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The Behavior of Streaming Instabilities in Dissipative Plasma

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Abstract

An approach is presented that allows getting detailed information on the behavior of streaming instabilities (SI) from the dispersion relation (DR). The approach is based on general assumptions and does not refer to any particular model and/or type of the stream interaction with background system (Cherenkov, cyclotron, etc.). The basis of the approach is transformation of the DR to an equation for slowly varying amplitude of the developing waveform. The solution of the equation actually presents results of the important problem of time evolution of initial perturbation and gives detailed information on the instability behavior. Most of the information is unavailable by other methods. For particular SI, only two parameters should be specified. The expression for the fields' structure shows that with increase in level of dissipation, SI gradually turns to dissipative streaming instability (DSI). Two new, previously unknown types of DSI are presented: DSI of overlimiting electron beam and DSI under weak beam-plasma coupling. Growth rates of these DSI depend on dissipation more critically than usual. Presented approach is valid for a large class of SI: beam-plasma instabilities of various types (Cherenkov, cyclotron, etc.) including over-limiting e-beam instabilities, the instability in spatially separated beam-plasma systems, Buneman instability, etc.

Keywords: streaming instability, dissipative instability, space-time evolution, slowly varying amplitude, transformation to dissipative instability

1. Introduction

Plasma is rich in instabilities. Many of them are a result of relative motion of plasma components. These, streaming instabilities (SI) are the most common in space and laboratory plasmas.

A well-known example is the beam-plasma instability [1], in which the directed motion of a small group of fast electrons passing through the background plasma excites potential oscillations with high growth rate near the plasma frequency. Close attention to this instability is due mainly to design of high power sources of electromagnetic radiation based on this instability. The sources have many advantages as compared to well-known vacuum devices [2, 3]. Another example (we mention these two only) is the Buneman instability [4], in which plasma electrons move with respect to ions. The instability plays an important role in many scenarios in space physics and geophysics. A striking example of plasma with relative electron-ion motion is current-carrying plasma. This object is often considered in plasma physics. The instabilities which are due to relative electron-ion motion play an important role in physics of controlled fusion also.

A clear understanding of physical nature of the SI, their role and influence on various processes in plasma requires substantial efforts. Physics of interaction of plasma components moving relatively to each other is essentially based on the concept of negative energy wave (NEW) [5]. This requires account of all factors which lead to NEW growth. Among them, dissipation plays an important role. Dissipation leads to energy losses for the growth of NEW. Influence of dissipation on the instabilities of streaming type is unique. Dissipation never suppresses the instabilities completely regardless on its level. Dissipation of high-level transforms the SI to dissipative streaming instability (DSI) [1]. These instabilities have a number of features: comparatively low growth rate, comparatively low level of excited oscillations, etc. For a few decades, DSI have been widely discussed, and it is supposed that they can be applied to explain various phenomena in space and laboratory plasma. Up to recently only one type of DSI was known, and it was believed that all types of electron stream instabilities (e.g., Cherenkov type, cyclotron type etc.) transform to the single known type of DSI. However, it turned out that other types of DSI also exist [6–8]. Changes in some basic physical parameters and/or system geometry lead to significant changes in physical nature of e-stream interaction with plasma. This changes result in two new, previously unknown types of DSI: DSI of over-limiting electron beam and DSI under weak coupling of the stream with the plasma. In both cases, the growth rate depends on dissipation more critically: $1/\nu$ instead of conventional $1/\sqrt{\nu}$ (here ν is the frequency of the collisions).

The transformation of the SI to dissipative type makes their behavior in the presence of dissipation of particular interest. In order to understand how instability turns to another type, it is necessary to investigate the evolution of its fields in space and time [9, 10]. Simultaneously, the expressions for fields' evolution give all available information on the SI: growth rates (spatial and temporal) under arbitrary level of dissipation, character of the instability (absolute/convective), range of unstable perturbations' velocities, influence of dissipation on the instability, etc. These details help to understand how the instability turns to DSI, how it transforms given equilibrium of background plasma, predict the level and/or scale of the changes, how nonlinear phenomena arise as well as predict possible saturation mechanisms, etc. In general, the character of the fields' development in space and time is one of the most important aspects of every instability.

