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Chapter

Equilibrium and Nonequilibrium Hydrodynamic Modes of a Nematic Liquid Crystal

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Abstract

We use a fluctuating hydrodynamics (*FH*) approach to study the fluctuations of the hydrodynamic variables of a thermotropic nematic liquid crystal (*NLC*) in a nonequilibrium steady state (*NESS*). This *NESS* is produced by an externally imposed temperature gradient and a uniform gravity field. We calculate analytically the equilibrium and nonequilibrium seven modes of the *NLC* in this *NESS*. These modes consist of a pair of sound modes, one orientation mode of the director and two visco-heat modes formed by the coupling of the shear and thermal modes. We find that the nonequilibrium effects produced by the external gradients only affect the longitudinal modes. The analytic expressions for the visco-heat modes show explicitly how the heat and shear modes of the *NLC* are coupled. We show that they may become propagative, a feature that also occurs in the simple fluid and suggests the realization of new experiments. We show that in equilibrium and in the isotropic limit of the *NLC*, our modes reduce to well-known results in the literature. For the *NESS* considered, we point out the differences between our modes and those reported by other authors. We close the chapter by proposing the calculation of other physical quantities that lend themselves to a more direct comparison with possible experiments for this system.

Keywords: fluctuating hydrodynamics, nonequilibrium fluctuations, hydrodynamic modes, thermotropic nematic liquid crystals

1. Introduction

When a fluid is in thermodynamic equilibrium, its state variables always present spontaneous microscopic fluctuations due to the thermal excitations of its molecules, producing deviations around the state of equilibrium. The theory of fluctuations for fluids in states close to equilibrium was initiated long ago by Einstein and Onsager, and it has been reformulated in several but equivalent ways. The first more systematic approach to introduce thermal fluctuations into the hydrodynamic equations was the fluctuating hydrodynamics (*FH*) of Landau and Lifshitz [1, 2]. It stems from the idea that the hydrodynamic equations are valid for any flow, including fluctuating changes in its state. Accordingly, stochastic currents are incorporated into the deterministic energy and momentum fluxes by adding fluctuating sources. This theory was put on a firm basis within the framework of the general theory of stationary Gaussian Markov processes by Fox and Uhlenbeck [3–5].

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This approach has matched the theory of Onsager and Machlup with that of Landau and Lifshitz for systems where the basic state variables do not possess a definite time reversal symmetry [6, 7]. However, in spite of the fact that the theory of fluctuations for nonequilibrium fluids was initiated in the late 1970s, and was pursued by many authors [8], still nowadays several questions concerning the nature of hydrodynamic fluctuations in stationary nonequilibrium states (*NESS*) are of current active interest. One of these issues is the long-range character of these fluctuations, especially far away from instability points [9]. Thermal fluctuations in an equilibrium fluid always give rise to short-range equal time correlation functions, except close to a critical point. But when external gradients are applied, equal-time correlation functions can develop long-range contributions, whose nature is very different from those in equilibrium. For many models and systems in nonequilibrium states, it has been shown theoretically that the existence of the so-called generic scale invariance is the origin of the long-range nature of the correlation functions [10, 11].

In the case of a simple fluid in a thermal gradient, the structure factor, which determines the intensity of the Rayleigh scattering, diverges as k^{-4} for small values of the wave number k. This amounts to an algebraic decay of the density-density correlation function, a feature that has been verified experimentally [12–14]. However, there are few similar studies for *NESS* of complex fluids. Among these, the enhancement of concentration fluctuations in polymer solutions under external hydrodynamic and electric fields [15], or the case of a polymer solution subjected to a stationary temperature gradient in the absence of any flow [16], has been discussed. Also, the behavior of fluctuations about some *NESS* has been analyzed in the case of thermotropic nematic liquid crystals. Specific examples are the nonequilibrium situations generated by a static temperature gradient [17], a stationary shear flow [18] or by an externally imposed constant pressure gradient [19, 20]. In the first two cases, it was found that the nonequilibrium contributions to the corresponding light scattering spectrum were small, but in the case of a Poiseuille flow induced by an external pressure gradient, the effect may be quite large. To our knowledge, however, at present, there is no experimental confirmation of these effects, in spite of the fact that for nematics, the scattered intensity is several orders of magnitude larger than for ordinary simple fluids.

When a hydrodynamic system relaxes from a state of thermodynamic equilibrium to another, almost all its degrees of freedom will return to that equilibrium value in a short, finite time τ determined by the microscopic interactions of the system. There are, however, some other degrees of freedom of collective character, the hydrodynamic modes, which will decay much more slowly. When $\tau \to \infty$, its characteristic frequencies $\omega \to 0$ ($\omega \sim 1/\tau$), when $k \to 0$. Such is the case, for example, of the propagation of sound waves and the conduction of heat in a simple fluid [21]. Hydrodynamics allows to describe these modes or degrees of freedom of greater duration, through the laws of conservation and balance of the system, and, as in the case of ordered systems, by the continuous breaking of symmetries [22, 23].

The central purpose of this work is to briefly review the general procedure developed by Fox and Uhlenbeck and show that it may be employed to treat fluctuating complex fluid systems like a thermotropic nematic liquid crystal (*NLC*) in a *NESS*. In particular, we describe the dynamics of the fluctuations of its hydrodynamic variables induced by a stationary temperature gradient and under the influence of gravity (a Rayleigh-Bénard system) on a nematic layer confined between two parallel horizontal plates in a steady state in a nonconvective regime [24–26]. Once the dynamics of fluctuation is established, we calculate the

time-dependent correlation functions in equilibrium between the fluctuating hydrodynamic variables, quantities that allow to obtain the transport properties of the system [27, 28]. One of these properties is the dynamic structure factor $S\left(\vec{k},\omega\right)$ of the system, which measures the magnitude of the changes in energy and momentum between the light beam and the fluid as functions of the wave vector $\vec{k}$ and ω.

For simple fluids with fixed $\vec{k}$, $S\left(\vec{k},\omega\right)$ consists of three well-separated Lorentzian features: a line or central peak (Rayleigh peak) located at $\omega = 0$ and two Brillouin peaks symmetrically located with respect to the central one [29, 30]. These three lines are directly related to the hydrodynamic modes of the simple fluid, and from them, it is possible to obtain relevant information about transport properties. For instance, the Rayleigh line, associated with a thermal diffusive mode, is due to the fluctuations of the entropy (or temperature) that diffuse in the fluid and its width is proportional to the thermal diffusivity. On the other hand, the Brillouin lines are related to two acoustic propagative modes and are the result of the coupled dynamics of the pressure fluctuations and a component of the flow velocity that are transmitted with the speed of sound in the medium. Their widths are proportional to the absorption of sound.

In the case of an anisotropic system like a *NLC*, fluctuating hydrodynamic theories have recently been proposed [22, 31] based on the methodology proposed by Landau and Lifshitz [1]. However, this analysis of the fluctuations of the nematic hydrodynamic variables is not precise, since it does not take into account the parity with respect to time reversal, so their description using the Onsager-Machlup formalism would be strictly inadequate. The correct theoretical framework should be the more general theory of Fox and Uhlenbeck [3–5, 20, 24]. However, although a *NLC* disperses light by several orders of magnitude more than an ordinary fluid [32], from both a theoretical and experimental point of view, the studies corresponding to the behavior of the fluctuations in these media around stationary states out of equilibrium are rather scarce. From the theoretical point of view, and only for the case of the transverse hydrodynamic variables [33], some studies of the behavior of orientational fluctuations have been carried out when analyzing the effect produced in the light scattering spectrum of a *NLC* in *NESS* induced by the presence of uniform temperature gradients [17] and by the action of a shear flow [18]. In both cases, it has been found that the effect of fluctuations in the light scattering spectrum is small, being difficult to detect experimentally. On the other hand, as far as we know, no theoretical study has been carried out on the behavior of the longitudinal variables of a nematic and much less on its spectrum of light scattering, both in states of thermodynamic equilibrium and outside of it. This is an open research topic. Nor have been performed analyzes of stationary states generated by other types of external gradients in these systems, with which could be obtained qualitatively and quantitatively much greater effects than those reported so far in the literature for simple fluids. It should be mentioned that although preliminary attempts have been made to calculate the transverse hydrodynamic modes of a nematic [34, 35], there are few studies that also involve the corresponding longitudinal modes [31]. Unfortunately, a clear and systematic method to derive the set of complete, transverse, and longitudinal hydrodynamic modes of a *NLC* is still lacking.

