Chapter

# Cosmological Constant and Particle Masses in Conformal Quantum Gravity

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## Abstract

It has been proposed that the equivalence principle of quantum gravity should be introduced as a fundamental symmetry in quantum gravity to reconcile quantum mechanics and general relativity. Such symmetry extends the equivalence principle of general relativity to the observer frames of reference which are in quantum mechanical motions. That means the quantum state of a particle is relative to the observer frame which can also be itself in a quantum mechanical state. As a consequence, all the physical laws apply not just to the frames of reference in any kind of motion as in the general relativity but also the same in the reference frames in quantum mechanical motions as well. The classical space-time concept therefore requires to be significantly modified. Because of such principle, the quantum gravity should be formulated in the quantum space-time-matter space with local conformal symmetry. In this book chapter, we explore the formulation of the quantum space-time-matter geometry with local conformal symmetry for discussing the relationship between the cosmological constant and quantum gravity as well as the mass spectrum of fundamental particles. The mathematical expressions of the fundamental particle masses and cosmological constant are discussed.


**Keywords:** cosmological constant, equivalence principle of quantum gravity, Higgs condensate, quantum space-time-matter geometry, local conformal symmetry

## 1. Introduction

The cosmological constant would be fundamentally related to the quantum nature of space-time. The author has proposed that the cosmological constant problem could be resolved by making the hypothesis that the space-time itself behaves as the phase of Higgs condensate (or say space-time condensate) and is discrete in nature. The estimated value of cosmological constant is in excellent agreement with the cosmological observations [1–4]. It has been further shown that the phase factor associated with the particle field in the discrete space-time could generate the CP violation in quark mixing system [5]. It is thus expected that the ultimate theory of quantum gravity would explain the cosmological constant problem.

However, there are fundamental inconsistencies between the general relativity and quantum mechanics in constructing a quantum theory of gravity. Actually, some important elements that are present in one theory are missing in the other. In the one hand, the general relativity does not involve the concepts of quantisation and probability as well as the uncertainty principle which are the important characteristics of quantum mechanics. On the other hand, quantum mechanics is not geometrical and there is no equivalence principle to make it independent with the quantum state of the observer frame of reference like the classical reference frame of general relativity. Such situation imposes fundamental difficulties in the unification of both theories. In order to achieve a more symmetrical treatment to bridge the gaps between both theories for their unification, it has been proposed that the equivalence principle of quantum gravity should be introduced in quantum gravity to reconcile the quantum mechanics and general relativity [6]. The equivalence principle of quantum gravity is that the laws of physics must be of such a nature that they apply to systems of reference in any kind of motion, both classical and quantum mechanical. Such symmetry extends the equivalence principle of general relativity to the observer frames of reference which are in quantum mechanical motions. That means the quantum state of a particle is relative to the observer frame of reference which can also be itself in a quantum mechanical state. As a consequence, all the physical laws apply not just to the frames of reference in any kind of classical motion as in general relativity but also the same in the reference frames in quantum mechanical motions as well. The classical space-time concept therefore requires to be modified significantly. Under such principle, the quantum gravity should be formulated in the quantum space-time-matter space with local conformal symmetry. The advantages of such treatment are that quantum mechanical motions of observers are introduced to extend general relativity from the classical to quantum mechanical domain. On the other hand, quantum mechanics can be made geometrical and relative to the quantum state of observer without any preferred frame. No preferred observer frame of reference is the essence of the principle of relativity.

In this book chapter, we explore the formulation of the quantum space-time-matter geometry with local conformal symmetry for discussing the relationship between the cosmological constant and quantum gravity as well as its connection with the Higgs condensate that would explain the nature of cosmological constant. Furthermore, such formulation implies that the mass spectrum of fundamental particles is related to the cosmological constant and the mathematical expressions of the fundamental particle masses and cosmological constant are discussed.

## 2. Quantum space-time-matter geometry

According to the principle of relativity, the space and time are defined by the coordinate system established respectively by the measuring-rods and synchronised clocks (or the equivalent measurement devices) at rest relative to the observer, which is known as the observer frame of reference, for the descriptions of physical events. In the theory of relativity, the measuring-rods and clocks are in classical motions and thus, within a specified measurement uncertainty, we can assume that their measuring results correspond to the points in the four-dimensional space-time coordinate system as space-time points. However, such classical concept of space-time can only be an approximation as the measuring-rods and clocks themselves are subject to quantum uncertainty and quantum mechanical motions, just the same as the observed matter particles. That means the measuring results of the measuring-rods and clocks should be probabilistic and subject to the uncertainty principle. When the interested scale of length and time are in microscopic level and the quantum uncertainties of the measuring-rods and clocks are large, we can imagine that no admissible coordinate system in classical sense can be established by such measuring-rods and clocks. On the other hand, if we require the quantum uncertainties of the measuring-rod and clock to be infinitely small, according to the uncertainty principle, the momentum and energy involved in the measurements, or say the observation or probing energy scale, should be infinitely large. However, when the energy and momentum is sufficiently large, according to the general relativity, a black hole with an event horizon comparable to the measuring scale will form and it will make the interested physical events hidden behind the event horizon and thus become inaccessible. That means it is not possible to achieve the measurement of space-time points. Such situations reveal that the classical space-time definition is obviously in trouble and modification to it is needed.

If we still adopt defining space-time by measuring-rods and clocks for describing the physical events, the quantum mechanical motions of such measuring-rods and clocks should be considered in the space-time definition. Therefore, the space-time cannot simply be a 4-dimensional coordinate system with space-time points but should be associated with the quantum states of measuring-rods and clocks specified by their state parameters. As the measuring-rod and clock are used for measuring the space and time, their quantum states should be characterised by the classical space-time vector $\vec{x}$; otherwise, they are not the suitable measuring devices for performing the space-time measurements. Furthermore, because of the constraint of measurement scale by the uncertainty principle as explained before, it is reasonable to introduce the quantum uncertainties associated with the space-time measurements as additional state parameters of the measuring-rod and clock. This treatment has the advantage that it is similar to specifying errors or uncertainties for experimental data but the major difference now is that the quantum uncertainty introduced should be an intrinsic spacetime property, rather than instrumental errors or uncertainties. This will lead to a very fundamental change on the spacetime concept. In this connection, let us denote the quantum state of the measuring-rod and clock, that is the quantum state of the spacetime by definition, as

$$\left|\vec{x}, \Delta\ \vec{x}\right\rangle \tag{1}$$

where $\vec{x}$ is the classical spacetime vector and $\Delta\vec{x}$ is the associated uncertainty. It should be noted that the above notation represents the quantum states of spacetime associated quantum uncertainties, which is the states of coordinate system established by measuring-rods and clocks, rather than the particle states. As the quantum uncertainties of the measuring-rod and clock depends upon their energy scales, the modified definition of spacetime should therefore be observation energy scale dependent.

