Chapter

Normal Boundary Intersection Applied to Controllers in Environmental Controls

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Abstract

Generally, the controllers currently used and implemented in the environmental field have certain set point values, which are pre-calibrated according to a specific process characteristic. However, instability in environmental processes is a difficult variable to fix. Thus, the use of numerous set points for specific process conditions may be a way of controlling instability. One way to obtain numerous setups within a working region is to use optimization algorithms for the construction of the Pareto frontier, each point of the boundary being represented by a different and at the same time optimum setup of operation. In this context, the construction of a Pareto frontier for a multiobjective and multivariate problem, established from an environmental problem, can be a way of getting around the problem of process instability. This chapter has a main objective to demonstrate the possibility of using the algorithm Normal Boundary Intersection (NBI), originally enunciated by Karna, as a precursor for the construction of the Pareto frontier, as well as the possibility of implementing the generated function for implementation in programmable logic systems.

Keywords: NBI, controllers, multivariate optimization, environmental, biosensors

1. Introduction

Comprehensively, much of the real industrial processes make use of several input variables (factors) at levels often unpredictable due to the instability displayed during operation in transient regime. The actual processes are very difficult to control, especially when it comes to numerous responses to be controlled.

Figure 1 shows in an illustrative way a real process where other factors that could directly influence the responses and interactions, called noise, were not considered. These noises can be related from the events of the environment where the process occurs, such as variations in the temperature of the medium, or events related to errors occurred by the operators. The use of complete second-order models to model processes should be restricted to only a certain interval specified by the levels presented for each of the factors analyzed. In the context of environmental processes, such as effluent decontamination in a treatment plant, the waste disposal parameters are defined according to country-specific standards and must be strictly followed. By using the effluent treatment plant as an example, it is practically impossible to maintain the constant input parameters such as the

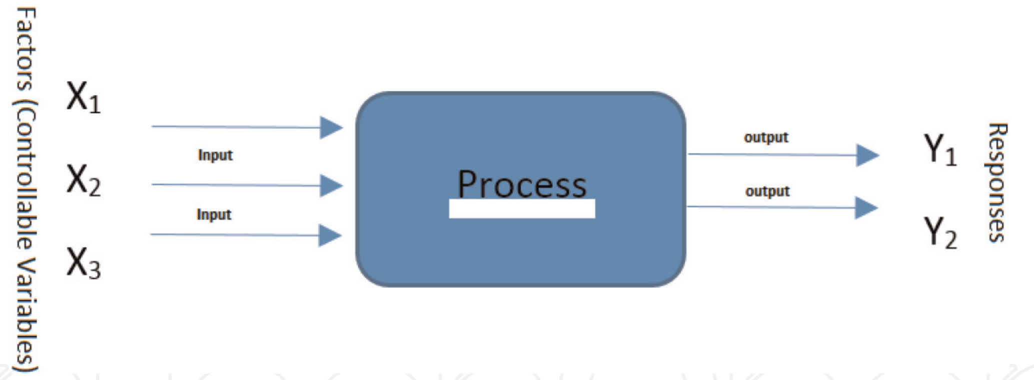

Figure 1.
General process diagram.

incoming organic load, heavy metals, and turbidity, among others. In order to keep the process running at steady state, with possible variations of input, it is necessary that the levels of the controllable factors be adequate in order to keep the responses at the exit within the pre-established parameters. This adjustment of levels can be achieved through sensors connected to programmable logic controllers, which usually operate through a set point.

These types of equipment are microprocessor computers that perform the function of control through specific software. One of the major problems encountered in using this device can be attributed to numerous, generally correlated, input conditions that may occur throughout the operation. Even with controller performance due to the set point, it no longer considers the possibility of interactions between these input parameters, which may compromise the permanence of the steady state. When the noise source is not discovered for later quantification, instability in the process can lead to desired responses outside of the predefined standards, leading to losses, associated cost, and environmental damage. Therefore, the controllers currently applicable cannot consider this instability generated by the noise industrially.

