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Accuracy of Hydrogeological Calculations and Forecasts

Mikhail M. Burakov

Abstract

Aquifer systems most often appear to be double-layered or multi-layered. The parameters of groundwater flow from adjacent horizons to the tested ones through separating low-permeable layers or the leakage of groundwater from the low-permeable overlapping layers are determined by the results of pumping. There are methods for determining the permeability parameters of tested horizons and flow parameters by the results of such pumping. However, the issue of assessment of flow parameter confidence remains current. This chapter proposes a method for performing such assessment. The method was tested on a specific example. The obtained error estimates for the parameters of a layered aquifer system are typical for groundwater filtration schemes in aquifers with overflow.

Keywords: aquifers, multilayer aquifer systems, geofiltration parameters, flow parameters through separating low-permeable sediments, accuracy of parameter determination

1. Introduction

There are quite a lot of examples of recording of groundwater (GW) flow from adjacent aquifers of layered formations in estimating the commercial reserves of deposits in artesian basins of various orders, solving other problems of GW filtration. The confidence of such assessments is entirely determined by two main factors, namely by the compliance of the design scheme with the natural conditions and the confidence of the established parameters of the main and adjacent aquifers and the flow parameters. All this shows the urgent need, on the one hand, of assessing the flow values only by direct methods, and on the other, revising the methods of experimental determination of the flow parameters in layered formations with assessments of random errors of such parameters.

2. Assessment of confidence of flow parameters determined by the results of pumping from the layered aquifer systems

2.1 Physical and mathematical models of groundwater filtration in multilayer systems with overflow

As is known, when conducting experimental filtration testing (EFT) of the layered formation to determine the parameters of low-permeable separating

sediments, the data on the decrease in the GW level mainly in the aquifer tested are used. At the same time, with a decrease in pressure in this aquifer during the process of cluster pumping test, two options of adjacent aquifer reaction are possible: (a) a decrease in the level determined by the pumping does not occur and (b) a decrease in pressure is recorded. In some cases, the elastic filtration in separating layers is significantly manifested.

GW filtration scheme, for which there is no reaction in the adjacent horizon during the EFT, GW movement in the tested horizon to the disturbing well operating with a constant flow rate, with an elastic filtration mode in separating low-permeable sediments, is described based on equation [1]. For a period of time that meets the condition

$$t < \frac{m_0 \mu_0^*}{10 k_z},$$

this equation is presented as

$$S = \frac{Q}{4\pi T} \ H(u, \beta), \quad u = \frac{r^2}{4\chi t}, \quad \beta = \frac{r}{4B} \ \sqrt{\frac{\mu_0^*}{\mu^*}}, \quad B = \sqrt{\frac{m_0 T}{k_z}}, \qquad (1)$$

where S is a decrease in piezometric level of GW at a distance r from the disturbance centre at a moment of time t from the start of pumping; Q is the constant flow rate of disturbance; $H(u, \beta)$ is the improper integral tabulated in [1]; $T = Km$, μ^*, K and m are water transmissibility, elastic water yield, permeability factor and the thickness of tested horizon, respectively; $\chi = \frac{T}{\mu^*}$ is the piezoconductivity of the tested horizon; k_z and μ_0^* are permeability factor and elastic water yield of separating low-permeable sediments, respectively; B is the flow factor.

In case of prolonged disturbances, when the estimated time values meet the following condition:

$$t > \frac{5 m_0 \mu_0^*}{k_z},$$

Eq. (1) is as follows [1]:

$$S = \frac{Q}{4\pi T} \ W\left(u \delta_1, \frac{r}{B}\right), \quad \delta_1 = 1 + \frac{\mu_0^*}{3\mu^*}, \qquad (2)$$

where $W\left(u \delta_1, \frac{r}{B}\right)$ is the well function for layered systems with the flow. The rest designations remain the same.

A special case arising from (2) is the equation obtained in [2], in which the volume of water in low-permeable sediments is assumed to be negligible, that is, $\mu_0^* \to 0$, and the filtration mode in low-permeable layer becomes hard. It is presented as

$$S = \frac{Q}{4\pi T} \ W\left(u, \ \frac{r}{B}\right). \qquad (3)$$

Here, as in (2), $W\left(u, \frac{r}{B}\right)$ is the improper integral tabulated in [3] (the Hantush function). The rest designations remain the same.

Eq. (3) is applicable only to aquifers with relatively small thickness [1], so that the following condition should be fulfilled:

$$\frac{m}{B} \leq 0.1.$$

This assumption is usually fulfilled [4], so accordingly, Eq. (3) is applicable for interpreting the EFT results in most practical cases.

Thus, to date, there is a well-developed theoretical base that provides the fundamental possibility of describing and studying GW filtration in layered systems with GW flow to the tested aquifers from adjacent ones. Based on this, the methods for the interpretation of the results of experimental cluster pumping from the layered aquifer systems were developed in order to determine the parameters of the tested horizon and the parameters of separating low-permeable sediments.

2.2 Formulation of problem of determining the random errors of the parameters

When conducting EFT, the parameters of the tested horizon and separating low-permeable sediments are not measured directly; they are calculated by equations, and these equations include directly measured characteristics of initiated disturbance of the aquifer system as arguments. The characteristics are measured with some errors so that in the parameters of the conducting medium determined by the results of experimental works, all possible errors of measurements of initial data and intermediate calculations appear.

In full accordance with the theory of errors in measurements and calculations in the process and interpretation of the results of test pumping from wells, there is a probability of distortion of the parameters to be determined (water transmissibility T, piezoconductivity χ and elastic water yield μ^* of the tested horizon and flow factor B and permeability factor of the separating low-permeable layer k_z), that is, introducing systematic and random errors [5–11].

Identification and elimination of systematic errors are an integral part of engineering calculations. The main approach to eliminate such errors is to make signed corrections in the results of measurements and calculations. The overwhelming majority of publications on the GW dynamics are devoted to estimates of systematic errors and methods of their accounting; in those publications, all types of errors are studied in sufficient detail.

