Chapter

Classifying the Existing Continuum Theories of Ideal-Surface Adhesion

Belov Petr Anatolyevich, Lurie Sergey Albertovich and Golovina Natalya Yakovlevna

Abstract

The chapter classifies the existing continuum theories of ideal-surface adhesion within the gradient theory of adhesion. Ideal surface herein means a defect-free surface, the deformed state of which is entirely defined by the displacement vector and its first (distortion) derivatives as well as its second (curvature) derivatives. Ideal surfaces have such kinematic variables as noncombined deformations and rotations. The classification is based on a formal quadratic form of potential surface energy, which comprises contracting the first-rank tensors (adhesive-force theory), second-rank tensors (adhesive-stress theory), and third-rank tensors (theory of adhesive couple stresses). To interpret the physical sense of the summands in the quadratic form of the potential-energy surface density, this research uses a rather common method of dividing the elastic solid into an internal solid plus a surface layer (adhesive, contact, boundary, or inter-phase layer). The formal structure of the adhesion-energy surface density is compared to the structure of the thickness-averaged potential energy of a selected 3D layer. The chapter establishes the most general structure of adhesive-moduli tensors for the surfaces of classical elastic solids. The adhesive modules specific to the surfaces of a solid in gradient elasticity theories are identified.

Keywords: continuum adhesion theories, adhesive moduli, adhesive interaction, scale effects, nonclassical physical parameters

1. Introduction

Recent investigations of adhesive properties of surfaces and interfaces in deformable solids, in the mechanics of heterogeneous structures, and in the mechanics of composites are developed in various publications and analyzed in detail [1–10]. The first adhesion continuum theories were developed in the framework of the classical theory of elasticity [1, 2, 11–14].

The theory of Gurtin-Murdoch [11], which has become classical, was called as the theory of elasticity of surfaces. A generalization of this theory is proposed in the paper [15].

The adhesion theories listed above determined the adhesion properties of ideal (defect-free) surfaces. Further generalization of the theory of adhesion on the surface of defective media is given in [16–20].

Belov and Lurie [19, 20] formulated a model in which the adhesive properties were attributed to the newly formed surface connected with a field of defects. A variational model that takes into account the adhesive interactions of perfect (not damaged) surfaces, surfaces damaged by defects, and their interaction was presented. The surface of the defective medium can be represented as a perfect surface and a defective surface. Each of them has its own adhesive properties, as well as the properties of interaction with each other.

Adhesive interactions between the inclusion and the matrix in fine composites [18] are of great interest, as they directly affect not only the stiffness, but also the strength properties of composites.

So, as the adhesive properties of the surface are determined not only by the tangential derivatives of the displacements but also the normal derivatives, the boundary value problems for a classical body can be redefined due to the presence of adhesive interactions proportional to the normal derivatives of the displacements. On the other hand, in gradient theories, in nonclassical boundary conditions, the inclusion of adhesive interactions gives new effects. In other words, the surface of the body consists of the surface of a classical body and the surface of a "gradient" body. They have different adhesion properties and can even interact with each other as well as a defect-free (ideal) surface and a surface damaged by defects.

Similarly, to the gradient theories of elasticity, which contain the quadratic form of the second derivatives of displacements in potential energy, there are adhesion theories that also take into account the quadratic form of the second derivatives of displacements in the potential adhesion energy. A gradient theory of second order, which can be considered as a generalization of the theory of Steigmann and Ogden [1, 2], is described in Belov and Lurie [20].

The purpose of this chapter is the sequential analysis of variational formulations of the theories of adhesive interactions and the classification of adhesion models by the degree of accuracy of accounted scale effects. Classification of theories of adhesion and gradient theories of elasticity in terms accounting for scale effects was proposed in the work [21].

We have the following statement regarding the general structure of the adhesion elastic moduli for the classical linearly elastic body [15, 17, 18].

In line with this, the Lagrangian L of the model is written as:

$$L = A - \iiint U_V dV - \oiint U_F dF \tag{1}$$

Here, $A = \iiint P_i^V u_i dV + \oiint P_i^F u_i dF$ is the work of the volumetric forces P_i^V and surface forces P_i^F during the displacements u_i; U_V is the potential-energy density; and U_F is the potential-energy surface density.

The difference between the potential surface energies of two solids in contact in each contact spot is what determines their adhesive interaction. This is why adhesion theories can be classified on the basis of the potential-energy surface density inherent in an isolated solid.