In the environmental area, due to the large number of parameters that must be monitored and pre-established as waste disposal control standards in receiving bodies, it is very common to maintain a certain operation for numerous responses. Thus, it is fundamental that the process can be previously known, modeled, and later optimized through algorithms already fomented by the literature, allowing the implementation of robust multiobjective optimization from the polynomial that describes all the responses, factors, and levels of the process in detail.

The concept of multiobjective robust optimization can be described as the set of nonlinear constrained programming (NLP) methods and algorithms that are intended to simultaneously optimize the mean and variance of multiple process characteristics that are in a way correlated output quantities that are reasonably well modeled by complete quadratic models. However, in effluent treatment processes that have multiple output characteristics are generally correlated.

Any process can be defined through a quadratic polynomial, if it is properly constrained within certain predefined intervals. The original concept of "robust" process was introduced by Genichi Taguchi in 1980 [1]. To this concept we can associate the original idea of RPD (robust parameter design), applied to generic processes. The more "robust" the details of the process are known, the more accurately it can be modeled and optimized. Therefore, there are several situations in which the multiple means of responses must be optimized and the multiple variances associated with each of the responses individually, minimized. This routine can be performed in order to reduce the interference attributed to the noise and to maintain a more stable process. As already mentioned, independently of the innumerable responses to be analyzed to a process, they are easily analyzed individually, even knowing the existence of a high associated positive correlation. Thus, when the responses have a very high correlation, mainly positive, very common in processes that involve chemical reactions, it becomes impracticable to perform the modeling of the multiple objective functions in an independent way, leading to the wrong responses.

In multiobjective optimization problems, the assignment of convex combinations of weights to the multiple responses leads to the agglutination of the objective functions that represent each response through weighted sums, thus generating a Pareto border or surface. Pareto border or surface is therefore a set of optimal values for multiple features obtained from a list of viable optimal points, obviously within a region of viable space. This agglutination of functions can be performed according to some methods: weighted sum and global criterion method (GCM). Both allow the construction of the Pareto border with some constraints attributed to the convexity of the objective function presented in the region of the viable space where the boundary is constructed. When there is a non-convex region in a certain objective function to be analyzed, the Pareto boundary cannot detect optimal points in this region.

Analyzing **Figure 2**, it is possible to verify a Pareto frontier for two responses, where each of the points represents different operating conditions. However, there is a discontinuity indicating no convex region of both functions representing the responses. One way to solve this problem is to use the algorithm Normal Boundary Intersection (NBI) to construct the Pareto frontier. This algorithm is able to determine points along the boundary, even in non-convex regions of space. The NBI algorithm considers two fixed points of the frontier ("best of the best and worst of the worst") known as utopia and nadir respectively. Between these fixed points, all others that make up the border are distributed.

One of the great possibilities in using this algorithm as a transfer function in control processes is precisely the possibility of choosing a number of different process setups, which consequently lead to optimized responses between the utopia and nadir points, which may be the limits of specification of the particular disposal