In order to have a proper representation space for the quantum state of particle, the spacetime state should satisfy the following relation

$$\left|\vec{x} + \delta\ \vec{x}, \Delta\vec{x}\right\rangle = \left|\vec{x}, \Delta\vec{x}\right\rangle + \delta\left|\vec{x}, \Delta\vec{x}\right\rangle \tag{2}$$

The quantum state $|\alpha\rangle$ of a particle can be projected to such quantum spacetime state to become a wave function $\phi\left(\vec{x}, \Delta\vec{x}\right)$ as

$$\phi\left(\vec{x}, \Delta\vec{x}\right) = \left\langle\vec{x}, \Delta\vec{x}\ |\alpha\right\rangle \tag{3}$$

From the above expression, one may find that, although such expression of wave function in terms of state kets is somehow similar to the usual expression in the ordinary quantum mechanics, the major differences between them are that the spacetime uncertainty should be specified in the quantum spacetime representation for the particle quantum state and the spacetime states are in general not a spacetime point. Furthermore, in the ordinary quantum mechanics, the expression $\left\langle \vec{x} \mid \alpha \right\rangle$ represents the projection of the particle quantum state $|\alpha\rangle$ to its position eigenstate $\left| \vec{x} \right\rangle$. The position eigenstate is simply the particle position quantum state as measured by the classical frame of reference. Whereas, the expression in Eq. (3) extends such meaning to the measurement of the particle position quantum state by an observer in quantum mechanical motion. The projection in the ordinary quantum mechanics therefore becomes a special case of it. The above treatment introduces quantisation and probability to the spacetime concept as required in the actual situation of physical measurements in microscopic scale and also, the meaning of particle wave function is extended to become the observation of the particle quantum state in a specific quantum spacetime state.

We can imagine that the particle quantum state can be observed in another quantum spacetime state $\left| \vec{y}, \Delta\ \vec{y} \right\rangle$ which can be expressed in general as linear combination of the eigenstate of $\left| \vec{x}, \Delta\ \vec{x} \right\rangle$. We can therefore express the transformation as

$$\left\langle \vec{y}, \Delta\ \vec{y} \mid \alpha \right\rangle = \int \left\langle \vec{y}, \Delta\ \vec{y} \mid \vec{x}, \Delta\ \vec{x} \right\rangle \left\langle \vec{x}, \Delta\ \vec{x} \mid \alpha \right\rangle \tag{4}$$

The integration sign represents the summation or integration of all the possible eigenstates of $\left| \vec{x}, \Delta\ \vec{x} \right\rangle$. Actually, we can omit such notation, when summing or integrating the states, by identifying the operator $\left| \vec{x}, \Delta\ \vec{x} \right\rangle \left\langle \vec{x}, \Delta\ \vec{x} \right|$ as the internal index of the summation or integration. That is similar to the Einstein's convention of omitting the summation sign when summing the internal index of the product of contravariant and covariant tensor in the general relativity. The expression in Eq. (4) is directly analogous to representing the projection of four vectors in different coordinate systems, which is associated with different frame of references, in special relativity. The above treatment allows us to bring the general relativity closer to quantum mechanics in constructing the theory of quantum gravity.

In order to bring the quantum mechanics closer to the general relativity, we need to introduce the equivalence principle and geometrical concept to the ordinary quantum mechanics in the theory of quantum gravity. As mentioned above, in the relativity, the measuring-rod and clock at rest relative to the observer defines the observer frame of reference. However, the measuring-rod and clock are subject to quantum mechanical motion and described by the quantum spacetime state $\left| \vec{x}, \Delta\ \vec{x} \right\rangle$, which means the observer frame of reference in different quantum mechanical motions should be described by different quantum spacetime states. As the equivalence principle of general relativity requires that all the physical laws apply to the frames of reference in any kind of classical motion but actually the observer frames of references can be in quantum mechanical motions, it is therefore natural and reasonable to extend the equivalence principle to the observer frames of references in quantum mechanical motions. As a consequence, all the physical laws apply not just to the frames of reference in any kind of classical motion as in general relativity but also the same in the reference frames in quantum mechanical motions as well. We call this the equivalence principle of quantum gravity [6]. Since different quantum mechanical observer frames are described by different quantum spacetime states, the equivalence principle of quantum gravity implies that the general laws of nature are required to be expressed by equations which hold good for all systems of quantum spacetime states which are covariant under the transformation in Eq. (4). Actually, this is an extension of the general covariance from the classical coordinate system to the to the quantum spacetime space.

In the general relativity, under the equivalence principle, we can always find a free fall frame, which is local in spacetime, in which there is no gravity acting on the observed particles. Similarly, based on the equivalence principle of quantum gravity, we may anticipate that we can always find a “quantum free” frame for a matter particle in which there is no observed quantum effect on the observed particle. However, the major difference between such arguments is that all the matter particles with different masses have the same acceleration under the gravity, whereas the quantum mechanical motions depend on the energy and momentum of the particles. That means the “quantum free” frame for one particle may not be valid for another, especially when two particles of different energy and momentum are observed together under the same quantum spacetime state. In the general relativity, when a particle is moving in a gravitational field, its equation of motion is governed by the geodestic equation given by the extremum of the following action

$$S = \int ds \tag{5}$$

However, as we know in relativistic mechanics, the equation of motion of free particle is given by the least action principle on the following action

