dot{\vartheta}, \quad \ddot{\sigma} = \frac{d\sigma}{d\vartheta} \ddot{\vartheta} + \frac{d^2 \sigma}{d\vartheta^2} \dot{\vartheta}^2 \quad (5)$$

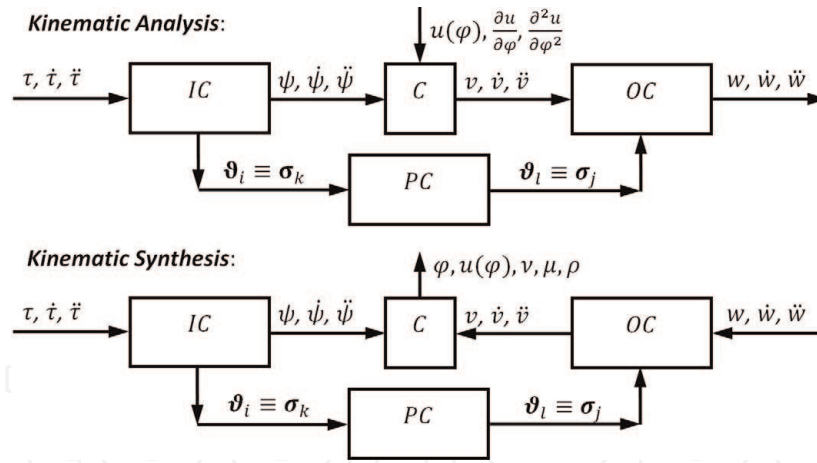
which expresses relations for calculating the velocity and the acceleration of the output member.

The individual linkage can be placed into so-called *chains* when the outputs of the given mechanism are also the inputs of the next mechanism. The algorithms of linkage solving are marked with transformation blocks  $T_k$  as shown in **Figure 3**.

In relation to solving the problems with the combined cam mechanisms, we have introduced the following nomenclature (designation) of positional and kinematic magnitudes of the main members. Variable  $\tau$  is assigned to the driving (input) member of a combined cam mechanism. The position of the driven (output) member is indicated with variable  $w$ . As a driven member, we usually consider such a working member or a body whose values are the result of the solution. The cam position with respect to the frame of the general cam mechanism is determined with an angular variable  $\psi$ , and the position of the jack is denoted by the generalized variable  $v$  (see **Figure 1**). The procedure of calculating a combined cam mechanism is shown through a block diagram, which is formed from transform blocks  $T_k$  and transform block  $C$  (see **Figure 4**). Transform block  $C$  expresses the algorithms of the solution of the general cam mechanism. From the view of the structure of a block diagram, it is advisable to divide the transform blocks into three groups. They are identified by this term *chain*:



**Figure 3.**  
A chain of transformation blocks of linkages.



**Figure 4.**  
Block diagrams of the computation of combined cam mechanisms.

- The input *chain* *IC* connects the input of the mechanical system to the input of block *C*.
- The output *chain* *OC* connects the output of block *C* to the output of the mechanical system.
- The parallel *chain* *PC* consists of blocks that do not belong to previous chains.

Indexes  $i, j, k$ , and  $l$  denote the  $i^{\text{th}}, j^{\text{th}}, k^{\text{th}}$ , and  $l^{\text{th}}$  linkage of the relevant chain.

The calculation of the positional and kinematic quantities of any member of a combined cam mechanism is designed as a kinematic analysis (see the block diagram in **Figure 4**). The input data are typical data (geometrical mass quantity and dimensions) on a mechanism, the procedure of computation of its chains, displacement function  $\tau = \tau(t)$  of the driving member, and the shape of the cam theoretical profile  $u(\varphi)$ . The output of the solution is the displacement of the selected member of the mechanical system, which is expressed by a triple of variables  $\mathbf{w} = (w, \dot{w}, \ddot{w})$ . Kinematic synthesis of a combined cam mechanism is used to the computation of polar coordinates  $\varphi, u$  of a radial cam or cylindrical coordinates  $\varphi, x, y$  of an axial cam. Furthermore, via synthesis, normal angle  $\nu$ , pressure angle  $\mu$ , and radius of curvature of the cam profile  $\rho$  are set. The input data of the task include data on a mechanical system, the calculation procedure of its chains, displacement function of driving member  $\tau = \tau(t)$ , and driven member of the system  $w = w(t)$ .

### 2.3 Displacement law

The shape of a cam contour is determined by the synthesis which is on the basis of the knowledge of a displacement law of the given combined cam mechanism and its dimensional parameters. The following observations on the displacement laws are presented in accordance with the knowledge in publication [1].

A function assigning time  $t$  to a position variable of a given member of a mechanical system is termed to a motion function  $w = f(t)$  of that member. The motion of the driving member is thus described by the independent motion function  $\tau(t)$  and the motion of the driven member by the dependent motion function  $w(t)$ . The displacement law expresses a functional dependency of the driven member motion on the driving member motion of a combined cam mechanism  $w = f(\tau)$ . Displacement laws  $w(\tau)$  of mechanical systems with the rotating input member are periodical functions with a period of  $2\pi$ . The period  $2\pi$  may be divided

into motion and dwell intervals. Displacements on each motion interval may be different to a maximum total rise  $W$  and an expression of the normalized form  $\eta = \eta(\xi)$  where the displacement and the range are in unity (see **Figure 5**). The variables  $\xi_k$  and  $\eta_k$  of  $k^{\text{th}}$  motion interval are in linear correlation with the original variables  $\tau$  and  $w$  and can be expressed as

$$\xi_k = \frac{\tau - \tau_{0k}}{T_k}, \quad \eta_k = \frac{w(t) - w_{0k}}{W_k} \quad (6)$$

The initial point  $O_k$  of displacement on each motion interval is defined by the coordinates  $\tau_{0k}, w_{0k}$ . The interval length of the independent variable  $\tau$  is given by the magnitude  $T_k > 0$ , and the maximum lift is expressed by the magnitude  $W_k \geq 0$ . The relationship between the original and normalized derivatives is

$$\frac{dw}{d\tau} = \frac{W}{T} \cdot \frac{d\eta}{d\xi} = \frac{W}{T} \cdot \eta'(\xi), \quad \frac{d^2w}{d\tau^2} = \frac{W}{T} \cdot \eta''(\xi) \quad (7)$$

Derivatives of the unity displacement to  $\xi$  will be denoted by primes, and index  $k$ , indexing the motion period, is neglected. When the variable  $w$  is a function  $t$ , then the derivatives will have the form