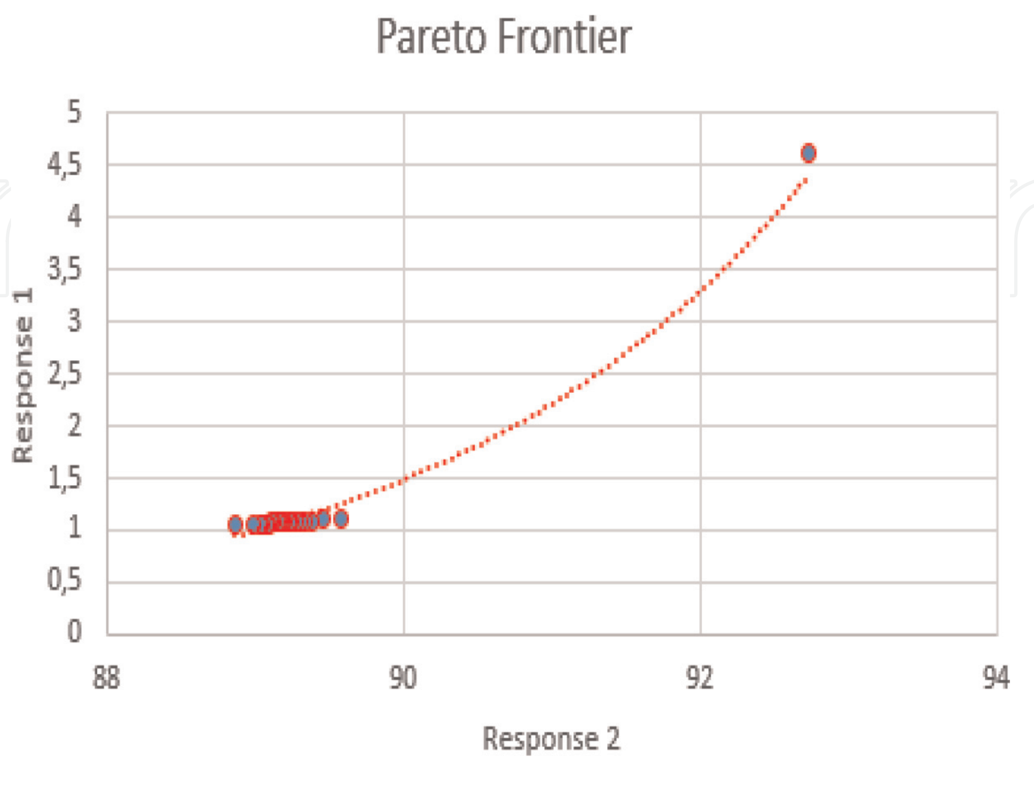

Figure 2.
Pareto frontier showing discontinuity.

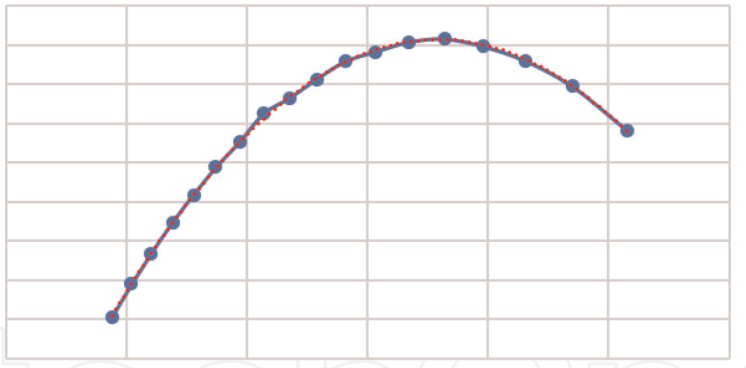

Figure 3.
Pareto border for bi-objective problem.

parameter in an effluent treatment plant, for example. The polynomial (**Figure 3**), which represents the Pareto frontier, can be used as a transfer function in scaling of a possible dynamic process controller for multiobjectives.

2. Description of the process NBI control

In order to facilitate the understanding of the possibility of implementing the NBI algorithm in controllers, let us take, for example, an industrial effluent treatment plant, which operates with a certain constant flow, due to the residence time necessary for part of the organic load to be degraded via bacteria and protozoa in an aerobic process. As a base of the input variables, we will work with initial organic load in terms of biochemical oxygen demand (BOD) and pH. As controllable factors, we will use the air or oxygen flow (aeration) and residence time. As desired responses, we will use as an illustration the removal of the organic load in terms of biochemical oxygen demand (BOD) and chemical oxygen demand (COD).

Modeling a typical problem processes, we could write that both responses have a direct relationship with the two factors presented as X_1 and X_2. However, keeping the process steady relative to the inputs becomes virtually impossible. By establishing, the two responses used, as an illustration of the application of the method, is it feasible to predict the aeration rate and residence time required. Certainly, the answer would be positive, if the entries were kept constant. However, if this standardization is not possible, how can we keep the responses within desirable patterns? Imagine in a situation of actual biological treatment process, where some changes can lead to periodic changes in the conditions of entry. For example, an increase in the rainfall rate may lead to the dilution of the organic matter present in the tributary and consequently the decrease of the initial BOD. The decrease of the initial BOD requires a lower concentration of dissolved oxygen so that bacteria and protozoa can decompose the organic matter in order to meet the exit standards, which would lead to the conclusion of shorter residence times required. There is a relationship as presented that can be considered a certainty. However, what is the relationship between them? What would be the best condition, to decrease aeration or increase residence time? These responses can only be met if we have this problem modeled. When working with models, we can easily predict the relationship of each of the factors to the expected response. This fact helps us reduce process costs and increase effectiveness in the targeted response. Through the use of models created from response surfaces, which have quadratic models, it is easily possible to determine local or global minimum or maximum points.