The situation is different with estimates of random errors of the parameters of aquifer systems to be determined. The latter is due to the cumulative manifestation of many factors, and the nature of the manifestation of each of these factors is not exactly reproduced in the repeated (and subsequent) testing of aquifer systems. Among random errors, the most significant is individual, instrumental, methodological, and model errors [8, 12]. These errors, unlike systematic ones, cannot be excluded from the results of measurements and calculations; therefore, their identification and estimation of the values of such errors are extremely important because, ultimately, they determine the confidence and reliability of the parameters to be set.

However, the issues of assessing the accuracy of primary measurements and the parameters of aquifer systems have not been studied enough. There are several works in the Russian language [9, 13–15], which basically represent the full list of publications with the reported results of the assessment of random, mainly instrumental, errors of the most important parameters, namely water transmissibility, piezoconductivity, and water yield of tested aquifers. In the publications mentioned, the accuracy of the primary measurements and the probable random errors of the parameters were analysed with reference to the methodological recommendations for interpreting the results of pumping from [16–18]. The main disadvantage of the latter is the use of single measurements of the level decrease in wells, which almost always leads to significant systematic methodological errors of the

established characteristics, which cannot be identified and eliminated. Estimates of random error parameters in such cases lose their significance.

Therefore, as a standard method for interpreting the EFT results of aquifers and layered systems, the graphic-analytical method, as well as reference curve method, which are largely devoid of this drawback [11, 19], were recommended. Accordingly, in some works [5, 12, 20–22], methodological approaches and results of assessing the reliability of parameters of aquifers, as well as layered aquifer systems with the flow, were described. In the works [23, 24], the results of assessing the individual random errors of the determined parameters and the detection of gross measurement errors are presented.

A well-known common equation is used to assess the confidence of EFT results in aquifers. Thus, if a differentiable function

$y = f(x_1, x_2, ..., x_n)$ is given, and if mean square error of x_i arguments σ_{x_i} (i = 1, 2, ..., n) is known (where n is the number of arguments), then the mean square error of this function is [7–10, 25]

$$\sigma_y \cong \sqrt{\sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i} \sigma_{x_i} \right)^2}. \qquad (4)$$

Measurement errors in most geological and geographical studies are estimated as limiting [6, 9], as well as in documents on measuring instruments used in hydrogeological practice, maximum permissible random errors are indicated. The latter is determined by the worst conditions that have arisen during measurements of any characteristic of the disturbance: all components of the errors are maximum in absolute value, and all of them are of the same sign. The probability of these errors is fixed and very small, and meanwhile, the need for error estimates with significantly higher probabilities often arises, since the actual errors of measurements and calculations are noticeably less than the limiting ones. Therefore, the guidelines on metrology (as, for example [7–10, 25]) recommend performing the calculations based on Eq. (4), which has a clearly expressed probabilistic nature. At the same time, for a normally distributed quantity, all spreads with an accuracy of fractions of a percent are within 3σ.

This implies that on the basis of (4), one can find out the maximum permissible errors of the parameters; however, for practical calculations, they are usually limited to the confidence probability of 0.954, and then the errors σ_{x_i} with a confidence probability of 0.683 are doubled [7, 9, 10, 25, 26]. Limiting permissible errors of measuring devices and primary measurements correspond to a confidence probability level of 0.997 and constitute $3\sigma_{x_i}$ [7, 9, 10, 25, 26]. Random errors of permeability parameters are estimated lower with a probability of 0.954.

2.3 Assessment of random error parameters of aquifer tested by test pumping

Let us consider the assessment of random errors of parameters of tested aquifer being a part of the multi-layered system, parameters of separating low-permeable sediments for the filtration scheme with the constant level in the adjacent horizon [21] on the example of cluster pumping from the pressure Upper Cretaceous (Mynkuduk) aquifer within the Suzak artesian basin [27].

The Mynkuduk horizon represents here the lowest part of the section of the Upper Cretaceous aquifer complex. The horizon is underlain by Palaeozoic poorly lithified silty-argillaceous sediments, which act as a regional confining layer. The water-bearing rocks of the Mynkuduk horizon are fine and medium grained sands with gravel and a low content of clay particles.

There are almost no persistent and extended regional confining layers between the Mynkuduk and above lying Inkuduk horizons or low-permeable interlayers within the horizon. All existing ones are lenses of various areas with a thickness ranging from 1–2 to 5–10 m. Low-permeable rocks are represented by clays, silts and clay sands. As a result, the adjacent aquifers of the Upper Cretaceous complex are hydraulically connected.

The disturbing well no. 2001c and observation wells no. 2002g and no. 2003g of test cluster within the Mynkuduk horizon and at the interface with Inkuduk horizon have penetrated several rather stably thick local clay interlayers with the thickness varying from 2 to 5–7 m. These interlayers determined the peculiarities of GW filtration in the disturbance region, which correspond to the regularities of their movement in multilayer aquifers in the presence of flow through the low-permeable separating sediments.

Pumping was conducted with a submersible pump at a constant average flow rate of Q = 11.10 m^3/h (266.4 m^3/day). The flow rate over the course of the experiment was measured by the volumetric method, that is, by the duration of the filling of the measuring tank with a volume of V = 100 dm^3. Time readings for measurements were made using stopwatch with a scale with a division value of 0.2 s.

The duration of the experimental pumping was 9 days. The depth down to the piezometric level in the wells during pumping was measured by two-contact electric level gauges with a measuring tape length of 50 m and a scale interval of 0.001 m.

It should be noted that the use of gauges with a measuring tape, on which millimetre divisions are applied, does not, in fact, solve the problem of raising the accuracy class of measuring equipment. The experience of using such devices in field conditions shows that the accuracy of the depth measurements to the GW level, in general, is about 0.5 cm (primarily due to the delayed action of the signal system), that is, accuracy remains the same with the traditionally used measuring devices [13, 15, 28]. In addition, as shown in [13, 15], with a relative maximum permissible random error of depth measurement to the GW level in the well by electrical level gauge with a measuring tape with millimetre divisions, in 0.06% of cases, 50 m depth is measured with the maximum absolute error of ±0.030 m, 40 m depth has ±0.024 m error and, finally, 10 m depth has ±0.006 m error. It clearly shows that the third digit after the decimal point in the depth down to GW level in well measured by such a level gauge does not imply any information and should be discarded; it is enough to have a scale with divisions of at least 0.005 m [13, 15].