By introducing an alternative set of state variables that takes into account the asymmetry presented by both, the velocity and the director fields due by their mutual coupling, two groups of fluctuating variables, namely, longitudinal and

transverse, can be clearly identified. Both set of variables are completely decoupled: there are five in the first and two in the second group. The longitudinal variables in turn can be separated into two mutually independent sets. The first is composed of two variables whose dynamics determine the existence of acoustic propagation modes; while the second, formed by three variables, giving rise to three hydrodynamic modes: one related to the orientation of the director and two more, the so-called visco-heat modes, that result from the coupling of the thermal diffusive and shear modes due by the presence of the gradient thermal and the gravitational field. As will be discussed later on, from the set of transverse variables, two hydrodynamic modes emerge: one due to the orientation of the director and another one due to shearing. Altogether, there are seven nematic hydrodynamic modes: five longitudinal and two transversal. As will be shown below, the applied gradient of temperature and gravitational field produce their greatest effect in the pair of visco-heat modes, which is quantified in them by means of the Rayleigh quotient R/R_c, where R is the number of Rayleigh and R_c the value that it reaches when in the nematic initiates the convection.

2. Liquid crystalline phases

The liquid crystal phase is a well-defined and specific phase of matter characterized by a remarkable anisotropy in many of their physical properties as solid crystals do, although they are able to flow. Liquid crystal phases that undergo a phase transition as a function of temperature (thermotropics) exist in relatively small intervals of temperature lying between solid crystals and isotropic liquids. Due to this intermediate nature, sometimes, these states are called also mesophases [32]. In general, liquid crystals are synthesized from organic molecules, some of which are elongated and uniaxial, so they can be represented as rigid rods; others are formed by disc-like molecules [35]. This molecular anisotropy is manifested macroscopically through the anisotropy of the mechanical, optical, and transport properties of these substances. The typical dimensions of the lengths of this type of structures are some tens of angstroms.

Liquid crystals are classified by symmetry. As it is well known, isotropic liquids with spherically symmetric molecules are invariant under rotational, $O(3)$, and

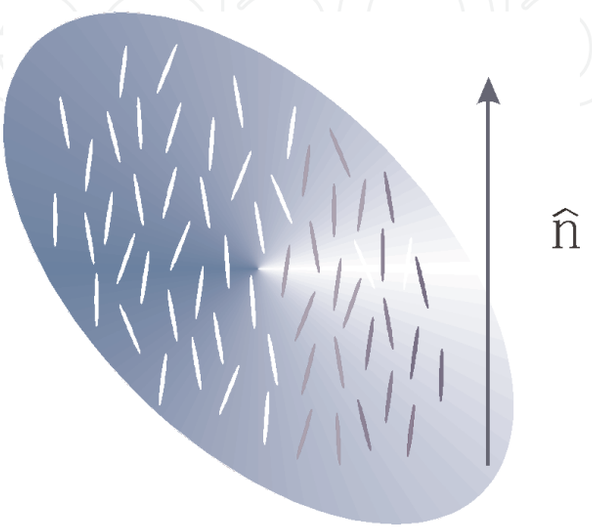


Figure 1.
Representation of the average orientation of the molecules of a thermotropic NLC by means of the director vector $\hat{n}$.

translational, $T(3)$, transformations. Thus, the group of symmetries of an isotropic liquid is $O(3) \times T(3)$. However, by decreasing the temperature of these liquids, the translational symmetry $T(3)$ is usually broken corresponding to the isotropic liquid-solid transition. In contrast, for a liquid formed by anisotropic molecules, by diminishing the temperature, the rotational symmetry $O(3)$ is broken, which leads to the appearance of a liquid crystal. The mesophases for which only the rotational invariance has been broken are called nematics. As shown, the centers of mass of the molecules of a nematic have arbitrary positions, whereas the principal axes of their molecules are spontaneously oriented along a preferred direction. If the temperature decreases even more, the symmetry $T(3)$ is also partially broken. The mesophases exhibiting the translational symmetry $T(2)$ are called smectics [36].

This preferential direction is described by a local unitary vector field, $\hat{n}$, called the director. This vector is easily distorted by the presence of electric and magnetic fields, as well as by the surfaces of the containers of the liquid crystals if they have been prepared properly [32]. With respect to *NLC*, it is important to point out that the director's orientation does not distinguish between the $\hat{n}$ and $-\hat{n}$ directions (nematic symmetry). A schematic representation of the order presented by the molecules in a nematic is shown in **Figure 1** .

3. Model

Consider a *NLC* thin layer of thickness d under the presence of a constant gravitational field $\vec{g} = -g\hat{z}$, where g denotes its magnitude and $\hat{z}$ the unit vector along the z axis. The initial configuration of the layer is homeotropic with a preferential orientation $\hat{n}_0$ along the z axis, as depicted in **Figure 2**. The nematic is confined between two parallel flat plates kept at fixed temperatures T_1 and T_2 ($T_1 < T_2$), so that a uniform temperature gradient $\nabla_z T \equiv -\alpha\hat{z}$ is established downward in the layer. The situation where the temperature gradient goes from bottom to top can also be considered, and in this case, $\nabla_z T \equiv \alpha\hat{z}$. The gravitational force induces a pressure gradient, $\nabla_z p = -\rho g\hat{z}$, where ρ is the mass density.

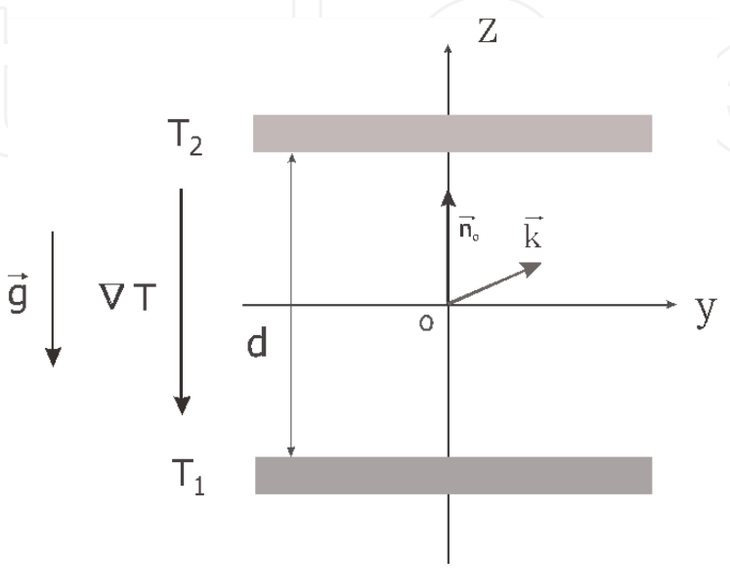


Figure 2.

The NLC cell subject to a constant gravitational field $\vec{g}$ and an external uniform temperature gradient ∇T. $\vec{k}$ is the scattering vector.