2.1 Stochastic response surface models

The response surface methodology (MSR) is a collection of mathematical and statistical techniques that allows modeling, analyzing, and optimizing problems whose response variables are influenced by many variables [2]. As mentioned earlier, there is great difficulty in knowing the behavior of independent and dependent variables in a process. Thus, the response surface allows the real approximation of the process from a quadratic model. The development through a Taylor polynomial, truncated in the quadratic term, takes what we call a second-order response surface:

$$Y(\mathbf{x}) = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j} \sum \beta_{ij} x_i x_j + \varepsilon \tag{1}$$

where β represents the coefficients of the model, k is the number of independent variables considered in the study, and ε is the error term.

The fact of using the response surface in a region close to high curvature of the model, presented according to local or global maxima or minima, according to convexity, does not effectively determine the best points or operation setups. However, what can be verified is a region of space that, depending on the levels of each of the independent variables, leads to better responses.

From the color gradient shown in **Figure 4**, it is possible to verify regions, delimited through the Cartesian axes representing the levels of each of the factors studied, leading to better responses. Thus, the construction of models through the surface response method becomes paramount for the application of later optimization algorithms. Among several optimization algorithms, the Normal Boundary Intersection (NBI) [3] has been used in several researches, in several different fields.

2.2 NBI algorithm

The NBI algorithm is developed in terms of an array that we call the payoff matrix Φ , which represents the optimal values of the multiple objective functions

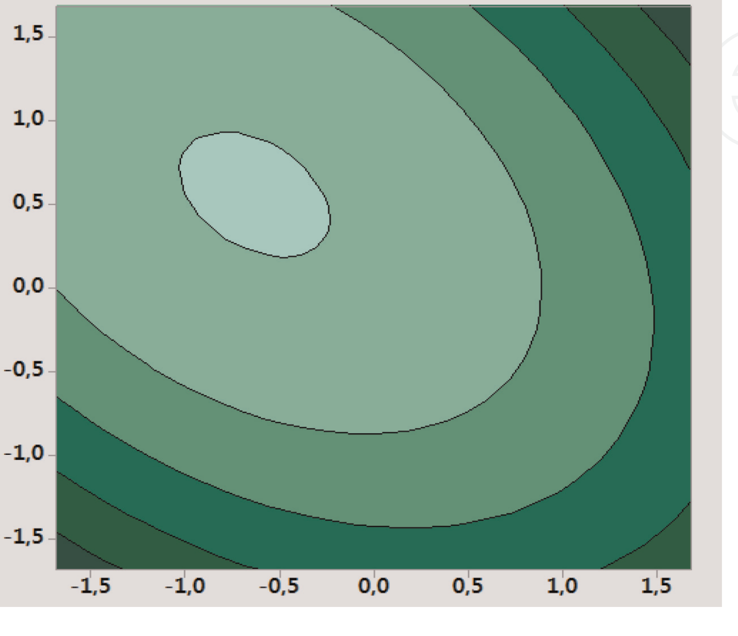

Figure 4.
Counter graphic.

minimized individually. The solution vector that minimizes the i-th objective function individually $f_i(x)$ is represented by x_i^* so that the minimum value of $f_i(x)$ at this point is $f_i^*(x_i^*)$. When replacing the individual optimum point x_i^* obtained in the optimization of objective function in the other functions, we have $f_i(x_i^*)$ which is therefore a nonoptimal value of this function. By repeating this algorithm for all functions, we can represent the payoff matrix as

$$\Phi = \begin{bmatrix} f_1^*(x_1^*) & \cdots & f_1(x_i^*) & \cdots & f_1(x_m^*) \\ \vdots & \ddots & & & \vdots \\ f_i(x_1^*) & \cdots & f_i^*(x_i^*) & \cdots & f_i^*(x_m^*) \\ \vdots & & & \ddots & \vdots \\ f_m(x_1^*) & \cdots & f_m(x_i^*) & \cdots & f_m^*(x_m^*) \end{bmatrix} \tag{2}$$