During the processing of the results of experimental pumping, a number of corrections for systematic errors were introduced into the experimental data. First, according to the results of inclinometry in the wells of the test cluster, the distances between the axes of the filters of the wells are specified. Accordingly, the geometric parameters of the test cluster are summarised in **Table 1**. Secondly, corrections for changes in the piezometric level in the wells depending on the changes in barometric pressure P are introduced into the measured decreases in the observation well levels of the test cluster. For this, before the start of pumping, the standard M-67 aneroid barometer was used to measure the atmospheric pressure for 3 days with an interval of 4 hours. At the same time points, the same two-contact electrical level gauge was used to measure the depth down to the GW level in the wells of the test cluster.

Processing of the observation results revealed a close correlational linear relationship of the fluctuations of the piezometric level with the fluctuations of the atmospheric pressure; the correlation factors characterising the closeness of this relationship for well nos. 2002g and 2003g were 0.9752 and 0.9799, respectively.

| Well no.	Distance between the axes of the wellheads of disturbing and observation wells *r*, m	Depth down to the middle part of filter axis, m	Shift of the middle part of filter axis relative to the wellhead, m		Actual distances from the centre of disturbance to the axes of filters of observation wells *r*, m
			X offset	*Y* offset	
2001c	0	500	−0.56	−2.20	—
2002g	24.81	512	−0.47	−1.11	25.18
2003g	125.19	512	−3.97	−4.09	127.54

Table 1.
The values of the horizontal shift of the filter axes in the wells of the test cluster.

From these equations, it follows that the depth down to the piezometric level z per change in atmospheric pressure *P* in 1 mm Hg changes by an average of 0.5 cm, and when the pressure drops, the depth z decreases and, conversely, as the pressure increases, *P* increases as well.

This correction is made further in the results of the level tracing. For this, throughout the experiment, barometric pressure was measured using the same aneroid barometer with 4 h intervals.

The temporal tracing graphs for the observation well nos. 2002g and 2003g presented in **Figure 1** clearly indicate the bends corresponding to the beginning of the GW flow from the adjacent parts of the Mynkuduk aquifer through low-permeable interlayers. Accordingly, the parameters of the tested part of the Mynkuduk aquifer were calculated only by the first asymptotic segments of the graphs corresponding to the filtration scheme in an isolated pressure horizon not limited in the plan [19, 29, 30]. The results of the parameter calculations are given in **Table 2**.

It is to be recalled that the computational formulae corresponding to the method of temporal tracing of a decrease in the water level in observation wells during
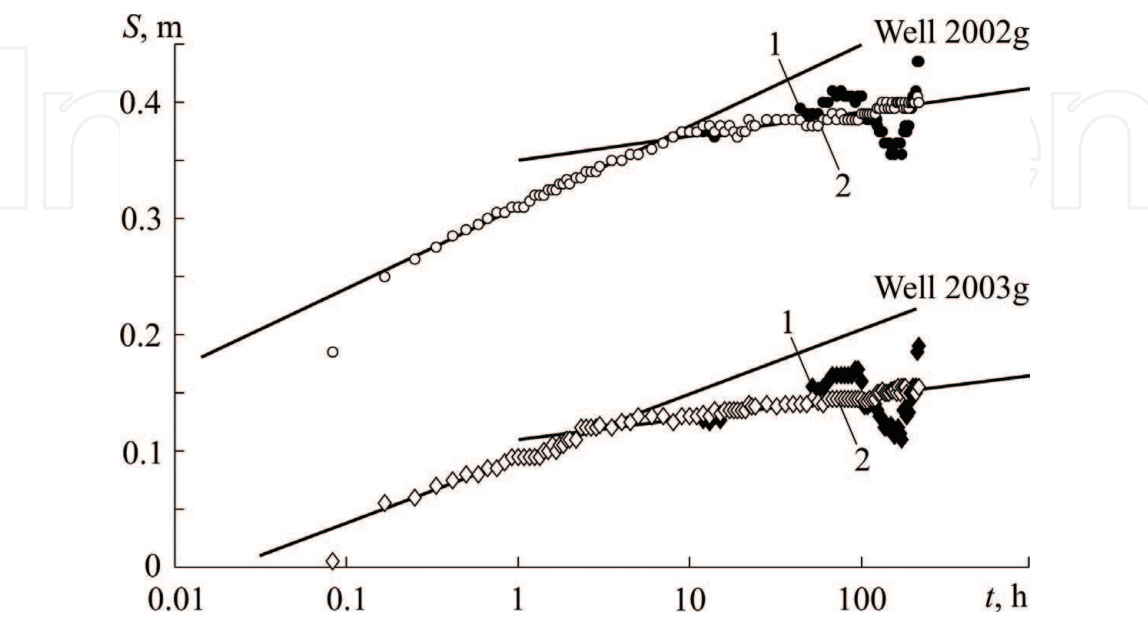
Figure 1.
Graphs of time tracking of the lowering the piezometric level in the observation wells of the experimental cluster 2002g and 2003g. Legend: 1—initial measurements of lowering the level; 2—lowering level adjusted for barometric pressure.

Parameters	Parameter values by temporal tracing data		By combined tracing data
	No. 2002g	No. 2003g	
Distance from the centre of disturbance to the axis of the filter of the observation well r, m	25.18	127.54	—
$2\sigma_r$, m	0.026	0.094	—
The number of experimental points on the calculated asymptote N	37	36	—
Slope ratio of the first calculated asymptote C, m	0.0690	0.0571	0.0684
Decrease in level A at a moment of time t_0, m	0.1890	0.0919	0.3797
Estimated pumping rate Q, m³/h	11.10	11.10	11.10
Water transmissibility T, m²/h	29.4	35.6	29.7
Standard error of the regression equation S_y, m	0.00242	0.00589	—
Normalised deviation of Student's distribution $t_{q,N-2}$	2.0301	2.0322	—
$2\sigma_C$, m	0.00407	0.00964	—
$2\sigma_A$, m	0.00185	0.00428	—
$2\sigma_Q$, m³/h	0.939	0.939	—
$2\sigma_T$, m²/h	3.04	6.72	—
Relative error of water transmissivity $2\delta_T$, %	10.33	18.87	—
B parameter value	2.731	1.610	—
Piezoconductivity χ, m²/h	$1.55 \cdot 10^5$	$2.95 \cdot 10^5$	$1.58 \cdot 10^5$
Elastic water yield μ^*	$1.9 \cdot 10^{-4}$	$1.2 \cdot 10^{-4}$	$1.9 \cdot 10^{-4}$
$2\sigma_B$	0.1638	0.2819	—
$2\sigma_\chi$, m²/h	$5.90 \cdot 10^4$	$1.93 \cdot 10^5$	—
Relative error of piezoconductivity $2\delta_\chi$, %	38.06	65.42	—
Relative error of elastic water yield $2\delta_{\mu*}$, %	39.44	68.01	—

