Chapter

Initial Evolution of the Moon

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Abstract

The problem of the origin of the Moon is of fundamental importance to understanding the mechanism of the planetary solar system's formation. It is important to know the mechanism of differentiation of substances in a growing planet. When planets are formed from a cold protoplanetary cloud, the matter of the inner regions of the Earth and the Moon remains at temperatures lower than the melting point of iron. The main volume of the matter of the protoplanet remains in its unmelted state, and its differentiation occurs in the formed planet. In this work, attention is paid to the most important internal sources of energy: the decay energy of short-lived isotopes, the dissipation of tidal friction energy, and thermal energy from accidental deposition of bodies and particles on a growing surface. Accounting for these sources provides a solution to the problem.

Keywords: moon matter, composition, age, growth dynamics, numerical models

1. Introduction

The study of the evolution of the moon is of exceptional interest for the knowledge of the processes which had occurred and occurred now on the Moon and the Earth, the formation of the internal regions of these bodies. Of undoubted interest is the study of the material composition of the surface of the planetary body nearest to the Earth, which makes it possible to clarify information about the processes in the protoplanetary cloud and the early stages of the accumulation of planets.

Before to the work of descent vehicles on the surface of Mars, the only direct information about the composition of the substance of external bodies was the results of a study of lunar soil, delivered to Earth by Soviet stations Luna-16 and Luna-20 and American astronauts. The geophysical measurements of the gravitational potential and the speed of propagation of seismic waves made it possible to build indirect models of the distribution of the density and velocity sections of the Moon [1]. For the no uniqueness of the geophysical interpretation, estimates of the density distribution and velocity of seismic waves for 1D models of the Moon that are very important for the knowledge of the current state are obtained. The lunar crust is fixed from the surface to a depth of about 55 km. The velocity of longitudinal seismic waves V_P increases here to a value of V_P = 5.8 km/s at a depth of 25 km, then sharply increases at this depth to a value of 6.8 km/s. Then the velocity increases slowly down to 7 km/s up to the 55 km border. This section is called the crust-mantle transition boundary. The surface is marked by a sharp increase in the velocity of V_P to the value of V_P = 8.1 km/s. Then there is a decrease in the velocities V_P and Vs to values of 7.8 km/s and 4.7 km/s, respectively, at a depth of 300 m. The layer in the depth interval (55–300) in km is called the upper mantle. In the range of

300–800 km, called the average mantle, the velocity is reduced to V_P = 7.5 km/s and V_S = (3.6–4.0) km/s.

In the lower part of the mantle, transverse waves are not recorded. At a depth of about 1500 km, the mantle-core transition region is fixed, and in the second one, there is a sharp decrease in the propagation velocity of the volume seismic waves to V_P = (4–5) km/s [2–5]. In spite of the no uniqueness of the geophysical interpretation, very important results of the density distribution and velocity of seismic waves for 1D models of the Moon had been obtained. The scheme of the seismic velocity model is presented in [1]. The nature of the propagation of seismic waves is very different from that observed in the Earth: the amplitude of oscillations increases sharply, and the decline is observed for (1–4) hours. The lunar crust differs significantly from the earth's crust in its elastic and viscous characteristics. The quality factor (the inverse of the attenuation coefficient) is estimated at 5000, while for the Earth, this estimate is in the range of 100–1000. The seismic activity of the moon is much smaller, about 1015 erg/year, whereas on Earth about 1024 erg/year.

Based on the mineralogical study of the delivered collection of lunar samples, it was established that, unlike the Earth, an early and extensive differentiation of matter took place on the Moon. At the same time, at the early stage of the formation of the Moon, the fractional crystallization of the substance, which formed, possibly, the entire Moon, followed by partial melting of the upper envelope, with a capacity of at least 250 km, occurred. Lunar magma was formed during the entire time of its cooling [6]. The question of the origin and composition of the substance of the moon is of fundamental importance for understanding the processes of formation of the planets of the solar system. Before the occurring of the mega-impact hypothesis, three main mechanisms for the formation of the Moon had been discussed in the literature: (1) the hypothesis of separation of the moon from the earth, (2) the capture hypothesis, and (3) hypothesis of the co-formation or co-accretion of the Earth and the Moon. The disadvantages of these hypotheses are considered in [7]. The idea of separating the substance of the Moon from the Earth was proposed by Darwin in 1880 [8]. Its inconsistency with the laws of celestial mechanics is considered in [9]. As the authors of [9] note, in the event of a rotational instability, which causes the separation of a part of a substance from a rotating body, a smooth separation of a satellite from the main body is impossible. The substance ejected as a result of rotational instability either flies away or returns back [9].