$$S = \int mds \tag{6}$$

The difference between them is that the rest mass is omitted in the particle action in general relativity. This is due to the fact that the inertial mass is the same as the gravitational mass under the equivalence principle of general relativity. However, when the quantum mechanical motion of matter particle is considered, the inertial mass cannot be neglected. We can understand this by considering the Schr*ö*dinger equation of a particle in gravitational field. As demonstrated by the COW experiment [7], the quantum interference pattern of neutrons induced by gravity depends upon the neutron inertial mass. Although it does not imply that the equivalence principle of general relativity is violated in the quantum mechanical particles under gravity, the experiment does show that the quantum mechanical motions of different particle masses under gravitational field cannot be made geometrical in spacetime coordinate system. This is another fundamental difficulty in unifying quantum mechanics with general relativity. In connection to this, the author has proposed that the spacetime should be merged with matter together to become the space-time-matter space in formulating the theory of quantum gravity [6]. But we cannot simply write the geometrical line element *mds* as the action for a particle in quantum gravity because of the quantum mechanical nature of matter particles as well as the above mentioned quantum spacetime concept. In this connection, let us introduce the quantum space-time-matter state $|\Psi\rangle$ by combining the quantum state of particle and spacetime as

$$|\Psi\rangle = |\phi\rangle \left| \vec{x}, \Delta\ \vec{x} \right\rangle \tag{7}$$

As the quantum space-time-matter space combines the quantum states of the matter particle and the spacetime, it should therefore be complex in nature. The quantum space-time-matter space also allows different quantum spacetime for describing different quantum particle. Then, by introducing the equivalence principle of quantum gravity, we can make the quantum mechanics become a geometrical theory in quantum space-time-matter space. Under the equivalence principle of quantum gravity, all the physical laws apply to the observer frames of reference in any classical and quantum mechanical motions, which means there is no preferred observer frame of reference in any classical and quantum mechanical motions. The particle states and the equation of motion should therefore be equally good for any observer frame of references with the same physical laws. As the general change of observer frame of reference should be associated with the change of the spacetime state, this requires the general covariance of the physical laws under the transformation of the spacetime state in Eq. (4). In the general relativity, the physical observations in accelerating observer frames are associated with the generalised spacetime coordinate system. The general covariance on spacetime coordinate system allows us to choose a local observer frame which is an inertial frame such that there is no acceleration on the observed particles. This is what we call the "free fall frame". The gravitational field, which provides universal acceleration on all matter particles, corresponds to the curvature of spacetime. Analogously, the extension of the physical observations in quantum observer frames is associated with the generalised quantum spacetime states system. The extended general covariance on the quantum spacetime states system allows us to choose a local quantum observer frame such that there is no observed quantum motion on the observed particles. We may call this the "quantum free" frame. The quantum mechanical motions then correspond to the curvature of the quantum space-time-matter space.

The equivalence principle of quantum gravity and the quantum spacetime concept therefore allows the interesting physical results that the quantum mechanical motion of a particle depends upon the quantum mechanical motion of the observer frame. That means the quantum mechanical motion is relative in nature. This can be further explained by the simple argument as follows. Let us suppose that there is an observer A and, with reference to its frame of reference, the observer B and observe C are in the same quantum mechanical states. Then, the state of observer C as observed by the observer B should be the same as the state of observer B as observed by the observer C since their states are the same and therefore should be symmetrical to each other. Analogous to the relativity that the observer is assumed to be at rest relative to the coordinate system, if we assume that the observer C can always find an appropriate energy scale to establish a coordinate system with a specified uncertainty that there is no observable quantum effect between the spacetime coordinates in the coordinate system. We can say that the space-time coordinates are free of quantum effect (mathematically speaking, this means that the observer as observed itself is an identity). Therefore, due to the symmetry between observer B and C, the observer B should be in a "quantum free" state as observed by observer C in such "quantum free" coordinate system and vice versa. As a result, we can always establish a "quantum free" frame for a quantum particle by choosing the quantum frame of reference at the same state of the observed particle.

As the quantum mechanical motion of a particle and the quantum spacetime state depends upon their energy scales, or say the term $mc/\hbar$, which means the changing of quantum spacetime state could be associated with the change of energy scale. As we understand in the relativistic quantum mechanics, the normalisation factor, say $N$, of a quantum state is related to the energy scale so that any change of the energy scale should be associated with the change of $N$. That means the transformation in Eq. (4) between different quantum spacetime states is not necessary a unitary transformation, which preserves the total probability of quantum states, due to the change of normalisation in different quantum observer frames. Such change of the normalisation of the quantum states, no matter the particle state or the quantum spacetime state, mathematically acts like applying a conformal factor to the state as the conformal transformation. Furthermore, as the quantum spacetime states are in the complex domain and the particle density is related to the modulus of $N$, the most general transformation between different quantum spacetime states could also involve a complex phase factor $e^{i\delta}$ multiplying to the conformal factor. This phase factor could have important physical meaning to the CP-violation problem [5] but we will limit our discussions in this chapter without considering such phase factor. As the quantum spacetime state is allowed to be changed with the spacetime variables, the conformal transformation should be local in nature. Since the equivalence principle of quantum gravity requires that there is no preferred quantum spacetime state for observation, that means the equation of motion should then be invariant under the local conformal transformation and the quantum space-time-matter space should therefore possess the local conformal symmetry.

In some sense, such local conformal symmetry behaves as a kind of gauge symmetry on the quantum space-time-matter space, rather than only on the spacetime. The local conformal transformation in quantum space-time-matter space acts on the quantum space-time-matter metric whereas the conformal transformation in spacetime acts on the spacetime metric. This make the local conformal transformation in the quantum space-time-matter space something different from the usual conformal transformation in spacetime. In contrast with the general relativity, of which its equivalence principle implies the general covariance of system of spacetime co-ordinates for expressing the physical laws, the equivalence principle of quantum gravity implies the general covariance with scale change of the system of co-ordinates of space-time-matter space. The incorporation of the local conformal symmetry in the theory make it behaves as Weyl like geometry but now with the spacetime replaced by the quantum space-time-matter space.