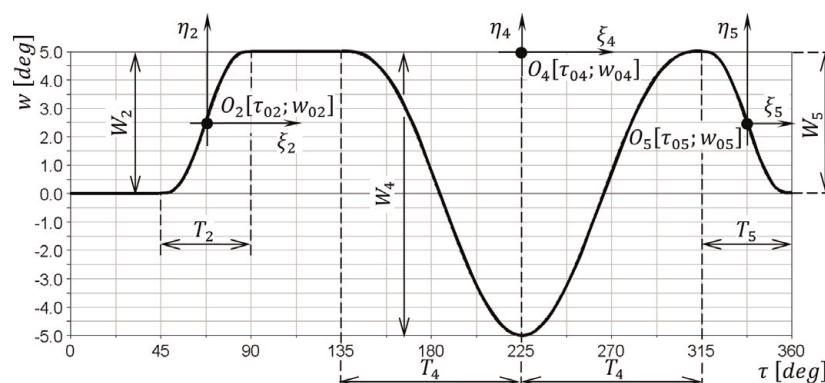
$$\frac{dw}{dt} = \frac{W}{T} \cdot \eta'(\xi) \cdot \frac{d\tau}{dt} = \frac{W}{T} \cdot \eta'(\xi) \cdot \dot{\tau},$$

$$\frac{d^2w}{dt^2} = \frac{W}{T} \cdot \left[ \frac{1}{T} \cdot \eta''(\xi) \cdot \left( \frac{d\tau}{dt} \right)^2 + \eta'(\xi) \cdot \frac{d^2\tau}{dt^2} \right] = \frac{W}{T} \cdot \left[ \frac{1}{T} \cdot \eta''(\xi) \cdot \dot{\tau}^2 + \eta'(\xi) \cdot \ddot{\tau} \right] \quad (8)$$

For the solution of problems related to the kinematic analysis and synthesis of cam systems, it is possible to use a broad set of displacement laws in a normalized form. These include, for example, polynomial, trigonometric, and exponential displacements and cycloidal, parabolic, and goniometric displacements (see [1]).

The choice of displacement law greatly has an influence over the dynamic properties and behavior of high-speed mechanical systems, and it should meet the following basic criteria within the specified conditions:

- In relation to the desired motion of the mechanism, the acceleration inertia forces, momentum, and performance of the cam mechanism should be always as small as possible.



**Figure 5.**  
An example of a displacement law.



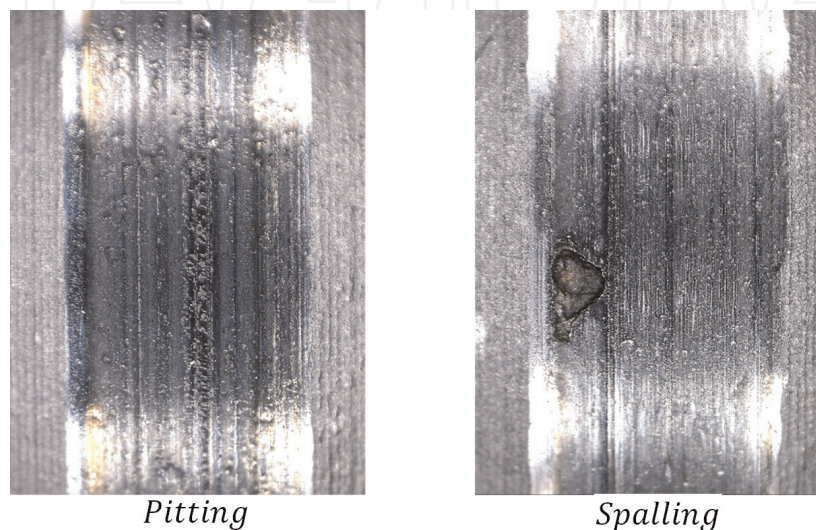
- The vibrations forced by the movement of the mechanical system should be kept at a minimum.

Both criteria lead to low dynamic strain on the members of the mechanical system due to dynamic effects. In addition, the second criterion is related to the accuracy of adherence to the prescribed working member positions and the elimination of any noise sources. Comprehensive and detailed information on the issues of the displacement law choice is provided in [1].

### 3. Contact of cam and cam follower

In this text section, we will focus mainly on the general kinematic pair formed by contact of a cam and a roller follower. The mentioned type of kinematic constraint in the technical practice is usually most often constituted by a cylindrical roller and a cam or a crowned roller and a cam. In terms of computational purposes, we can substitute both mentioned contacts for the contact of cylindrical bodies with parallel axes and the contact of an elliptical body with a cylindrical one.

The contact area of the general kinematic pair is subjected to cyclic loading within the working cycle, while the contact surfaces are primarily in rolling contact in combination with a small percentage of mutual sliding. Thus, the transmission of normal and tangential forces is realized. These phenomena cause deformation of both bodies in the contact and cause contact stress in them. The state of stress on the working surfaces and under it is characterized by the principal stresses, which are transient and have the character of pulses with a period of  $2\pi$ . Fatigue damage of the cam and follower contact surfaces may occur after a certain number of cycles in the operation of cam mechanisms, as long as a certain limit value of this stress is exceeded at any point of the contact area. This damage is in the form of cavities (pitting), which develop from cracks on the working surface. For cams with a hardened surface, this layer can be broken and then peels off (spalling) (see **Figure 6**). Both types of damage occur due to the contact stress that can be described by the theory of contact mechanics (see [5]). In terms of estimating the lifetime of the contact areas of the general kinematic pair, the distribution of the reduced stress in the surface areas and at a certain depth under it is therefore an important criterion. Thus, in the area of cam mechanisms, it is primarily a matter of determining the service life of the cam and follower contact surfaces depending on the conditions of their force loading.