3.1 Stationary state

The external gradients drive the nematic layer into a nonequilibrium steady state. We shall assume that the temperature difference $T_1 - T_2$ amounts only to a few degrees, so that there are no nematic layer flows $\left(v_i^{st} = 0\right)$, nor instabilities of the Rayleigh-Bénard type. In this *NESS,* we choose as the nematodynamic variables the set $\Psi = \{\rho, s, v_i, n_i\}$, where $s\left(\vec{r}, t\right)$ is the specific entropy density (entropy per unit mass), the hydrodynamic velocity is $v_i\left(\vec{r}, t\right)$ and $n_i\left(\vec{r}, t\right)$ is the director field. It is to be expected that in this steady state, the changes in Ψ^{st} will only occur in the z direction, so that $\Psi^{st} = \Psi[p(z), T(z)]$, where p is the local pressure. We assume that Ψ^{st} admits an expansion of the Taylor series around an equilibrium state $\left(T_0, p_0\right)$ at $z_0 = 0$, and we consider only first-order terms in the gradients. Thus, by setting the values of the temperature at the plates, $T_1 = T(z = -d/2)$ and $T_2 = T(z = d/2)$, the steady temperature profile is completely determined by:

$$T^{st} = T(z) = T_0 + \frac{dT}{dz} z = T_0 \left(1 - \frac{\alpha}{T_0} z\right), \tag{1}$$

where $T_0 \equiv T^{st}(z = 0) = (T_1 + T_2)/d$ and $\alpha \equiv \Delta T/d$, with $\Delta T \equiv T_1 - T_2$. In what follows, we shall only consider $T_0 \approx 3 \times 10^2 K$, and it will be convenient to introduce the *effective temperature gradient* $\nabla_z T^{st} \equiv X\hat{z}$ as [37],

$$X \equiv -\alpha + \frac{g\beta T_0}{c_p}, \tag{2}$$

which contains explicitly the contributions of both external forces. In Eq. (2), c_p is the specific heat at constant pressure, β is the thermal expansion coefficient, which satisfies the relationship $\beta^2 \equiv (\gamma - 1)c_p/T_0 c_s^2$, where c_s is the adiabatic sound velocity in the nematic, $\gamma \equiv c_p/c_v = c_s^2/c_T^2$, being c_v the specific heat at constant volume and c_T the isothermic sound velocity in the nematic.

4. Nematodynamic equations

The geometry of the proposed model allows us to separate the hydrodynamic variables into transverse (T) and longitudinal (L) variables with respect to $\hat{n}_0$ and $\vec{k}$, [33]. The former set is $\Psi^T\left(\vec{r}, t\right) \equiv \{v_x, n_x\}$, while the latter reads $\Psi^L\left(\vec{r}, t\right) \equiv \left\{p, v_y, v_z, s, n_y\right\}$. We want to describe the stochastic dynamics of the spontaneous thermal deviations (fluctuations) $\delta\Psi\left(\vec{r}, t\right) = \Psi\left(\vec{r}, t\right) - \Psi^{st}$ around the above defined stationary state. A complete set of stochastic equations for the space-time evolution of the fluctuations is obtained by linearizing the general nematodynamic equations [20, 22, 24], and by using the *FH* formalism. This starting set of equations is given explicitly by Eqs. (19)–(22) in Ref. [25]. However, since for the nematic mesophase, the rotational invariance has been broken, it is convenient to rewrite these nematodynamic equations in a representation which takes into account that a symmetry breaking has occurred along the z axis.

In order to take into account the effect of the intrinsic anisotropy of the fluid in the dynamics of the fluctuations, as well as to facilitate the calculation of the nematic modes and the spectrum of light scattering, it is convenient to introduce a

new state variables. In the case of the present model, owing to the initial orientation of the director $\hat{n}_i^{st}$, the *NLC* exhibits several symmetries: rotational invariances around the z axis, symmetry under inversions with respect to both, the xy plane and with respect to reflections on planes containing the z axis. A proper set of variables for this purpose was proposed long ago [38, 39], in terms of the variables $\{\delta p, \delta\varphi, \delta s, \delta\xi, \delta f_1, \delta\psi, \delta f_2\}$, defined in detail in Eqs. (6)–(10) in Ref. [26] (or Eqs. (53)–(57) in [25]). In this new representation, the complete set of stochastic hydrodynamic equations for the fluctuations takes an alternative form given by Eqs. (11)–(17) in Ref. [26] (or Eqs. (58)–(64) in [25]). The matrix representation of the Fourier transformation of this set of equations is given by:

$$\frac{\partial}{\partial t}\delta\vec{X}\left(\vec{k},t\right) = -M\delta\vec{X}\left(\vec{k},t\right) + \vec{\Theta}\left(\vec{k},t\right), \tag{3}$$

where $\delta\vec{X}\left(\vec{k},t\right) = \left(\delta\vec{X}^L, \delta\vec{X}^T\right)^t$ with $\delta\vec{X}^L\left(\vec{k},t\right) = \left(\delta\widetilde{p}, \delta\widetilde{\varphi}, \delta\widetilde{s}, \delta\widetilde{\xi}, \delta\widetilde{f_1}\right)^t$ and $\delta\vec{X}^T\left(\vec{k},t\right) = \left(\delta\widetilde{\psi}, \delta\widetilde{f_2}\right)^t$. The superscript t denotes the transpose, while L and T indicate, respectively, the longitudinal and transverse sets of variables. In Eq. (3), M stands for a 7×7 hydrodynamic matrix which is diagonal in the $5\times 5\, N^L$ and the $2\times 2\, N^T$ blocks. The explicit form of these matrices is not necessary in our discussion; however, they are given explicitly by Eqs. (21) and (22) in Ref. [26] (see also Eqs. (72) and (73) in Ref. [25]). The stochastic terms, $\vec{\Theta}\left(\vec{k},t\right)$, in Eq. (3) are given by the column vector $\vec{\Theta}\left(\vec{k},t\right) = \left(\vec{\Theta}^L, \vec{\Theta}^T\right)^t$ which explicit form of its components can be found in Eqs. (32) and (33) in Ref. [26] (or Eqs. (84) and (85) of Ref. [25]). It is important to emphasize that as a consequence of this change of representation, in this last system, it can be clearly seen how the nematic variables are separated in two sets completely independent: the five longitudinal $\left\{\delta\widetilde{p}, \delta\widetilde{\varphi}, \delta\widetilde{s}, \delta\widetilde{\xi}, \delta\widetilde{f_1}\right\}$ and the two transverse $\left\{\delta\widetilde{\psi}, \delta\widetilde{f_2}\right\}$.

However, in order to facilitate the calculation of the hydrodynamic modes, we define a new set of variables having the same dimensionality, $\left[\delta z_j\left(\vec{k},t\right)\right] = M^{1/2}L^{-1/2}t$ $(j = 1,\ldots,7)$: $z_1 \equiv \left(\rho_0 c_s^2\right)^{-1/2}\delta\widetilde{p}$, $z_2 \equiv \left(\rho_0 k^{-2}\right)^{1/2}\delta\widetilde{\varphi}$, $z_3 \equiv \left(\rho_0 T_0 c_p^{-1}\right)^{1/2}\delta\widetilde{s}$, $z_4 = \left(\rho_0 k^{-4}\right)^{1/2}\delta\widetilde{\xi}$, $z_5 \equiv \left(\rho_0 c_s^2 k^{-2}\right)^{1/2}\delta\widetilde{f_1}$, $z_6 \equiv \left(\rho_0 k^{-2}\right)^{1/2}\delta\widetilde{\psi}$, $z_7 \equiv \left(\rho_0 c_s^2 k^{-2}\right)^{1/2}\delta\widetilde{f_2}$. In terms of these new variables, the system of equations (3) is rewritten in the more compact form as:

$$\frac{\partial}{\partial t}\vec{Z}\left(\vec{k},t\right) = -N\vec{Z}\left(\vec{k},t\right) + \vec{\Xi}\left(\vec{k},t\right), \tag{4}$$

where $\vec{Z}\left(\vec{k},t\right) = \left(\vec{Z}^L, \vec{Z}^T\right)^t$ with $\vec{Z^L}\left(\vec{k},t\right) = (z_1, z_2, z_3, z_4, z_5)^t$ and $\vec{Z}^T\left(\vec{k},t\right) = (z_6, z_7)^t$. In Eq. (4), N stands for a 7×7 hydrodynamic matrix which is diagonal in the $5\times 5\, N^L$ and the $2\times 2\, N^T$ blocks. Again, the explicit form of these matrices is not necessary in our discussion, but they are given explicitly by Eqs. (39)–(41) in Ref. [26] (see also Eqs. (94)–(96) in [25]). In Eq. (4), $\vec{\Xi}\left(\vec{k},t\right) = \left(\vec{\Xi}^L, \vec{\Xi}^T\right)^t$ is the stochastic term, composed by the longitudinal $\vec{\Xi}^L\left(\vec{k},t\right) = (\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5)^t$ and transverse $\vec{\Xi}^T\left(\vec{k},t\right) = (\zeta_6, \zeta_7)^t$ noise vectors. The

explicit form of the components ζ_m, $m = 1...7$, as well as their fluctuation-dissipation relations (*FDR*), can be found in Eqs. (169)–(175) and Eqs. (176)–(186), respectively, in Appendix A of [25].

