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# Guaranteed Performance Consensus for Multi-Agent Systems

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## Abstract

The guaranteed performance of the consensus control for multi-agent systems with Lipschitz nonlinear dynamics and directed interaction topologies is investigated, where the directed interaction topology contains a spanning tree. By a special matrix transformation, guaranteed performance consensus problems are transferred into guaranteed performance stabilization problems. Then, the criteria of guaranteed performance consensus for nonlinear multi-agent systems with directed interaction topologies are obtained, and an upper bound of the introduced performance function is given. A numerical simulation is given to demonstrate the effectiveness of the proposed results. Finally, some possible topics about the guaranteed performance consensus problem for nonlinear multi-agent systems are proposed.

**Keywords:** guaranteed performance consensus, multi-agent system, nonlinear dynamic

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## 1. Introduction

In the past decades, many researchers focused on consensus problems for multi-agent systems due to their wide applications, including formation control of mobile agents [1], synchronization in wireless sensor networks [2], distributed automatic generation control for cyber-physical micro-grid system [3], and rendezvous [4] or flocking [5] of multiple vehicles. After Olfati-Saber and Murray [6] proposed a theoretical framework for the consensus problem, a lot of remarkable conclusions for linear multi-agent systems were presented in the literature, respectively (see the survey papers [7–12] and the references therein). In fact, many control systems in practical applications are nonlinear. Consensus problems for multi-agent systems with nonlinear dynamic have been investigated in existing works. It should be pointed out that

existing works about nonlinear consensus problem focused on the consensus condition under a control protocol, but the consensus regulation performance was not considered by a performance index.

With the development of the consensus control theory, the guaranteed performance consensus for the multi-agent by the guaranteed performance control approach has received more and more attentions. In the guaranteed performance consensus problems, the consensus regulation performance was explicitly considered by the guaranteed performance function. By the constraint of the performance index, the consensus control can be seen as an optimal or suboptimal problem, and the control process is more affected by choosing appropriate control parameters. In existing literatures about the guaranteed performance consensus problem, such as [11–15], the dynamic characteristic of each agent in the multi-agent systems was linear. For the linear multi-agent systems, the state-space decomposition approach was widely used to decompose the consensus and disagreement dynamics of multi-agent system, and the disagreement dynamics is the key of guaranteed performance consensus control. Moreover, the guaranteed performance consensus with other control methods has been studied, such as sampled-data control [16], fault-tolerant control [17], event-triggered control [18], tracking control [19], and impulsive control [20].

For the consensus problems of nonlinear multi-agent systems, the intercoupling relationship between the consensus and disagreement dynamics because of the nonlinear dynamic. Then, the state-space decomposition approach is not able to deal with the nonlinear consensus problems. To the best of our knowledge, there are very few research works about the guaranteed performance consensus for nonlinear multi-agent systems. Moreover, the interaction topologies in most of the existing works were undirected, and there were few works about guaranteed performance consensus problem with directed interaction topologies.

In the current chapter, the guaranteed performance consensus for multi-agent systems with nonlinear dynamics is studied by introducing a performance function. By a special matrix transformation, the guaranteed performance consensus problems are transferred into guaranteed performance stabilization problems, and some conclusions about guaranteed performance consensus for nonlinear multi-agent systems are obtained.

## 2. Preliminaries and problem descriptions

In the current paper, the interaction topology among all agents of multi-agent systems can be modeled by a directed graph  $G = (\mathcal{V}, \mathcal{E})$ , where  $\mathcal{V} = \{1, 2, \dots, N\}$  and  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  represent the agent set and the directed edge set, respectively. A directed edge in a directed graph denoted as  $(i, j)$  which means that agent  $i$  can obtain information from agent  $j$ . Define  $\mathcal{W} = [w_{ij}] \in \mathbf{R}^{N \times N}$  with  $w_{ij} \geq 0$  and  $w_{ii} = 0$  is the adjacency matrix, where  $w_{ij} > 0$  if there is a directed edge between agent  $i$  and agent  $j$ ,  $w_{ij} = 0$  otherwise. If  $w_{ij} > 0$ , agent  $j$  is called the neighbor of agent  $i$ , and all the neighbors of agent  $i$  consist of the neighboring set of agent  $i$ . A directed path is a sequence of ordered edges of the form  $(i_1, i_2), (i_2, i_3), \dots$ . The Laplacian

matrix of the interaction topology  $G$  is defined as  $L = [l_{ij}] \in \mathbf{R}^{N \times N}$ , where  $l_{ii} = \sum_{j \neq i} w_{ij}$  and  $l_{ij} = -w_{ij}$ . In the current paper, it is assumed that the directed graph  $G$  has a directed spanning tree. The following lemma shows basic properties of the Laplacian matrix of a directed interaction graph.