Analogous to the general relativity, in constructing the geometry of general covariance for a curved quantum space-time-matter space, we may consider a general small line element on a small region of quantum space-time-matter space. Let us define the length of such line element $dL$ as the inner product of $\delta|\Psi\rangle$ in a small region of quantum space-time-matter space as

$$dL^2 = \delta|\Psi\rangle \cdot \delta|\Psi\rangle = \delta\left(|\phi\rangle\left|\vec{x}, \Delta\vec{x}\right\rangle\right) \cdot \delta\left(|\phi\rangle\left|\vec{x}, \Delta\ \vec{x}\right\rangle\right) \tag{8}$$

where the inner product can be defined as the usual inner product used for quantum mechanical states and thus $dL^2$ has a real value. In fact, it is reasonable to take real value for the inner product of the projection of a state onto itself. However, because of the local conformal symmetry, $dL^2$ should not be an invariant under the transformation in Eq. (4) due to the change of the observer frame of reference. Let us also introduce the operator $\widehat{\delta\vec{x}_{\Delta x}}$, which can extract the spacetime length from the quantum spacetime state, for small changes on spacetime state with the property as

$$\widehat{\delta\vec{x}_{\Delta x}}\delta\left|\vec{x}, \Delta\vec{x}\right\rangle = \delta\vec{x}_{\Delta x}\delta\left|\vec{x}, \Delta\vec{x}\right\rangle \tag{9}$$

Suppose that we do not consider any mixing between the quantum matter particle state and the quantum spacetime state, of which they respectively correspond to the object and observer, by any symmetry operation, we may define another line element $dl$ of quantum space-time-matter space, which combines the inner product of the quantum space-time-matter state with the extracted spacetime vector, as

$$dl^2 = \delta|\phi\rangle\widehat{\delta\vec{x}_{\Delta x}}\delta\left|\vec{x}, \Delta\ \vec{x}\right\rangle \cdot \delta|\phi\rangle\widehat{\delta\vec{x}_{\Delta x}}\delta\left|\vec{x}, \Delta\ \vec{x}\right\rangle \tag{10}$$

The element $dl^2$ is also real and can be expressed in generalised coordinates $\xi_m^\mu$ in the quantum space-time-matter space as

$$dl^2 = G_{\mu\nu}^{mn} d\xi_m^\mu d\xi_n^\nu \tag{11}$$

where $G_{\mu\nu}^{mn}$ is the combined metric of the quantum space-time-matter space and the extracted spacetime length and the index $\mu$ and $\nu$ are the usual spacetime index in general relativity while $m$ and $n$ are the index associated with the inner product of the quantum spacetime and matter states. We have assumed the expansion of the quantum space-time-matter states by the discrete eigenstates such that the discrete indices can be used above as analogous to the general relativity. But actually, expansion on continuous eigenstates can be used with the integration, rather than summation, as well in the expression without changing the formulation. If we formulate the curved quantum space-time-matter space with local conformal symmetry, let us define the local conformal operator $\Omega$ on the quantum space-time-matter state as

$$\delta|\Psi\rangle \to \Omega\delta|\Psi\rangle \tag{12}$$

As mentioned, we will not discuss the phase factor which may appear with the conformal operator in the transformation of the quantum spacetime. In view of the local conformal symmetry, we can make use of the Weyl geometry for the quantum space-time-matter space. The closest admissible action analogously to the action for Einstein equation under the local conformal symmetry would be taken as [8].

$$S = \int \mathfrak{R}^2 \sqrt{-G} d^n \xi \tag{13}$$

where $\mathfrak{R}$ is the curvature scalar defined for the quantum space-time-matter space in analogous to Weyl geometry and $n$ is the number of dimensions of the quantum space-time-matter space. This is a generalised action of quantum gravity in the formulation of the quantum space-time-matter space of which the generalised coordinates of spacetime and matter are combined together. Such action determines the combined scale of the matter state and the associated spacetime energy scale. The variation of the action gives

$$\delta S = \delta \int \mathfrak{R}^2 \sqrt{-G} d^n \xi = \int \left(2\mathfrak{R}\delta\mathfrak{R}\sqrt{-G} + \mathfrak{R}^2 \delta\sqrt{-G}\right) d^n \xi = 0 \tag{14}$$

As in Weyl geometry, the curvature scalar can be expressed as

$$\mathfrak{R} = \tilde{\mathfrak{R}} - (n-1)(n-2)\frac{(\partial\Phi)^2}{(\Phi)^2} + 2(n-1)\frac{1}{\sqrt{-G}}\left(\sqrt{-G}\frac{\partial^\alpha\Phi}{\Phi}\right)_{|\alpha} \tag{15}$$

where $\tilde{\mathfrak{R}}$ is the metric $G_{\mu\nu}^{mn}$ dependent part of the curvature scalar in Weyl geometry. The last term is the boundary term which can be neglected in the variation of the action. Actually, the $\Phi$ field is the scale of metric $G_{\mu\nu}^{mn}$ which determines the relative quantum effect of a particle as observed in a frame of reference in quantum mechanical motion. If we impose the gauge $\mathfrak{R} \sim 4\Lambda$ for fixing the scale of the quantum space-time-matter space, where $\Lambda$ is a constant, Eq. (13) becomes

$$S = \int \left( \tilde{\mathfrak{R}} - 2\Lambda - (n-1)(n-2)\frac{(\partial\Phi)^2}{(\Phi)^2} \right) \sqrt{-G} d^n \xi \tag{16}$$

In order to make the expression of the action simpler, as analogous to the Weyl geometry of spacetime, we can perform a local conformal transformation on the metric as $G^{mn}_{\mu\nu} \rightarrow \Phi^2 G^{mn}_{\mu\nu}$ to remove the $\Phi$ field in the action before imposing the gauge $\mathfrak{R} \sim 4\Lambda$. The action can be reduced to

$$S = \int \left( \tilde{\mathfrak{R}} - 2\Lambda \right) \sqrt{-G} d^n \xi \tag{17}$$

The equation shows that the term $\Lambda$, which determines the scale of the quantum space-time-matter space, behaves mathematically like the cosmological constant in general relativity but it is now in the quantum space-time-matter space rather and its physical meaning is different.