A general expression for the potential-energy surface density U_F for an ideal surface is written as:

$$2U_F = A_{ij} u_i u_j + A_{ijmn} u_{i,j} u_{m,n} + A_{ijkmnl} u_{i,jk} u_{m,nl} + \ldots \tag{2}$$

where u_i, $u_{i,j}$, and $u_{i,jk}$ are the displacement vector, its first derivatives, and second derivatives, respectively; A_{ij}, A_{ijmn}, A_{ijkmnl} are tensors of the rank-specific adhesive moduli, which are transversely isotropic to the unit normal vector to the surface n_i. According to Green's formulas, each summand in (2) corresponds to a specific set of adhesive-force factors: adhesive forces a_i, adhesive stresses a_{ij}, or adhesive couple stresses a_{ijk}, etc.

$$\begin{aligned} a_i &= \frac{\partial U_F}{\partial u_i} = A_{ij} u_j \\ a_{ij} &= \frac{\partial U_F}{\partial u_{i,j}} = A_{ijmn} u_{m,n} \\ a_{ijk} &= \frac{\partial U_F}{\partial u_{i,jk}} = A_{ijkmnl} u_{m,nl} \end{aligned} \tag{3}$$

Accordingly, adhesive-moduli tensors are structured as follows [20]:

$$A_{ij} = a_n n_i n_j + a_s \delta^*_{ij} \tag{4}$$

$$\begin{aligned} A_{ijmn} = {} & \lambda^F \delta^*_{ij} \delta^*_{mn} + \mu^F \left(\delta^*_{im} \delta^*_{jn} + \delta^*_{in} \delta^*_{jm} \right) + \alpha^F \left(n_i n_n \delta^*_{jm} + n_m n_j \delta^*_{in} \right) \\ & + \beta^F \left(n_i n_j \delta^*_{mn} + n_m n_n \delta^*_{ij} \right) + \delta^F n_i n_m \delta^*_{jn} + B^F \delta^*_{im} n_j n_n + A^F n_i n_j n_m n_n \end{aligned} \tag{5}$$

$$\begin{aligned} A_{ijkmnl} = {} & A_1 \Big(\delta^*_{ij} \delta^*_{km} \delta^*_{nl} + \delta^*_{mn} \delta^*_{li} \delta^*_{jk} + \delta^*_{ij} \delta^*_{kn} \delta^*_{ml} + \delta^*_{mn} \delta^*_{lj} \delta^*_{ik} \\ & + \delta^*_{ij} \delta^*_{kl} \delta^*_{mn} + \delta^*_{ik} \delta^*_{jm} \delta^*_{nl} + \delta^*_{ml} \delta^*_{ni} \delta^*_{jk} + \delta^*_{in} \delta^*_{km} \delta^*_{jl} + \delta^*_{mj} \delta^*_{li} \delta^*_{nk} \\ & + \delta^*_{in} \delta^*_{lk} \delta^*_{jm} + \delta^*_{ik} \delta^*_{jn} \delta^*_{ml} + + \delta^*_{il} \delta^*_{km} \delta^*_{nj} + \delta^*_{im} \delta^*_{kj} \delta^*_{nl} + \delta^*_{im} \delta^*_{lj} \delta^*_{nk} \\ & + \delta^*_{im} \delta^*_{nj} \delta^*_{kl} \Big) + A_2 \Big(n_i n_m \delta^*_{kj} \delta^*_{nl} + n_i n_m \delta^*_{lj} \delta^*_{nk} + n_i n_m \delta^*_{nj} \delta^*_{kl} \Big) \\ & + A_3 \Big(n_i n_j \delta^*_{km} \delta^*_{nl} + n_m n_n \delta^*_{li} \delta^*_{jk} + n_i n_j \delta^*_{kn} \delta^*_{ml} + n_m n_n \delta^*_{lj} \delta^*_{ik} \\ & + n_i n_j \delta^*_{kl} \delta^*_{mn} + n_m n_n \delta^*_{lk} \delta^*_{ij} \Big) + A_4 \Big(n_i n_n \delta^*_{km} \delta^*_{jl} + n_m n_j \delta^*_{li} \delta^*_{nk} \\ & + n_i n_n \delta^*_{ml} \delta^*_{jk} + n_m n_j \delta^*_{ik} \delta^*_{nl} + n_i n_n \delta^*_{lk} \delta^*_{jm} + n_m n_j \delta^*_{kl} \delta^*_{ni} \Big) \\ & + A_5 \left(n_j n_n \delta^*_{ik} \delta^*_{ml} + n_j n_n \delta^*_{il} \delta^*_{km} \right) + A_6 n_j n_n \delta^*_{im} \delta^*_{kl} + A_7 \delta^*_{kl} n_i n_j n_m n_n \end{aligned} \tag{6}$$