*Lemma 1* [21]: Let  $L = [l_{ij}] \in \mathbf{R}^{N \times N}$  be the Laplacian matrix of a directed interaction graph  $G$ , then (i)  $L$  at least has a zero eigenvalue, and  $\mathbf{1}_N$  is an associated eigenvector; that is,  $L\mathbf{1}_N = 0$ . (ii) 0 is a simple eigenvalue of  $L$ , and all the other nonzero eigenvalues have positive real parts if and only if  $G$  has a directed spanning tree, i.e.,  $0 = \lambda_1 < \text{Re}(\lambda_2) \leq \dots \leq \text{Re}(\lambda_N)$ .

In the current chapter, consider the guaranteed performance consensus problem of a group of  $N$  nonlinear agents with the dynamics of  $i$ th agent given by

$$\dot{x}_i(t) = Ax_i(t) + Mf(x_i(t)) + Bu_i(t), \quad (1)$$

where  $i = 1, 2, \dots, N$ ,  $A \in \mathbf{R}^{d \times d}$ ,  $B \in \mathbf{R}^{d \times p}$ ,  $M \in \mathbf{R}^{d \times m}$  are constant real matrices,  $x_i(t)$  and  $u_i(t)$  are the state and the control input of the  $i$ th agent, respectively, and the function  $f(\cdot) : \mathbf{R}^n \times [0, +\infty) \rightarrow \mathbf{R}^m$  is a continuously differentiable vector-valued function representing the nonlinear dynamics of the  $i$ th agent which satisfies the following condition:

$$\|f(\xi_1(t)) - f(\xi_2(t))\| \leq \alpha \|\xi_1(t) - \xi_2(t)\|, \quad (2)$$

where  $\forall \xi_1(t), \xi_2(t) \in \mathbf{R}^d$ ,  $t \geq 0$  and  $\alpha > 0$  is a constant scalar. In the current paper, a directed graph  $G$  is used to describe the interaction. Based on local relative state information of the neighbor agents, consider the following distributed consensus protocol:

$$u_i(t) = K \sum_{j=1}^N w_{ij} (x_j(t) - x_i(t)), \quad (3)$$

where  $w_{ij}$  is the weight of the edge between agents  $j$  and  $i$ . Let  $x(t) = [x_1^T(t), x_2^T(t), \dots, x_N^T(t)]^T$  and  $g(x(t)) = [f(x_1(t))^T, f(x_2(t))^T, \dots, f(x_N(t))^T]^T$ , and then multi-agent system Eq. (1) with consensus protocol Eq. (3) can be rewritten in a vector form as

$$\dot{x}(t) = ((I_N \otimes A) - (L \otimes BK))x(t) + (I_N \otimes M)g(x(t)). \quad (4)$$

Define  $\varepsilon(t) = [\varepsilon_1^T(t), \varepsilon_2^T(t), \dots, \varepsilon_{N-1}^T(t)]^T$  with  $\varepsilon_i(t) = x_i(t) - x_{i+1}(t)$ , then existing matrix

$$H = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & -1 & 0 \\ 0 & 0 & 0 & \dots & 1 & -1 \end{bmatrix} \in \mathbf{R}^{(N-1) \times N},$$

such that  $\varepsilon(t) = (H \otimes I_d)x(t)$ . Thus, a performance function can be defined as follows:

$$J_C = \sum_{i=1}^{N-1} \int_0^{+\infty} \varepsilon_i^T(t) Q \varepsilon_i(t) dt, \quad (5)$$

where  $Q$  is a given symmetric positive matrix with appropriate dimension. The following lemmas are introduced:

*Lemma 2* [22]: For a Laplacian matrix  $L \in \mathbf{R}^{N \times N}$  of the graph  $G$  and the matrix  $H$ , there exists a matrix  $U \in \mathbf{R}^{N \times (N-1)}$  such that  $L = UH$ .