We can then connect the formulation of quantum space-time-matter space to the general relativity by further specifying the relationship between the spacetime element $ds$ in the general relativity and the quantum spacetime. Also, for simplicity, we will assume that the matter field is a scalar field. In fact, the argument can be extended to the fermionic field as well as the tetrad $e^I_\mu$ formalism of gravity [9, 10]. Firstly, let us write

$$\left\langle \delta\phi^2 ds^2 \right\rangle = \delta|\phi\rangle \widehat{\delta\vec{x}_{\Delta x}} \delta\left|\vec{x}, \Delta\ \vec{x}\right\rangle \cdot \delta|\phi\rangle \widehat{\delta\vec{x}_{\Delta x}} \delta\left|\vec{x}, \Delta\ \vec{x}\right\rangle \tag{18}$$

where $\left\langle \delta\phi^2 ds^2 \right\rangle$ is the combined inner product of the quantum space-time-matter state with the extracted spacetime vector for a small region of quantum space-time-matter space. It acts like the expectation value of quantum probability weighted spacetime and matter element and then Eq. (10) above becomes

$$dl^2 = \left\langle \delta\phi^2 ds^2 \right\rangle \tag{19}$$

If we consider the special case that the quantum-space-time-matter space is a linear space, which is the metric $G^{mn}_{\mu\nu}$ does not vary with the associated generalised coordinates $\xi_m$ of the quantum space-time-matter space, the above expression can be simplified as

$$dl^2 = \left\langle \phi^2 ds^2 \right\rangle \tag{20}$$

Actually, we can express the curved quantum spacetime in generalised coordinates analogous to Weyl geometry as

$$\left\langle \phi^2 ds^2 \right\rangle = \left\langle \phi^2 \overline{g}(\Delta x)_{\mu\nu} \right\rangle dx^\mu dx^\nu \tag{21}$$

where $\left\langle \phi^2 \overline{g}(\Delta x)_{\mu\nu} \right\rangle$ is the quantum probability weighted spacetime metric, which is specified with measurement uncertainty, combined with the matter field. If the quantum spacetime state is the eigenstate of the $\overline{g}(\Delta x)_{\mu\nu}$, that is the observer frame and observation energy scale for the quantum spacetime state is the same as the spacetime metric specified with measurement uncertainty, Eq. (20) can be written as

$$dl^2 = \left\langle \phi^2 \overline{g}(\Delta x)_{\mu\nu} \right\rangle dx^\mu dx^\nu = \left\langle \phi^2 \right\rangle \overline{g}(\Delta x)_{\mu\nu} dx^\mu dx^\nu \tag{22}$$

As we assume that the quantum space-time-matter space is a linear space, the metric $G_{\mu\nu}^{mn}$ does not vary with $\xi_m$ and it is not necessary to consider the variation of the $\xi_m$ coordinates in the action in Eq. (17). The Weyl geometry of the quantum space-time-matter space can be reduced to that only the curvature on the spacetime metric is described. The number of dimension $n$ of the action in Eq. (17) can therefore be reduced to four. Although the equation resembles the Einstein action with the cosmological constant in general relativity, it actually describes the dynamics of the quantum space-time-matter space rather than simply the spacetime. The dependence on the quantum probability and uncertainty parameter of $\left\langle \phi^2 \overline{g}(\Delta x)_{\mu\nu} \right\rangle$ provides a freedom for the spacetime metric to change scale with the matter field. For instance, when we need to probe into the microscopic scale by using high energy scale observation, the metric will change due to the uncertainty required changes. This let us to connect the metric in macroscopic scale with the microscopic scale observation. Physically, it means that when the spacetime observation is performed by quantum mechanical devices, the probability of observation of existence of spacetime state should be taken into account in the metric of quantum spacetime. This echoes with the mentioned requirements of introduction of the quantum uncertainty into the spacetime states when we need to consider the quantum mechanical nature of measuring-rods and clock to bring classical general relativity closer to quantum mechanics. One may find that when the probability of quantum spacetime metric tends to unity, $\overline{g}(\Delta x)_{\mu\nu} = g_{\mu\nu}$, which is the metric of the general relativity. For simplicity, let us write $\overline{g}(\Delta x)_{\mu\nu}$ as $\overline{g}_{\mu\nu}$. Then, the action in Eq. (17) now can be expressed in terms of $\overline{g}_{\mu\nu}$ and $\phi$ as

$$S = \int \left( \phi^2 \overline{R} - 2\Lambda\phi^4 + 6(\partial\phi)^2 \right) \sqrt{-\overline{g}} d^4x \tag{23}$$

The action behaves like the conformal coupling of a scalar field to the gravitation field. Actually, we can divide the action by the factor 1/12 to align the term $(\partial\phi)^2$ as the kinetic term of the $\phi$ field and gives

$$S' = \int \left( \frac{1}{12}\phi^2 \overline{R} - \frac{1}{6}\Lambda\phi^4 + \frac{1}{2}(\partial\phi)^2 \right) \sqrt{-\overline{g}} d^4x \tag{24}$$

If the Ricci scalar $\overline{R}$ of the spacetime is a constant, we will show that it relates to the mass term of the scalar matter field. Applying the spontaneous symmetry breaking on the $\phi$ field of such Lagrangian, the minimum of the potential gives the relation

$$\frac{\overline{R}}{4\Lambda} = \phi_0^2 \tag{25}$$

We later will know that selecting the value of $\phi_0$ determines the relative scale between the matter field and spacetime metric. In fact, when selecting the scale in the variation of the Weyl action in Eq. (13), the local conformal symmetry of the quantum space-time-matter space is broken and the combined scale between the matter density and the four dimensional spacetime energy scale is fixed. But it does not mean that the local conformal symmetry on the spacetime and matter field which relates to the changing energy scale of observation is broken and actually, it is not. Let us define the conformal operator $\omega$, which is associated with the change of scale of observation, in the spacetime on the metric and scalar field as