**Figure 6.**  
*Some common types of fatigue damages of cams and rollers.*

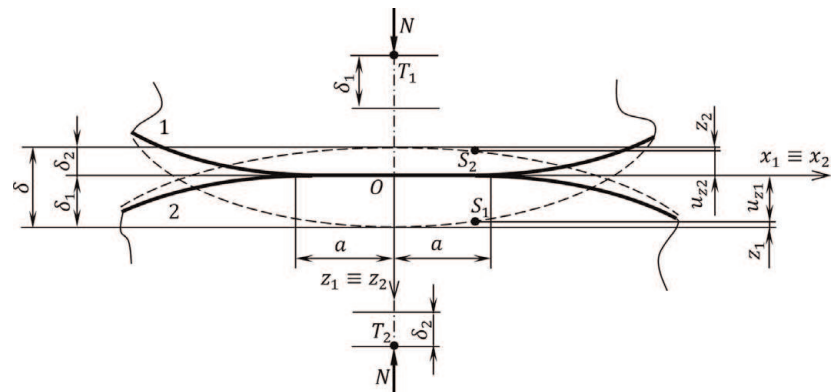
The state of deformation and stress existing between the two elastic bodies in contact under load can be established both based on the contact mechanics and based on the use of the finite element method (FEM). The contact mechanics deals with the study of stress and deformation of solids being in contact at one point or along a line, acting under normal and also tangential forces. Physical and mathematical relationships are formulated on the basis of knowledge of continuum mechanics as well as mechanics of materials with the focus on elastic, viscoelastic, and plastic bodies in static or dynamic contact. The principles of contact mechanics are used to solve the problems of contact of rolling bodies (balls, rollers, barrels, needles, tapered rollers) and roller bearing rings, the contact of teeth in gearings, the contact of railway wheels and rails, mechanical constraints, and, last but not least, the contact of the cam and cam follower. The result of the calculations is also Hertzian contact stress, where there is local stress in the contact area, being caused by the contact of two curved areas, whereas these are slightly deformed due to the acting load. Hertzian contact stress is a fundamental quantity in formulating the equations for determining the carrying capacity and fatigue lifetime of cam mechanisms, bearings, gearings, and all objects in general, whose surfaces are in contact. Comprehensive and detailed information on the issues of contact mechanics is provided in [5].

### 3.1 Contact stress

When two three-dimensional bodies are brought into contact, they touch initially at a single point (*contact of the convex crowned roller with the cam*) or along a line (*contact of the cylindrical roller with the cam*). Under the action of the slightest load  $N$ , they will deform, and contact is made over a finite area which is small, compared with the dimensions of both bodies. During the compression distant points of the two bodies ( $T_1$  and  $T_2$ ) are approached the each other by a distance  $\delta$ . Points  $S_1$  and  $S_2$  on the approaching contact surfaces are elastically displaced by amounts  $u_{z1}$  and  $u_{z2}$ , as shown in **Figure 7**. The shape of each surface in the contact region can be described by a homogeneous quadratic polynomial in two variables [5]:

$$z_i = \mp \frac{1}{2} \left( \frac{1}{\rho_{xi}} x^2 + \frac{1}{\rho_{yi}} y^2 \right), \quad i = 1, 2 \tag{9}$$

where  $\rho_{xi}$  and  $\rho_{yi}$  are the principal radii of curvature of the surface at the rectangular coordinate system origin (see **Figure 8**). Contact stresses and deformations satisfy the differential equations for the stress and strain of homogeneous,



**Figure 7.**  
*The contact of two nonconforming bodies after elastic deformation.*

isotropic, and elastic bodies in equilibrium. The pressure distribution on the contact area is given by the equation [5]:

$$p(x, y) = p_H \sqrt{1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2} \quad (10)$$

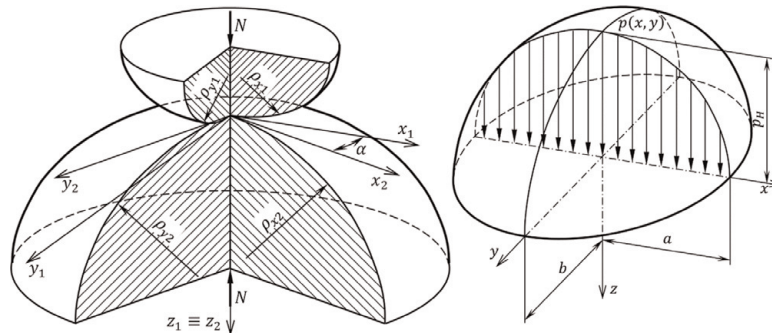
where  $a$  and  $b$  are respective major and minor semi-axes of the elliptical contact area and a maximum value  $p_H$  is called Hertzian pressure. The elasticity characteristics of the bodies in contact are introduced with the effective modulus of elasticity [5]:

$$\frac{1}{E^*} = \sum_i \frac{1 - \nu_i^2}{E_i}, \quad i = 1, 2 \quad (11)$$

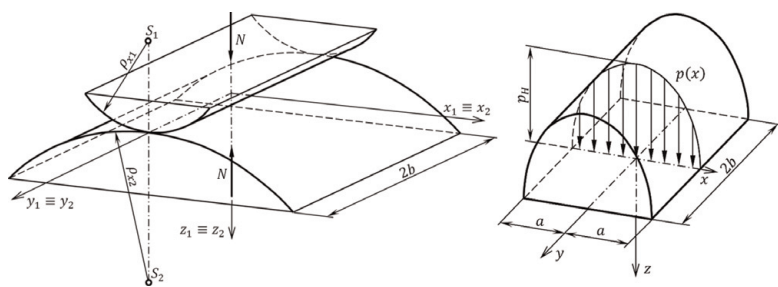
where  $E_i$  and  $\nu_i$  are the respective Young's modulus of elasticity and Poisson's ratio of the individual solids.

The contact stress is highly concentrated in the vicinity of the contact area and decreases rapidly with an increasing distance from it. Thus, the stress area is close to the body contact. Since the contact surfaces are dimensionally small compared to the rest of the bodies, the stresses around the contact area are not too much dependent neither on the shape of the bodies in the contact nor on the way of mounting the bodies. This hypothesis simplifies the definition of boundary conditions and allows applying the theory of elasticity of large bodies.