*Lemma 3* [22]: If  $G$  has a directed spanning tree and  $U$  is full column rank, the real part of the eigenvalue of the matrix  $HU$  satisfies  $\text{Re}(\lambda(HU)) > 0$ , and there exist a symmetric and positive definite matrix  $W \in \mathbf{R}^{(N-1) \times (N-1)}$  and a positive scalar  $\gamma$  such that

$$(HU)^T W + WHU > \gamma W,$$

where  $0 < \gamma < 2\min\{\text{Re}(\lambda(HU))\}$ .

*Lemma 4* [23]: For any given  $\forall \xi_1(t), \xi_2(t) \in \mathbf{R}^d$  and matrices  $S$  of appropriate dimensions, one has

$$2\xi_1^T(t) S \xi_2(t) \leq \xi_1^T(t) \xi_1(t) + \xi_2^T(t) S^T S \xi_2(t).$$

In the sequel, the definitions of the guaranteed performance consensus and consensualization are given, respectively.

*Definition 1*: Multi-agent system (Eq. (4)) is said to achieve guaranteed performance consensus with the performance function (Eq. (5)) if  $\lim_{t \rightarrow +\infty} (x_j(t) - x_i(t)) = 0$ , and there exists a scalar  $J_C^* > 0$  such that  $J_C \leq J_C^*$ , where  $J_C^*$  is said to be an upper bound of the performance function (Eq. (5)).

*Definition 2*: Multi-agent system (Eq. (1)) is said to be guaranteed performance consensualizable by the consensus protocol (Eq. (2)) with the performance function (Eq. (5)), if there exists a gain matrix  $K$  such that Eq. (4) achieves guaranteed performance consensus.

Due to the definition  $\varepsilon_i(t) = x_i(t) - x_{i+1}(t)$ , one has  $\lim_{t \rightarrow +\infty} \varepsilon(t) = 0$  if and only if  $\lim_{t \rightarrow +\infty} (x_i(t) - x_{i+1}(t)) = 0$  with  $i = 1, 2, \dots, N-1$ ; that is,  $x_1(t) = x_2(t) = \dots = x_N(t)$  when  $t \rightarrow +\infty$ . Then, it is obtained that  $\varepsilon(t)$  is the disagreement vector of multi-agent system (Eq. (4)). Thus,  $J_C$  in Eq. (5) represents the guaranteed performance of the consensus control for multi-agent system (Eq. (4)).

### 3. Analysis of guaranteed performance consensus

By  $\varepsilon(t) = (H \otimes I_d)x(t)$ , multi-agent system (Eq. (4)) can be transformed into

$$\dot{\varepsilon}(t) = ((I_{N-1} \otimes A) - (HU \otimes BK))\varepsilon(t) + (I_{N-1} \otimes M)\zeta(x(t)), \quad (6)$$

where

$$\zeta(x(t)) = (H \otimes I_d)g(x(t)) = \begin{bmatrix} f(x_1(t)) - f(x_2(t)) \\ f(x_2(t)) - f(x_3(t)) \\ \vdots \\ f(x_{N-1}(t)) - f(x_N(t)) \end{bmatrix} \quad (7)$$

and the matrix  $U$  satisfies Lemmas 2 and 3. Moreover, from Eq. (3), one has  $u(t) = -(L \otimes K)x(t)$  with  $u(t) = [u_1^T(t), u_2^T(t), \dots, u_N^T(t)]^T$ . By  $\varepsilon(t) = (H \otimes I_d)x(t)$  and Lemma 2, one has

$$u(t) = -(U \otimes K)\varepsilon(t) \quad (8)$$

For  $J_C$ , the performance function (Eq. (5)) can be rewritten in a vector form as

$$J_C = \int_0^{+\infty} \varepsilon^T(t)(I_{N-1} \otimes Q)\varepsilon(t)dt. \quad (9)$$

The following result presents a sufficient condition for nonlinear multi-agent system (Eq. (4)) to achieve guaranteed performance consensus and designs the consensus control gain matrix for the distributed consensus protocol (Eq. (3)).