Here, n_i is the unit normal vector to the surface and $\delta^*_{ij} = \left(\delta_{ij} - n_i n_j \right)$ is the planar Kronecker tensor $\left(\delta^*_{ij} n_j = \delta^*_{ij} n_i = 0, \delta^*_{ij} \delta^*_{ij} = 2 \right)$.

2. Theory of adhesion with adhesive forces

The first summand in Eq. (2) identifies the contribution made by the "spring" adhesion model [22]. The model derives its name from the specific nature of the corresponding adhesive forces a_i. In the spring theory of adhesion, adhesive forces are proportional to displacement, which enables comparing them to the response of the Winkler foundations from the classical theory of elasticity. In the spring model, provided that the surface properties are isotropic, there are two adhesive parameters per (4): the stiffness of the normal spring a_n and that of the tangential spring a_s.

There is an approach based on comparing the adhesive properties to the properties of a fictitious finite-thickness surface layer; this approach can be reduced to the spring theory. The algorithm of reducing a 3D surface layer to a spring model consists in finding its thickness such that the deformations in the real contact surface and in the surface layer are equivalent [23]. The disadvantage here is that the algorithm cannot explain the adhesive properties of 2D structures such as graphene or single-wall nanotubes, since such structures feature no thickness and therefore no surface layer.

The algorithm can be demonstrated by a simple example. Let an elastic solid be presented as an internal solid plus a surface layer, or Skin, the thickness h whereof is so small compared to the total size of the solid that the deformed state of this layer can be deemed homogeneous. Then, the distribution of displacements in this layer can be deemed linear across its thickness, which is equivalent to Timoshenko's kinematic hypotheses [24]:

$$\begin{cases} u = u_0 + u_1\left(\dfrac{z}{h} - \dfrac{1}{2}\right) \\ v = v_0 + v_1\left(\dfrac{z}{h} - \dfrac{1}{2}\right) \\ w = w_0 + w_1\left(\dfrac{z}{h} - \dfrac{1}{2}\right) \end{cases} \tag{7}$$

Note that $w_1 = w(h) - w(0)$ is the relative normal displacement of the corresponding points on the opposite sides of the "surface" layer; $u_1 = u(h) - u(0)$ and $v_1 = v(h) - v(0)$ are the projections of the tangential relative displacements; and $\sqrt{u_1 u_1 + v_1 v_1}$ is the magnitude of tangential relative displacements. The classical-medium Lagrangian expression can be rewritten to obtain the following equalities:

$$\begin{aligned} L = A - \iiint\limits_{V} U_V dV = A - \iiint\limits_{V-Skin} U_V dV - \iiint\limits_{Skin} U_V dV = \\ = A - \iiint\limits_{V-Skin} U_V dV - \oiint\left(\int\limits_0^h U_V dz\right) dF = \\ = A - \iiint\limits_{V-Skin} U_V dV - \oiint U_F dF \end{aligned} \tag{8}$$

For a classical elastic solid, the volumetric portion of the "surface-layer" potential energy is written as:

$$\begin{aligned} \int\limits_0^h U_V dz = \frac{1}{2}\{ & 2\mu\left[\left(u_{0,x}u_{0,x} + v_{0,y}v_{0,y}\right)h + \left(u_{1,x}u_{1,x} + v_{1,y}v_{1,y}\right)h/12\right] \\ & + \lambda\left[\left(u_{0,x} + v_{0,y}\right)^2 h + \left(u_{1,x} + v_{1,y}\right)^2 h/12\right] + \mu\left[\left(u_{0,y} + v_{0,x}\right)^2 h \right. \\ & \left. + \left(u_{1,y} + v_{1,x}\right)^2 h/12\right] + \mu\left[\left(w_{0,x}w_{0,x} + w_{0,y}w_{0,y}\right)h \right. \\ & \left. + \left(w_{1,x}w_{1,x} + w_{1,y}w_{1,y}\right)h/12\right] + 2\mu\left(w_{0,x}u_1 + w_{0,y}v_1\right) \\ & + 2\lambda\left(u_{0,x} + v_{0,y}\right)w_1 + \mu(u_1u_1 + v_1v_1)/h + (2\mu + \lambda)w_1w_1/h \} \end{aligned} \tag{9}$$