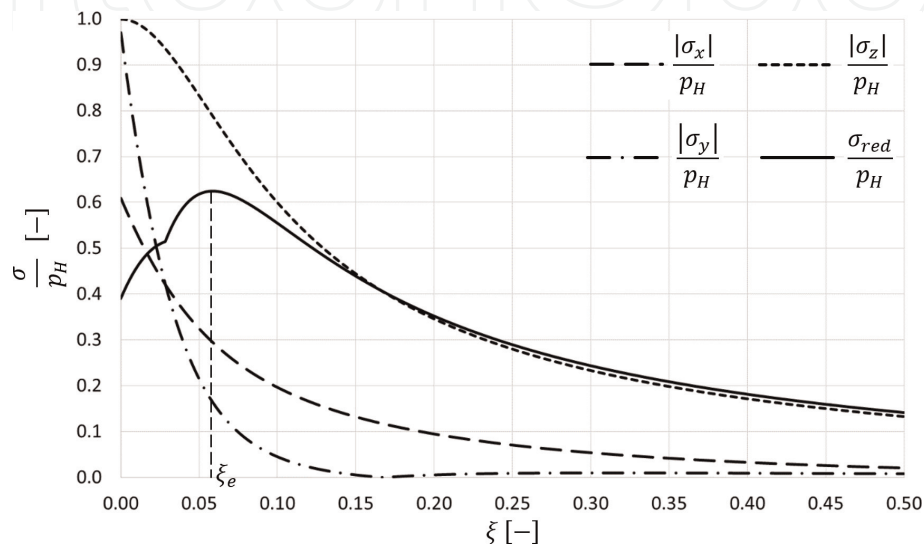
In the case of the contact of *the convex crowned roller with the cam* and *the cylindrical roller with the cam*, it is clear that the abovementioned assumptions are satisfied. Thus, the results of Hertzian contact stress theory can be used to determine the stress state in their contact areas and at a certain depth under the surface (see [5]) or to use the results related to these contact types being given in publications [3, 6, 7], or we can directly calculate the contact stress for point or line contact by using the computational algorithms available on the webpage [8]. In **Figure 8**, there is a schematic presentation of the contact of the convex crowned roller and the cam (left) and distribution of the contact stress in the contact area (right). A similar case is shown in **Figure 9** with the only difference that there is the contact of the cylindrical roller and the cam. The stress state is in these cases expressed by principal stresses  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$  and reduced stress  $\sigma_{red}$ . These are universally compressive stresses, and their absolute value decreases with the distance from the surface. **Figure 10** generally describes the courses of individual stresses in the symmetry plane  $xz$  in dependence on depth  $z$  under the surface. From the course of the reduced stress  $\sigma_{red}$ , it is obvious that its maximum value is at a certain depth of  $z_e$ , which is expressed in unit form  $\xi_e$ . The principal stresses  $\sigma_x$ ,  $\sigma_z$  reach their extreme



**Figure 8.**  
Point contact.



**Figure 9.**  
 Line contact.



**Figure 10.**  
 The stress state in a symmetry plane *xz*.

values on the surface of the bodies in the contact area and equal to the Hertzian pressure value.

Another possible way of determining the stress in the contact areas of bodies is to use the finite element method. To achieve the relevant results, a dense finite element mesh in the contact area is required. This requirement leads to a large number of solved linear algebraic equations. The computational body contact algorithm is based on a numerical iteration of finding the elements in contact, so the numerical solution of the equations of the assignment takes place in several steps.

### 3.2 Reduced stress

In order to estimate the lifetime of the contact areas, it is first necessary to determine the magnitude of the reduced stress  $\sigma_{red}$ , which is compared with the value of the stress limit  $\sigma_h$ . The permissible stress determined on the basis of yield tensile strength  $R_{p0.2}$  or ultimate tensile strength  $R_m$  refers to uniaxial stress. It is therefore necessary to have a criterion for comparing uniaxial stress and multiaxial stress in order to assess the strength at the multiaxial stress. These criteria provide hypothesis of strength. In the case of malleable materials, the failure occurs in the maximum shear stress plane at the tensile stress. The Guest’s hypothesis is used (see, e.g., [9]). According to this hypothesis, the stress is judged from the point of view of the highest shear stress, which is proportional to the maximum of differences in the principal stresses:



$$\sigma_{red} = \max\{|\sigma_x - \sigma_y|, |\sigma_z - \sigma_x|, |\sigma_z - \sigma_y|\} \quad (12)$$

The principal stress components  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$  in the condition Eq. (12) are determined depending on the kind of contact of the general kinematic pair. These components must be established over the whole operation cycle of a cam mechanism to determine their maximum values. **Figure 10** shows that the highest value of the reduced stress is at a certain distance under the surface. In order to calculate the principal stresses, it is possible to use conclusions of contact mechanics for the respective contact of two elastic bodies, but it is also possible to utilize the method of finite elements (see [6, 7]).

In the case of general kinematic pairs, the contact load on the contact surface of the bodies and under it has the periodic course. At the contact areas of the general kinematic pair and below these points, contact stress becomes a periodical magnitude related to the angular cam displacement  $\psi$ . These transitory stresses are characterized by pulses with a periodicity  $2\pi$ . The strength conditions acting under a variable loading are shown in Smith's diagram (see [10]), where the mean stress is determined by the equation:

$$\sigma_m = \frac{1}{2} [\max\sigma(\psi) + \min\sigma(\psi)], \quad \psi \in \langle 0, 2\pi \rangle \quad (13)$$

and its amplitude by the equation:

$$\sigma_a = \frac{1}{2} [\max\sigma(\psi) - \min\sigma(\psi)], \quad \psi \in \langle 0, 2\pi \rangle \quad (14)$$

In Eqs. (13) and (14), the variable  $\psi$  presents the rotation angle of the cam. Strength conditions under a variable load are given by the stress limit  $\sigma_h$  which is in agreement with the disturbance caused by the transitory stress. The reduced stresses  $\sigma_{red}(\psi, z)$  are limited by the actual strength condition, written as [1]:

$$\max\sigma_{red}(\psi, z) < \sigma_h, \quad \psi \in \langle 0, 2\pi \rangle, \quad z \geq 0 \quad (15)$$

where variable  $z$  is the depth under the contact surface of the cam or roller. Based on the tests of steels with ultimate tensile strength  $R_m \in \langle 500, 1500 \rangle \text{ MPa}$ , the fatigue limit of the material loaded by the tension is supposed as  $\sigma_C \approx 0.35R_m$  and the yield strength as  $R_{p0.2} \approx (0.55 \div 0.8)R_m$  (see [1]). Since the limit, the value of the transient stress, can be approximately determined according to  $\sigma_h \approx \sigma_C \approx 0.7R_m$ , then it is possible to replace the condition Eq. (15) by the inequality [1]:

$$\max\sigma_{red}(\psi, z) < R_{p0.2}, \quad \psi \in \langle 0, 2\pi \rangle, \quad z \geq 0 \quad (16)$$

Conditional inequality Eq. (16) describes the fact that during the operation of the cam mechanisms, no destructive action of elastic deformation occur in the general kinematic pair.

