Chapter

Introductory Chapter: Nonlinear Optical Phenomena

Boris I. Lembrikov

1. Introduction

The number of publications concerning different aspects of nonlinear optics is enormous and hardly observable. We briefly discuss in this chapter the fundamental nonlinear optical phenomena and methods of their analysis. Nonlinear optics is related to the analysis of the nonlinear interaction between light and matter when the light-induced changes of the medium optical properties occur [1, 2]. The nonlinear optical effects are weak, and their observation became possible only after the invention of lasers which provide a highly coherent and intense radiation [2]. A typical nonlinear optical process consists of two stages. First, the intense coherent light induces a nonlinear response of the medium, and then the modified medium influences the optical radiation in a nonlinear way [1]. The nonlinear medium is described by a system of the dynamic equations including the optical field. The optical field itself is described by Maxwell's equations including the nonlinear polarization of the medium [1, 2]. All media are essentially nonlinear; however, the nonlinear coupling coefficients are usually very small and can be enhanced by the sufficiently strong optical radiation [1, 2]. For this reason, to a first approximation, light and matter can be considered as a system of uncoupled oscillators, and the nonlinear terms are some orders of magnitude smaller than the linear ones [2]. Nevertheless, the nonlinear effects can be important in the long-time and long-distance limits [2]. Generally, the light can be considered as a superposition of plane waves $A \exp i\left[\left(\vec{k} \cdot \vec{r}\right) - \omega t\right]$ where $\vec{k}, \omega, \vec{r}, t$ are the wave vector, angular frequency, radius vector in the space, and time, respectively [1, 2]. The medium oscillators can be electronic transitions, molecular vibrations and rotations, and acoustic waves [2]. Typically, only a small number of linear and nonlinear oscillator modes are important that satisfy the resonance conditions [1–3]. In such a case, the optical fields can be represented by a finite sum of discrete wave packets $\vec{E}(z,t)$ given by [1–3]

$$\vec{E}(z,t) = \frac{1}{2}[A(z,t) \exp i(kz - \omega t) + c.c.] \tag{1}$$

where $c.c.$ stands for the complex conjugate and $A(z,t)$ is the slowly varying envelope (SVE) such that [1–3]

$$\left|\frac{\partial^2 A}{\partial z^2}\right| \ll \left|k \frac{\partial A}{\partial z}\right|; \left|\frac{\partial^2 A}{\partial t^2}\right| \ll \left|\omega \frac{\partial A}{\partial t}\right| \tag{2}$$

Here we for the sake of definiteness consider the one-dimensional case. The evolution of the waves (1) is described by the system of the coupled equations in the so-called SVE approximation (SVEA) when the higher-order derivatives of the SVE can be neglected according to conditions (2) [1–3]. The typical nonlinear optical phenomena are self-focusing, self-trapping, sum- and difference-frequency generation, harmonic generation, parametric amplification and oscillation, stimulated light scattering (SLS), and four-wave mixing (FWM) [1].

During the last decades, optical communications and optical signal processing have been rapidly developing [1–4]. In particular, the nonlinear optical effects in optical waveguides and fibers became especially important and attracted a wide interest [1–4]. The nonlinear optical interactions in the waveguide devices have been investigated in detail in Ref. [3]. Nonlinear fiber optics as a separate field of nonlinear optics has been reviewed in Ref. [4]. The self-phase modulation (SPM), cross-phase modulation (XPM), FWM, stimulated Raman scattering (SRS), stimulated Brillouin scattering (SBS), pulse propagation, and optical solitons in optical fibers have been considered in detail [4]. Silicon photonics, i.e., integrated optics in silicon, also attracted a wide interest due to the highly developed silicon technology which permits the combination of the photonic and electronic devices on the same Si platform [5]. The nonlinear optical phenomena in Si nanostructures such as quantum dots (QD), quantum wells (QW), and superlattices had been discussed [6]. It has been shown that the second harmonic generation (SHG) in silicon nanostructures is possible despite the centrosymmetric structure of Si crystals [6].

Nonlinear dynamics in complex optical systems such as solid-state lasers, CO_2 lasers, and semiconductor lasers is caused by the light-matter interaction [7]. Under certain conditions, the nonlinear optical processes in such optical complex systems result in instabilities and transition to chaos [7].