*Theorem 1:* Assume that  $G$  has a directed spanning tree. Nonlinear multi-agent system (Eq. (4)) achieves guaranteed performance consensus if there exists a symmetric and positive definite matrix  $P \in \mathbb{R}^{d \times d}$  such that  $\Xi < 0$ , where

$$\Xi = \begin{bmatrix} \Xi_{11} & PM & \alpha I_d & Q \\ * & -\omega_{\max}^{-1} I_d & 0 & 0 \\ * & * & -\omega_{\min} I_d & 0 \\ * & * & * & -\omega_{\min} Q \end{bmatrix}$$

with

$$\begin{aligned} \Xi_{11} &= A^T P + PA - \gamma P B B^T P, \\ \omega_{\min} &= \min\{\lambda_i(W), i = 1, 2, \dots, N-1\}, \\ \omega_{\max} &= \max\{\lambda_i(W), i = 1, 2, \dots, N-1\}. \end{aligned}$$

In this case, for the distributed consensus protocol (Eq. (3)), the consensus control gain matrix  $K = B^T P$ .

*Proof:* Consider the following Lyapunov functional candidate:

$$V(t) = \varepsilon^T(t)(W \otimes P)\varepsilon(t), \quad (10)$$

where  $P \in \mathbb{R}^{d \times d}$  is a symmetric and positive definite matrix and  $W$  satisfies Lemma 3. Then, the time derivation of the Lyapunov functional candidate  $V(t)$  along the trajectory of Eq. (6) is

$$\dot{V}(t) = 2\varepsilon^T(t)((I_{N-1} \otimes A) - (HU \otimes BK))^T(W \otimes P)\varepsilon(t) + 2\zeta^T(x(t))(W \otimes M^T P)\varepsilon(t). \quad (11)$$

Let  $K = B^T P$ , and then Eq. (11) can be transformed into

$$\begin{aligned} \dot{V}(t) = & \varepsilon^T(t) \left( W \otimes (A^T P + PA) - \left( (HU)^T W + WHU \right) \otimes PBB^T P \right) \varepsilon(t) \\ & + 2\zeta^T(x(t))(W \otimes M^T P)\varepsilon(t). \end{aligned} \quad (12)$$

By Lemma 4 and Eq. (2), one can see that

$$2\zeta^T(x(t))(W \otimes M^T P)\varepsilon(t) \leq \zeta^T(x(t))\zeta(x(t)) + \varepsilon^T(t)(W^2 \otimes PMM^T P)\varepsilon(t), \quad (13)$$

By Eqs. (2) and (7), one can see that

$$\zeta^T(x(t))\zeta(x(t)) \leq \alpha^2 \varepsilon^T(t)\varepsilon(t), \quad (14)$$

From Lemma 3 and Eq. (13),  $\dot{V}(t)$  satisfies

$$\dot{V}(t) \leq \varepsilon^T(t) \left( W \otimes (A^T P + PA - \gamma PBB^T P + \alpha^2 \omega_{\min}^{-1} I_d + \omega_{\max} PMM^T P) \right) \varepsilon(t), \quad (15)$$

where the fact that

$$\omega_{\min} I_{N-1} \leq W \leq \omega_{\max} I_{N-1}$$

is used.  $\omega_{\min}$  and  $\omega_{\max}$  are the minimum and the maximum eigenvalues of  $W$ , respectively.

Define

$$\dot{\mathfrak{S}}(t) = \dot{V}(t) + \tilde{J}_C, \quad (16)$$

where  $\tilde{J}_C \geq 0$  and

$$\tilde{J}_C = \varepsilon^T(t)(I_{N-1} \otimes Q)\varepsilon(t).$$

It should be pointed out that if  $\dot{\mathfrak{S}}(t) \leq 0$ , then  $\dot{V}(t) \leq 0$ . Then, one can see that

$$\dot{\mathfrak{S}}(t) \leq \varepsilon^T(t) \left( W \otimes (A^T P + PA - \gamma PBB^T P + \alpha^2 \omega_{\min}^{-1} I_d + \omega_{\max} PMM^T P + \omega_{\min}^{-1} Q) \right) \varepsilon(t). \quad (17)$$

By the Schur complement, if  $\Xi < 0$ , one has

$$\dot{\mathfrak{S}}(t) \leq 0$$

and  $\dot{\mathfrak{S}}(t) = 0$  if and only if  $\varepsilon(t) \equiv 0$ . Then, by  $\tilde{J}_C \geq 0$ , if  $\Xi < 0$ ,  $\dot{V}(t) \leq 0$  and  $\dot{V}(t) = 0$  if and only if  $\varepsilon(t) \equiv 0$ . Thus,  $\lim_{t \rightarrow +\infty} \varepsilon(t) = 0$  holds; that is,

$$x_1(t) = x_2(t) = \dots = x_N(t)$$



when  $t \rightarrow +\infty$ . Therefore, if there exists  $P$  that satisfies  $\Xi < 0$ , then guaranteed performance consensus for multi-agent system (Eq. (4)) with  $K = B^T P$  is achieved.