5. Hydrodynamic modes

In order to find the hydrodynamic modes, or decay rates [37], we need the Fourier transform of the linear system (4), which yields an algebraic system of equations in terms of the variables $\vec{k}$ and ω. The hydrodynamic modes are obtained by calculating its eigenvalues $\lambda = i\omega$, given by the roots of the characteristic equation $p(\lambda) = p^L(\lambda)p^T(\lambda) = 0$, where $p^L(\lambda)$ and $p^T(\lambda)$ are the characteristic polynomials of fifth and second order in λ of the matrices N^L and N^T, respectively. These roots are calculated below.

5.1 Longitudinal modes

Following the method proposed by [13] for a simple fluid, it can be shown that longitudinal variables can be separated in turn and within a very good approximation, into two completely independent sets of variables, $\vec{Z}^L_X = (z_1, z_2)^t$ and $\vec{Z}^L_Y = (z_3, z_4, z_5)^t$, as it is shown in the Subsection 3.1 of Ref. [25], or in more detail in [24]. This approximation allows us to rewrite the characteristic polynomial of longitudinal variables as $p^L(\lambda) = p^L_{XX}(\lambda)p^L_{YY}(\lambda)$. It should be mentioned that $p^L_{XX}(\lambda)$ and $p^L_{YY}(\lambda)$ are polynomials of second and third degree in λ, and explicitly are given by the Eqs. (44) and (45) in Ref. [26] (or Eqs. (117) and (118) in [25]).

While there is no analytical difficulty to solve the quadratic and cubic equations $p^L_{XX}(\lambda)$ and $p^L_{YY}(\lambda)$, the explicit form of their exact roots can be quite complicated. However, it is possible to estimate them following a procedure based partially on a method suggested in Ref. [40], which allows to identify the following quantities in the equation for $p^L_{YY}(\lambda)$, namely, $(\gamma - 1)D_T k^2$, $\sigma_1 k^2$, $k^2 c_s^2$ and $g^2 k_\parallel^2/(c_s^2 k^2)$. They depend on the anisotropic coefficients of diffusivity D_T and on the viscosity σ_1. The former quantity is a function of the parallel $\chi_\parallel$ and perpendicular $\chi_\perp$ components of thermal diffusivity, while the latter depends on the nematic viscosity coefficients ν_i $(i = 1, ..., 5)$ (see Eqs. (23) and (24) in Ref. [26], or Eqs. (74) and (75) in [25]). In the same way, in the equation for $p^L_{YY}(\lambda)$, the following quantities can be identified, $g\alpha\beta\frac{k_\perp^2}{k^2}$, $gX\beta\frac{k_\perp^2}{k^2}$, $D_T k^2$, $\Omega\chi_a k^2$, $\sigma_3 k^2$, $\frac{K_1}{\gamma_1}k^2$, $\frac{\Omega^2 K_I}{\rho_0}k^4$, which depend on the anisotropic coefficients of viscosity σ_3, of elasticity K_I, symmetry Ω (see, respectively, Eqs. (26), (28), and (30) in [26]), as well as the anisotropy $\chi_a = \chi_\parallel - \chi_\perp$ and the torsional viscous coefficient γ_1. We now compare all these quantities with $\overline{\omega} \equiv c_s k$, by introducing the (small) reduced dimensionless quantities:

$$
\begin{gathered}
a_0 \equiv \frac{g\alpha\beta}{\overline{\omega}^2}\frac{k_\perp^2}{k^2}, \quad a_0' \equiv \frac{gX\beta}{\overline{\omega}^2}\frac{k_\perp^2}{k^2}, \quad a_0'' \equiv \frac{g^2 k_\parallel^2}{\overline{\omega}^2 c_s^2 k^2}, \quad a_1 \equiv \frac{D_T k^2}{\overline{\omega}}, \quad a_1' \equiv \frac{\Omega\chi_a k^2}{\overline{\omega}}, \\
a_2 \equiv \frac{\sigma_1 k^2}{\overline{\omega}}, \quad a_3 \equiv \frac{\sigma_3 k^2}{\overline{\omega}}, \quad a_5 \equiv \frac{K_I k^2}{\gamma_1 \overline{\omega}}, \quad a_6 \equiv \frac{\Omega^2 K_I k^4}{\rho_0 \overline{\omega}^2}.
\end{gathered}
\tag{5}
$$

The relevant point for our purpose is to realize that for most nematics at ambient temperatures, ρ_0 and Ω are of order of magnitude 1, $\gamma_1 \sim 10^{-1}$, χ_i and ν_i are of order 10^{-2}–10^{-3}, $K_i \sim 10^{-6}$–10^{-7}, while $\beta \sim 10^{-4}$ [32]. If we consider that $\alpha \lesssim 1$ and

$g \sim 10^3$, and knowing that in a typical light scattering experiments $k = 10^5 \text{cm}^{-1}$ and $c_s = 1.5 \times 10^5 \text{cms}^{-1}$ [41], the quantities given in Eq. (5) have the following orders of magnitude: $a_0 \sim 10^{-21}$, $a_0' \sim 10^{-21}$, $a_0'' \sim 10^{-24}$, $a_1 \sim 10^{-3}$, $a_1' \sim 10^{-3}$, $a_2 \sim 10^{-2}$, $a_3 \sim 10^{-2}$, $a_5 \sim 10^{-5}$ and $a_6 \sim 10^{-6}$. We now follow the method of Ref. [40] and the solutions of the polynomial $p_{YY}^L(\lambda)$ may be obtained by a perturbation approximation in terms of these small quantities. However, in what follows, we improve this approximation by using its exact roots and by expressing them in terms of the reduced quantities (Eq. (5)) of order k^2 [24].

5.1.1 Sound longitudinal modes

They are the roots of the characteristic equation $p_{XX}^L(\lambda) = 0$. Its roots are complex conjugate and are given by (see Eqs. (47) and (48) in [26], or Eqs. (128) and (129) in [25]):

$$\lambda_1 \simeq \Gamma k^2 + ic_s k, \quad \lambda_2 \simeq \Gamma k^2 - ic_s k, \tag{6}$$

where $\Gamma \equiv \frac{1}{2}[(\gamma - 1)D_T + \sigma_1]$ is the anisotropic sound attenuation coefficient of the *NLC*. This result shows that the sound propagation modes, λ_1 and λ_2, are in complete agreement with those already reported in the literature for *NLC* [31, 34].

5.1.2 Visco-heat and director longitudinal modes

These modes are the roots of the characteristic equation $p_{YY}^L(\lambda) = 0$. In Ref. [26] (or in [25]), it is shown that, up to first order in the small quantities (Eq. (5)), these roots can be written approximately as:

$$\begin{aligned} \lambda_{3,4} = & \frac{1}{2}\left(D_T k^2 + \sigma_3 k^2 - \frac{\Omega^2 K_I k^4}{\rho_0 \sigma_3 k^2}\right) \\ & \mp \frac{1}{2}\sqrt{\left(D_T k^2 + \sigma_3 k^2 - \frac{\Omega^2 K_I k^4}{\rho_0 \sigma_3 k^2}\right)^2 - 4 D_T k^2 \sigma_3 k^2 \left(1 - \frac{R}{R_c}\right)}, \end{aligned} \tag{7}$$

and

$$\lambda_5 \simeq \frac{K_I k^2}{\gamma_1} + \frac{\Omega^2 K_I k^4}{\rho_0 \sigma_3 k^2}, \tag{8}$$

with

$$\frac{R\left(\vec{k}\right)}{R_c} \equiv -\frac{g\beta \hat{k}_\perp^2}{D_T \sigma_3 k^4}\left[X + \frac{\alpha \Omega \chi_a}{D_T \sigma_3}(\sigma_3 + D_T)\right], \tag{9}$$

where $\hat{k}_\perp^2 \equiv k_\perp^2/k^2$. In Eq. (7), $R \equiv \frac{\beta g \Delta T d^3}{\sigma_3 \chi}$ is the Rayleigh number and R_c denotes its critical value above which convection sets in. It should be emphasized that our results are expressed in terms of the ratio $R\left(\vec{k}\right)/R_c$ and are, therefore, independent of the value of the separation d between the plates. However, the appropriate value of d in an experiment should be chosen with an experimental criterion [42].