Table 2.
Porosity and permeability parameters of the Mynkuduk aquifer, their random errors and errors of intermediate calculations (with a confidence probability of 0.954) established by the test cluster.

pumping, for which the solution of the problem of inflow to the well arising from the initial differential filtration equation, are approximated by the equation [19, 29, 30].

$$S = A + C \lg t, \tag{5}$$

and are presented in the form of [12, 21].

$$T = \frac{0.183 \ Q}{C}, \quad C = \frac{S_2 - S_1}{\lg t_2 - \lg t_1}, \quad \chi = 0.445 \ \frac{r^2}{t_0} \ 10^B, \quad B = \frac{A}{C}, \tag{6}$$

where C is the slope ratio of the calculated asymptotic segment in the temporal tracing graph; S_1 and S_2 are the decrease of the level in the assigned observation well at time points t_1 and t_2 from the start of pumping, respectively; A is the decrease in the level at time point t_0; r is the distance between the axes of the filters of the

disturbing and the observation wells. The values S_1, S_2 and A are removed from the asymptotic part of the tracing graph, and the time t_0 is set to 1 in the selected units, so lg $t_0 = 0$.

Conversion of equations from (6) in accordance with (3) gives [12, 20].

$$\sigma_T \cong \sqrt{\left(\frac{0.183 \ \sigma_Q}{S_2 - S_1} \lg \frac{t_2}{t_1}\right)^2 + \left[\frac{0.183 \ Q}{(S_2 - S_1)^2} \lg \frac{t_2}{t_1}\right]^2 \left(\sigma_{S_1}^2 + \sigma_{S_2}^2\right)}, \qquad (7)$$

$$\sigma_\chi \cong \sqrt{\left(0.890 \frac{r}{t_0} 10^B \sigma_r\right)^2 + \left(1.036 \frac{r^2}{t_0} 10^B \sigma_B\right)^2}, \qquad \sigma_B \cong \sqrt{\left(\frac{\sigma_A}{C}\right)^2 + \left(\frac{A \ \sigma_C}{C^2}\right)^2}. \qquad (8)$$

Errors in the time readings do not exceed a few seconds (in the extreme case, the first tens of seconds); therefore, when processing the experimental data, the terms of sum in (3) containing σ_t can be neglected [12, 20]; they are omitted in formulae (7) and (8).

When formulating the problem of assessing the accuracy of parameters, a priori, it was assumed that the measured decreases of the level unambiguously fall on the asymptotes of the temporal tracing graphs; this assumption is clearly manifested in Eq. (7). In this case, the fact that the experimental set of points was dispersed relative to the true asymptote, which corresponds to the actual parameters of the conducting medium, was disregarded. And yet, in a point cloud, even with a visible regression connection, in each specific case, there can be several averaging options (performing computational asymptotes), which is, among other things, the cause of individual errors. The results of the study of such individual random errors are set forth, in particular, in [23].

As is known, one of the most important representations of the macroscopic phenomenological theory of filtration is the constancy of parameters of medium over time. With reference to the graph-analytical method, this means the existence of asymptotic segments with constant slope ratio on the temporal tracing graphs. The dispersion of measured values of GW level decrease relative to these asymptotes is due to the realisation of measurement errors (instrumental error), fluctuations in the disturbance flow rate around its average value taken as the calculated one (methodical error) and the influence of chaotic geofiltration inhomogeneity of higher order or effective one (model error). Therefore, the experimentally recorded deviations of the level drops from the approximating asymptotes (which determine the final random errors of the parameters) should always exceed such deviations only due to instrumental errors.

Thus, the task of estimating the random errors of transmissibility, piezoconductivity and water yield of water-bearing sediments is reduced to find the calculated asymptotes that best match the true asymptotes and compare them with the asymptotes for the extreme variants of the drawing. Estimates of the discrepancies between them are maximal and complexly contain all possible random errors of permeability parameters [5, 12, 20].

The necessity of comparing the chosen calculated asymptote with the true one is a serious problem: within the framework of the physical and mathematical model of nonstationary GW filtration, and there are no criteria determining the preferred choice of any of the asymptotes. A very fruitful idea to get around this problem seems to be an involvement of the true (more precisely, the closest to the true) asymptotes of the regression analysis to justify this choice. Analysis of parameters A and C in (5) as regression coefficients and their confidence intervals for a selected level of significance provides the required estimate of the total random errors of the

parameters. Optimization of studies of this kind is achieved on the basis of the maximum likelihood method, which, assuming a normal distribution of experimental data, is reduced to the least squares method [5, 12, 20, 21]. In [24], when justifying the method for detecting gross errors in measuring level decreases, the distribution of the deviations of these measured decreases from the calculated asymptotes justified by the least squares method was proved as complying with the normal law.

As it was noted before, the confidence intervals of the regression coefficients A and C to the selected level of significance are identified with the random errors of these coefficients to the same level of significance, and taking into account the latter, the errors of the parameters are established. Random errors A at any level of significance q were determined by the formulae [31].

$$\Delta_A = \pm \frac{t_{q,\ N-2}\ S_y}{\sqrt{N-2}} \sqrt{1 + \frac{N\left(\overline{\lg\ t}\right)^2}{\sum\limits_{j=1}^{N}\left(\lg\ t_j - \overline{\lg\ t}\right)^2}}, \quad S_y = \sqrt{\sum\limits_{j=1}^{N}\left(\frac{S_j - \tilde{S}_j}{N-2.}\right)^2} \qquad (9)$$

Here $\overline{\lg\ t}$ is the average value of the variable $\lg\ t_j$; $t_{q,\ N-2}$ is the normalised deviation of the Student's distribution that depends on the level of significance q and the number of degrees of freedom $N-2$, the values of the deviation can be found, for example, in [26]; N is the number of measured values of the level decrease on the calculated asymptote; S_y is the standard error of the regression equation; $\tilde{S}_j$ is the value of variable S, calculated by the regression equation.