The "adhesive properties" of this layer depend on many factors, including the relative displacement of corresponding points on its fronts. Note that the summands in the first four lines of the expression (9) are proportional to h^{+1}, the summands in the fifth line are proportional to h^0, while the summands in the last line are proportional to h^{-1}. Provided a sufficiently thin layer, these summands must make the greatest contribution to the potential-energy expression as long as all the layer deformations belong to the same order. These components are the energy of springs resisting the relative normal and tangential displacement of the layer front points; they determine the spring stiffness:

$$a_n = (2\mu + \lambda)/h \tag{10}$$

$$a_s = \mu/h \tag{11}$$

The stiffness values are found via Young's modulus, the surface-layer shear, and the unknown thickness h.

Equations (10) and (11) indicate that in order for the spring stiffness values to be independent, it is necessary to abandon the condition that the "surface layer" modules must be the same as the modules inside a solid. Indeed, excluding the "surface-layer" thickness from (10), (11) means that the springs are proportional stiffness-wise:

$$a_s = a_n\mu/(2\mu + \lambda) \tag{12}$$

this contradicts the initial assumption that the adhesive properties of an isolated-solid surface (1) do not depend on the internal mechanical properties of the same solid; it also contradicts the empirical data, according to which one and the same solid may have different adhesive properties depending on what chemical or physicochemical method was used to activate its surface [25].

Thus, expression (1) *is entirely different from the adhesive surface layer model* (8), (9) *even in the context of the simplest spring model*. Potential adhesion energy in (1) will have the following structure for the spring theory of adhesion:

$$2U_F = A_{ij}u_iu_j \tag{13}$$

while the tensor of second-rank adhesive moduli A_{ij} is structured as:

$$A_{ij} = a_n n_i n_j + a_s\delta^*_{ij} \tag{14}$$

3. Adhesion theories with adhesive stresses

As is outlined above, *expressions* (8) and (9) *are not directly applicable to the potential energy of adhesion*; they only enable structuring the potential energy of adhesion in (2) and finding some physical analogies and interpretations. In particular, the first summand in (2) can be interpreted as the potential energy of springs and the stiffness whereof has the same physical dimensionality [Pa/m] as shown in the ratios (10) and (11). In doing so, it is stated that the surface-layer deformations belong to the same order. Let us now assume the opposite. Assume that the traverse deformations are negligible compared to longitudinal deformations. Then the summands of the last and last-but-one lines in the expression (9) can be ignored. Let us study such expression (9) for the potential "surface-layer" energy further to find analogy to the second summand in (2).

4. Gurtin-Murdoch theory (surface elasticity theory)

The first, the third, the fifth, the tent, and the twelfth summands in (9) determine the potential tension-compression and shear energy in their corresponding planes of the surface layer [11]. If Timoshenko's static hypothesis (the noncompressibility hypothesis) is introduced to the kinematic hypotheses (7), then the kinematic variable w_i can be expressed in terms of the changing area $\left(u_{0,x} + v_{0,y}\right)$; then, the summands above are reduced to being written in terms of the potential energy of the layer during tension or compression (Line 1) or in the case of shear (Line 2):

$$\int_0^h U_V dz|_{M-G} = \frac{1}{2}h\left\{4\mu\frac{(\mu+\lambda)}{(2\mu+\lambda)}u_{0,x}u_{0,x} + 2\frac{2\mu\lambda}{(2\mu+\lambda)}u_{0,x}v_{0,y}\right.$$
$$\left. + 4\mu\frac{(\mu+\lambda)}{(2\mu+\lambda)}v_{0,y}v_{0,y} + \mu\left(u_{0,y} + v_{0,x}\right)^2\right\}$$

Accordingly, the potential adhesion energy in (1) will have the following structure for the Gurtin-Murdoch theory of adhesion [12]:

$$2U_F = A_{ijmn}u_{i,j}u_{m,n} \tag{15}$$

while the tensor of fourth-rank adhesive moduli A_{ijmn} is structured as:

$$A_{ijmn} = \lambda^F\delta^*_{ij}\delta^*_{mn} + \mu^F\left(\delta^*_{im}\delta^*_{jn} + \delta^*_{in}\delta^*_{jm}\right) \tag{16}$$

This interprets the first to modules of the fourth-rank tensor in (5). Apparently, the adhesive moduli λ^F and μ^F are adhesive analogs to the Lame coefficients for the planar stress-state problem in the classical theory of elasticity. What must not be forgotten is that adhesive moduli have a different dimensionality: [Pa/m] rather than [Pa].