When the nonlinear multi-agent system (Eq. (4)) achieves guaranteed performance consensus, the performance of consensus control is described by the performance function (Eq. (5)). Then, an upper bound of the performance function (Eq. (5)) is able to determine.

*Theorem 2:* Assume that  $G$  has a directed spanning tree. If nonlinear multi-agent system (Eq. (4)) with a symmetric and positive definite matrix  $P \in \mathbb{R}^{d \times d}$  achieves guaranteed performance consensus, then the performance function (Eq. (5)) has an upper bound:

$$J_C^* = x^T(0)(H^T W H \otimes P)x(0).$$

*Proof:* From the proof of Theorem 1, it is obtained that

$$\tilde{J}_C \leq -\dot{V}(t) \quad (18)$$

when nonlinear multi-agent system (Eq. (4)) achieves guaranteed performance consensus. For Eq. (18), integrating both sides along with  $t \in [0, +\infty)$  gives

$$J_C = \int_0^{+\infty} \tilde{J}_C dt \leq - \int_0^{+\infty} \dot{V}(t) dt. \quad (19)$$

Since  $\lim_{t \rightarrow +\infty} V(t) = 0$ , one has  $J_C \leq V(0)$ . Thus,  $J_C^* = V(0)$  is an upper bound of the quadratic performance function (Eq. (5)). From Eq. (10) and  $\varepsilon(t) = (H \otimes I_d)x(t)$ , the result of Theorem 2 is obtained.

In the existing works [11–20], the guaranteed performance consensus problems for linear multi-agent systems have been studied. Theorem 1 gives a sufficient condition for nonlinear multi-agent system (Eq. (4)) to achieve guaranteed performance consensus. Moreover, the directed topology is considered in the current chapter, but the topologies in [11–19] were undirected, and the directed case problem was dealt with by the sampled-data control in [20].

## 4. Design of guaranteed performance consensus

*Theorem 3:* Multi-agent system (Eq. (1)) is said to be a guaranteed performance consensus-able by consensus protocol (4) if there exists  $d$ -dimensional matrix  $\tilde{P} = \tilde{P}^T > 0$  such that  $\tilde{\Xi} < 0$ , where

$$\tilde{\Xi} = \begin{bmatrix} \tilde{\Xi}_{11} & M & \alpha \tilde{P} & \tilde{P}Q \\ * & -\omega_{\max}^{-1} I_d & 0 & 0 \\ * & * & -\omega_{\min} I_d & 0 \\ * & * & * & -\omega_{\min} Q \end{bmatrix}$$

with



$$\begin{aligned}\tilde{\Xi}_{11} &= \tilde{P}A^T + A\tilde{P} - \gamma BB^T, \\ \omega_{\min} &= \min\{\lambda_i(W), i = 1, 2, \dots, N-1\}, \\ \omega_{\max} &= \max\{\lambda_i(W), i = 1, 2, \dots, N-1\}.\end{aligned}$$

In this case, the control gain matrix satisfies  $K = B^T\tilde{P}-1$ , and the guaranteed performance function has an upper bound:

$$J_C^* = x^T(0)(H^TWH \otimes \tilde{P}-1)x(0).$$

*Proof:* The method of changing variables is used to determine  $K$ . Pre- and post-multiplying  $\Xi < 0$  by  $\Pi = \text{diag}\{P^{-1}, I_d, I_d, I_d\}$  and  $\Pi^T = \text{diag}\{P^{-T}, I_d, I_d, I_d\}$ , respectively, one has

$$\begin{bmatrix} \bar{\Xi}_{11} & M & \alpha P^{-1}I_d & P^{-1}Q \\ * & -\omega_{\max}^{-1}I_d & 0 & 0 \\ * & * & -\omega_{\min}I_d & 0 \\ * & * & * & -\omega_{\min}Q \end{bmatrix} < 0$$

with  $\bar{\Xi}_{11} = P^{-1}A^T + AP^{-T} - \gamma BB^T$ . Setting  $\tilde{P} = P^{-1}$ , one has  $\tilde{\Xi} < 0$ . From Theorem 1, if  $\tilde{\Xi} < 0$  are feasible, then multi-agent system (Eq. (4)) can achieve consensus. Therefore, by Definition 2 and Theorem 2, the conclusion of Theorem 3 can be obtained.