### 3.3 Lifetime of contact areas

The criterion of damage of the loaded contact areas of the cam and the follower in cam mechanisms is the formation of cracks and cavities, so-called pitting. The problem of the pitting formation on the surfaces of the bodies in contact is with that the initiation and propagation of cracks in the loaded material is completely unpredictable because the crack nuclei that form are distributed randomly in the



material. Such crack nuclei are inclusions in material or surface irregularity caused by the production and treatment of this material. Therefore, it is difficult to predict exactly the stress state at the point of the contact area in which the damage occurs due to the load. This problem can be solved by introducing a criterion whereby the value of the highest principal compressive stress is determined in the contact area of the bodies, and this is brought into relation with the ultimate strength of the respective material. In the case of pure rolling contact, its magnitude is equal to the maximum value of the contact pressure—Hertzian pressure  $p_H$ .

In the previous part of the article, we briefly discussed ways of determining the stress in the surface and subsurface parts of the contact areas of the bodies. The following text will mention two theoretical approaches, one of which is presented in [1] and the other in [2]. Both procedures result from the knowledge of the contact mechanics, and based on this, the Hertzian pressure value  $p_H$  in the body contact area has been determined. Both methods require further the knowledge of some or several parameters that characterize the material from which the cam is made. These characteristic parameters are not available for all materials; therefore it is necessary to use experimental procedures. Not all materials are available for these materials, so further experimental procedures are required.

In the course of the operation cycle of the cam mechanism, no damage caused by the formation of pits is acceptable on the contact surface under load. Referring to [1], such fatigue damage will not occur if Hertzian pressure is given by the Niemann empirical relation in the form:

$$p_H \leq \frac{K}{N^{1/6}} f(H), \quad K = 4777 \text{ MPa} \quad (17)$$

where the variable  $N$  is the lifetime in millions of cycles,  $H$  is the surface hardness in contact, and the function  $f(H)$  represents the influence of the surface hardness on the permissible Hertzian pressure  $p_H$  (MPa). The factor  $K$  is a constant being determined empirically. The surface hardness  $H$  related to the maximum Hertzian pressure limit  $p_{H\text{Max}}$  in the course of the operation cycle is denoted as  $HB$  for the Brinell and  $HRC$  for the Rockwell scales. Then, for the function  $f(H)$ , the following empirical relations are formulated:

$$f(H) = \frac{HB}{1000}, \quad f(H) = 0.251 + \frac{HRC}{100} \left[ \frac{HRC}{100} \left( 2.74 - 1.22 \frac{HRC}{100} \right) - 0.6 \right] \quad (18)$$

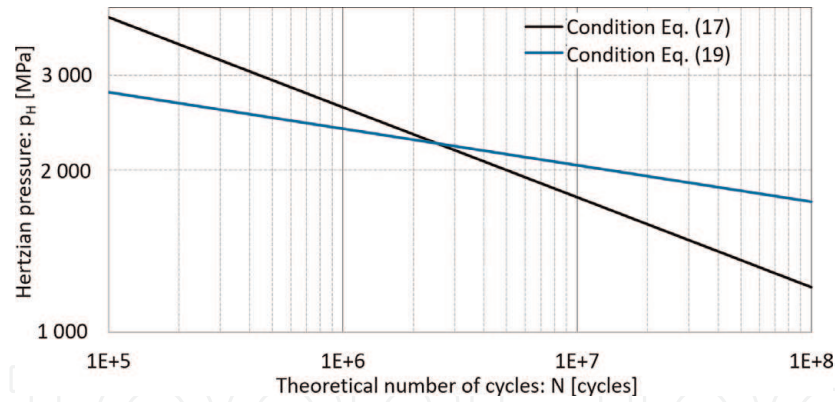
This criterion Eq. (18) is very simple because it is dependent on the only material parameter  $H$ .

Based on conclusions introduced in [2], a condition for the level of stress can be derived, in which the contact surface of the body will not be damaged:

$$p_H \leq \sqrt{\frac{E^*}{145.03789 \cdot \pi}} \left( \frac{10^\zeta}{N} \right)^{\frac{1}{\lambda}} \quad (19)$$

in which the constant  $E^*$  characterizes material elasticity of bodies in contact that is defined by Eq. (11). In Eq. (19) the strength factors  $\zeta$  and  $\lambda$  express the slope and the intercept with the stress axis of the so-called  $S$ - $N$  diagram in logarithmic coordinates for the surface fatigue limit of the respective material.  $S$ - $N$  diagram is determined by regression analysis from a large number of test data.

Both conditions according to relations Eqs. (17) and (19) are illustrated graphically for selected steel C22 (1020, 1.0402) in **Figure 11**. The graphical



**Figure 11.**  
Load-life relationships for steel C22.

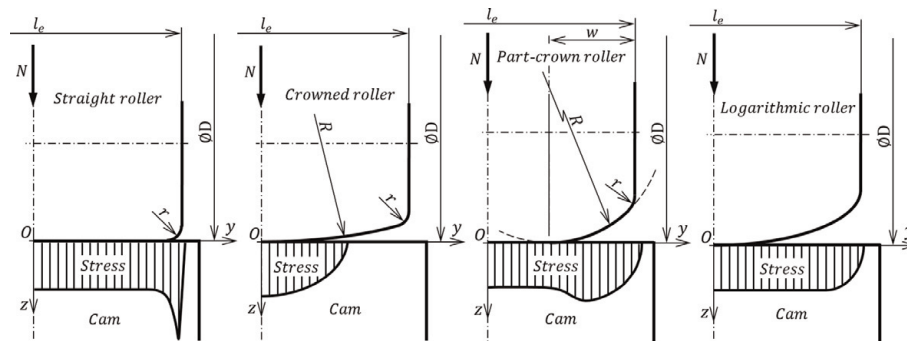
representation shows a considerable difference between the mentioned theories in determining the lifetime of bodies in contact. In the region of limited lifetime, which is given by the number of cycles to damage  $N = 10^7$  cycles, the conditions for the lifetime determination are according to condition Eq. (19) are usually more safety-related than condition Eq. (17). In the region of an unlimited lifetime, i.e.,  $N > 10^7$  cycles, it is the opposite case. Similar conclusions in both approaches to the determination of the lifetime of bodies in contact were also proved for other steels.

#### 4. Effect of the roller crown shape on the cam stress

The geometrical shape of the roller crown itself has a significant effect on the stress distribution, due to the load and inertia effects in the contact areas of the general kinematic pair, which is usually formed in practice by a cylindrical roller and a cam or a crowned roller and a cam (see **Figure 12**).

It has been proven that the contact stresses in the vicinity of the shape, discontinuities in the contact area of the bodies in contact, are considerably higher than those reached outside the area of their immediate influence as described in the publication [2]. For this reason, the contact surfaces are more stressed, and their fatigue lifetime is reduced.