The Rayleigh-number ratio $R\left(\vec{k}\right)/R_c$ contains two contributions: the first term is due to the presence of the effective temperature gradient X, which depends on both, the temperature gradient α and the gravity field g. The second term is entirely a contribution due to α and the nematic anisotropy χ_a. For typical nematics and conventional light scattering experiments, both contributions are approximately of order 10^{-16}.

The decay rates λ_3 and λ_4 for an inhomogeneous nematic given by Eq. (7) are called visco-heat modes, because they are composed of the coupling between the thermal $D_T k^2$ and shear $\sigma_3 k^2 - \frac{\Omega^2 K_I k^4}{\rho_0 \sigma_3 k^2}$ diffusive modes through the ratio $R\left(\vec{k}\right)/R_c$. The nature of these modes may be propagative or diffuse, as will be shown below.

5.1.3 Values of $R\left(\vec{k}\right)/R_c$

The three nematic modes (7) and (8) could be two propagative and one diffusive, or all of them completely diffusive; its nature depends on the values assumed by the ratio $R\left(\vec{k}\right)/R_c$. For simple fluids, these features have been predicted theoretically and corroborated experimentally, but to our knowledge, not for an *NLC*. In this sense, the following results suggest that it might be feasible to be also verified experimentally for nematics.

5.1.3.1 Propagative and diffusive modes

If we take into account the orders of magnitude of the small quantities (Eq. (5)), the nematic modes (7) and (8) in general are real and different. Nevertheless, it may happen that these modes may be transformed into one real and two complex conjugate roots. This occurs if $R\left(\vec{k}\right)/R_c < R_0$, where

$$R_0 \equiv -\frac{\left[\left(\sigma_3 - \frac{\Omega^2 K_I}{\rho_0 \sigma_3}\right) - D_T\right]^2}{4 D_T \sigma_3}, \tag{10}$$

which is always negative. Thus, if we consider the orders of magnitude of the involved quantities and typical light scattering experiment values of k, $D_T k^2 \sim 10^7$, $\sigma_3 k^2 \sim 10^8$, and $\frac{\Omega^2 K_I k^4}{\rho_0} \sim 10^{14}$, then $R_0 \cong -10^1$ and the visco-heat modes, Eq. (7), will be propagative when $R\left(\vec{k}\right)/R_c \lesssim -10^1$. This situation corresponds to the propagation region indicated in **Figure 3**. The decay rate λ_5, Eq. (8), remains to be real. It is worth emphasizing that this case corresponds to overstabilized states, where out of the three decay rates, two are propagative visco-heat modes and the other one is completely diffusive. According to Eq. (9), this occurs if the α contained in the effective temperature gradient X changes its sign and increases by several orders of magnitude, situation that may be achieved by reversing the direction in which the temperature gradient is applied, i. e., when heating from below, and by increasing its intensity. As far as we know, there are no theoretical analyses nor experimental evidence for the existence of visco-heat propagating modes in *NLC* under the presence of a temperature gradient and a uniform gravitational field. Given that in simple fluids, under these conditions, there are analytical [8, 37, 38] and experimental [43] studies that support the presence of visco-heat

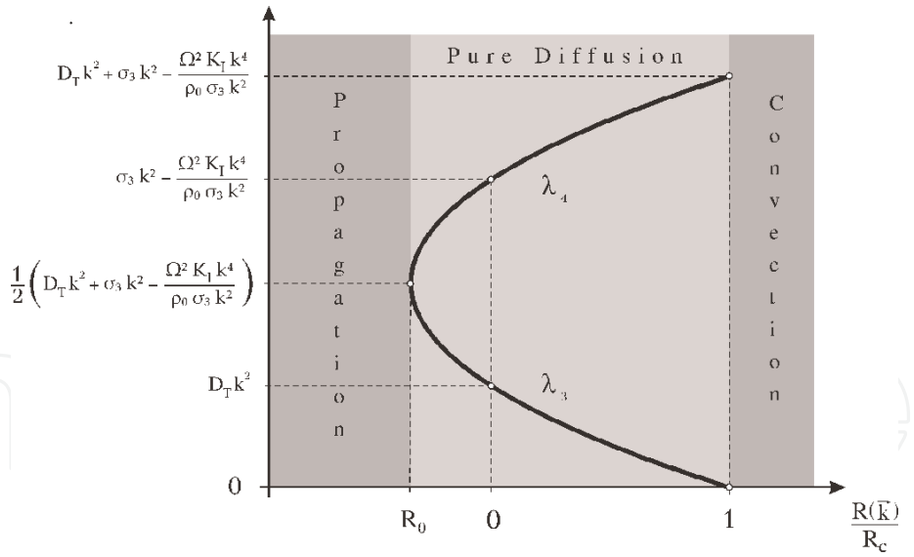


Figure 3.

The real part of the nematic visco-heat modes λ_3 and λ_4 as a function of the Rayleigh ratio $R\left(\vec{k}\right)/R_c$. When $R\left(\vec{k}\right)/R_c < R_0$, both modes are propagative; if $R_0 \le R\left(\vec{k}\right)/R_c \le 1$, both are completely diffusive. For $R\left(\vec{k}\right)/R_c = R_0$, both modes are equal. In equilibrium, $R\left(\vec{k}\right)/R_c = 0$, and the onset of convection occurs for $R\left(\vec{k}\right)/R_c = 1$.

propagation modes, this prediction suggests that it may be worth to design experiments to corroborate this phenomenon for nematics.

5.1.3.2 Pure diffusive modes

When $R_0 \le R\left(\vec{k}\right)/R_c \le 1$, the two visco-heat modes preserve the same form as in Eq. (7) and the other one remains identical to Eq. (8), but all are real and completely diffusive. In this regime, the following cases are of special interest. For instance, if $R\left(\vec{k}\right)/R_c = R_0$, then the visco-heat modes (7) reach the same value, and consequently, the three decay rates are:

$$\lambda_{3,4} = \frac{1}{2}\left(D_T k^2 + \sigma_3 k^2 - \frac{\Omega^2 K_I k^4}{\rho_0 \sigma_3 k^2}\right), \tag{11}$$

and λ_5, that takes the same form as in Eq. (8). These visco-heat modes are identified at the vertex of the parabola in **Figure 3**.

Since for nematics, $\sigma_3 - \frac{\Omega^2 K_I}{\rho_0 \sigma_3}$ is usually greater than D_T, it can be seen from Eq. (7) that, as $R\left(\vec{k}\right)/R_c$ grows and approaches 1, the magnitude of the heat diffusive mode decreases, whereas the one of the shear mode increases. At the onset of convections regime, $R\left(\vec{k}\right)/R_c = 1$, i. e., when R reaches its critical value R_c and the two visco-heat modes (7) are simplified to:

$$\lambda_3 = 0, \tag{12}$$

$$\lambda_4 = D_T k^2 + \sigma_3 k^2 - \frac{\Omega^2 K_I k^4}{\rho_0 \sigma_3 k^2}, \tag{13}$$

while the third, λ_5, is identical to Eq. (8). This behavior for the decay rates λ_3 and λ_4 is also shown in **Figure 3**.