Later Ringwood [10] attempted to modify Darwin's hypothesis by assuming that the material from the Earth's mantle was ejected into the Moon's orbit by strikes of large meteorites. The hypothesis of a joint formation of the Earth and the Moon is considered in [11]. Schmidt [11] assumed that the Moon had accumulated in the vicinity of the growing Earth from a near-Earth swarm of bodies, which was continuously increased from a protoplanetary cloud. As the authors of [12] note, "the hypothesis of O. Yu. Schmidt is based on processes that necessary must take place during the accumulation of the Earth and from the mechanical point of view it seems to be the most promising [12]." However, within the framework of this model, it was not possible to explain the difference in the chemical composition of the Earth and the Moon. The main problem that arose in connection with the thermal state of the Moon is the need to substantiate the source of the early differentiation of matter at the stage of their accumulation. To overcome the difficulties of the three main hypotheses of lunar formation, the mega-impact hypothesis [13, 14] was proposed.

In our work, the task was set to construct a numerical model of the Moon accumulation process, in which we would be able to reconcile the experimental obtained data. Taylor [15] believes that if the Earth-Moon system is unique, it is possible that its genesis is unusual and this unusual variant is the hypothesis of the formation of the Moon as a result of the mega impact, the Earth's collision with a space body of planetary size (with mass of Mars or more). A number of geochemical contradictions that are incompatible with the mega-impact hypothesis are considered by Galimov in [7].

We were faced with the task: using the model of accumulation of the terrestrial planets proposed in papers [16–18], to conduct numerical simulations of the temperature distribution in the interior of the planet for successively increasing with time values of the body radius in the 3D environment and in contrast to the results [18], explore the features of the evolution of primary heterogeneities depending on the rate of accumulation of the second largest body in the "feeding" zone of a growing Earth in a 3D process model. In works [16–18], the proposed model of the heterogeneous accumulation of the Earth, based on a two-stage mechanism for the formation of the pre-planetary of the planet, is presented. It assumes that in the first stage, primary pre-planets are formed, the central part of which consists of the most high-temperature condensates close in composition to the CAI—Ca-Al inclusions found in the Allende meteorite. The middle envelope of these pre-planets is represented by an iron-nickel material, which condenses from the gaseous phase following high-aluminum condensates. In the process of growth of primary pre-planets, they were heated, as a result of the decay of short-lived radioactive isotopes, the main of which is ^{26}Al with a half decay of $\tau = 7.38 \times 10^5$ years [19]. The ratio of $^{26}Al/^{27}Al$ in protoplanetary matter is estimated to be 5×10^{-5} [20]. With this content of ^{26}Al, as the mass of the pre-planetary grows, the temperature of their central regions increases, and in the center of the pre-planetary with a radius of more than 200 km, it can reach 2200 K [17]. This is quite enough to melt the Ca-Al material in the central part of the core, the melting point of which is 1830 K [1], and the iron-nickel mixture in its middle envelope. The outer envelope of the pre-planetary, which transfers heat to the space, will remain solid. Further development of the process of formation of the planets is as follows. In accordance with the accumulation model of Safronov [21], the number of cores formed at the initial stage of the agglomeration process of condensation products is large, and they will often collide with each other. The collision of two pre-planetary with similar sizes, molten aluminosilicate core and middle envelope, folded with iron and a solid silicate outer envelope will lead to their destruction. Medium, molten envelopes in a collision will merge, forming a new pre-planet, the core of which consists of an iron-nickel alloy. The substance of the aluminosilicate core of the primary core will be extruded from their centers and thrown into the feeding zone for new core formed as a result of the collision. The outer hard envelope, the lower part of which could consist of a substance close to the composition of pallasites or ordinary chondrites, will be destroyed, and part of the fragments will also be thrown out of the feeding area of the growing planet. In this way, the metallic core of the Earth and the Moon is formed, and the separation of their chemical reservoirs of the core and the mantle takes place.