In general, we can expect to achieve a longer lifetime of the general kinematic pair of any cam mechanism by using the cylindrical profile of the roller crown. However, in the case of a conventional straight roller, there are discontinuities at the intersection of the roller cylindrical profile with the cam profile. These are caused by the fact that one contact area is axially shorter than the other and also by



**Figure 12.**  
Some types of roller profiles in contact with a cam.

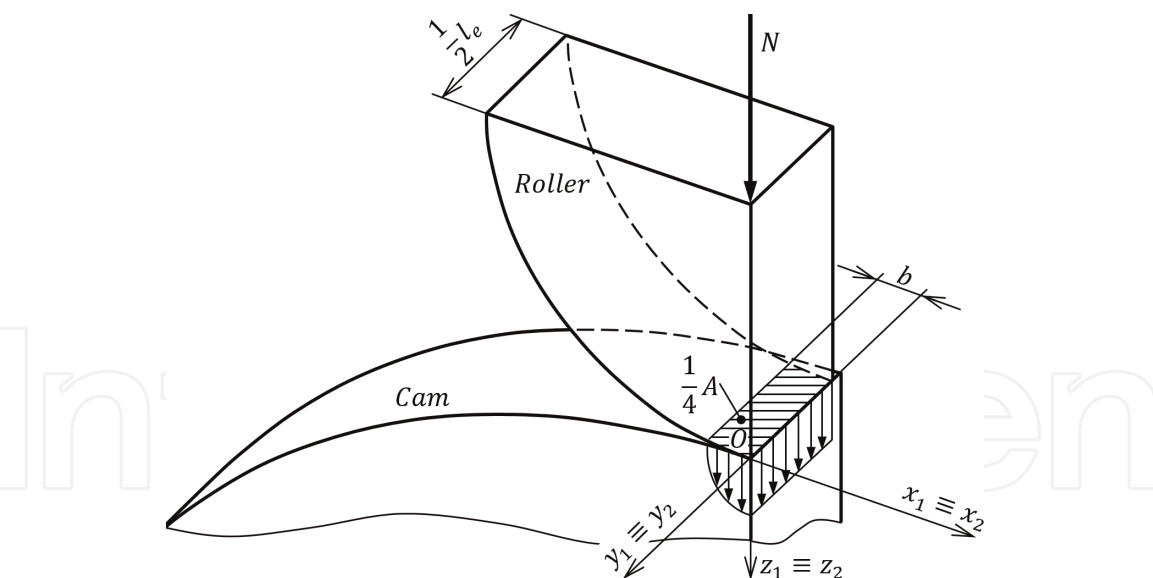
chamfering the roller edges. In the vicinity of those profile discontinuities, the contact between the roller and cam cannot be considered as simply a straight line contact but for a more complex three-dimensional contact type. Therefore, in this case, it is not possible to apply the conclusions of Hertz's theory of contact to the calculation of the contact stress distribution. These discontinuities cause a high concentration of contact pressure at the appropriate point in the contact area. In fact, these local increases in the distribution of contact pressure can exceed the strength limit of the given material and thus cause plastic deformation in the contact area, the formation of residual stresses in the material, or hardening of steel. The area in question will further be more prone to fatigue damage to the contact surfaces such as pitting or spalling.

To ensure a more even distribution of contact stress in the contact area of the roller and cam, it is necessary to modify the shape of the axial cross-section of the roller crown. This is one of the reasons for the practical application of crowned rollers, when the radius of curvature of the crown profile is far greater than the radial radius of the roller (see **Figure 12**). Furthermore, we can easily compensate with their use misalignment between the roller and cam, without causing a fundamental change in the contact stress distribution. However, the largest concentration of contact stress distribution is achieved in the middle of the contact area with all the consequences as in the case of the straight roller.

The reduction of excessive contact stress in the vicinity of straight roller edges is achieved by such a shape of the axial cross-section of the crown, which includes straight line and tangent circular arcs that are connected to it on each side (see **Figure 12**). However, this shape of the roller crown leads to a certain concentration of contact stress in the transition from the cylindrical segment of the roller to the crowned one. Uniform distribution of stress for different levels of loading of the general kinematic pair contact areas is achieved by the logarithmic profile of the roller crown (see [11, 12]). This type of the roller crown is characterized by a monotonously decreasing profile from its center to the edge according to the logarithmic function (see **Figure 12**).

These facts will be demonstrated on examples of cam contacts with a cylindrical, crowned, and part-crown roller, which were defined using the finite element method (see [3, 13]). The formation of the model of a general kinematic pair using the finite element method is based on assumptions on the basis of which Hertz's theory of contact of two elastic bodies is derived (see [5]). The main assumption is that the contact area is continuous and much smaller than the characteristic dimensions of the bodies in contact. Therefore, the stresses in the vicinity of the contact area are not so dependent on the shape of the bodies in contact nor on how these bodies are fixed. Furthermore, it is assumed that the contact stress is very concentrated in the vicinity of the contact region and rapidly decreases with an increasing distance from it. The region of stress acting is therefore in the vicinity of the contact of the bodies. Through these basic assumptions, the definition of boundary conditions is simplified, and the application of the theory of elasticity of large bodies with sufficiently small deformations is allowed.

When creating a finite element model of a general kinematic pair of a cam mechanism, the contact of the roller with the cam will be replaced by the contact of two segments of solids of revolution. One of the solids of revolution represents the roller with the desired forming profile, and the other with a cylindrical profile replaces the cam. The radius of the cylinder is identical to the radius of curvature of the cam at the point of its contact with the roller. For the purposes of the computational analysis, we will use one eighth of each of them, assuming the parallelism of the axes of both replacement solids (see **Figure 13**). In the  $O_{xz}$  and  $O_{yz}$  planes in



**Figure 13.**  
A schematic drawing of a roller in contact with a cam.

		Straight roller	Crowned roller	Part-crown roller
Nominal diameter	$D$ [mm]	35.0	35.0	35.0
Effective width	$l_e$ [mm]	18.0	18.0	18.0
Fillet radius	$r$ [mm]	0.6	0.6	0.6
Crown radius	$R$ [mm]	—	500	200
Crown width	$w$ [mm]	—	—	6.0