The calculation of the errors of the slope ratio C is performed on the basis of the equation for the limits of its confidence interval [31], in which the value of the confidence interval at any level of significance is as follows:

$$\Delta_C = \pm \frac{t_{q,\ N-2}\ S_y}{\sqrt{N-2}} \sqrt{\frac{N}{\sum\limits_{j=1}^{N}\left(\lg\ t_j - \overline{\lg\ t}\right)^2}}. \qquad (10)$$

The method of processing experimental data provides for a constant pumping rate throughout its duration. However, in fact, its value fluctuates around a certain average value taken as a calculated one. In the works [9, 13, 15], information is given on the instrumental errors of pumping flow rate measurement by methods used in hydrogeological practice. In particular, devices for establishing flow rate by the volumetric method provide measurements with limiting relative errors of 1–6%.

In fact, the total random errors in measuring the flow rate are noticeably bigger than instrumental ones, since the recorded fluctuations Q are also due to the unstable in time operation of the water-lifting equipment. Due to the equal probability of deviation in one direction or another from the average measured values of the flow rate, their distribution can be considered normal, and then the standard quadratic deviation Q is equal to

$$\sigma_Q = \sqrt{\frac{\sum\limits_{j=1}^{N}\left(Q_j - Q\right)^2}{N-1}}. \qquad (11)$$

Here, Q_j and Q are the measured and average values of flow rate, respectively.

Eq. (7) can be changed taking into account the second in (6), (10) and (11) as follows:

$$\sigma_T \cong \sqrt{\left(\frac{0.183 \; \sigma_Q}{C}\right)^2 + \left(\frac{0.183 \; Q \; \sigma_C}{C^2}\right)^2}. \tag{12}$$

The absolute random error of the piezoelectric conductivity is still estimated by the ratio (8).

Now, there are dependencies necessary for calculating all the errors of primary measurements and intermediate calculations for calculating random errors of permeability parameters. The results of calculations of the parameters of the Mynkuduk aquifer, their random absolute and relative errors and errors of intermediate calculations performed in accordance with the methods developed in [5, 12, 20–22] shown herein, are summarised in **Table 2**.

According to these results, the following remarks should be made. Random errors of transmissibility and piezoconductivity, established according to the level tracing data in well no. 2003g, significantly (approximately twice) exceed those for the parameters of well no. 2002g. This fact has a simple and quite logical explanation. With a sufficiently large depth down to the piezometric level in the observation wells (~28–30 m) and the same degree of dispersion of experimental points relative to the calculated asymptotes on the level tracing graphs in both wells of the test cluster, the error of the parameters is greater than for the well for which lower absolute values of level decrease are recorded. This is well no. 2003g, located at a greater distance from the centre of the disturbance.

The logarithmic approximation of the Theis formula as (5) and (6) holds for the conditions of a quasi-stationary filtration flow [19]. There is an analytical criterion (control time) t_K used to find the plot of the graph that meets the quasi-stationary mode:

$$t_K = \frac{r^2}{0.4\chi}. \tag{13}$$

All designations here remain the same.

The control time of the onset of the quasi-stationary filtration mode, the assessment of which is made on the basis of the parameters presented in **Tables 1** and **2**, in accordance with (13) is as follows: $t_K \cong 0.01$ h (36 s) for well no. 2002g, the filter of which is located at a distance of r_1 = 25.18 m from the axis of the filter of the disturbing well; $t_K \cong 0.26$ h (15 min) for well no. 2003g, the filter of which is located at a distance of r_2 = 127.54 m from the axis of the filter of the disturbing well.

The calculated values of the control time clearly showed that the selected rectilinear asymptotic segments on the temporal tracing graphs in the observation wells of the test cluster fully satisfy the applicability condition of a logarithmic approximation of the Theis formula for processing and interpreting experimental data. Accordingly, the parameters of the test aquifer, calculated for these sections, are representative.

A kind of quality control performed by the EFT for the layered aquifer system under conditions of GW flow into the test horizon from the adjacent one through separating low-permeable formation provides a combined way of processing the tracing data of the piezometric level decrease in the observation wells of the test cluster. A characteristic feature of indicator graphs of combined level tracing, as well as temporal tracing graphs, is the presence of bends that limit rectilinear asymptotic segments with different rates of change in the rate of level decrease (**Figure 2**). At the same time, the second asymptotic segments of the observation wells, which are located at different distances from the disturbance centre, deviate from the first asymptote common to the same observation wells. Thus, the match of

the first asymptotes, their confluence, is an indicator of the quality of the EFT of the Mynkuduk aquifer under conditions of GW flow.

The results of calculations of the parameters of the tested Mynkuduk aquifer according to the interpretation of the combined tracing graphs are presented in **Table 2**. These parameters are in excellent agreement with the parameters established according to the interpretation of temporal tracing of the level.

2.4 Estimates of random error parameters of groundwater overflow

The results of the test pumping ensured the determination of the parameters of the GW flow from the adjacent aquifer to the tested one through low-permeable sediments. Their assessment was carried out on the basis of the filtration scheme in a multi-layered formation with a constant level in the feeding aquifer [1–4, 19, 32].

As it was noted before, the temporal tracing graphs of the piezometric level in the observation wells Nos. 2002g and 2003g, two asymptotic segments are clearly distinguished. According to the first, the parameters of the tested Mynkuduk aquifer are estimated (see **Table 2**). The second, final ones ensured the calculation of the flow factor B and the permeability factor of the separating low-permeable layer.