Each line of Φ is composed of minimum and maximum values of $f_i(x)$. In the NBI method, these values can be used to normalize the objective functions, especially when they are represented by different scales or units. In a similar way, writing the set of individual optimums in a vector, we have

$$f^U = \left[f_1^*(x_1^*)\ \ldots,\ f_i^*(x_i^*)\ \ldots,\ f_m^*(x_m^*)\right]^T \tag{3}$$

This vector is called utopia point. In the same way, by grouping the maximum (nonoptimal) values of each objective function, we have

$$f^N = \left[f_1^N \quad \ldots, \quad f_i^N \quad \ldots, \quad f_m^N\right]^T \tag{4}$$

This vector is called nadir points.

Using these two sets of extreme points, the normalization of the objective functions can be obtained as

$$\bar{f}(x) = \frac{f_i(x) - f_i^U}{f_i^N - f_i^U}, i = 1, \ldots, m \tag{5}$$

This normalization therefore leads to the normalization of the payoff matrix, $\overline{\Phi}$. The convex combinations of each line of the payoff matrix, $\overline{\Phi}$, form the "convex hull of individual minima" (CHIM) or the utopia line.

Figure 5 illustrates the main elements associated with multiobjective optimization. The anchor points represent the individual solutions of two functions. Points a and b are calculated from the stepped payoff matrix, $\overline{\Phi}\, w_i$. Considering a set of convex values for the weights, w, one has to $\Phi\, w_i$ represent a point on the utopia line, making $\hat{n}$ denote a unit vector normal to the line at point's utopia $\Phi\, w_i$ in the direction of origin; at the time, $\Phi\, w + D\,\hat{n}$ with $D \in \mathrm{R}$ will represent the set of points in that normal.

The point of intersection of this normal with the boundary of the viable region that is closest to the origin will correspond to the maximization of the distance between the utopia line and the Pareto border. Thus, the NBI method can be written as a constrained nonlinear programming problem such that

$$\begin{aligned} &\underset{(\mathbf{x},t)}{\text{Max}} && D \\ &\textit{subject to}: && \overline{\Phi}w + D\hat{n} = \overline{F}(\mathbf{x}) \\ & && \mathbf{x} \in \Omega \end{aligned} \tag{6}$$