**Table 1.**  
Nominal dimensions of rollers.

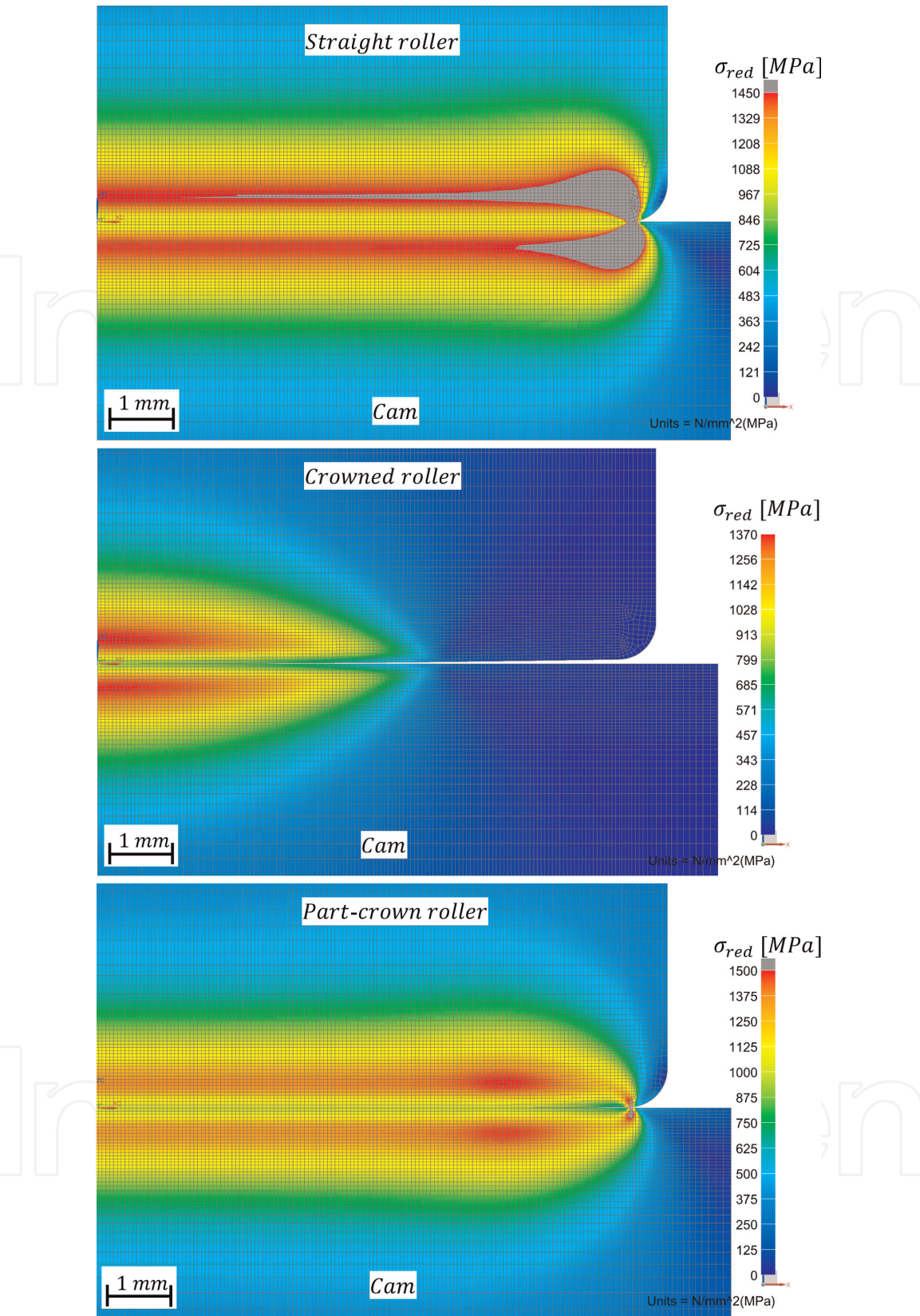
		Roller: 100Cr6	Cam: 16MnCr2
Young’s modulus of elasticity	$E$ [GPa]	210	206
Shear modulus	$G$ [GPa]	81	79
Poisson’s ratio	$\nu$ [—]	0.3	0.3038

**Table 2.**  
Material characteristics of steels.

			Straight	Crowned	Part-crown
Load	$N$ [N]		40,000	15,000	35,000
Hertzian pressure	$p_H$ [MPa]		2500	2400	2400
Maximum reduced stress	Hertz theory	$\max(\sigma_{red})$	1440	1370	1480
	MKP	[MPa]	1970	1360	
Major contact radii	$a$ [mm]		—	5.555	—
Half width of contact/minor contact radii	$b$ [mm]		0.586	0.538	—
Depth of maximum reduced stress	$z_e$ [mm]		0.42	0.38	0.38

**Table 3.**  
Summary of results.





**Figure 14.**  
The stress state in the symmetry plane  $xz$  determined by FEM.

accordance with **Figure 13**, there are then defined boundary conditions of the symmetry of the solved task accordingly. The lower surface of the body, which has the meaning of the cam, is fixed, i.e., all translations and rotations are prevented for all nodes of finite element mesh of the body in this area. Through the definition of the boundary conditions, the upper surface of the roller part is allowed to displace in the direction of the applied force  $N$ , i.e., in the direction of the  $z$ -axis

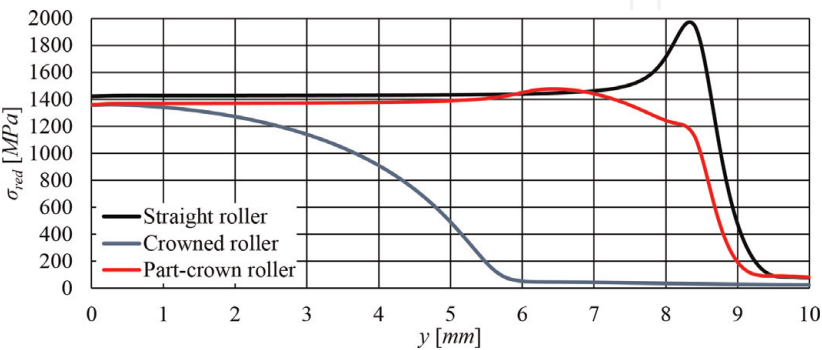


of the  $Oxyz$  coordinate system. Also the acting of force  $N$  is evenly distributed over this area.

To achieve the relevant results, it is important that a uniform and dense finite element mesh be used to the discretization of the contact area and its vicinity of both bodies. The size of the elements can gradually increase with an increasing distance from the contact areas. For example, the size and shape of the contact area can be predicted by calculating based on Hertz's contact theory applied to contact of cylindrical bodies with parallel axes or to contact of a body with general profile and cylindrical body. Using this theory, we calculate the components of deformations and stresses in the contact area and its vicinity of both bodies in contact. Furthermore, the shape and size of the contact region are determined depending on the load size. This issue has been dealt with in articles [3, 6, 7] or is published in detail in [5]. Then, on the basis of the data thus determined, we define the space to create an acceptable finite element mesh density of the analyzed bodies with respect to the corresponding results compared to the real state.