2. Mathematical modeling of the temperature distribution in the moon at the stage of its accumulation

The model [21] on the formation of planets and their satellites from a protoplanetary cloud is used as the initial one. The results of numerical modeling obtained by us [16, 17] showed that already at an early stage of the Earth's accumulation process, heat release during decay by short-lived naturally radioactive elements and above all ^{26}Al is enough to exceed in the protoplanetary dimensions (50–100 km); a molten central region and a relatively thin, solid, predominantly silicate upper envelope could be formed. This creates the conditions for the mechanism of differentiation of a substance into a chemical reservoir of the future predominantly iron core and a silicate reservoir of the Earth's mantle to be realized at small relative velocities of collisions of small bodies—pre-planets. The molten parts were combined, and the solid, mostly silicate, could not yet be held by the weak gravitational field of the planet and returned to the "feed zone." It is this material component that provided the chemical reservoir, from which the second largest body of the Earth-Moon system was formed. Fractionation went on in small bodies and was mostly completed in less than 10 million years, while the formation of the structure of the core and the mantle of the Earth continued for about 100 million years. The union of the liquid internal parts of the colliding bodies occurred as a result of inelastic collision; therefore, most of the potential gravitational energy was converted into heat through the kinetic energy of the collision. The process continued until the core reached a large part of the modern mass. At the final stage of the growth of the pre-planet, the mass of the Earth's pre-planet is already sufficient to hold an ever-increasing proportion of the silicate envelope of the falling out bodies. And the composition of the growing area is increasingly enriched with an admixture of silicates. The process of a completely inelastic collision of accumulated bodies with a high degree of potential energy conversion by gravitational interaction into heat, gradually turned into a mechanism of almost solid-state collisions, in which only a small part of the kinetic energy is converted into heat absorbed by the planet's pre-planet.

The carried out mathematical modeling of the thermal evolution of a growing planet implements the described process scheme. For the growth rate of the pre-planet of the planet, the Safronov model is used in the variant [21]:

$$\frac{\partial m}{\partial t} = 2(1+2\theta)r^2\omega\left(1-\frac{m}{M}\right)\sigma \tag{1}$$

where ω is the angular velocity of the orbital motion, σ is the surface density of the substance in the "feeding" zone of the planet, M is the modern mass of the planet, r is the radius of the growing pre-planet, and θ is a statistical parameter taking into account the distribution of particles by mass and velocity in the "feeding" zone. Usually, the study of changes in PT conditions in growing satellites of the planets is not paid attention. In this work, on the contrary, we aim to study how strongly these conditions can depend on the rate of increase in body weight. Let us use the estimate for the mass of the largest-growing body m and the next largest body m_1 obtained in the paper [12, 22] for the parameters of bodies in the Earth's group:

$$\frac{m_1}{m} \approx \frac{1}{3}\left(1-0.6\left(\frac{m}{M}\right)^{0.3}\right) \tag{2}$$

The mathematical description of mass-energy transport in a growing self-gravitating body of variable radius consists in setting boundary value problems for a system of equations for the balance of momentum, energy, conservation of mass of matter, and the Stefan problem at the boundaries of regions with melt zones [16–18]:

$$\rho\left[\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}\,\nabla\right)\vec{V}\right] = -\nabla P + \eta\Delta\vec{V} + \left(\frac{\eta}{3}+\xi\right)\nabla\left(\nabla\,\vec{V}\right) - \rho\nabla W \tag{3}$$

$$\rho T\left[\frac{\partial S}{\partial t}+\left(\vec{V}\,\nabla\right)S\right]=\lambda\Delta T+Q \tag{4}$$

$$\Delta W_1=-4\pi\gamma\rho \quad W=W_1+W_2 \tag{5}$$

$$\frac{\partial\rho}{\partial t}+\nabla\left(\rho\,\vec{V}\right)=0 \tag{6}$$

$$L\frac{\partial\,\vec{\psi}}{\partial t}=\vec{q}\Big|_{\xi+0}-\vec{q}\Big|_{\xi-0} \tag{7}$$