The character of space–time evolution of given instability is an important issue in many branches of physics. In plasma physics, we firstly note theory of amplifiers and oscillators in the microwave range based on interaction of e-beam with wave, where obvious progress is achieved [2, 3]. These studies are also important for research on plasma instabilities associated with research on nuclear fusion, astrophysics, etc.

The mathematical solution of the problem of initial perturbation evolution reduces to calculation of the integral with a complete dispersion relation (DR) in the denominator of the integrand. An overall view on the character of the instability may be obtained by investigation of the asymptotic behavior of the Green's function. In order to derive analytical expression for the fields' space–time distribution, the DR should be specified and solved before integration. In this way, essential difficulties appear which usually cannot be overcome. One must apply approximate methods to obtain results. Presented here (see also [11]) approach is similar to traditional approach in many respects, but, in the same time, advantageously differs from it. Representation of the fields in form of wave train with slowly varying amplitude (SVA) allowed to overcome the difficulties and to obtain the space–time structure of the fields without reference on any particular model. Thereby, the approach singles out intrinsic peculiarities of various types of SI. The results show that all types of the beam-plasma instabilities (Cherenkov, cyclotron, etc.) have similar dynamics of development. By specifying only two parameters in the unified expression one can investigate given particular case of beam instability. With increase in level of dissipation all SI gradually turn to DSI.

This review considers all these aspects: getting detailed information on SI, their space–time evolution and transformation to DSI. Presented approach shows that the DR which usually describes given SI can serve not only for solution of the well-known (and very simplified) initial and boundary problems. Its application is much wider. It can give much more information on the instability. Namely, it actually gives the solution of the well-known (and very important [9]) problem of time evolution of initial perturbation. The DR can give space–time structure of the fields at the instability development. In its turn, the fields' structure contains complete information on the instability. Most of this information is unavailable by other methods. The expressions for fields' evolution also show in detail the transformation of SI to dissipative type. Two new, previously unknown types of DSI are presented.

Large variety of SI characterize by various types of the interaction with background systems (plasma-filled or not), various values of streaming currents, etc. From this follows various types of their DR and ensuing equation for SVA. They are considered separately. In Section 2, the evolution of various types of beam instabilities (Cherenkov, cyclotron, and the instability in periodical structure) are considered. All they characterize by small contribution of the beam in DR and this fact allowed generalizing the consideration. Section 3 gives the evolution of over-limiting e-beam instability. Due to influence of the beam space charge, the instability of such beams has other physical nature as compared to instability of conventional e-beams. In Sections 4 and 5, the instability in spatially separated beam-plasma system and the Buneman instability are considered. The peculiarity of last case is in the role of plasma ions.

2. The behavior beam-plasma instabilities in dissipative plasma

2.1. Equation for slowly varying amplitude

Consider an electrodynamical system of arbitrary geometry (plasma filling is not obligatory) and let a monoenergetic relativistic electron beam penetrate it. The general form of the dispersion relation (DR) of such system is

$$D_0(\omega, \mathbf{k}) + D_b(\omega, \mathbf{k}) = 0 \quad (1)$$

where ω is the frequency of perturbations and \mathbf{k} is the wave vector. $D_0(\omega, \mathbf{k}) = 0$ is the “cold” DR describing proper frequencies of the systems in the absence of the beam (its main part), and $D_b(\omega, \mathbf{k})$ is the beam contribution. We also assume that the beam density is small enough to satisfy the condition $|D_b(\omega, \mathbf{k})| \ll |D_0(\omega, \mathbf{k})|$. In following consideration, we will not specify the form of $D_0(\omega, \mathbf{k})$. Beam electrons interact with proper oscillations of background system and this interaction leads to instability. The interaction may be of various types: Cherenkov, cyclotron, interaction with periodical structure, etc. The general form of $D_b(\omega, \mathbf{k})$ may be written as