$$\bar{g}_{\mu\nu} \to \omega^2 \bar{g}_{\mu\nu} \tag{26}$$

and

$$\phi \to \omega^{-1}\phi \tag{27}$$

As mentioned, we will not discuss the phase factor which may appear with the conformal operator in the transformation of the quantum spacetime. We may find that the conformal transformation, which is the change of scale of observation, in spacetime does not change the action in Eq. (24), which is a conformal invariant. Actually, the change of scale of observation should preserve the uncertainty relation $\Delta x \Delta p \sim \hbar$, since such change of scale is governed by the uncertainty relation. The local conformal symmetry of spacetime is broken when the scale of observation is fixed. This is the case when we apply the spontaneous symmetry breaking condition in Eq. (25) to the action. One may be aware that we need to fix two degree of freedoms in determining the scale of the whole theory in quantum gravity. This point is very important since we will find that we can change the scale in one space and then compensate by the other to make it invariant. This property let the quantum gravity possess an interesting double conformal structure. Actually, the selection of scale in the variation of the Weyl action determines the scale of $\Lambda$. Then with the relationship of the spontaneous symmetry breaking in Eq. (25), all three parameters $\overline{R}$, $\Lambda$ and $\phi_0$ can be fixed so as the whole scale of the observer and matter space. Actually, if the electroweak energy density scale is chosen for $\Lambda\phi^4$ or $\overline{R}\phi^2$ in the variation of the Weyl action, the equation becomes a Higgs potential like Lagrangian. That means it is possible to interpret the Higgs potential as the broken Weyl action of the quantum space-time-matter space with an observation scale dependent metric. It provides the physical explanation of the Higgs potential and its relationship with quantum gravity. Furthermore, the interesting thing is that even the energy density scale is fixed at electroweak scale, we still have the freedom to choose the scale of observation by the said conformal transformation on spacetime. This will lead to the relationship between the cosmological constant and Higgs potential as well as the fundamental particle masses as discussed in the next section.

## 3. Fundamental particle masses

If we consider that the conformal symmetry is spontaneously broken in the action of Eq. (24), just as breaking the gauge symmetry in Higgs mechanism, we may consider the shift field $h$ around the minimum of the potential as $\phi = \phi_0$. If we first consider the symmetry breaking at an energy scale $\phi_0^2 = 6m'^2$ related to the fundamental interaction, say electroweak scale, the equation becomes

$$S = \int \left( \frac{1}{2} m'^2 \overline{R} - 3\Lambda m'^4 - \frac{1}{2}(\partial h)^2 + \frac{1}{2}\overline{R}h^2 + \dots \right) \sqrt{-\bar{g}}\, d^4x \tag{28}$$

If we now change the scale of observation by applying conformal transformation on the spacetime as

$$\bar{g}_{\mu\nu} \to \left( \frac{m'}{M_p} \right)^2 \bar{g}_{\mu\nu} \tag{29}$$

and

$$\phi \rightarrow \left(\frac{m'}{M_p}\right)^{-1}\phi \tag{30}$$

where the mass scale $M_p$ is reduced Planck mass. Then we can write the equation as

$$S = \int \left(\frac{1}{2}M_p^2\overline{R'} - 3\Lambda M_p^4 - \frac{1}{2}\left(\partial h'\right)^2 + \frac{1}{2}\overline{R'}h'^2 + \dots\right)\sqrt{-\overline{g}}d^4x \tag{31}$$

We have to note that the Planck mass is an energy scale introduced here to the Lagrangian through the relative ratio with $m'$ in the conformal transformation. As a result, the action become the gravitational action with cosmological constant and the excitation field become the matter field that coupled to the gravitational field and the square of Planck mass become the coupling constant between matter and gravity. The term $\overline{R'}h'^2$ acts as the mass term of the excitation field $h$. Unexpectedly, it also indicates that the spontaneous symmetry breaking on electroweak energy scale is associated with the symmetry breaking that generate the gravitational action. Since $\overline{R'}$ is the conformal transformation of the related to the vacuum energy of spacetime in empty space and since $\overline{R}\sqrt{-\overline{g}}$ should be transformed as $\overline{g}$ and therefore $\overline{R'}$ should be equal to $\left(m'/M_p\right)^2\overline{R}$ which is the particle mass square term. This implies the existence of a particle with mass $\left(m'/M_p\right)\overline{R}^{\frac{1}{2}}$. The calculation results mean that the particle mass of the matter field is related to the vacuum energy of the spacetime, which is the cosmological constant.

On the other hand, due to the double conformal symmetry structure of the quantum space-time-matter space, we may change the observer frame of reference for observing such particle and this will lead to the change on the broken scale of the quantum space-time-matter space. Recalling that the scale of the quantum space-time-matter space is fixed by the gauge $\mathfrak{R} \sim 4\Lambda$ and let the factor for the associated scale change is $\Omega^4$, the scale of $\Lambda$ is therefore changed as

$$\Lambda \rightarrow \Omega^4\Lambda \tag{32}$$

The scale change of $\Lambda$ implies that the spontaneous symmetry breaking condition in Eq. (25) requires to be varied so that the mass value $\left(m'/M_p\right)\overline{R}^{\frac{1}{2}}$ in Eq. (31) of the said particle will be changed also. However, we still have not established the reference for the values of mass and energy and, actually, mass value is just the mass ratio relative to the mass standard reference which could fundamentally be the Planck mass $M_p$. Recalling that, in Eq. (31), the Planck mass is introduced only through the conformal transformation of spacetime. That means, in changing of frame of reference, the Planck mass is required to be re-determined in the new observer frame of reference. The freedom of choosing the observation scale by conformal transformation on the spacetime and matter field in the new frame of reference let us have the freedom to do the transformation such that the mass value as reference to the mass standard reference of a particle remained unchanged. This point is important in the equivalence principle of quantum gravity since it ensures that there is no preferred observer frame of reference such that the observer is not able to distinguish the nature of the frames of reference that he is sitting in for making his observations and measurements. As the mass value is determined by the ratio $\overline{R}\sqrt{-\overline{g}}/\phi^2$, for the mass remains unchanged in the new frame, we need to have

$$\frac{\overline{R}\sqrt{-\overline{g}}}{\phi^2} = \frac{m'}{M_p} = unchanged \tag{33}$$