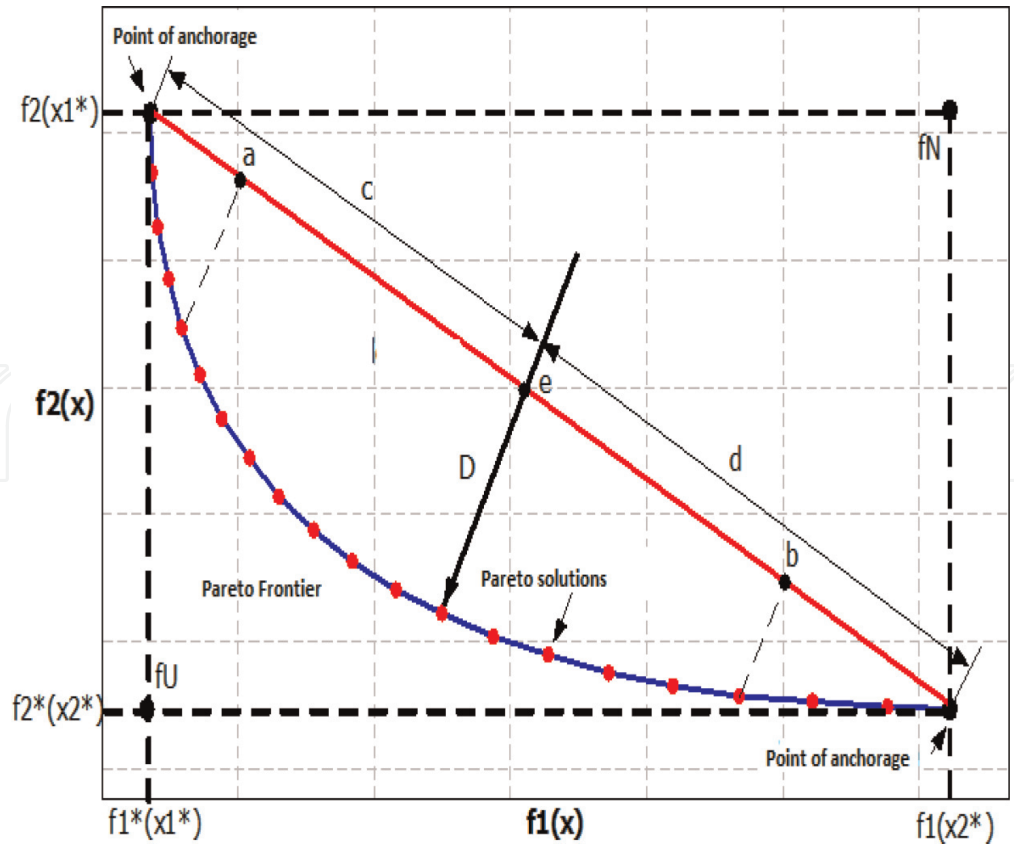

Figure 5.
Normal to intersect method (NBI).

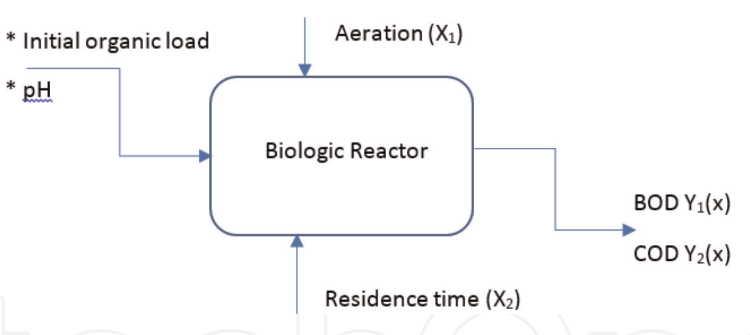

Figure 6.
General scheme of the process.

2.3 Implementation of the NBI control system

For the process described as an example, there are two controllable factors represented by the aeration rate (x_1) and residence time (x_2). However, according to **Figure 6**, there are also two input variables that cannot be measured, mainly due to the instability of a biological treatment plant, according to initial organic charge z_1 and pH z_2. The first artifice presented will be the transformation of each of these variables into known values, from experiments carried out on a smaller scale.

Thus, we will have the following factors: aeration rate (x_1), residence time (x_2), initial organic load (x_3), and pH (x_4). From a surface of response called central composite design (CCD), it is possible to construct a quadratic model, executing 31 experiments in laboratory scale:

$$Y1_x = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{11} {x_1}^2 + \beta_{22} {x_2}^2 + \beta_{33} {x_3}^2 + \beta_{44} {x_4}^2 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{14} x_1 x_4 + \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 + \beta_{34} x_3 x_4 + \varepsilon \tag{7}$$

$$Y2_x = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{11} {x_1}^2 + \beta_{22} {x_2}^2 + \beta_{33} {x_3}^2 + \beta_{44} {x_4}^2 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{14} x_1 x_4 + \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 + \beta_{34} x_3 x_4 + \varepsilon \tag{8}$$

Each of the coefficients presented in the two equations, represented by β_i, β_{ii}, and β_{ij}, is determined by the ordinary least square (OLS) regression algorithm where x_1, x_2, x_3, and x_4 are the factors already stated. With the models presented, it is possible to propose an optimization of both responses from the NBI algorithm for the four factors (**Figure 7**).

The Pareto frontier constructed from the optimum of both responses can now, from each of the setups assigned to each point, serve as the basis for implementation in controllers.