The application of the above procedure will be demonstrated on the analysis of the contact stress distribution in the contact area of the cam, of which nominal dimensions are the width  $l_{Cam} = 20\text{ mm}$  and the curvature radius  $R_{Cam} = 50\text{ mm}$  in the vicinity of the contact point with the respective roller. We will consider three types of rollers, in which the nominal dimensions are listed in accordance with **Figures 12 and 13** and **Table 1**. The aim of the analysis of the contact stress of the cam is the determination of the maximum possible value of the reduced stress  $\sigma_{red}$  in the considered part of the cam depending on the greatest possible load of the roller by force  $N$ . Knowledge of this stress value is crucial with respect to fatigue damage of the working surfaces of the cam and the follower. The characteristic material parameters of the cam and the rollers are stated in **Table 2**.

Significant results are summarized in **Table 3**. Based on the size of the contact areas and the depth of the maximum reduced stress, the area was defined to create a quality network of elements in the vicinity of the contact of three roller and cam types in the creation of appropriate models using the FEM. In the case of cam and roller contact with the crown with convex segments, only the results from the FEM analysis are shown in the table. **Figure 14** shows the distribution of the reduced stress induced by contact of the said cam roller types with the cam. **Figure 15** shows the course of the maximum reduced stress in a depth of  $z_e$  according to the cam width for all rollers types in contact with the cam. Depth  $z_e$  expresses the distance from the contact area of the cam surface where the maximum value of the reduced stress is just obtained.



**Figure 15.**  
The course of the maximum reduced stress depending on the cam width.

From **Table 3** and **Figures 14** and **15**, it is evident that there is an increase in the size of the reduced stresses in the vicinity of the profile discontinuities. This feature is particularly evident in the case of a cylindrical roller, where the contact between the roller and the cam cannot be regarded as merely straight but rather as a more complex three-dimensional type of contact. Therefore, Hertz's theory of contact cannot be applied to this type of contact around the shape discontinuities. Based on this theory, there are very good results compared to the FEM in contact of the general body with the cylindrical one and inside the contact area of the two cylindrical bodies with parallel axes. Furthermore, it is clear that a uniform distribution of stress can be achieved by such a shape of a roller crown whose profile includes the straight and two circular portions according to the schematic representation in **Figure 12**. This roller crown profile is advantageous in terms of load transfer capacity and process of its manufacture.

## 5. Conclusions

The presented chapter gives basic information on the stress problems related to a general kinematic pair of cam mechanisms. This type of kinematic pair is formed with at least one cam and a follower. The general cam mechanism is a very simple three-member mechanical system, which can implement the required working movements very accurately. Therefore, they are widely used in the design of various machines and equipment of the manufacturing industry. With the increasing pressure on the size and quality of machinery production of the manufacturing industry, the demand for its increased performance, reliability, and service life is growing. This fact is closely related to the detailed knowledge of the dynamic properties and behavior in the machinery during its operation. Thus, a dynamic response induced in the general kinematic pair is dependent both on the dynamic properties of all mechanical systems and on the prescribed displacement law. The choice of displacement law should be in conformity with the main requirements, which are, for example, reduced natural vibration, low dynamic load, high positional accuracy, and noiseless action.

Due to the effects of inertia and working forces, there are induced force ratios in the general kinematic pair that are the cause of contact strain. If a certain limit on this strain is exceeded, fatigue damage of the cam and follower contact surfaces may occur in the operation of the cam mechanisms. So knowledge of the distribution of the contact stress and its size are necessary when designing cam mechanisms. Contact stress expressed in Hertz pressure or the principal stresses becomes the criterion for determining the lifetime of the working areas of the mentioned kinematic pair. The lifetime itself depends on the choice of materials from which the individual parts are made and their physical and mechanical properties and the way of material processing and the production technologies of individual parts or the way and intensity of loading. There is currently a wide choice of materials for cams and cam followers of cam mechanisms. In technical practice, however, the most commonly used cams are made from steel. The surface of the cam or cam follower can be heat treated or chemically heat treated. The aim of the treatment is to achieve the desired mechanical or physical—chemical properties of the contact areas of the cam mechanisms. The purpose of this procedure is to increase the hardness and resistance of the contact surface against wear and to keep a resilient core of the respective component. The results from the abovementioned show that it is necessary to know various characteristic

parameters describing material properties, heat treatment, or another technological processing for the lifetime estimation using theoretical methods. Therefore, experimental methods are an integral part of determining the lifetime of the working surface of the cam and the follower.

The working surfaces of the general kinematic pair of the cam mechanism frequently operate under extreme conditions, which are high loads, high sliding speeds, and poor lubricating conditions. Thus, this fact can lead to wear or excessive friction and thereby reduce the service lifetime and efficiency. These effects may be reduced by the application of coating on the working surfaces of the cam and follower. Coating is a technological process consisting of the fact that a very thin layer (the order of thousands of millimeters) is applied to the surface of an object, which has a relatively high hardness and strength compared to the underlying material. The thin layer of the coating forms a so-called barrier of the surface layers of the respective component against their chemical and physically mechanical wear. Coatings are generally used to improve hardness and tribological properties, wear resistance, and oxidation of exposed surface layers of the components. The coatings extend the lifetime of the sliding and rolling surfaces and help reduce the required power consumption while increasing performance. This decreases the use of lubricants and allows the use of new material combinations in the implementation of the relevant machinery. In some cases, coatings are even a necessary structural element for higher mechanical and thermal loads.

An effective way of analysis of the dynamic contact strain of the general kinematic pair of the cam mechanism is further presented here, which consists in the interaction of the knowledge and conclusions of Hertzian contact theory between two bodies and the advantages of using the finite element method. Using Hertz's contact theory, we predict the shape and size of the contact region of bodies in contact. In this way, we define the region to create an even and fine mesh of elements of an appropriate size in the vicinity of contact. Taking into account the assumptions of the Hertzian theory, the definition of the FEM model of a general kinematic pair is considerably simplified, and this reduces the number of algebraic equations needed to solve this problem; thereby, the computational time is reduced. This method leads to the achievement of relevant results compared to the real state. The method is presented in the task of determining the effect of the roller crown shape on the cam stress.

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