As under the change of scale in Eq. (31), the term $\overline{R}\sqrt{-\overline{g}}$ will be transformed as

$$\frac{\overline{R}\sqrt{-\overline{g}}}{\phi^2} \rightarrow \frac{\Omega^4\overline{R}\sqrt{-\overline{g}}}{\phi^2} \tag{34}$$

and the conformal transformation $\omega$ on the spacetime and matter field induce the transformation as

$$\frac{\Omega^4\overline{R}\sqrt{-\overline{g}}}{\phi^2} \rightarrow \frac{\omega^4\Omega^4\overline{R}\sqrt{-\overline{g}}}{\phi^2} \tag{35}$$

Therefore, the unchanged mass ratio requires that $\omega = 1/\Omega$. Once the particle mass value remains unchanged in the transformed frame and since the observer is not able to distinguish the frames of reference that he makes the observations and measurements, in order to explore the structure of the quantum space-time-matter space under broken symmetry, we can repeatedly perform the above processes of changing observer frame such that in every changed frame the mass of the particle are still equal to $m'/M_p$, although the $M_p$ varies with the transformed frame of reference. We can therefore find that a set of scale factors is allowed for the quantum space-time-matter space scale

$$(\Omega)^{2n} = \left(\frac{m'}{M_p}\right)^2 \Rightarrow \Omega = \left(\frac{m'}{M_p}\right)^{\frac{1}{n}} \tag{36}$$

where $n$ is the number of the said transformation to achieve the mass factor $m'/M_p$ in a frame. The relation means that, for observing a particle with mass factor $m'/M_p$ in a frame, it is possible that it can be due to a number of conformal transformations on the quantum space-time-matter space to bring to the observed mass value in a frame of reference. That means, in an observer frame of reference, there is a set of possible scale factors $\Omega$ for the broken scale of quantum space-time-matter space. As in an observer frame of reference the Planck mass is fixed, the set of possible broken scales of quantum space-time-matter space implies that there is a set of possible mass states with the mass factors as in Eq. (36). Suppose $m'/M_p \sim 10^{-15}$, that is $m' = 2.435\,TeV$, a set of mass factors can be found for different values of $n$, for instance $n$ = 1 to 5, as follows

$$\Omega_{m1} = \frac{m'}{M_p} = \frac{2.435\,GeV}{2.435 \times 10^{18}\,GeV} = 10^{-15} \tag{37}$$

$$\Omega_{m2} = \left(\frac{m'}{M_p}\right)^{\frac{1}{2}} = \left(\frac{2.435\,GeV}{2.435 \times 10^{18}\,GeV}\right)^{\frac{1}{2}} = 3.16 \times 10^{-8} \tag{38}$$

$$\Omega_{m3} = \left(\frac{m'}{M_p}\right)^{\frac{1}{3}} = \left(\frac{2,435\,GeV}{2.435 \times 10^{18}\,GeV}\right)^{\frac{1}{3}} = 1.0 \times 10^{-5} \tag{39}$$

$$\Omega_{m4} = \left(\frac{m'}{M_p}\right)^{\frac{1}{4}} = \left(\frac{2,435\,GeV}{2.435 \times 10^{18}\,GeV}\right)^{\frac{1}{3}} = 1.778 \times 10^{-4} \tag{40}$$

$$\Omega_{m5} = \left(\frac{m'}{M_p}\right)^{\frac{1}{5}} = \left(\frac{2,435\,GeV}{2.435 \times 10^{18}\,GeV}\right)^{\frac{1}{5}} = 1 \times 10^{-3} \tag{41}$$

As mentioned above, since the particle mass $m_i$ is equal to $\Omega_{mi}\overline{R}^{\frac{1}{2}}$, if $\overline{R}^{\frac{1}{2}} = 10\,TeV$, the following mass values $m_i$ can be found and when compared with the lepton masses data of the Particle Data Group [11]:

$$m_1 = 10^{-15}\overline{R'}^{\frac{1}{2}} = 10^{-15} \times 10\,TeV = 0.01\,eV \sim m_\nu? \tag{42}$$

$$m_2 = 3.16 \times 10^{-8}\overline{R'}^{\frac{1}{2}} = 3.16 \times 10^{-8} \times 10\,TeV = 316.23\,keV \sim m_e \tag{43}$$

$$m_3 = 1.0 \times 10^{-5}\overline{R'}^{\frac{1}{2}} = 1.0 \times 10^{-5} \times 10\,TeV = 100\,MeV \sim m_\mu \tag{44}$$

$$m_4 = 1.778 \times 10^{-4}\overline{R'}^{\frac{1}{2}} = 1.778 \times 10^{-4} \times 10\,TeV = 1.778\,GeV \sim m_\tau \tag{45}$$

$$m_5 = 1 \times 10^{-3}\overline{R'}^{\frac{1}{2}} = 1 \times 10^{-3} \times 10\,TeV = 10\,GeV \tag{46}$$

where $m_\nu$, $m_e$, $m_\mu$ and $m_\tau$ are respectively the mass of neutrinos, electron, muon and tau lepton. Although the mass values of the three neutrino mass states are so far not experimentally determined, the mass value of $m_1$ state is consistent with the cosmological constraint on the sum of the neutrino masses. The actual neutrino masses might not be a necessary degenerate as the radiative correction of different neutrino mass states is not considered in the above calculation and that would lead to small differences between the actual neutrino masses and the calculated mass state value. For $m_2$ state, it is of the same order of magnitude of the electron mass. The difference between them is about 38% and could be due to the radiative correction to the QED vacuum. When the value of the mass states become relatively large, we find that the mass of $m_3$ state is equal to the experimental mass value of muon up to about 5% and the $m_4$ state is even equal to the experimental mass value of tau lepton up to about 0.06%. For the mass states with $n>4$, there are many so far not experimentally observed particle states. They could be the dark matter particles or the mass resonance states. For instances, the mass value of $m_5$ state is consistent with the dark matter candidate with a mass of 10 GeV as proposed by some researchers [12].