The inversed problem that is the estimation of $W(z)$ function is solved by the known values of Q, S_K (where S_K is the estimated level decrease in the observation well at the selected time) and T:

$$W\left(u, \frac{r}{B}\right) = \frac{4\pi T S_K}{Q}. \tag{14}$$

Then, using the established values of $W(z)$ and u from the tables of the Hantush function given, for example, in [3], $\frac{r}{B}$ ratio value is selected, from which the flow factor B is calculated. Taking into account the values of the latter and if the layer thickness of low-permeable sediments m_0 is known, the value of their permeability factor k_z is estimated using the formula [1–3, 19] as follows:
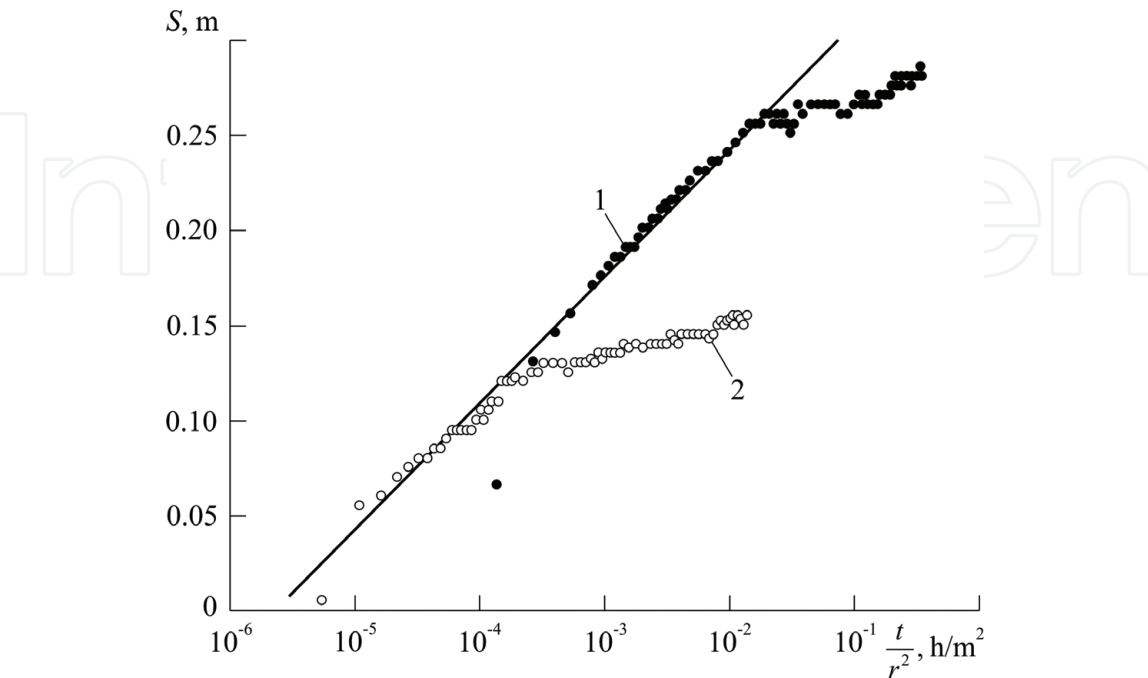
Figure 2.
Graphs of the combined tracking of the lowering the piezometric level in the observation wells of the experimental cluster 2002g and 2003g, taking into account the corrections introduced. Legend: 1—lowering the level in the well no. 2002g; 2 – lowering the level in the well no. 2003g.

$$k_z = \frac{T m_0}{B^2}. \tag{15}$$

The results of the calculations of the flow factor and the permeability factor of low-permeable sediments in the observation wells of the test cluster are summarised in **Table 3** [21]. The values of the flow factor and permeability factor of the separating layer, averaged over the observation wells of the test cluster, are $B \cong 1810$ m and $k_z \cong 7.1 \cdot 10^{-5}$ m/h.

Applying formula (3) to Eq. (14) gives the following equation for the root mean square deviation of the function $W(z)$ [21]:

Parameters	Parameter values by observation wells of the cluster	
	No. 2002g	No. 2003g
Average pumping rate Q, m³/h	11.10	11.10
Distance from the centre of disturbance to the axis of the filter of the observation well r, m	25.18	127.54
Estimated moment of time from the start of pumping t, h	20	20
Estimated level decrease S_K, m	0.257	0.135
Number of experimental points on the calculated (second) asymptote N	63	65
Standard error of the regression equation S_y, m	0.003862	0.002150
Normalised deviation of Student's distribution $t_{t,\ N-2}$	1.9994	1.9986
$2\ \sigma_{S_K}$, m	0.003254	0.001668
Value of Hantush function $W(z)$	8.554	5.441
$2\ \sigma_{W(z)}$	1.148	1.127
$2\ \delta_{W(z)}$, %	13.42	20.71
Estimated value of u parameter	$5.11 \cdot 10^{-5}$	$6.89 \cdot 10^{-4}$
$2\ \sigma_u$	$1.95 \cdot 10^{-5}$	$4.52 \cdot 10^{-4}$
$2\ \delta_u$, %	38.16	65.60
$\frac{r}{B}$ ratio	0.0137	0.0722
Relative error $2\delta_{\frac{r}{B}}$, %	40.45	68.79
$2\sigma_{\frac{r}{B}}$	0.0055	0.0497
Value of flow factor B, m	1840	1770
$2\sigma_B$, m	740	1220
$2\delta_B$, %	40.22	68.93
Thickness of the layer of low-permeable sediments m_0, m	7.0	7.0
$2\sigma_{m_0}$, m	0.50	0.50
Permeability factor for the separating layer k_z, m/h	$6.1 \cdot 10^{-5}$	$8.0 \cdot 10^{-5}$
$2\sigma_{k_z}$, m/h	$4.9 \cdot 10^{-5}$	$1.1 \cdot 10^{-5}$
$2\delta_{k_z}$, %	80.33	137.50

Table 3.
Values of flow factor and porosity and permeability parameters of low-permeable sediments and their random errors (with a confidence probability of 0.954).

$$\sigma_{W(z)} \cong \sqrt{\left(\frac{4\pi S_K \sigma_T}{Q}\right)^2 + \left(\frac{4\pi T \sigma_{S_K}}{Q}\right)^2 + \left(\frac{4\pi T S_K \sigma_Q}{Q^2}\right)^2}. \qquad (16)$$

As it was mentioned before, the error of the calculated decrease taken from the asymptote for any lg t_K at any selected time point with a confidence level of 0.683 is expressed by the following equation:

$$\sigma_{S_K} = \pm \frac{t_{q,\ N-2}\ S_y}{\sqrt{N-2}} \sqrt{1 + \frac{N\left(\lg t_K - \overline{\lg t}\right)^2}{\sum\limits_{j=1}^{N}\left(\lg t_j - \overline{\lg t}\right)^2}}. \qquad (17)$$

Eq. (17) is the basis of experimental estimates of σ_{S_K}.