where $\vec{V}$ is fluid velocity, P is pressure, S is entropy, W_1 is gravity potential, W_2 is centrifugal potential, ρ is density, η and ξ are coefficients of the first and second viscosity, λ is thermal conductivity coefficient, γ is gravitational constant, Q is total the power of internal energy sources per unit volume, L is the heat of phase transition, $\frac{\partial\vec{\psi}}{\partial t}$ is the velocity of displacement of the interface, $\vec{q}\Big|_{\xi+0}$ *and* $\vec{q}\Big|_{\xi-0}$ the heat flux density, respectively, before and after the phase boundary, and ∇ and Δ are the operators Nabla and Laplace.

The main difficulties are connected with the solution of the boundary value problem for the Navier-Stokes equation (Eq. 2). Even in the approximation with constant viscosity coefficients, as used in [16], finding a numerical solution in a 3D spherical layer is a significant problem. In addition, within the framework of equation (Eq. 2), it is difficult to describe the forced convective mixing of a substance near the surface of a growing body when individual bodies fall. The temperature distribution in the body of increasing radius is found from the numerical solution of the boundary value problem for the heat equation, taking into account the possibility of a melt appearing without explicitly highlighting the position of the crystallization front and parametric accounting for convective heat transfer in the melt [23]:

$$c_{ef}\rho\frac{\partial T}{\partial t}=\nabla\left(\lambda_{ef}\nabla T\right)+Q \tag{8}$$

where c_{ef}, λ_{ef} is the effective values of heat capacity and thermal conductivities, which take into account the heat of melting in Stefan's [24] problem and the presence of convective heat transfer, T is the sought temperature at a point at time t, and Q is the volume power of internal heat sources. The problem was solved by the finite difference method using an implicit, monotonic, conservative scheme. In Eqs. (1)–(8), the steps on the temporal and spatial grids are the same. The time grid step is variable and, with the density distribution chosen, as a function of depth, is calculated from Eqs. (1)–(2). Using these equations, at each time step, the mass of the growing planet and the distribution of lithostatic pressure in internal regions are calculated. For each value of the achieved size of a growing planet, the melting temperature distribution is calculated. In the core, the dependence of the melting point of mainly iron composition is calculated according to [25]. In the region of the predominantly forming silicate mantle, the dependence of the melting point on pressure is used [26]. The zone of complete and partial melting was determined for each time layer by comparing the calculated temperature distribution with the distribution of the melting temperature at a given depth.

Conditions are set on the surface of the pre-planet to ensure the balance of the incoming part of the potential energy of the gravitational interaction of bodies, the expenses of heat for heating the incoming substance and the heat flow re-emitted into the space taking into account the transparency of the external environment:

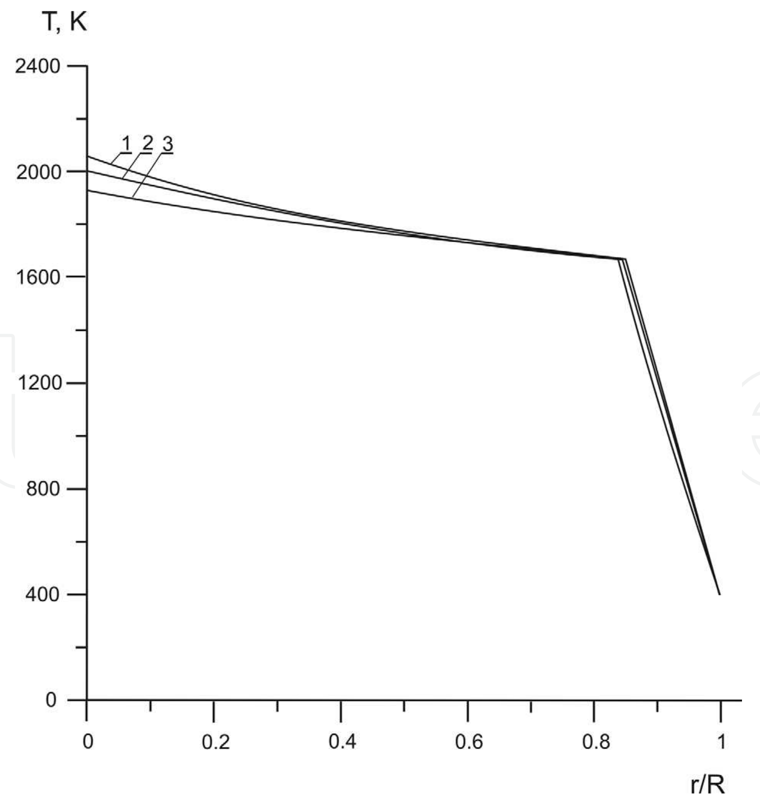

Figure 1.
The distribution of the temperature in the model of pre-planetary body. Its radius is (1) 400 km, (2) 300 km, and (3) 250 km [16].

$$k\rho \frac{\gamma M}{r}\frac{dr}{dt} = \varepsilon\sigma\left[T^4 - T_1^4\right] + \rho c_P[T - T_1]\frac{dr}{dt} \tag{9}$$

where ρ is the density of matter, γ is the gravitational constant, M is the mass of the growing planet, and r is its radius. T and T_1 are, respectively, the body temperature at the boundary, the external environment ε is the coefficient of transparency of the medium, c_p is the specific heat, k is the fraction of the potential energy converted to heat, and σ is the Stefan-Boltzmann constant. In Eq. (9), just as in Eqs. (1)–(8), the steps on the temporal and spatial grids are used the same.

The qualitative difference between the obtained variants of the results of numerical simulation in a 3D model is that it was possible to trace the occurrence of thermal and density heterogeneities. The occurrence of these heterogeneities is due to the random distribution of bodies and particles by mass and velocity, which is described by parameter θ in Eq. (1) and is taken into account in Eq. (9) using a random number generator that determines M when calculating the left side of this equation inside the layer, on which is an increase in the radius of the body over time. On **Figure 1** we show the results of calculating the temperature for a one-dimensional spatial model of a growing body.

3. Results and discussion

The results of the numerical solution of the problem for the 3D model of the environment [16, 17] are obtained, which allow us to trace the formation and further dynamics of three-dimensional anomalous in temperature and composition areas resulting from the fall of bodies and particles on the surface of a growing planet when they are randomly distributed over masses. Quantitative estimates of the parameters that determine the solution of the problem are extremely difficult.

One can only hope that further mineralogical and geophysical research results will reduce this uncertainty. One of the results obtained for the temperature distribution over the cross section of the globular sector of the growing Moon model is shown in **Figure 2a**. As can be seen from the above results, with the rapid growth of the Moon, the concentration of short-lived radioactive elements in the center of the growing body is significant, and their contribution to the energy balance can provide temperatures of 2500–3000 K in the region of R < 300 km. For large values of the lunar radius, numerous melt inclusions are recorded, which, with further evolution of the body, tend to unite. Thus, it becomes possible to trace the formation of the "ocean of magma" [27], the presence of which, according to modern concepts, is necessary for the formation of a powerful lunar crust of an anorthosite composition. **Figure 2b** presents the numerical results of the possible temperature distribution for a hypothetical body, the rate of increase in mass which is set significantly less compared to the rate of increase in the mass of the moon when it accumulates at (Eq. 1). This simulates the accumulation of the next mass in the Earth-Moon system. For this option, a slow increase in the volume of the nucleus leads to the fact that most of the decay energy of short-lived radioactive elements has time to dissipate into space and the temperature of the central region barely exceeds the melting point of pure iron. For large values of the radius in the variation of the rate of growth of body weight for the conditions of **Figure 2b**, the distribution of the current temperature values is lower than the melting temperature at a given lithostatic pressure. This eliminates the possibility of endogenous formation of a powerful anorthosite lunar crust and thereby imposes restrictions on the models of the formation of the moon. Common to the physical conditions considered in the variants of **Figure 2a** and **b** is that, due to the small magnitude of the gravitational acceleration, convective heat and mass transfer at the considered depths remains weak and for the analyzed period of time, the randomly distributed thermal heterogeneities caused by the falling of bodies still locally persist. Their presence predetermines the further dynamics of the bowels of the growing Moon.