In fact, the above argument can be extended to the case that $n$ is allowed to be half-integer, in the formulation of the gravitational fields by tetrads $e^I_\mu$ for considering its coupling with fermions. In such case, we can find more mass states which are associated with half integer $n$ values, for instance, $n = 1.5$, $n = 2.5$ etc. For $n = 4.5$ and $n = 8.5$, we can find that the mass value are respectively $m_{4.5} = 4.64GeV$ and $m_{8.5} = 171.9GeV$. The values are very close to the mass values of bottom and top quarks. That means the half integer values of n may somehow correspond to the quark masses. However, it is possible that, due to the QCD vacuum, the calculated values of the quark masses which are comparable with the QCD vacuum energy would have greater discrepancy with the actual current quark masses.

## 4. Cosmological constant

As we explained before, Eqs. (24) and (31) is related by the conformal transformation of spacetime and the matter field and spontaneous symmetry breaking which is determined by the observation scale. Actually, Eq. (24) resembles the following action of the Higgs potential when $\overline{R'}$ is a constant in one hand

$$S' = \int \left(\frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4 + \frac{1}{2}(\partial\phi)^2\right)\sqrt{-\overline{g}}d^4x \tag{47}$$

On the other hand, as shown above, it can become the action of gravitational field coupled to matter field after the local conformal transformation and spontaneous symmetry breaking as in Eq. (31). By comparing Eqs. (24) and (47), we can relate the coefficients between them as follows

$$\frac{\overline{R}}{12} = \frac{\mu^2}{2} \tag{48}$$

$$\frac{\Lambda}{6} = \frac{\lambda}{4} \tag{49}$$

Furthermore, by comparing Eq. (31) with the gravitational action, we can find that

$$3\Lambda M_p^2 = \Lambda_{cc} \tag{50}$$

where $\Lambda_{cc}$ is the cosmological constant. By eliminating $\Lambda$ in the above equations, we get

$$\Lambda_{cc} = 4.5\lambda M_p^2 \tag{51}$$

The calculated cosmological constant is of the order of Planck scale. It is due to the fact that the observation scale is in the microscopic scale or say high energy scale of which the metric $\overline{g'}_{\mu\nu}$ is quantum mechanically uncertain in nature. Thus, it is not the scale of our cosmological observation. Actually, we can apply a conformal transformation on the quantum space-time-matter space metric to change the observer frame of reference which is under the same factor as the one in obtaining the fundamental particle masses but with $n = 2$ as

$$\Omega = \left(\frac{m'}{M_p}\right)^2 \tag{52}$$

Since the cosmological constant transforms as the square of the metric, that is conformal factor $\Omega^4$, with respect to the conformal factor of $\Omega^2$ on the metric, the cosmological constant value becomes

$$\Lambda_{cc} = 4.5\lambda \frac{m'^4}{M_p^2}\left(\frac{m'}{M_p}\right)^4 \tag{53}$$

By putting $\lambda = 0.258$, which is based on the 125 GeV Higgs particle mass and 246 GeV electroweak VEV value,

$$\Lambda_{cc} = 6.49 \times 10^{-66}\, eV^2 \tag{54}$$

This calculated cosmological constant value is in very good agreement with the observation value of $4.33 \times 10^{-66}\, eV^2$ of Planck CMB probe with just about 50% difference when connecting to the fundamental particle masses.

## 5. Discussion and conclusions

We have formulated the quantum space-time-matter geometry with the equivalence principle of quantum gravity for discussing the relationship between the cosmological constant and quantum gravity as well as the mass spectrum of fundamental particles. Because of the freedoms allowed for changing the quantum observer frames under the equivalence principle of quantum gravity and the energy scale specific observation required under the quantum properties of the spacetime measuring devices, the quantum space-time-matter space possesses a double conformal symmetry geometrical structure. In such structure, we need two parameters to determine the scale of quantum space-time-matter space so as to describe the physical world.

In our calculation, it is fascinating that, once such parameters are determined, the associated conformal factor $\Omega = \left(\frac{m'}{M_p}\right)$ with different exponent $n$ values gives the fundamental particle mass values and the observed cosmological constant value with very good agreement. It indicates that the fundamental particle masses is connected to the cosmological constant and some fundamental physical meaning are behind such factor, particularly the meaning of the value of 2.435 TeV. It also demonstrates that the quantum space-time-matter space has a complicated conformal structure which is related to the fundamental particles when applying the equivalence principle of quantum gravity to it.

Actually, we can extend the theory by incorporating the gauge symmetry of the fundamental interactions to it so as to discuss the relationship between the mass spectrum of fundamental particles and the standard model under the conformal nature of quantum gravity. As indicated in our calculation, it can be anticipated that the gauge symmetry operation for the flavour states of lepton would in general not be commutable with the conformal symmetry operation of the mass states. The mass states are under a conformal symmetry of the quantum space-time-matter space whereas the flavour states are under the gauge symmetry. This is consistent with the fact that the neutrino flavour eigenstates are not the same as their mass states. Our formulation indicates that the mass states can even be changed from one to another when observing under different observer frame of reference in quantum mechanical motions. As we can see, the underlying symmetry that associated with particle masses would require the combination of the local conformal symmetry with the gauge symmetry into the local conformal gauge symmetry.

Furthermore, the above mass formula allows the existence of some mass states that are so far not experimentally observed. This might provide new opportunities for discussing whether such mass states are related to the dark matter as well as the possibilities of discovering such particles experimentally, although some of the mass states may be forbidden by some so far unknown physical rules, just as the forbidden rules of atomic spectra.

One of the key underlying physical meanings of the above calculation is that there is no absolute existence of spacetime. Similar to the quantum nature of the matter fields, the existence of spacetime is probabilistic in nature. And it is due to such reason, the vacuum energy can be very small in macroscopic scale but very large in microscopic scale. Actually, there is an underlying fundamental symmetry between the quantum spacetime states and quantum matter states that we may call it quantum spacetime matter symmetry, or object-observer symmetry. In fact, an observer could be an object to another observer or vice versa and this actually is the essence of the principle of relativity.

Finally, it is expected that, given that the equivalence principle of quantum gravity and the energy scale dependent quantum spacetime concept are introduced under the double conformal symmetry of the quantum space-time-matter geometry, it is reasonable that other mathematical formalism can be used as well to arrive at the same conclusions. The mathematical treatment is not necessary restricted to the approach introduced in this book chapter.

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