So, now there is a possibility of finding random errors of the values of the Hantush function (used later to determine the flow factor B and the permeability factor of low-permeable sediments k_z), and, consequently, the errors of the flow parameters (see **Table 3**).

A few comments should be made concerning the order of calculations.

The calculation of the random errors of $\frac{r}{B}$ parameter based on the original equation of the Hantush function from [3] is impossible in principle. To implement this procedure, the following technique was proposed [21].

Random errors of the Hantush function are quite simply calculated from Eq. (16). The errors of the parameter u, one of the arguments of the Hantush function, are estimated from the following equation (neglecting the term of sum containing σ_t).

$$\sigma_u \cong \sqrt{\left(\frac{r\sigma_r}{2\chi t}\right)^2 + \left(\frac{r^2 \sigma_\chi}{4\chi^2 t}\right)^2},$$

where all the designations remain the same.

The ratio $\frac{r}{B}$, as provided by the method [19], is calculated using the Hantush function tables based on the following equation:

$$\frac{r}{B} = Inf\ W(z). \qquad (18)$$

In the presence of the calculated absolute values of functions $W(z)$ and u, as well as their random errors $\sigma_{W(z)}$ and σ_u, it is easy to determine the relative errors of functions $\delta_{W(z)} = \frac{\sigma_{W(z)}}{W(z)}$ and $\delta_u = \frac{\sigma_u}{u}$ for the same level of confidence probability. Accordingly, regardless of the type of function $W(z)$ in (18), the relative random error of the ratio $\frac{r}{B}$ can be represented as follows:

$$\delta_{\frac{r}{B}} \cong \sqrt{\delta_{W(z)}^2 + \delta_u^2},$$

whence it is easy to estimate $\sigma_{\frac{r}{B}}$.

Having denoted the calculated value of ratio $\frac{r}{B}$ as G, and assigning the random error calculated above, that is, $\sigma_{\frac{r}{B}} = \sigma_G$ to the last parameter, the equation for the random error B can be presented as follows:

$$\sigma_B \cong \sqrt{\left(\frac{\sigma_r}{G}\right)^2 + \left(\frac{r\sigma_G}{G^2}\right)^2}. \qquad (19)$$

The calculation of the random error σ_B by formula (19) is provided with all the necessary intermediate values (**Table 3**) and does not cause difficulties.

Based on the ratio (15), it is possible to immediately calculate the random errors of the permeability factor of low-permeable sediments. If formula (3) is applied to Eq. (15), then [21]

$$\sigma_{k_z} \cong \sqrt{\left(\frac{m_0 \sigma_T}{B^2}\right)^2 + \left(\frac{T \sigma_{m_0}}{B^2}\right)^2 + \left(\frac{2 T m_0 \sigma_B}{B^3}\right)^2}. \tag{20}$$

In the latter ratio, only the value of the error in measuring the thickness of the layer of low-permeable sediments σ_{m_0} remains undetermined. If assumed that the thickness of the layer is calculated as the difference between two measurements of the depth to its bottom z_Π and its top z_K (for example, drill stems and geophysical instrument cable) with a measuring tape that provides the maximum permissible random error of 0.1% with the rounding of the measurement result to 1 cm, then.

$$m_0 = z_\Pi - z_K.$$

Then the random instrumental error of the layer thickness with a confidence level of 0.683 will be as follows [21]:

$$\sigma_{m_0} \cong \sqrt{\left(\frac{1}{3} z_\Pi \, 0.001 + 0.01\right)^2 + \left(\frac{1}{3} z_K \, 0.001 + 0.01\right)^2}. \tag{21}$$

If a layer of a relatively small thickness ($\sim$5–8 m) is located at a depth of about 500 m, then $2\sigma_{m_0} \cong 0.50$ m.

2.5 Estimates of systematic errors of parameters of the tested aquifer and separating low-permeable sediments

The basis of the above methods for calculating the parameters of the tested aquifer and separating low-permeable sediments is a rather artificial assumption that the characteristics of the first asymptotic segments of the temporal and combined level tracing graphs do not show the GW flow from the adjacent horizon; therefore, this flow does not affect the values of the determined parameters, test aquifer and separating low-permeability sediments. In fact, the flow begins to manifest itself almost immediately with the start of pumping. Therefore, in fact, certain parameters contain systematic errors significant in absolute values [33, 34].

To determine the parameters from the values of which such systematic errors are excluded, original methods were proposed in [33, 34]. Here, the probable values of the indicated systematic errors established by comparing the calculated parameters of the tested Mynkuduk aquifer and separating low-permeable layer obtained using one of these methods from [33, 34] are considered. The method uses the ratio of the piezometric level decreases in combination with the method of selection given in **Tables 1** and **2**.

In accordance with this method, the primary processing of temporal tracing graphs in the observation wells of the test cluster is performed in the same way as recommended in [5, 12, 20, 21]. As a result of this processing, the logarithmic trends of the second asymptotic segments of the level tracing graphs in the observation wells are established. Accordingly, in the subsequent processing of the experimental

data, either the calculated values of the decrease in the piezometric level at the designated moments of time, calculated on the basis of the equations of these trends, or the values of the decrease, recorded directly from the second asymptotic segments, are used.

Sternberg [35] obtained an alternative representation of Eq. (3) [32] as follows:

$$S = \frac{Q}{2\pi T} \, K_0(z) = \frac{Q}{2\pi T} \, K_0 \left(.r \, \sqrt{\frac{1}{2\chi t} + \frac{1}{B^2}} \right). \tag{22}$$

Here $K_0(z)$ is the modified Bessel function of the second type of zero order. The rest designations remain the same.