$$D_b(\omega, \mathbf{k}) = -\frac{\omega_b^2 A(\omega, \mathbf{k})}{\gamma^3 (\omega - \mathbf{k}\mathbf{u} - f)^2} \quad (2)$$

where \mathbf{u} is the velocity of the beam electrons, ω_b is the Langmuir frequency of streaming electrons, γ is the relativistic factor of the beam electrons, and $A(\omega, \mathbf{k})$ is a polynomial with respect to ω and \mathbf{k} of degree no higher than two. The expression for f depends on the type of the beam interaction with plasma:

$$f = \begin{cases} 0, & \text{if the interaction is of Cherenkov type} \\ n\Omega/\gamma, & \text{if the interaction is of cyclotron type} \\ k_{\text{cor}}u, & \text{if the beam interacts with periodical structure} \end{cases}, \quad (3)$$

where Ω is the cyclotron frequency, $n = 1, 2, 3, \dots$, $k_{\text{cor}} = 2\pi/l_0$, l_0 is the spatial period of the structure. Below, we will show that properties of the instabilities follow from the general form (2) and do not depend on the expression for f .

Let an initial perturbation arises in point $z = 0$ (electron stream propagates in the direction $z > 0$) at instant $t = 0$ and the instability begins developing. Our aim is to obtain shape of the perturbation (i.e., space–time structure of the fields) at arbitrary instant t and based on the expression, investigate the behavior of the instability. In following consideration, we interest in longitudinal structure of the field (their dependence on z and t only). We single out two arguments: the frequency ω and longitudinal wavelength k . Other arguments play no part in following. To avoid overburdening of the formulas below, they are omitted. The transversal structure of the fields may be obtained in regular way by expansion on series of eigenfunctions of given system.

The development of wave pulse in its linear stage obeys the DR (1). The beam instability reveals itself most effectively on frequencies, closely approximating to roots of the main part of Eq. (1) and simultaneously to the beam proper oscillations (e.g., space charge wave). This means that following two conditions must be satisfied:

$$D_0(\omega, k) = 0 \quad ; \quad \omega - ku - f = 0. \quad (4)$$

Therefore, it would appear reasonable to assume that developing fields form a wave train of following type

$$E(z, t) = E_0(z, t) \exp \{-i\omega_0 t + ik_0 z\}, \quad (5)$$

where the carrier frequency ω_0 and k_0 satisfy the conditions (4). We also assume that the amplitude $E_0(z, t)$ is slowly varying as compared to ω_0 and k_0 that is,

$$\left| \frac{\partial E_0}{\partial t} \right| \ll \omega_0 E_0 \quad ; \quad \left| \frac{\partial E_0}{\partial z} \right| \ll k_0 E_0 \quad (6)$$

In such formulation, the problem of the instability evolution reduces to determination of the slowly varying amplitude (SVA) $E_0(z, t)$. As the fields vary near ω_0 and k_0 , one can use following formal substitutions to derive an equation for SVA

$$\omega \rightarrow \omega_0 + i \frac{\partial}{\partial t} \quad ; \quad k \rightarrow k_0 - i \frac{\partial}{\partial z}, \quad (7)$$

Expanding the DR (1) in power series near ω_0 and k_0 , one can obtain the equation for SVA

$$\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial z} \right)^2 \left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} + \nu \right) E_0(z, t) = i|\delta_0|^3 E_0(z, t) \quad (8)$$

where

$$\delta_0 = \left\{ \frac{\omega_b^2 A(\omega, k)}{\partial D_0 / \partial \omega} \right\}_{\omega = \omega_0, k = k_0}^{1/3} ; \quad \nu = \left\{ \frac{\text{Im } D_0}{\partial D_0(\omega, k) / \partial \omega} \right\}_{\omega = \omega_0, k = k_0} ; \quad v_0 = - \left\{ \frac{\partial D_0(\omega, k) / \partial k}{\partial D_0(\omega, k) / \partial \omega} \right\}_{\omega = \omega_0, k = k_0}$$

$\text{Im } \delta_0$ is the maximal growth rate of the beam instability, ν describes dissipation in the system (its coincidence to collision frequency is not obligatory), and v_0 is the group velocity of resonant wave in the "cold" system.