Theorem 3 presents the LMI conditions for controller design of guaranteed performance consensus. The feasibility of these LMI conditions can be checked by using the MATLAB's LMI Toolbox.

## 5. Simulations

A nonlinear multi-agent system composed of four agents is analyzed to demonstrate the effectiveness of the proposed approach, where all agents are labeled from 1 to 4. The dynamics of each agent is described in Eq. (1) with

$$\begin{aligned}A &= \begin{bmatrix} 0 & 1 \\ -1.5 & -0.6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ M &= \begin{bmatrix} 0.7 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad f(x_i(t)) = \begin{bmatrix} 0.1 \sin x_i(t) \\ 0.04 \sin x_i(t) \end{bmatrix}.\end{aligned}$$

It can be seen that  $\alpha = 0.1$  in Eq. (2). The initial states of all agents are

$$x_1(0) = \begin{bmatrix} -2.2 \\ -1.5 \end{bmatrix}, \quad x_2(0) = \begin{bmatrix} -1.2 \\ -0.7 \end{bmatrix}, \quad x_3(0) = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, \quad x_4(0) = \begin{bmatrix} 1.6 \\ 0.3 \end{bmatrix}.$$

In the performance function in Eq. (5),

$$Q = \begin{bmatrix} 0.45 & 0 \\ 0 & 0.3 \end{bmatrix}$$

are given. A directed interaction topology  $G$  is given in **Figure 1**, where the weights of edges of the interaction topology are 1 and the Laplacian matrix of  $G$  is

$$L = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 2 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

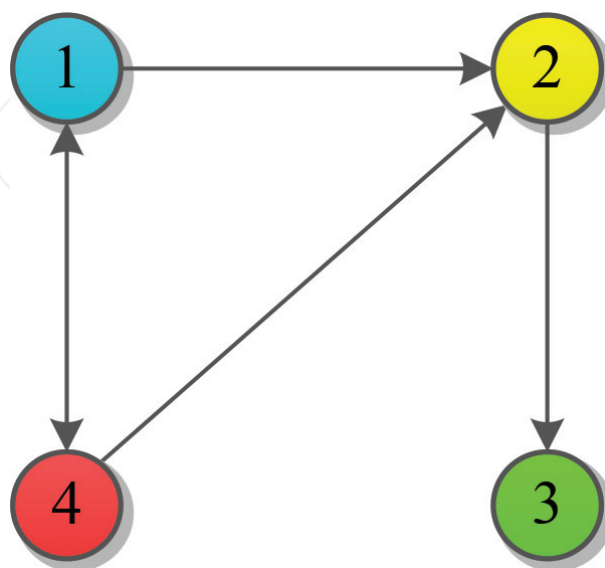
By the definition of the matrix  $H$ , it is obtained that

$$H = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

Then, the matrix

$$U = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

and  $\min\{\text{Re}(\lambda(HU))\} = 1$  satisfy the conditions of Lemma 2. In this simulation,  $\gamma = 1.6$  is chosen. By Lemma 3 and LMI Toolbox of MATLAB, the matrix



**Figure 1.** The interaction topology  $G$ .

$$W = \begin{bmatrix} 1.0433 & 0.0812 & -0.4787 \\ 0.0812 & 0.4320 & 0.1325 \\ -0.4787 & 0.1325 & 0.9919 \end{bmatrix}.$$