In this chapter we briefly describe the basic nonlinear optical phenomena. The detailed analysis of these phenomena may be found in [1–7] and references therein. The chapter is constructed as follows. Maxwell's equations for a nonlinear medium and nonlinear optical susceptibilities are considered in Section 2. The mechanisms and peculiarities of the basic nonlinear effects mentioned above are discussed in Section 3. Conclusions are presented in Section 4.

2. Maxwell's equations for a nonlinear medium and nonlinear optical susceptibilities

All electromagnetic phenomena are described by macroscopic Maxwell's equations for the electric and magnetic fields $\vec{E}\left(\vec{r},t\right)$ and $\vec{H}\left(\vec{r},t\right)$ [1–8]. They have the form [4]

$$\nabla\cdot\vec{B}=0 \tag{3}$$

$$\nabla\cdot\vec{D}=\rho_{free} \tag{4}$$

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \tag{5}$$

$$\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t} \tag{6}$$

Here ρ_{free} is the free charge density consisting of all charges except the bound charges inside atoms and molecules; $\vec{J}$ is the current density; the electric induction is given by $\vec{D}=\varepsilon_0\vec{E}+\vec{P}$; the magnetic induction (magnetic flux density) has the form $\vec{B} = \mu_0 \vec{H} + \vec{M}$; ε_0, μ_0 are the free space permittivity and permeability, respectively; and $\vec{P}$, $\vec{M}$ are the induced electric and magnetic polarizations, respectively. For nonmagnetic media $\vec{M} = 0$. Equations (3)–(6) describe the vectors averaged over the volumes which contain many atoms but have linear dimensions smaller than substantial variations of the applied electric field [8]. Combining Eqs. (3)–(6) we obtain the wave equation for the light propagation in a medium. It has the form [1–8]

$$\nabla \times \nabla \times \vec{E} + \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = -\mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} \tag{7}$$

Here c is the free space light velocity. The polarization $\vec{P}$ is a complicated nonlinear function of $\vec{E}$ [1]. In the general nonlinear case, the polarization $\vec{P}$ as a function of the electric field $\vec{E}$ can be expanded into a power series of $\vec{E}$ as follows [1, 2]:

$$\begin{aligned}\frac{1}{\varepsilon_0}P_j\left(\vec{r},t\right) = &\int_{-\infty}^{\infty} \chi_{jk}^{(1)}\left(\vec{r}-\vec{r}',t-t'\right)E_k\left(\vec{r}',t'\right)d\vec{r}'\,dt' \\ &+\int_{-\infty}^{\infty} \chi_{jkl}^{(2)}\left(\vec{r}-\vec{r}_1,t-t_1;\vec{r}-\vec{r}_2,t-t_2\right)E_k\left(\vec{r}_1,t_1\right)E_l\left(\vec{r}_2,t_2\right)d\vec{r}_1\,dt_1 d\vec{r}_2\,dt_2 \\ &+\int_{-\infty}^{\infty} \chi_{jklm}^{(3)}\left(\vec{r}-\vec{r}_1,t-t_1;\vec{r}-\vec{r}_2,t-t_2;\vec{r}-\vec{r}_3,t-t_3\right)E_k\left(\vec{r}_1,t_1\right)E_l\left(\vec{r}_2,t_2\right) \\ &\times E_m d\vec{r}_1\,dt_1 d\vec{r}_2\,dt_2 d\vec{r}_3\,dt_3 + \dots\end{aligned} \tag{8}$$

Here, $\chi^{(1)}\left(\vec{r},t\right)$ is the linear susceptibility; $\chi^{(n)}\left(\vec{r},t\right)$, $n > 1$ is nth-order nonlinear susceptibility [1]. Suppose that the electric field is a group of monochromatic plane waves given by [1]

$$\vec{E}\left(\vec{r},t\right) = \sum_n \vec{E}_{0n}\left(\vec{k}_n,\omega_n\right)\exp\left[i\left(\vec{k}_n\cdot\vec{r}\right) - i\omega_n t\right] \tag{9}$$