5. Belov-Lurie theory (theory of ideal adhesion)

The Belov-Lurie theory of ideal adhesion [15] can be formulated by adding a summand similar to the seventh summand $\mu h\left(w_{0,x}w_{0,x} + w_{0,y}w_{0,y}\right)$ in (9) to the summands that define the Gurtin-Murdoch theory in (2). The potential energy of adhesion per (1) is structured as follows for this adhesion theory:

$$2U_F = A_{ijmn}u_{i,j}u_{m,n} \tag{17}$$

while the tensor of fourth-rank adhesive moduli A_{ijmn} is structured as:

$$A_{ijmn} = \lambda^F\delta^*_{ij}\delta^*_{mn} + \mu^F\left(\delta^*_{im}\delta^*_{jn} + \delta^*_{in}\delta^*_{jm}\right) + \delta^F n_i n_m \delta^*_{jn} \tag{18}$$

Paper [15] shows that the adhesive modulus δ^F is the Laplace capillary pressure constant if the expressions (17) and (18) apply to liquids. This modulus is critical for the adhesive properties of surfaces in solids, as it takes into account the adhesion effects of the same order as in the Gurtin-Murdoch theory. The latter is incomplete in this respect. Paper [26] compares the Gurtin-Murdoch theory against the Belov-Lurie theory in terms of modeling the adhesive interactions between the edges of cracks that differ in mode. It shows that the statements of boundary problems for mode II and mode III cracks will coincide for both theories. However, for mode I cracks, the ideal-adhesion theory takes into account the adhesive properties of the crack edges, while the classical statement or Gurtin-Murdoch statement does not.

6. Theory of adhesion surfaces of classical media

It shows that the adhesive properties of surfaces of a solid are closely related to the models of the solid itself. Indeed, if the internal properties of a solid are defined by the classical theory of elasticity, i.e., $U_V = U_V(u_{i,j}) = C_{ijmn}u_{i,j}u_{m,n}$, then (1) means that:

$$\delta L = \delta A - \iiint \sigma_{ij}\delta u_{i,j}dV - \oint\!\!\oint (a_i\delta u_i + a_{ij}\delta u_{i,j} + a_{ijk}\delta u_{i,jk})dF =$$

$$= \iiint (\sigma_{ij,j} + P_i^V)\delta u_i dV + \oint\!\!\oint \Big\{ \Big[P_i^F - \sigma_{ij}n_j - a_i$$

$$+ \left(a_{ij} - a_{ijk,q}\delta^*_{kq}\right)_{,p}\delta^*_{pj}\Big]\delta u_i - \left(a_{ij} - a_{ijk,q}\delta^*_{kq}\right)n_j\delta\dot{u}_i \Big\} dF = 0 \quad (19)$$

For simplicity, consider an edge-less solid, i.e., a solid that has a smooth rather than piecewise-smooth surface. Analysis of the variational Eq. (19) shows that the boundary problem for the classical theory of elasticity is redefined, as a correctly formulated system will have three boundary conditions in each nonsingular surface point. At the same time, the variational Eq. (19) provides six boundary conditions in each non-singular surface point. To remove this contradiction, it must be required that the force factors always equal zero on the surface of classical elastic solids when varying the $\delta\dot{u}_i$. This requirement is equivalent to the conditions of adhesion-moduli tensors:

$$A_{ijmn}n_j = \alpha^F n_m\delta^*_{in} + \beta^F n_i\delta^*_{mn} + B^F\delta^*_{im}n_n + A^F n_i n_m n_n \equiv 0 \quad (20)$$

$$A_{ijkmnl}n_j = A_3\left(n_i\delta^*_{km}\delta^*_{nl} + n_i\delta^*_{kn}\delta^*_{ml} + n_i\delta^*_{kl}\delta^*_{mn}\right)$$

$$+A_4\left(n_m\delta^*_{li}\delta^*_{nk} + n_m\delta^*_{ik}\delta^*_{nl} + n_m\delta^*_{kl}\delta^*_{ni}\right) + A_5\left(n_n\delta^*_{ik}\delta^*_{ml} + n_n\delta^*_{il}\delta^*_{km}\right)$$

$$+A_6 n_n\delta^*_{im}\delta^*_{kl} + A_7\delta^*_{kl}n_i n_m n_n \equiv 0 \quad (21)$$