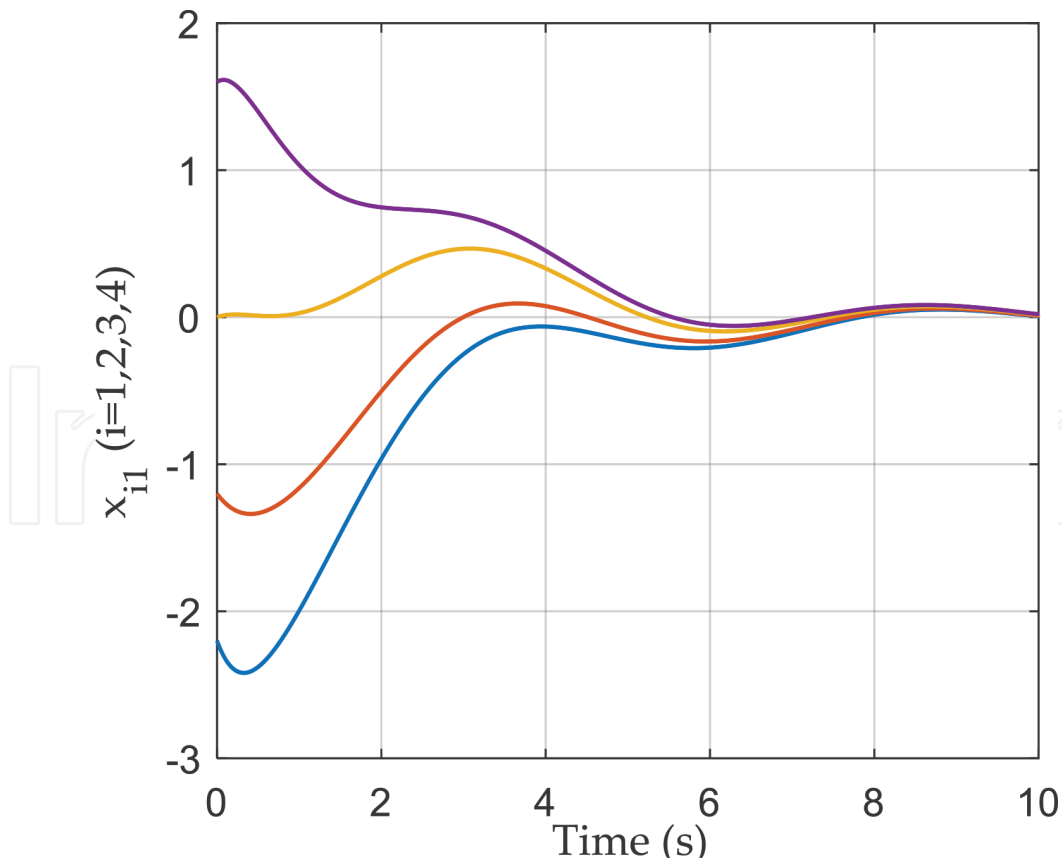
Thus, according to Theorem 3, one has

$$\tilde{P} = \begin{bmatrix} 0.5534 & -0.0481 \\ -0.0481 & 0.7672 \end{bmatrix}$$