It should be noted that our expressions for these three decay rates are not in agreement with those reported for an *NLC* in the literature [44, 45]. In these works, the director mode tends to zero, the shear mode does not change and there is an additional mode which is the sum of the thermal and director modes. In contrast, we have found that the thermal mode λ_3 vanishes, the director mode λ_5 is virtually unchanged, while λ_4 has contributions from the thermal and shear diffusive modes. We know that this phenomenon also occurs in the simple fluid, where there are two diffusive modes, the thermal mode also vanishes and the other one has contributions from the shear and thermal modes. In other words, our results reduce to the corresponding one for a simple fluid as R reaches its critical value R_c. Because for a simple fluid, these features have been predicted theoretically, our results suggest that it might be feasible to verify them experimentally also for nematics [8, 37, 38].

5.2 Transverse modes

As mentioned earlier, $p^T(\lambda)$ is the characteristic polynomial of second order in λ of the matrix N^T. The corresponding transverse hydrodynamic modes are the roots of this equation $p^T(\lambda) = 0$.

5.2.1 Shear and director transverse modes

Accordingly, the shear and director transverse modes are the roots of $p^T(\lambda) = 0$, and are given by Eq. (63) in Ref. [26] (or by Eq. (157) in [25]). Following again the approximate method of small quantities used previously, the quantities σ_4, $K_{II}k^2/\gamma_1$ and $\lambda_+ K_{II}k^2k_\parallel^2/\rho_0$, may be identified in this equation. In terms of them, we have another set of anisotropic coefficients given by the viscosity σ_4, the elasticity K_{II}, and symmetry λ_+ (see, respectively, Eqs. (27), (29), and (31) in [26]). We also define the small or reduced dimensionless quantities, analogous to those defined in Eq. (5), namely, $a_4 \equiv \frac{\sigma_4 k^2}{\overline{\omega}}$, $a_5' \equiv \frac{K_{II}k^2}{\gamma_1\overline{\omega}}$, $a_6' \equiv \frac{\lambda_+^2 K_{II}}{\rho_0\overline{\omega}^2}k^2k_\parallel^2$, where again $\overline{\omega} \equiv c_s k$. It should be noted that the viscous coefficient σ_4 only depends on the viscous coefficients ν_2, ν_3, while the elastic coefficient K_{II} depends on the two Frank elastic constants K_2 and K_3. Since for typical nematics $\lambda_+ \sim 1$, $\gamma_1 \sim 10^{-1}$, $\sigma_4 \sim 10^{-2}$, $K_{II} \sim 10^{-6}$ [32], and also by taking into account that $c_s \sim 10^5$, $k \sim 10^5, g \sim 10^3$, the quantities a_4, a_5' and a_6' have the orders of magnitude $a_4 \sim 10^{-2}$, $a_5' \sim 10^{-5}$, and $a_6' \sim 10^{-6}$. According to Eqs. (64) and (65) in Ref. [26] (or Eqs. (167) and (168) in [25]), up to first order in such small amounts, these two roots can be written as:

$$\lambda_6 = \sigma_4 k^2 - \frac{\lambda_+^2 K_{II}k^2k_\parallel^2}{\rho_0\sigma_4 k^2}, \qquad \lambda_7 = \frac{K_{II}k^2}{\gamma_1} + \frac{\lambda_+^2 K_{II}k^2k_\parallel^2}{\rho_0\sigma_4 k^2} \tag{14}$$

It should be noted that these shear and director diffusive transverse modes also match completely with those already reported for nematics [22, 31, 32].

6. The equilibrium and simple fluid limits

From the hydrodynamic modes calculated for an *NLC* in a *NESS* determined by a Rayleigh-Bénard system, it is possible to obtain, as limit cases, the corresponding

modes of a nematic in the state of equilibrium and those of a simple fluid under the same nonequilibrium regime. Both situations are of physical interest and are discussed below.

6.1 Nematic in equilibrium

It has been found that for an *NLC* in a *NESS*, the effects of the external gradients α and g are only manifested in the coupling of the thermal diffusive and shear longitudinal modes, which gives rise to the visco-heat modes $\lambda_{3,4}$ indicated, respectively, by means of Eq. (7). If the nematic layer is in a state of homogeneous thermodynamic equilibrium, $g = 0$ and $\alpha = 0$, and therefore $X = 0$ and $R\left(\vec{k}\right)/R_c = 0$. Thus, the hydrodynamic modes of a nematic, in the state of equilibrium (denoted by the superscript e), are composed of five longitudinal and two transverse modes. The longitudinal modes are integrated by the two acoustic propagatives λ_1^e and λ_2^e given by Eq. (6); as well as by the three diffusives, which consist of one thermal:

$$\lambda_3^e = D_T k^2, \tag{15}$$

another of shear:

$$\lambda_4^e = \sigma_3 k^2 - \frac{\Omega^2 K_I k^2}{\rho_0 \sigma_3} \tag{16}$$

and one more of the director, λ_5^e, which is the same as Eq. (8). The longitudinal diffusive modes (15) and (16) are obtained precisely from Eq. (7), since in this, the Rayleigh ratio, given by Eq. (9), is zero if α and g vanish. Moreover, the pair of transverse modes consist of the shear and director modes λ_6^e and λ_7^e which are equal to the Eq. (14). It is necessary to mention that the decay rates λ_i^e $(i = 1...7)$ are well known in the literature [22, 31, 46]. Note that λ_3^e and λ_4^e are shown in the middle part of **Figure 3**.

6.2 Simple fluid in a Rayleigh-Bénard system

Given that in the isotropic limit (simple fluid limit), the degree of nematic order goes to zero, n_i is no longer a hydrodynamic variable, and the elastic constants K_i (for $i = 1, 2, 3$) and the kinetic parameters γ_1, λ vanish. Also, $\chi_\perp$ and $\chi_\parallel$ are reduced to the coefficient of thermal diffusivity χ and $\chi_a = 0$. On the other hand, the nematic viscosities are reduced in the following way: $\nu_1 \to \eta$, $\nu_2 \to \eta$, $\nu_3 \to \eta$, $\nu_4 \to \zeta + \frac{1}{3}\eta$, $\nu_5 \to -\frac{2}{3}\eta + \zeta$, where η and ζ denote, respectively, the shear and volumetric viscosities of the simple fluid. As a result, from Eqs. (23)–(31) in Ref. [26] (or Eqs. (74)–(82) in [25]), it follows that in the isotropic limit $D_T \to \chi$, $\sigma_1 \to \frac{1}{\rho_0}\left(\frac{4}{3}\eta + \zeta\right)$, $\sigma_2 \to 0$, $\sigma_3 \to \nu$, $\sigma_4 \to \nu$, where $\nu \equiv \eta/\rho_0$ is the kinematic viscosity, whereas $K_I \to 0$, $K_{II} \to 0$, and $\Omega \to 0$. Consequently, by making the identifications indicated above, the corresponding hydrodynamic modes of a simple fluid can be obtained when it is in a Rayleigh-Bénard system. Thus, according to Eq. (6), a simple fluid has the two acoustic propagative modes:

$$\lambda_1 \simeq \Gamma' k^2 + i c_s k, \quad \lambda_2 \simeq \Gamma' k^2 - i c_s k, \tag{17}$$

where c_s corresponds to the adiabatic velocity of the sound in this medium and $\Gamma' \equiv \frac{1}{2}\left[(\gamma-1)\chi + \frac{1}{\rho_0}\left(\frac{4}{3}\eta+\zeta\right)\right]$ is the corresponding coefficient of sound attenuation. On the other hand, according to the Eq. (7), the longitudinal visco-heat modes are:

$$\lambda_{3,4} \simeq \frac{1}{2}(\chi+\nu)k^2 \mp \frac{1}{2}\sqrt{(\chi+\nu)^2k^4 - 4\chi\nu k^4\left(1-\frac{R}{R_c}\right)}. \tag{18}$$

In the isotropic limit of the simple fluid, $\lambda_5 = \lambda_7 = 0,$ so that, according to the Eq. (14), the only transverse mode of this substance in a Rayleigh-Bénard system is:

$$\lambda_6 = \nu k^2. \tag{19}$$

In Eq. (18), the ratio $R\left(\vec{k}\right)/R_c$ is defined as:

$$\frac{R\left(\vec{k}\right)}{R_c} \equiv -\frac{g\beta X\hat{k}_\perp^2}{\chi\nu k^4}, \tag{20}$$

which, in this limit case, can be derived from Eq. (9). It should be pointed out that Eq. (20) coincides with the Eq. (2.21) of reference [37]. The modes (17)–(19) are in complete concordance with those analytically calculated in [8, 37, 38].