Then, the Fourier transform of the nonlinear polarization (1) yields [1]

$$\vec{P}\left(\vec{k},\omega\right) = \vec{P}^{(1)}\left(\vec{k},\omega\right) + \vec{P}^{(2)}\left(\vec{k},\omega\right) + \vec{P}^{(3)}\left(\vec{k},\omega\right) + \dots \tag{10}$$

where

$$\begin{aligned}P_j^{(1)}\left(\vec{k},\omega\right) &= \chi_{jk}^{(1)}\left(\vec{k},\omega\right)E_k\left(\vec{k},\omega\right); \\ P_j^{(2)}\left(\vec{k},\omega\right) &= \chi_{jkl}^{(2)}\left(\vec{k}=k_n+k_m,\omega=\omega_n+\omega_m\right)E_k\left(\vec{k}_n,\omega_n\right)E_l\left(\vec{k}_m,\omega_m\right); \\ P_j^{(3)}\left(\vec{k},\omega\right) &= \chi_{jkls}^{(3)}\left(\vec{k}=k_n+k_m+k_p,\omega=\omega_n+\omega_m+\omega_p\right) \\ &\quad\times E_k\left(\vec{k}_n,\omega_n\right)E_l\left(\vec{k}_m,\omega_m\right)E_s\left(\vec{k}_p,\omega_p\right)\end{aligned} \tag{11}$$

and

$$\chi^{(n)}\left(\vec{k}=\vec{k_1}+\vec{k_2}+\ldots+\vec{k_n};\omega=\omega_1+\omega_2+\ldots+\omega_n\right)$$
$$=\int_{-\infty}^{\infty}\chi^{(n)}\left(\vec{r}-\vec{r_1},t-t_1;\ldots;\vec{r}-\vec{r_n},t-t_n\right)$$
$$\times\exp\left\{-i\left[\left(\vec{k_1}\cdot\left(\vec{r}-\vec{r_1}\right)\right)-\omega_1(t-t_1)+\ldots+\left(\vec{k_n}\cdot\left(\vec{r}-\vec{r_n}\right)\right)-\omega_n(t-t_n)\right]\right\}$$
$$\times d\vec{r}_1dt_1\ldots d\vec{r}_ndt_n \tag{12}$$

The linear and nonlinear optical properties of a medium are described by the linear and nonlinear susceptibilities (12), and the nth-order nonlinear optical effects in such a medium can be obtained theoretically from Maxwell's Eqs. (3)–(6) with the polarization determined by Eq. (8) [1]. We do not present here the analytical properties of the nonlinear susceptibilities which are discussed in detail in Ref. [1].

In some simple cases, the nonlinear susceptibilities can be evaluated by using the anharmonic oscillator model [1, 8]. It is assumed that a medium consists of N classical anharmonic oscillators per unit volume [1]. Such an oscillator may describe an electron bound to a core or an infrared-active molecular vibration [1]. The equation of motion of the oscillator in the presence of an applied electric field with the Fourier components at frequencies $\pm\omega_1,\ \pm\omega_2$ is given by [1]

$$\frac{d^2x}{dt^2}+\Gamma\frac{dx}{dt}+\omega_0^2x+ax^2=\frac{q}{m}\left[E_1\left(e^{-i\omega_1t}+e^{i\omega_1t}\right)+E_2\left(e^{-i\omega_2t}+e^{i\omega_2t}\right)\right] \tag{13}$$

Here x is the oscillator displacement; Γ is the decay factor; ω_0 is the oscillator frequency; q, m are the oscillator charge and mass, respectively; and the anharmonic term ax^2 is small and can be considered as a perturbation in the successive approximation series given by [1, 8]

$$x=x^{(1)}+x^{(2)}+x^{(3)}+\ldots \tag{14}$$

The nonlinear terms become essential when the electromagnetic power is large enough in such a way that a medium response cannot be considered linear anymore [8]. We limit our analysis with quadratic and cubic nonlinearities proportional to x^2 and x^3, respectively [1–8]. The induced electric polarization P can be expressed by using the solutions (13) and (14) as follows: $P=Nqx$ [1]. In general case, the microscopic expressions for nonlinear susceptibilities of a medium are calculated by using the quantum mechanical approach. In particular, the density matrix formalism is a powerful and convenient tool for such calculations [1, 2, 7, 8].