For each point referring to the specific response condition, a different setup is considered. For the chosen point 1 according to **Figure 8**, there is a BOD of 33.2 and a COD of 67, and under these conditions, we have the levels of each of the factors:

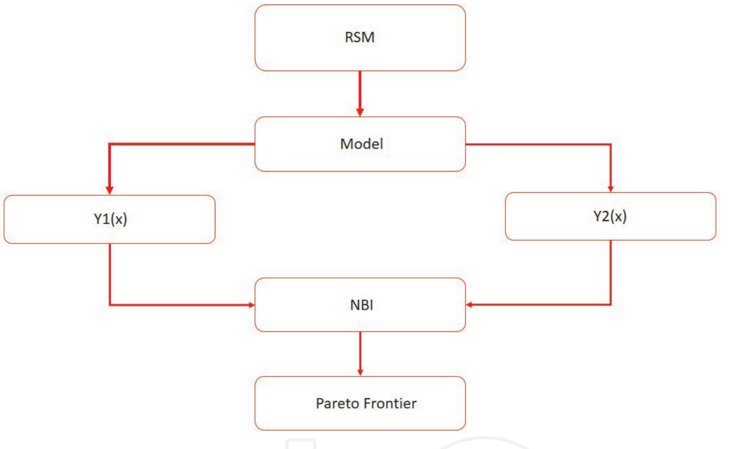

Figure 7.
Modeling and optimization flowchart.

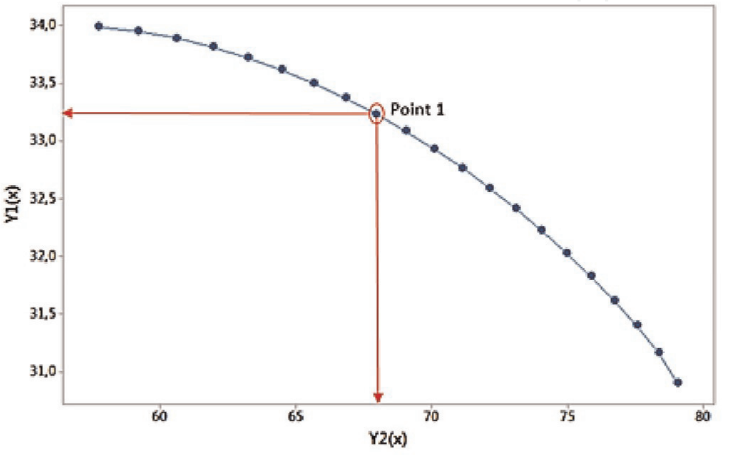

Figure 8.
Pareto frontier with sample choice point.

$$\begin{cases} \text{Aeration rate} = x_1 \\ \text{Residence time} = x_2 \\ \text{Initial organic load} = x_3 \\ \text{pH} = x_4 \end{cases} \tag{9}$$

In the conditions of this chosen point, replacing in (Eqs. (6) and (7)) the response surface, we have two quadratic equations, one referring to Y_1 (x) and Y_2 (x).

The implementation of the transfer function in the control will be done according to **Figure 9**.

The two responses provided in the example, enter into a multiprocessor system according to pre-established parameters. The multiprocessing system introduces