Thus, the adhesion theories for the surfaces of classical elastic media are limited by the following structure for the potential adhesion energy:

$$2U_F = A_{ij}u_i u_j + A_{ijmn}u_{i,j}u_{m,n} + A_{ijkmnl}u_{i,jk}u_{m,nlj} \quad (22)$$

In the context of (4) and (18), the tensors of adhesive moduli will have the following structure that is maximally general for a surface that confines a classical medium:

$$A_{ij} = a_n n_i n_j + a_s\delta^*_{ij} \quad (23)$$

$$A_{ijmn} = \lambda^F\delta^*_{ij}\delta^*_{mn} + \mu^F\left(\delta^*_{im}\delta^*_{jn} + \delta^*_{in}\delta^*_{jm}\right) + \delta^F n_i n_m\delta^*_{jn} \quad (24)$$

$$A_{ijmnkl} = A_1\Big(\delta^*_{ij}\delta^*_{km}\delta^*_{nl} + \delta^*_{mn}\delta^*_{li}\delta^*_{jk} + \delta^*_{ij}\delta^*_{kn}\delta^*_{ml} + \delta^*_{mn}\delta^*_{lj}\delta^*_{ik}$$

$$+\delta^*_{ij}\delta^*_{kl}\delta^*_{mn} + \delta^*_{ik}\delta^*_{jm}\delta^*_{nl} + \delta^*_{ml}\delta^*_{ni}\delta^*_{jk} + \delta^*_{in}\delta^*_{km}\delta^*_{jl} + \delta^*_{mj}\delta^*_{li}\delta^*_{nk}$$

$$+\delta^*_{in}\delta^*_{lk}\delta^*_{jm} + \delta^*_{ik}\delta^*_{jn}\delta^*_{ml} + \delta^*_{il}\delta^*_{km}\delta^*_{nj} + \delta^*_{im}\delta^*_{kj}\delta^*_{nl} + \delta^*_{im}\delta^*_{lj}\delta^*_{nk}$$

$$+\delta^*_{im}\delta^*_{nj}\delta^*_{kl}\Big) + A_2 n_i n_m\left(\delta^*_{kj}\delta^*_{nl} + \delta^*_{lj}\delta^*_{nk} + \delta^*_{nj}\delta^*_{kl}\right) \quad (25)$$

This seems to be the first time to state the above.

7. Steigmann-Ogden theory (instantaneous adhesion theory)

Transforming the surface-layer potential energy expression (9) according to Kirchhoff's rather than Timoshenko's hypotheses, the energy is written as the potential plate-bending energy:

$$\int_0^h U_V dz|_{S-0} = \frac{1}{2}\{Dw_{0,xx}w_{0,xx} + 2vDw_{0,xx}w_{0,yy} + Dw_{0,yy}w_{0,yy} + 2(1-v)Dw_{0,xy}w_{0,xy}\} \tag{26}$$

In this case, the summands in (27) have a structure that corresponds to the particular case (26), where $A_1 = 0$. In this case, the potential adhesion energy depends only on the second tangent derivatives of the surface deflections [22]. In the general case, it must also depend on the second tangent derivatives of the displacement projections tangential to the surface of a solid. Thus, the Steigmann-Ogden theory [2] is a special case of the gradient adhesion theory [27], which is defined by the structure of the sixth-rank adhesive-moduli tensor (6) for Toupin's gradient media, or by the tensor of moduli (26) for classical elastic solids.