3. Nonlinear optical effects

Electromagnetic waves in a medium interact through the nonlinear polarization (8) [1]. Typically, a nonlinear optical effect that occurs due to such an interaction is described by the coupled wave equations of the type (7) with the nonlinear susceptibilities (12) as the coupling coefficients [1]. In general case, the coupled wave method can also include waves other than electromagnetic [1]. For instance, in the case of SBS process, the acoustic waves are taken into account, and in the case of

SRS process, the molecular vibrations are typically considered [1, 2, 4]. The coupled wave equations are usually solved by using SVEA (2) [1]. In this section, we discuss some important nonlinear optical phenomena caused by the quadratic and cubic susceptibilities $\chi^{(2)}$ and $\chi^{(3)}$, respectively. It should be noted that $\chi^{(2)} = 0$ in the electric dipole approximation for a medium with inversion symmetry [1].

We start with the sum-frequency, difference-frequency, and second harmonic generation. These phenomena are based on the wave mixing by means of the quadratic susceptibility $\chi^{(2)}$. The three coupled waves are $\vec{E}\,(\omega_1)$, $\vec{E}\,(\omega_2)$, and $\vec{E}\,(\omega_3)$ where $\omega_3 = \omega_1 + \omega_2$ in the cases of sum-frequency [1]. The second-order nonlinear polarization with a sum-frequency ω_3 in such a case has the form [1]

$$P_j^{(2)}(\omega_3 = \omega_1 + \omega_2) = \varepsilon_0 \chi_{jkl}^{(2)}(\omega_3 = \omega_1 + \omega_2) E_k(\omega_1) E_l(\omega_2) \tag{15}$$

Similarly, in the case of the difference-frequency generation, we obtain [1]

$$P_j^{(2)}(\omega_2 = \omega_3 - \omega_1) = \varepsilon_0 \chi_{jkl}^{(2)}(\omega_2 = \omega_3 - \omega_1) E_k(\omega_3) E_l^*(\omega_2) \tag{16}$$

where the asterisk means the complex conjugation. Consider the particular case of equal frequencies $\omega_1 = \omega_2 = \omega$. In such a case, the nonlinear polarization (15) has the form $P_j^{(2)}(\omega_3 = 2\omega)$, and the second harmonic generation (SHG) takes place [1]. The efficient nonlinear wave mixing can occur only under the phase-matching conditions. The phase mismatch Δk between the coupled waves is caused by the refractive index dispersion $n(\omega_i)$. The collinear phase matching $\Delta k = 0$ can be realized in the medium with an anomalous dispersion or in the birefringent crystals [1]. The detailed analysis of the sum-frequency generation, difference-frequency generation, and SHG in different configurations may be found in [1, 3, 6]. It can be shown that the efficient sum-frequency generation can be realized under the following conditions [1]. The nonlinear optical crystal without the inversion symmetry or with the broken inversion symmetry should have low absorption at the interaction frequencies $\omega_{1,2,3}$ and a sufficiently large quadratic susceptibility $\chi^{(2)}$ and should allow the collinear phase matching. The particular phase-matching direction and the coupled wave polarizations should be chosen in order to optimize the effective nonlinear susceptibility $\chi_{eff}^{(2)}$. The length of the nonlinear crystal must provide the required conversion efficiency. The efficient SHG can be realized with the single-mode laser beams focused into the nonlinear optical crystal [1].

Sum-frequency generation, difference-frequency generation, and SHG can be also carried out in the waveguide nonlinear optical devices [3]. Typically, a thin film of a nonlinear material such as ZnO and ZnS, ferroelectric materials $LiNbO_3$ and $LiTaO_3$, and III-V semiconductor materials GaAs and AlAs can be used as a waveguiding layer [3]. The output power $P^{(2\omega)}(L)$ of the second harmonic (SH) mode under the no-pump depletion approximation is given by [3]

$$P^{(2\omega)}(L) = \left(P_0^{(\omega)}\right)^2 k^2 L^2 \left(\frac{\sin \Delta L}{\Delta L}\right)^2 \tag{17}$$

where $2\Delta = \beta^{(2\omega)} - \left(2\beta^{(\omega)} + K\right)$; $K = 2\pi/\lambda$; $P_0^{(\omega)}$ is the input pump power; k is the coupling constant; L is the device length; Δ is the phase mismatch; λ is the pump wavelength; $\beta^{(\omega)}, \beta^{(2\omega)}$ are the propagation constants of the pump and SH waves, respectively; and Λ is the period of the quasi-phase matching (QPM) grating. Waveguide SHG devices can be used in optical signal processing such as laser printer, laser display, optical memory, short pulse, multicolor, and ultraviolet light generation [3].