It is interesting to compare the resulting numerical modeling of the structure of the lunar crust, mantle, and core with the crust, mantle, and core at the stage of the Earth's accumulation. The distributions of hydrostatically varying pressures, as well as melting points in these structures, lead to their qualitative change. Thus, the relative

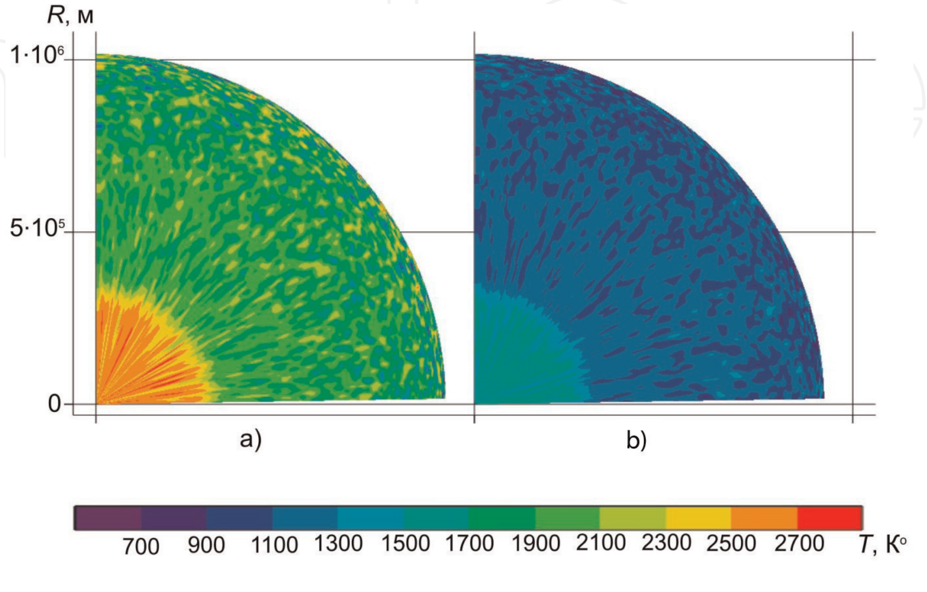

Figure 2.
The dependence of temperature distribution in the three-dimensional sector of the growing moon on the growth rate of its mass by the time the radius reaches R = 1000 km. (a) a model with the physical parameters of the moon; (b) slow growth rate by the ratio (Eq. 2).

proportion of the lunar mantle, as can be seen from **Figure 2a** and **b**, is much larger than the fraction of the Earth's mantle (**Figures 3** and **4**) [18]. The emerging boundaries in the process of accumulation of the crust-mantle of the Moon and the crust-mantle of the Earth are much more irregular, which is apparently due to significantly different pressure values at their boundaries. The same can be noted for the boundaries of the core-mantle of the Moon and the Earth. For the Earth, there is a vast solid core and an external molten core, whereas for the Moon, the inner core is either completely absent or does not appear significantly. In the future, already at the geological stage of development, this may lead to a significant difference in the mineral composition of the moon's crust from the crust of the earth. The lunar crust can be predominantly or even exclusively basalt composition.