Having presented Eq. (22) for $S^{(1)}$ and $S^{(2)}$, which are the decreases in GW level in observation wells as applied to the second asymptotic segments of the tracing graphs in the observation wells of the test cluster located at the distances r_1 and r_2 from the disturbance centre at the selected time point t_i, and having taken their ratio after simple transformations (provided that the tested aquifer is assumed to be homogeneous and isotropic, that is, $T = const$), the following is obtained [33, 34]:

$$\frac{S^{(1)}}{S^{(2)}} = \frac{K_0 \left(r_1 \, \sqrt{\frac{1}{2\chi t_i} + \frac{1}{B^2}} \right)}{K_0 \left(r_2 \, \sqrt{\frac{1}{2\chi t_i} + \frac{1}{B^2}} \right)}, \tag{23}$$

The rest designations remain the same.

Based on (23) in [33, 34], an algorithm was developed for calculating the water transmissibility T of the test aquifer, the piezoconductivity χ and the flow factor B, implemented on the basis of the probabilities in the MS Excel spreadsheet.

Figure 3.
Spreadsheet page with the results of the processing of groundwater level tracking data in observation wells of the pilot cluster using the drop ratio method.

Moreover, in [33, 34], the calculation of the Bessel function $K_0(z)$ was performed using its approximation by polynomials [36].

Figure 3 shows a spreadsheet page with the calculation of the flow factor B, water transmissibility T and elastic water yield μ^* from observation wells of the test cluster. The initial data on the piezometric level decreases were recorded by the second asymptotic segments of the temporal tracing graphs, and the value of $\chi = 1.7 \cdot 10^6$ m²/day was used as the piezoconductivity, close to that calculated by the results of processing and interpreting the first asymptotic segments of the tracing graphs for the GW level decrease in the observation wells of the test cluster.

The selected value of the flow factor B is equal to $B = 647.8$ m, and the calculated values of the water transmissibility T and the elastic water yield μ^* are $T = 554.6$ m²/day and $\mu^* \cong 3.3 \cdot 10^{-4}$, respectively (**Figure 3**). If the thickness of the separating layer $m_0 = 7.0$ m [21] is calculated by formula (15), then the value of its permeability factor is $k_z = 9.25 \cdot 10^{-3}$ m/day (or $3.85 \cdot 10^{-4}$ m/h).

3. Discussion of the obtained results

The relative random errors of water transmissibility T and piezoconductivity χ determined by the results of temporal level tracing in the observation wells of the test cluster and equal, respectively, to 10.33–18.87% and 38.06–65.42% with a confidence probability of 0.954, generally correspond to the level of such errors of these parameters determined from the results of EFT in aquifers [12, 20]. At the same time, the random errors of water transmissibility and piezoconductivity, established by the data of level tracing in well no. 2003g, significantly (approximately twice) exceed those for the parameters of well no. 2002g. This fact has a simple and quite logical explanation. With a sufficiently large depth down to the piezometric level in the observation wells ($\sim$28–30 m) and the same degree of dispersion of experimental points relative to the calculated asymptotes on the level tracing graphs in both wells of the test cluster, the error of the parameters is greater than for the well for which lower absolute values of level decrease are recorded. This is well no. 2003g, located at a greater distance from the centre of the disturbance.

Random errors of water transmissibility and especially piezoconductivity very significantly affect the reliability of the determined parameters of the flow through the thickness of low-permeable sediments and represent flow factor B and the permeability factor k_z. The relative random errors of the first of these parameters for the observation wells of the test cluster are 40.22–69.93%, that is, the values of the parameters set are certainly significant for both observation wells, although they are of unequal accuracy. The second parameter for well no. 2002g, which is closest to the centre of the disturbance, is also significant; its relative random errors are about 80.33%. For well no. 2003g, which is located at a greater distance from the centre of disturbance, these parameters are not significant at first glance, since their relative random errors exceed the established values of these parameters and represent about 137.50%. The reason for this, as in the case of piezoconductivity, seems to be the smaller absolute value of the level decrease in well no. 2003g, than in well no. 2002g.

The application of the method of the ratio of the GW level decrease in the observation wells of the test cluster ensured the identification of systematic errors

in the parameters of the tested aquifer and the parameters of the separating low-permeable formation determined by traditional methods. Thus, the value of the water transmissibility T of the test aquifer obtained by the method of the level decrease ratio is T = 554.6 m^2/day and is significantly differs by 27.23–54.06% from the values determined according to the temporal and combined level tracing in the observation wells of the test cluster [21] (respectively, $T_{2002g} = 705.6$ and $T_{2003g} = 854.4$ m^2/day for temporal tracing, and T = 712.8 m^2/day for combined tracing). Even more significant are the differences in the values of the flow factor established by different methods; the following values are given in [21]: B_{2002g} = 1840 m and B_{2002g} = 1770 m. These values are 2.84 times and 2.73 times greater than the values determined by the method of level decrease ratio, respectively.

4. Conclusion

As in the case of a two-layered formation [21, 22], a paradoxical situation is created—the model correctly represents and describes the actual process of GW filtration in a layered formation, and the effective parameters of low-permeable separating sediments included in this model are set with errors exceeding their absolute values. Nevertheless, as in the case of two-layered formation, such parameters should be recognised as significant. With relative random errors exceeding 100%, the parameters of low-permeable sediments cannot take on a zero or negative value, since this contradicts the original physical and mathematical model. Accordingly, the errors should be presented only in the form of relative, expressed in multiplicity values relative to the most probable values of the set parameters.

The application of the method of the ratio of the GW level decrease MW in the observation wells of the test cluster ensured the identification of systematic errors in the parameters of the tested aquifer and the parameters of the separating low-permeable formation. It is clear that as a calculation and the most likely parameters, the parameters determined by the method of the ratio of level decrease should be used. In this case, the above random errors of these parameters should be used as estimates of the random errors of the parameters, determined, among other things, by the method of the ratio of level decreases.

Such peculiarities of the ratio between the parameters of the tested aquifers and the separating low-permeable sediments in layered aquifer systems established according to the results of their EFT and their random errors appear to be characteristic of all types of the structure of these systems.

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