Consider the nonlinear optical effects related to the cubic susceptibility $\chi^{(3)}$. These phenomena are much weaker than the second-order ones. However, they can exist in centrosymmetric media where $\chi^{(2)} = 0$ and may be strongly pronounced under the high enough optical intensity pumping. We briefly discuss self-focusing, SPM, third harmonic generation (THG), SBS, SRS, and FWM.

Self-focusing is an induced lens effects caused by the self-induced wavefront distortion of the optical beam propagating in the nonlinear medium [1]. In such a medium, a refractive index n has the form [1]

$$n = n_0 + \Delta n\left(|E|^2\right) \tag{18}$$

Here n_0 is the refractive index of the unperturbed medium, $\Delta n\left(|E|^2\right)$ is the optical field-induced refractive index change, and E is the optical beam electric field. Typically, the field-induced refractive index change can be described as $\Delta n = n_2|E|^2$ like in the case of the so-called Kerr nonlinearity [1, 3]. If $\Delta n > 0$, the central part of the optical beam with a higher intensity has a larger refractive index than the beam edge. Consequently, the central part of the beam travels at a smaller velocity than the beam edge. As a result, the gradual distortion of the original plane wavefront of the beam occurs, and the beam appears to focus by itself [1]. The self-focusing results in the local increase of the optical power in the central part of the beam and possible optical damage of transparent materials limiting the high-power laser performance [1].

SPM is also caused by the positive refractive index change (18). It is the temporal analog of self-focusing which leads to the spectral broadening of optical pulses [4]. In optical fibers, for short pulses and sufficiently large fiber length L_f, the combined effect of the group velocity dispersion (GVD) and SPM should be taken into account [4]. The GVD parameter β_2 is given by [4]

$$\beta_2 = \frac{1}{c}\left(2\frac{dn}{d\omega} + \omega\frac{d^2n}{d\omega^2}\right) \tag{19}$$

In the normal-dispersion regime when $\beta_2 > 0$, the combined effect of the SPM and GVD leads to a pulse compression. In the opposite case of the anomalous-dispersion regime $\beta_2 < 0$, SPM and GVD under certain conditions can be mutually compensated [4]. In such a case, the pulse propagates in the optical fiber as an optical soliton, i.e., a solitary wave which does not change after mutual collisions [4]. The solitons are described with the nonlinear Schrödinger equation (NLS) which can be solved with the inverse scattering method [4]. The fundamental soliton solution $u(\xi, \tau)$ has the form [4]

$$u(\xi, \tau) = \eta[\cosh(\eta\tau)]^{-1}\exp\left(i\eta^2\xi/2\right) \tag{20}$$

Here η is the soliton amplitude; $\tau = (t - \beta_1 z)/T_0$; $\xi = z/L_D$; $\beta_1 = 1/v_g$; v_g is the light group velocity in the optical fiber; L_D is the dispersion length; and T_0 is the initial width of the incident pulse. The optical solitons can propagate undistorted over long distances, and they can be applied in fiber-optic communications [4].

Consider now THG. Unlike SHG, it is always allowed [1]. The third harmonic $\vec{E}(3\omega)$ is caused by the third-order nonlinear polarization given by [1, 2]

$$P_j^{(3)}(3\omega) = \varepsilon_0 \chi_{jklm}^{(3)}(3\omega) E_k(\omega) E_l(\omega) E_m(\omega) \tag{21}$$

The cubic susceptibility $\left|\chi^{(3)}\right|$ is usually small compared to the $\left|\chi^{(2)}\right|$ [1]. For this reason, the laser intensity required for the efficient THG is limited by the optical damage in crystals [1]. The phase matching for the THG is difficult to achieve which results in low efficiency of the THG process [1, 4]. THG can be realized in highly nonlinear optical fibers where the phase matching can be accomplished [4].