Moreover, if in the coefficient matrix M of the stochastic system given by Eq. (20) in Ref. [26], the simple fluid limit is taken, it reduces to a matrix that is a generalization of the one given by the Eq. (6) in [38]. Additionally, if in the corresponding matrix M found for the simple fluid, the equilibrium limit is now considered, i. e., when α and g vanish, the resulting matrix is also reduced to that given by Eq. (4) of [38].

6.2.1 Values of $\mathrm{R}\left(\vec{k}\right)/\mathrm{R_c}$

The two visco-heat mode, as in the nematic, could be propagative or diffusive. These characteristics depend on the values assumed by the ratio $R\left(\vec{k}\right)/R_c$. For simple fluids, these have been predicted theoretically and corroborated experimentally.

6.2.1.1 Propagative modes

If $R\left(\vec{k}\right)/R_c < R_0$, where $R_0 \equiv -(\nu-\chi)^2/(4\chi\nu) < 0,$ the visco-heat modes (18) will be propagative. According to Eq. (20), this occurs again if the α contained in X changes its sign and increases by several orders of magnitude, a situation that is achieved by inverting the temperature gradient (when heated from below and its intensity is increased). There are analytical [8, 37, 38] and experimental [43] studies that report, for simple fluids in these conditions, the presence of visco-heat propagative modes.

6.2.1.2 Pure diffusive modes

When $R_0 \leq R\left(\vec{k}\right)/R_c \leq 1$, the visco-heat modes preserve the form (Eq. (18)), they are real and completely diffusive. In this regime, there are again three cases of

special interest. If $R\left(\vec{k}\right)/R_c = R_0$, then both visco-heat modes (18) are identical and equal to:

$$\lambda_{3,4} = \frac{1}{2}(\chi + \nu)k^2. \tag{21}$$

On the other hand, if the simple fluid is in a state of homogeneous thermodynamic equilibrium, $g = 0$ and $\alpha = 0$, so that $X = 0$ and $R\left(\vec{k}\right)/R_c = 0$; consequently, in this equilibrium state (identified by the superscript e), there is a thermal diffusive mode:

$$\lambda_3^e = \chi k^2 \tag{22}$$

and the shear mode:

$$\lambda_4^e = \nu k^2. \tag{23}$$

These decay rates are well known in the literature [8, 37, 38]. Finally, because in a simple fluid, commonly ν is greater than χ, according to Eq. (18), and as $R\left(\vec{k}\right)/R_c$ grows and approaches to 1, the magnitude of the thermal diffusive mode decreases, while the shear mode grows. At the threshold of the convective regime (when $R\left(\vec{k}\right)$ reaches its critical value R_c), $R\left(\vec{k}\right)/R_c = 1$, and the two visco-heat modes (18) acquire the values:

$$\lambda_3 = 0 \tag{24}$$

and

$$\lambda_4 = (\chi + \nu)k^2. \tag{25}$$

These three cases are consistent with those obtained in analytical studies already reported for simple fluids in this regime [8, 37, 38]. Schematically, its behavior is very similar to that illustrated in **Figure 3**, and this can be seen in **Figure 1** of the reference [37].

7. Conclusions

In this work, we have used the standard formulation of *FH* to describe the dynamics of the fluctuations of a *NLC* layer in a *NESS* characterized by the simultaneous action of a uniform temperature gradient α and a constant gravitational field g, which corresponds to a Rayleigh-Bénard system. The analysis carried out takes into account only the nonconvective regime. The most important results are the analytic expressions for the seven nematic hydrodynamic modes. The explicit details of several of the calculations can be found in Refs. [25, 26]. To summarize the results obtained in this work and to put them into a proper context, the following comments may be useful.

First, in our analysis, the symmetry properties of the nematic are taken into consideration, and this allowed us to separate its hydrodynamic variables into two completely independent sets: one longitudinal, composed of five variables, and the

other transverse, consisting of only two variables. From the equations that govern the dynamics of the variables in these sets, the corresponding hydrodynamic modes were calculated. The longitudinal modes are two acoustic, λ_1 and λ_2, modes (6), as well as the triplet formed by the visco-heat pair λ_3 and λ_4, modes (7), and the director λ_5, mode (8). In addition, the transverse ones are given by the shear λ_6 and the director λ_7, modes (14). We find that the influence of the temperature gradient α and the gravitational field g occurs only in the longitudinal modes, being greater its effect $\left(\sim 10^{-9}\right)$ on the visco-heat pair λ_3 and λ_4. This effect is quantified by means of the Rayleigh ratio $R\left(\vec{k}\right)/R_c$, Eq. (9), where R is the Rayleigh number and R_c is its critical value above which convection sets in. The developed analysis corresponding to the nonconvective regime was carried out under the condition $R\left(\vec{k}\right)/R_c \leq 1$.

The analytical expressions calculated for the hydrodynamic modes of a nematic in the *NESS* considered exhibit behaviors that are of great interest in the following particular situations. First, if the isotropic limit of the simple fluid is taken, the *NLC* hydrodynamic modes reduce to those in the same state out of equilibrium, modes (17)–(19), [8, 37, 38]. If $R = 0$, that is, in the absence of the uniform temperature gradient and the constant gravitational field, our expressions are simplified and reduce to those already reported for a nematic in the state of thermodynamic equilibrium, modes (6), (8), (14), (15), and (16), [22, 31, 46]. In this case, if we also consider the limit of the simple fluid, they agree with those of this system in equilibrium, modes (17), (19), (22), and (23), [41, 47, 48]. When $R = R_c$, that is, at the threshold of convection, from the triplet of longitudinal λ_3, λ_4 and λ_5, the visco-heat λ_3 vanishes, and λ_4 is the sum of the thermal and shear modes, modes (12) and (13); while that of director λ_5 is identical to mode (8) [37, 38]. Moreover, if in this nematic threshold of convection, the limit of the simple fluid is considered, the modes of this system are recovered: one is zero, mode (24), and the other is the sum of the thermal and shear modes, mode (25), [37, 38]. Also, if $R\left(\vec{k}\right)/R_c < R_0\left(\widetilde{k}\right)$, where $R_0\left(\vec{k}\right)$ is the reference value (10), our results predict that the visco-heat pair λ_3 and λ_4, modes (7), become propagative; in the limit of the simple fluid, under similar conditions, the corresponding modes (18) are also propagative. The latter have been predicted theoretically [8, 37, 38] and verified experimentally [43].

However, it should be mentioned that our hydrodynamic modes λ_3, λ_4, and λ_5 do not coincide with those reported in the literature for an *NLC* in the same *NESS* considered here [44, 45], which consist in one mode due to the director, another more product of the coupling of the thermal and director modes, and a shear mode. The effect of external forces α and g is only manifested in the first two modes. This triplet is reduced to the corresponding director, thermal, and shear longitudinal modes of an *NLC* in the state of thermodynamic equilibrium, as well as to the thermal and shear modes of a simple fluid in such state. It should be noted that from the analytical expressions of these modes, the existence of nematic propagative modes cannot be predicted; much less, in this *NESS*, in the simple fluid. In addition, when the threshold of convection in the nematic is considered, the director mode is canceled, another one is the sum of the thermal and director modes, and the shear mode remains unchanged; consequently, when the limit of the simple fluid is taken, they are reduced to thermal and shear modes. This last result differs completely from the already reported [37, 38] for the hydrodynamic modes of a simple fluid at the threshold of convection, where one is zero, mode (24), and the other the sum of the thermal with the shear, mode (25).