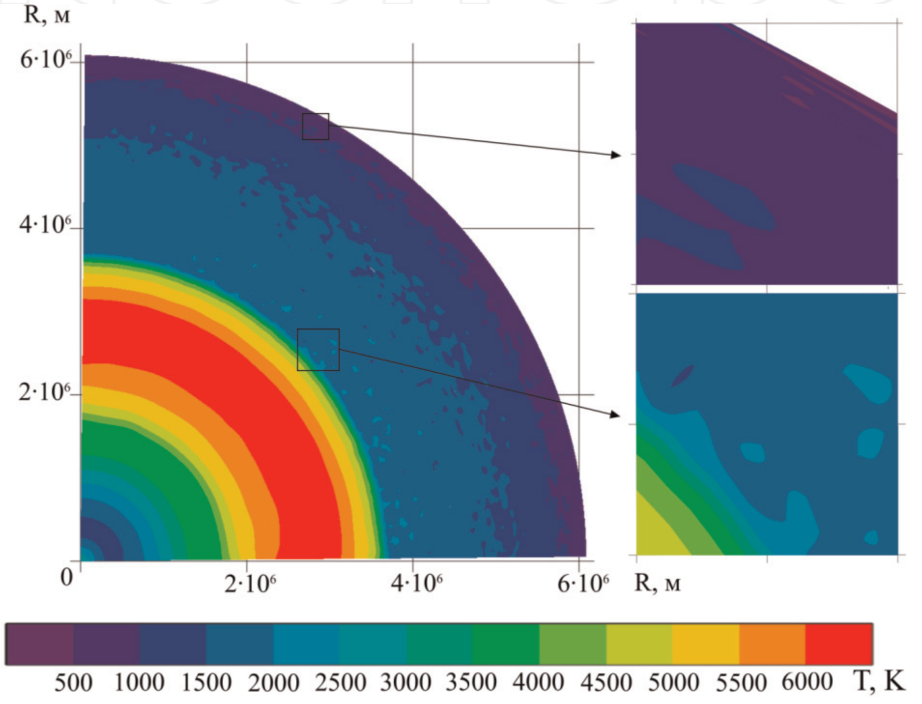

Figure 3.
The temperature distribution of the Earth's interior to the end of accumulation along the section of the 3D sector (without taking into account the heat dissipation energy of tidal friction) [18].

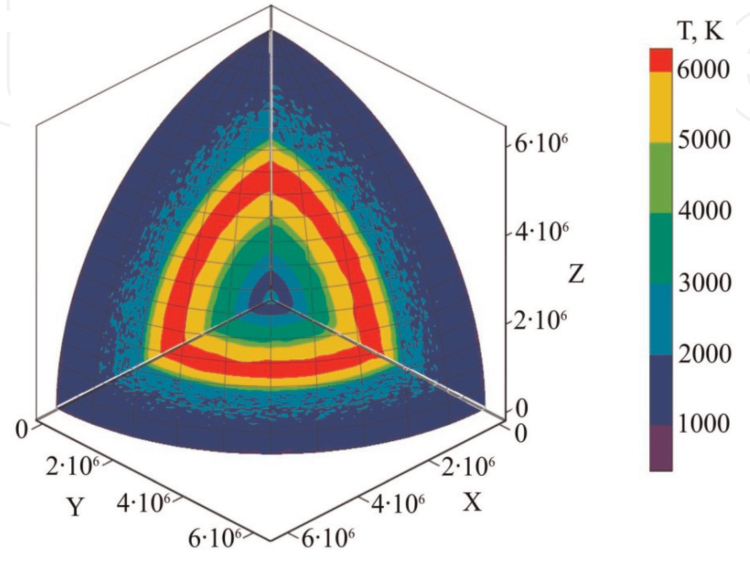

Figure 4.
An example of the temperature distribution and initial thermal heterogeneities in the protoplanet up to the end of its accumulation, without taking into account the heat of tidal friction [18].

4. Conclusions

A numerical solution of the problem is obtained for successively varying temperature and mass distributions over the cross sections of the three-dimensional spherical sector of the Moon model at the stage of its accumulation. It is shown that solutions can be obtained for temperature distribution, based on modern estimates of the physical parameters of the Moon, which provide the endogenous origin of a powerful anorthosite lunar crust. The random distribution of heterogeneities in the inner parts and on the surface of the Moon, caused by the fall of bodies on the growing surface during the accumulation process, controls the initial conditions of the planet's dynamics.

The initial conditions for the Moon and the Earth are taken for a body with a radius of 10 km, the average composition of which corresponds to the matter of carbonaceous chondrites [17] from the common feeding zone of the Earth and the Moon. With their further growth in numerical simulation on the 3D surface of the sphere and at each step of the time grid, which means the radius value, the changing boundary conditions are calculated in accordance with Eq. (5) [18]. These conditions reflect the random distribution of the accumulated bodies in size, composition, and velocity of impact with both the growing Moon and the Earth, which leads to different composition and structure of their internal regions (**Figures 2** and **3**).

In the results presented in **Figure 2a** and **b**, in the initial conditions, the concentration of Al_2O_3 is different: **Figure 2a** reflects the increased content of Al_2O_3, and **Figure 2b** shows the reduced content of Al_2O_3 [17]. To determine the preferred numerical simulation variant, additional geochemical and geophysical space–time data are required.

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