8. Gradient-media surface adhesion theories

Let the internal properties of a solid be determined by Toupin's gradient theory, i.e.,

$$U_V = U_V(u_{i,j}, u_{i,jk}) = (C_{ijmn}u_{i,j}u_{m,n} + C_{ijkmnl}u_{i,jk}u_{m,nl})/2$$

Then (1) means that:

$$\begin{aligned}\delta L = \delta A - \iiint(\sigma_{ij}\delta u_{i,j} + \sigma_{ijk}\delta u_{i,jk})dV - \oiint(a_i\delta u_i + a_{ij}\delta u_{i,j}\\ + a_{ijk}\delta u_{i,jk})dF = \iiint(\sigma_{ij,j} - \sigma_{ijk,kj} + P_i^V)\delta u_i dV\\ + \oiint\Big\{\Big[P_i^F - \sigma_{ij}n_j + \sigma_{ijk,k}n_j + (\sigma_{ijk}n_k)_{,p}\delta^*_{pj}\\ -\Big(a_i - \big(a_{ij} - a_{ijk,q}\delta^*_{kq}\big)_{,p}\delta^*_{pj}\Big)\Big]\delta u_i - \Big[\sigma_{ijk}n_jn_k\\ +\big(a_{ij} - a_{ijk,q}\delta^*_{kq}\big)n_j\Big]\delta\dot{u}_i\Big\}dF = 0\end{aligned} \tag{27}$$

As in the case of deriving the variational Eq. (19), for simplicity, consider an edgeless solid, i.e., a solid that has a smooth rather than piecewise-smooth surface. Analysis of the variational Eq. (28) shows that the boundary problem for Toupin's gradient theory is correct, as the formulated system will have six pairs of alternative boundary conditions in each non-singular surface point. Note that the requirements (20) that define the structure of adhesive tensors for classical-solid surfaces also specify a group of adhesive moduli that only manifest on the surfaces of Toupin's solids. It can therefore be stated that the adhesive moduli $\alpha^F, \beta^F, B^F, A^F$ and A_3, A_4, A_5, A_6, A_7 are specific only to the gradient theories of elasticity as generalized by Toupin's theory. This also seems to be the first time to state the above.

9. Graphene-type surface adhesion theories

Consider the ratio (1) in its extreme case, in which the volumetric portion of the potential energy of the solid is negligible compared to the surface energy:

$$L = A - \oiint U_F dF \tag{28}$$

The potential-energy surface density structure is the same as in (2). Note that the normal derivative to the surface is not defined for kinematic variables in 2D structures. This is why the structure (29) can be the most general structure for the adhesive-moduli tensors [30].

Paper [28] considers the properties of graphene-like structures without the "spring" portion of potential energy. It has been found that graphene features the bending properties of Timoshenko's plates, while its cylindrical stiffness is determined by the adhesive modulus A_2 whereas the shear stiffness is determined by the adhesion modulus δ^F. Therefore, the mechanical tension-compression and planar shear properties are determined by the remaining moduli λ^F, μ^F and A_1.

Paper [29–31] considers the properties of single-walled nanotubes (SWNT), the potential energy of which is written specifically as (2).

$$2U_F = A_{ijmn} u_{i,j} u_{m,n} + A_{ijkmnl} u_{i,jk} u_{m,nl}$$

The resultant finding is that the mechanical properties of single-walled nanotubes are determined by a nonclassical modification of the cylindrical-shell theory equations.

10. Conclusion

This chapter presents an attempt to classify the existing continuum theories of ideal-surface adhesion within the gradient theory of adhesion [20, 28, 31]. The classification is based on a formal quadratic form of potential surface energy (2), which comprises contracting the first-rank tensors (adhesive-force theory), second-rank tensors (adhesive-stress theory), and third-rank tensors (theory of adhesive couple stresses). To interpret the physical sense of the summands in the quadratic form of the potential-energy surface density, this research uses a rather common method of dividing the elastic solid into an internal solid plus a surface layer (6) (adhesive, contact, boundary, or inter-phase layer). The formal structure of the adhesion-energy surface density (2) is compared to the structure of the potential energy averaged over the thickness of the selected subsurface 3D layer (7). There has been found the most general structure of adhesive-moduli tensors for the surfaces of classical elastic solids (20). The chapter identifies the adhesive moduli specific to the surfaces of solids in gradient theories of elasticity (18).

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Author details

Belov Petr Anatolyevich[1], Lurie Sergey Albertovich[1]
and Golovina Natalya Yakovlevna[2]*

1 Institute of Applied Mechanics of RAS, Moscow, Russia

2 Industrial University of Tyumen, Surgut, Russia

*Address all correspondence to: ntgolovina@rambler.ru

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