SBS is a nonlinear optical effect related to parametric coupling between light and acoustic waves [1]. It is described by the coupled wave equation (7) for the coupled counterpropagating light waves $\vec{E}_{1,2}(\omega_{1,2})$ and the acoustic wave equation for the mass density variation $\Delta\rho(\omega_a = \omega_1 - \omega_2)$ [1, 2, 4]. The nonlinear coupling between light and acoustic waves is caused by the electrostrictive pressure $p \sim \rho_0 \frac{\partial \varepsilon_r}{\partial \rho} \left(\vec{E}_1(\omega_1) \cdot \vec{E}_2^*(\omega_2) \right)$ where ρ_0, ε_r are the equilibrium medium mass density and permittivity, respectively. The acoustic wave enhanced by the interacting pump and signal (Stokes) wave modulates the mass density of the medium which in turn modulates the refractive index [1, 3, 4]. For the typical values of the attenuation coefficient and the acoustic frequency shift of about 5 GHz, the acoustic wave excitation is overdamped, and the signal Stokes wave $\vec{E}_2(\omega_2)$ would grow in the backward direction $-z$ under the conditions that $\operatorname{Im} \chi_B^{(3)} > 0$, $\omega_1 \gg \omega_a = \omega_1 - \omega_2 > 0$, and the optical gain is larger than the optical wave damping constant [1]. The pumping wave $\vec{E}_1(\omega_1)$ is decaying in the forward direction z [1]. SBS has been successfully demonstrated in optical fibers, and the SBS gain in a fiber can be used for the amplification of the weak signal with the frequency shift equal to the acoustic frequency ω_a [4]. Brillouin fiber amplifiers may be used for applications where the selective amplification is needed [4].

Consider now the SRS process. SRS can be described in the framework of the quantum mechanics as a two-photon process where one photon with energy $\hbar\omega_1\left(\vec{k}_1\right)$ is absorbed by the system and another photon with energy $\hbar\omega_2\left(\vec{k}_2\right)$ is emitted [1]. The system itself makes a transition from the initial state with the energy E_i to the final state with the energy E_f, and the energy conservation takes place: $\hbar(\omega_1 - \omega_2) = E_f - E_i$ [1].

In the framework of the coupled wave description, SRS is a third-order parametric generation process where the optical pump wave $\vec{E}_1(\omega_1)$ generates a Stokes wave $\vec{E}_2(\omega_2)$ and a material excitation wave [1]. The nonlinear polarization $\vec{P}^{(3)}(\omega_{1,2})$ related to SRS in such a case takes the form [1, 2]

$$\vec{P}^{(3)}(\omega_1) = \varepsilon_0 \chi_{R1}^{(3)} |E_2|^2 \vec{E}_1(\omega_1), \quad \vec{P}^{(3)}(\omega_2) = \varepsilon_0 \chi_{R2}^{(3)} |E_1|^2 \vec{E}_2(\omega_2) \tag{22}$$

where $\chi_{R1,2}^{(3)}$ are the third-order Raman susceptibilities coupling the optical waves and providing SRS process [1]. They can be evaluated by using the quantum mechanical methods [1]. Typically, the material excitation wave in the SRS process is considered as molecular vibrations or optical phonons [1, 2, 4]. The specific feature of SRS is the so-called Stokes-anti-Stokes coupling [1, 2]. Indeed, the mixing of the pump wave with the frequency ω_1 and the Stokes wave with the frequency ω_2 results in the generation of the anti-Stokes wave $\vec{E}_a(\omega_a = 2\omega_1 - \omega_2)$ at the anti-Stokes frequency $\omega_a = 2\omega_1 - \omega_2 > \omega_1$ [1]. Consequently, the coupled wave analysis of SRS should include the equations for the pump wave, Stokes wave, anti-Stokes wave, and the material excitation wave [1, 2]. The analysis of this problem can be found in Refs. [1, 2]. Usually, the anti-Stokes wave is attenuated [2]. SRS in optical fibers can be used for the development of Raman fiber lasers and Raman fiber amplifiers [4].