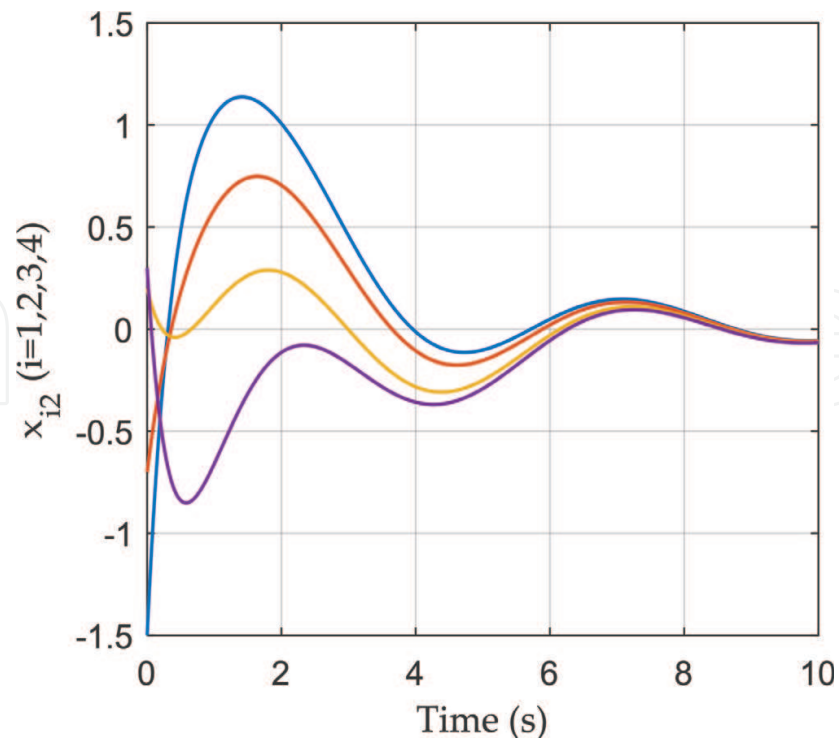
and

$$K = [0.1139 \ 1.3105].$$

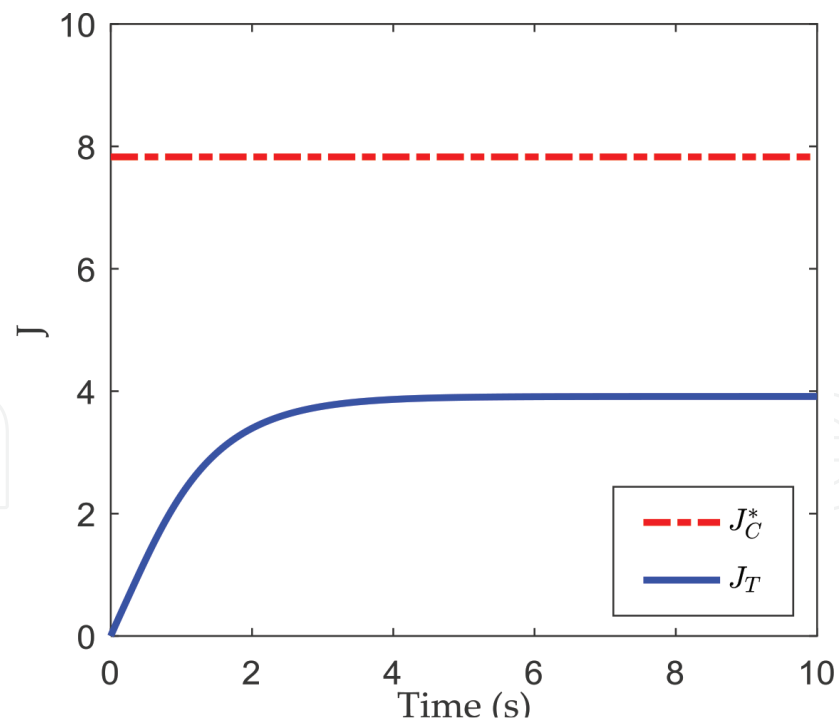
In **Figures 2** and **3**, the state trajectories of the nonlinear multi-agent system are shown, and one can see that the state of all agents is convergent. By Theorem 3, an upper bound of the guaranteed performance function is  $J_C^* = 7.8284$ . **Figure 4** presents the guaranteed performance function and the upper bound. From **Figure 3**, one can see that there exists conservatism induced by the approach taken to compute the upper bound on the performance, where the actual performance  $J_T$  and then the conservatism can be depicted by  $\Delta J = J_C^* - J_T$ . It is clear



**Figure 2.** State trajectories of  $x_{i1}(t)$  ( $i = 1, 2, 3, 4$ ).



**Figure 3.** State trajectories of  $x_{i2}(t)$  ( $i = 1, 2, 3, 4$ ).



**Figure 4.** Trajectories of the performance function.

that the multi-agent system achieves guaranteed performance consensus although there exists conservatism.

## 6. Conclusions

In this chapter, the guaranteed performance consensus problems for nonlinear multi-agent systems with directed interaction topologies were studied. A special matrix transformation was introduced, and guaranteed performance consensus problems were transferred into guaranteed performance stabilization problems. Sufficient conditions for guaranteed performance consensus control were obtained, and an upper bound was given.

The directed topology was assumed to be fixed and connected in the guaranteed performance consensus problem, and then the application of conclusions of the current paper is limited. Therefore, the influence of the general switching topologies for the guaranteed performance consensus problem is the possible topic. The existing work [24, 25] assumed that the switching topologies were strongly connected and balanced, but the joint-connected switching topology cases have not been studied.

The analysis method was used to analyze the guaranteed performance consensus problem with Lipschitz-type nonlinearities in the chapter, but this method cannot be directly applied to the problem with the other kind of nonlinearities. Therefore, the analysis approach for the guaranteed performance consensus problem with general nonlinear dynamic should be given in future works.

Sliding mode control (SMC) technique has a strong robustness for external noises, such as [26, 27], and then the SMC technique is a possible method for guaranteed performance consensus problems to improve the robustness of the obtained consensus controller in future works.

Moreover, it is usually desirable to design a controller which not only achieves formation but also ensures an adequate level of performance in many practical formation control problems. Then, the idea of guaranteed performance control can be introduced into formation control problems for multi-agent systems. To this end, the guaranteed performance consensus algorithm is an effective approach, such as the guaranteed performance formation control in [28].

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## Conflict of interest

No potential conflict of interest was reported by the authors.

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