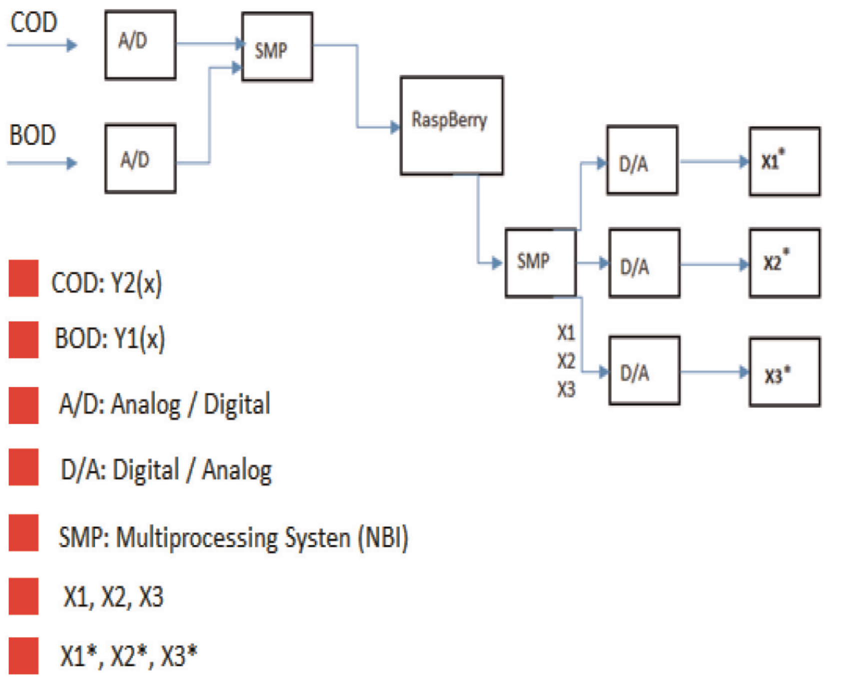

Figure 9.
Proposed arrangement for implementation.

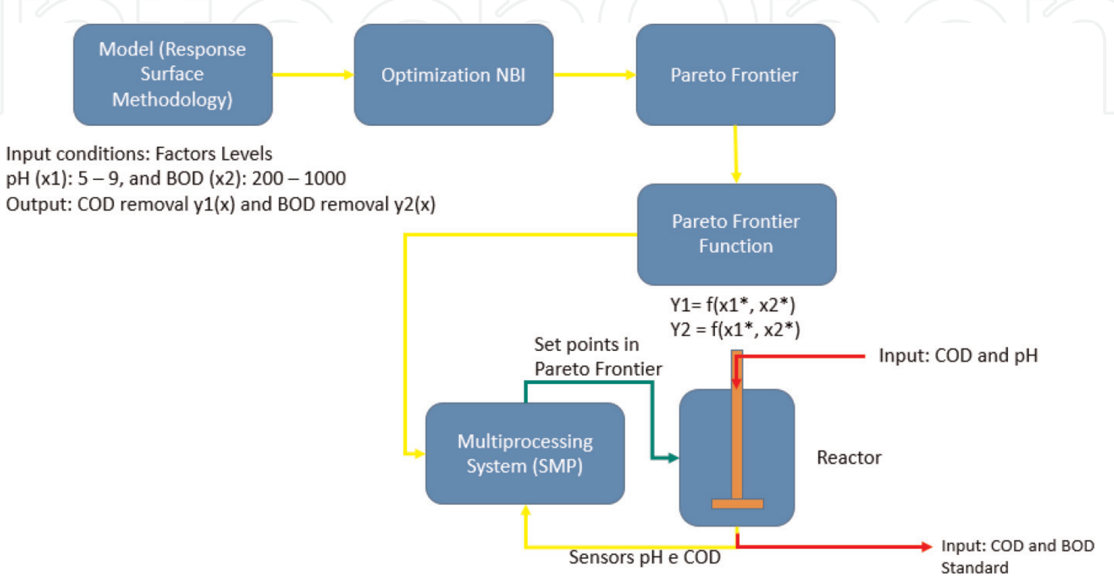

Figure 10.
Flow sheet of implementation algorithm.

polynomials referring to each different setup that consisted of the Pareto frontier, and for each setup, there are specific values of COD and BOD in mgO_2L^{-1}. From these inputs, the factors can be determined in optimized terms, X_1 *, X_2 *, and X_3 *.

An example of implementation for pH 5–9 and BOD values between 200 and 1000 mgL^{-1} follows the flow sheet (**Figure 10**).

As already mentioned, one of the advantages of the method is the correction of the input parameters belonging to the Pareto frontier, consisting of innumerable set points within an optimal solution space.

3. Conclusions

Although it has not yet been implemented in controllers, the use of algorithms such as NBI can facilitate the operation of this equipment, as well as lower costs of implementation and operation of environmental systems.

Conflict of interest

The author certify that they have no affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements) or nonfinancial interest (such as personal or professional relationships, affiliations, knowledge, or beliefs) in the subject matter or materials discussed in this chapter book.

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