FWM is the nonlinear process with four interacting electromagnetic waves [1]. FWM is a third-order process caused by the third-order nonlinear susceptibility $\chi^{(3)}$. It can be easily observed by using the high-intensity lasers, and it has been demonstrated experimentally [1]. FWM is a complicated nonlinear phenomenon because it exhibits different nonlinear effects for different combinations of the coupled wave frequencies, wave vectors, and polarizations. The analysis of FWM is based on the general theory of optical wave mixing [1, 2, 4]. For three input pump waves with frequencies $\omega_{1,2,3}$, the singly resonant, doubly resonant, and triply resonant cases can occur [1]. They correspond to the situations when one, two, or three input frequencies or their algebraic sums approach medium transition frequencies [1]. In such cases the third-order susceptibility $\chi^{(3)}$ can be divided into a resonant part $\chi_R^{(3)}$ and a nonresonant part $\chi_{NR}^{(3)}$ [1]. The FWM process has some important applications. Due to the wide range of the mixed frequencies, FWM can be used for the generation of the waves from the infrared up to ultraviolet range [1]. For instance, the parametric amplification can be realized when two strong pump waves amplify two counterpropagating weak waves [1]. The frequency degenerate FWM occurs when the frequencies of the four waves are the same. It is used for the creation of a phase-conjugated wave with respect to one of the coupled waves [2]. In such a case, the phase of the output wave is complex conjugate to the phase of the input wave [1, 2]. FWM in optical fibers can be used for signal amplification, phase conjugation, wavelength conversion, pulse generation, and high-speed optical switching [4].

4. Conclusions

We briefly discussed the fundamentals of nonlinear optics. The nonlinear optical phenomena are caused by the interaction between light and matter. Generally, all media are nonlinear. However, optical nonlinearity is extremely weak, and the observation of the nonlinear optical effects became possible only after invention of lasers as the sources of the strong enough coherent optical radiation. The nonlinear optical processes are described by Maxwell's equations with the nonlinear polarization of the medium. The coupled equations for the interacting electromagnetic and material waves are usually solved by using SVEA. Typically, the second- and third-order polarizations are considered. The nonlinear polarization and the optical field in the medium are related by the nonlinear susceptibilities which in general case can be evaluated by the quantum mechanical methods. In some simple cases, the classical model of anharmonic oscillator also can be used. We briefly discussed the fundamental nonlinear phenomena related to the second- and third-order susceptibilities. The former exists only in the media without the inversion symmetry, while the latter exists in any medium.

The typical nonlinear optical phenomena related to the second-order susceptibility are the sum-frequency generation, difference-frequency generation, and SHG. The typical nonlinear optical phenomena related to the third-order susceptibility are self-focusing, SPM, optical soliton formation and propagation, different types of SLS such as SBS and SRS, and FWM. SBS involves the acoustic waves. SRS involves the material excitations such as molecular vibrations. We also discussed some peculiarities of nonlinear optical processes in optical fibers. The nonlinear optical effects are widely used in optical communications and optical signal processing.

Author details

Boris I. Lembrikov
Department of Electrical Engineering and Electronics, Holon Institute of Technology (HIT), Holon, Israel

*Address all correspondence to: borisle@hit.ac.il

References

[1] Shen YR. The Principles of Nonlinear Optics. Hoboken, New Jersey, USA: Wiley; 2003. 563 p. ISBN: 0-471-43080-3

[2] Moloney JV, Newell AC. Nonlinear Optics. Boulder, Colorado, USA: Westview Press; 2004. 436 p. ISBN: 0-8133-4118-3

[3] Suhara T, Fujimura M. Waveguide Nonlinear-Optic Devices. Berlin, Heidelberg, Germany: Springer; 2003. 321 p. ISBN: 3-540-01527-2

[4] Agrawal G. Nonlinear Fiber Optics. London, New York: Academic Press; 2013. 629 p. ISBN: 978-0-12397-023-7

[5] Reed GT, Knights AP. Silicon Photonics. Chichester, England: Wiley; 2004. 255 p. ISBN: 0-470-87034-6

[6] Aktsipetrov OA, Baranova IM, Evtyukhov KN. Second Order Non-Linear Optics of Silicon and Silicon Nanostructures. Boca Raton, FL, USA: CRC Press; 2016. 581 p. ISBN: 978-1-4987-2415-9

[7] Otsuka K. Nonlinear Dynamics in Optical Complex Systems. Dordrecht, The Netherlands: Kluwer Academic Publishers; 1999. 295 p. ISBN: 07923-6132-6

[8] Meystre P, Sargent M III. Elements of Quantum Optics. Berlin, Heidelberg, New York: Springer; 1998. 432 p. ISBN: 3-540-64220-X