Nevertheless, our calculated expressions for the visco-heat λ_3, λ_4, and director λ_5 modes predict both the existence of propagative modes and the form that this triplet acquires in the convection threshold, and moreover, they reduce to the corresponding modes in all the different limit cases already mentioned. In this respect, we believe that they are more general than those reported in the literature [44, 45]. As far as we know, the diffusive or propagative nature of the modes λ_3 and λ_4, depending on the values taken by the ratio $R\left(\vec{k}\right)/R_c$, was not known; therefore, its derivation represents a relevant contribution of this work. Since in simple fluids, the existence of propagative modes has been predicted and verified experimentally, our predictions about the existence of this phenomenon in the modes of an *NLC* suggest the realization of new experiments.

Finally, it should be noted that this theory can be useful, since the description of some characteristics of our model lend themselves to establish a more direct contact with the experiment. Actually, physical quantities, such as director-director and density-density correlation functions, memory functions or the dynamic structure factor $S\left(\vec{k},\omega\right)$, may be calculated from our *FH* description. In Ref. [49], an application of this nature was developed by calculating the Rayleigh dynamic structure factor for the *NLC* under the *NESS* already mentioned, and its possible comparison with experimental studies is discussed; a preliminary analysis can be consulted in Ref. [50]. Another studies of the dynamic structure factor for an *NLC* in a different *NESS*, such as that produced by the presence of an external pressure gradient, were published in the references [19, 20].

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Conflict of interest

The authors declare that they have no conflict of interests.

Nomenclature

k	wave number
ω	angular frequency
τ	relaxation time of almost all degrees of freedom
$S\left(\vec{k},\omega\right)$	dynamic structure factor
$\vec{k}$	wave vector
R	Rayleigh number
R_c	Rayleigh number at the convection threshold
R/R_c	Rayleigh ratio
$O(3)$	orientation symmetry group
$T(3)$	translation symmetry group
$\hat{n}$, $\vec{n}$ or n_α	director field
d	thickness of the nematic layer

$\vec{g}$	constant gravitational force of magnitude g
$\hat{x}, \hat{y}, \hat{z}$	Cartesian unitary vectors
x, y, z	Cartesian coordinates
T	temperature
α	temperature gradient of magnitude $\nabla_z T$
p	hydrostatic pressure
$\nabla_z p$	pressure gradient
ρ	volumetric density of mass
$\vec{v}$	flow velocity
s	specific density of entropy (entropy per unit mass)
$\vec{r}$	position vector
ΔT	temperature difference between the plates of the cell
X	effective temperature gradient
β	coefficient of thermal expansivity
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
γ	ratio of specific heats
c_s	adiabatic sound velocity
c_T	isothermic sound velocity
Ψ	set of nematodynamic variables
$\delta\varphi$	divergence of $\delta\,\vec{v}$
$\delta\psi$	component z of the rotational of $\delta\,\vec{v}$
$\delta\xi$	component z of the double rotational of $\delta\,\vec{v}$
δf_1	divergence of $\delta\,\vec{n}$
δf_2	component z of the rotational of $\delta\,\vec{n}$
t	as superscript, indicates transpose matrix
$\delta\vec{X}\left(\vec{k},t\right)$	vector whose components are the spatial Fourier transform of the variables δp, $\delta\varphi$, δs, $\delta\psi$, $\delta\xi$, δf_1 and δf_2
$\delta\vec{X^L}\left(\vec{k},t\right)$	longitudinal component of $\delta\vec{X}\left(\vec{k},t\right)$
$\delta\vec{X^T}\left(\vec{k},t\right)$	transverse component of $\delta\vec{X}\left(\vec{k},t\right)$
M	coefficient matrix of the linear system for $\delta\vec{X}\left(\vec{k},t\right)$
M^L and M^T	longitudinal and transverse submatrices of M
$\vec{\Theta}\left(\vec{k},t\right)$	stochastic vector of the linear system for $\delta\vec{X}\left(\vec{k},t\right)$
$\vec{\Theta^L}\left(\vec{k},t\right)$	longitudinal component of $\vec{\Theta}\left(\vec{k},t\right)$
$\vec{\Theta^T}\left(\vec{k},t\right)$	transverse component of $\vec{\Theta}\left(\vec{k},t\right)$
$z_i\left(\vec{k},t\right)$	variables of same dimensionality ($i = 1,\ldots, 7$)
$\vec{Z}\left(\vec{k},t\right)$	vector of the variables $z_i\left(\vec{k},t\right)$
$\vec{Z^L}\left(\vec{k},t\right)$	longitudinal component of $\vec{Z}\left(\vec{k},t\right)$
$\vec{Z^T}\left(\vec{k},t\right)$	transverse component of $\vec{Z}\left(\vec{k},t\right)$
N	coefficient matrix of the linear system for $\delta\vec{Z}\left(\vec{k},t\right)$
N^L and N^T	longitudinal and transverse submatrices of N
$\vec{\Xi}\left(\vec{k},t\right)$	noise vector of the linear system for $\vec{Z}\left(\vec{k},t\right)$

$\vec{\Xi}^L\left(\vec{k},t\right)$	longitudinal component of $\vec{\Xi}\left(\vec{k},t\right)$
$\vec{\Xi}^T\left(\vec{k},t\right)$	transverse component of $\vec{\Xi}\left(\vec{k},t\right)$
ζ_i	noise components of $\vec{\Xi}^L$ $(i = 1,..., 5)$ and $\vec{\Xi}^T$ $(i = 6, 7)$
$p(\lambda)$	characteristic polynomial of the matrix N
$p^L(\lambda)$	characteristic polynomial of the submatrix N^L
$p^T(\lambda)$	characteristic polynomial of the submatrix N^T
λ	eigenvalues of $p(\lambda)$
$\vec{Z}^L_X\left(\vec{k},t\right)$ and $\vec{Z}^L_Y\left(\vec{k},t\right)$	components of the vector $\vec{Z}^L\left(\vec{k},t\right)$
$p^L_{XX}(\lambda)$ and $p^L_{YY}(\lambda)$	polynomials in which $p^L(\lambda)$ is broken down
D_T	anisotropic thermal coefficient
σ_1, σ_2, σ_3, and σ_4	anisotropic viscous coefficients
$\chi_\parallel$ and $\chi_\perp$	thermal diffusivities parallel and perpendicular to $\vec{n}$
χ_a	anisotropic thermal diffusivity
ν_i	nematic viscosities $(i = 1, ..., 5)$
Ω and λ_+	anisotropic adimensional nematic coefficients
K_1, K_2 and K_3	elastic coefficients of Frank
K_I and K_{II}	anisotropic elastic coefficients
γ_1	torsion viscosity
$\overline{\omega}$	auxiliary parameter
a_0, a'_0, and a''_0	small dimensionless longitudinal quantities
a_1, a'_1, and a_2	small dimensionless longitudinal quantities
a_3, a_5, and a_6	small dimensionless longitudinal quantities
λ_1 and λ_2	acoustic propagative longitudinal modes
Γ	anisotropic sound attenuation coefficient
λ_3 and λ_4	visco-heat longitudinal modes
λ_5	director diffusive longitudinal mode
$k_\perp$and $k_\parallel$	components of $\vec{k}$ perpendicular and parallel to $\hat{n}_0$
$\hat{k}_\perp \equiv k_\perp/k$	unit vector of $\hat{k}_\perp$
R_0	reference value of the Rayleigh ratio below which visco-caloric modes are propagative
a_4, a'_5, and a'_6	small dimensionless transverse quantities
λ_6 and λ_7	shear and director diffusive transverse modes
λ^e_i	nematic modes in the state of equilibrium $(i = 1, ..., 7)$
χ	thermal diffusivity of a simple fluid
η and ζ	shear and volumetric viscosities of a simple fluid
ν	kinetic viscosity of a simple fluid
Γ'	attenuation coefficient of sound in a simple fluid

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