The Eq. (8) describes the evolution of SVA $E_0(z, t)$ in space and time for all systems those may be described by the DR in form (1). Eq. (8) may be solved by using Fourier transformation with respect to spatial coordinate z and Laplace transformation with respect to time t . The corresponding equation for the transform $E_0(\omega, k)$ is

$$\left\{ (\omega - ku)^2 (\omega - kv_0 + i\nu) - |\delta_0|^3 \right\} E_0(\omega, k) = J(\omega, k) \quad (9)$$

$$E_0(\omega, k) = \int_0^\infty dt \int_{-\infty}^\infty dz E_0(z, t) \exp(i\omega t - ikz) : \quad (10)$$

where the function $J(\omega, k)$ is determined by initial conditions. Its power with respect to ω and k is no higher than the power of the origin equation. The specific form of this function is not essential for following. It is only necessary that $J(\omega, k)$ be smooth and not equal to zero identically. The amplitude of the wave train can be obtained by inverse transformation

$$E_0(z, t) = \frac{1}{(2\pi)^2} \int_{C(\omega)} d\omega \int_{-\infty}^\infty \frac{dk J(\omega, k) \exp(-i\omega t + ikz)}{(\omega - ku)^2 (\omega - kv_0 + iv) - |\delta_{Bn}|^3} \quad (11)$$

Here, $C(\omega)$ is the contour of integration over ω . For given case, it is a straight line that lies in the upper half plane of the complex plane $\omega = \text{Re}\omega + i\text{Im}\omega$ and passes above all singularities of the integrand. Thus, the problem has been reduced to the integration in Eq. (11). It is convenient to transform the variables ω and k to another pair ω and $\omega' = \omega - ku$. The first integration (over ω) may be carried out by residue method and the integration contour must be closed in the lower half plane. The pole is

$$\omega_1(\omega') = \left(1 - \frac{v_0}{u}\right)^{-1} \left(\frac{|\delta_0|^3}{\omega'^2} - \omega' \frac{v_0}{u} - iv\right) \quad (12)$$

The second integration (over ω') cannot be carried out exactly, and we are forced to restrict ourselves by approximate, steepest descend method. That is, Eq. (11) will be worked out in asymptotic limit of comparatively large t . In this case, the integration contour should be deformed in order to pass through the saddle point in needed direction. The saddle point is

$$\omega'_s = \delta_0 \left\{ \frac{2(ut - z)}{(z - v_0 t)} \right\}^{1/3} \exp(2\pi i/3) \quad (13)$$

As a result of the integration, we obtain following expression for the SVA [11].

$$E_0(z, t) = \frac{J_0}{2\sqrt{\pi}} \frac{\exp\{\chi_v^{(\text{und})}(z, t)\}}{\sqrt{(u - v_0)f(z, t)}} \exp\{i\varphi(z, t)\} \quad (14)$$

$$\chi_v^{(\text{und})}(z, t) = \chi_0^{(\text{und})}(z, t) - v \frac{z - v_0 t}{u - v_0} \quad ; \quad \chi_0^{(\text{und})}(z, t) = \frac{3\sqrt{3}}{4} \frac{\delta_0}{u - v_0} \left\{ 2(ut - z)(z - v_0 t)^2 \right\}^{1/3}$$

$$f(z, t) = 3\delta_0^3(ut - z) \quad ; \quad \varphi(z, t) = \frac{\chi(z, t)}{\sqrt{3}} + \frac{\pi}{4}$$

and J_0 is the value of $J(\omega, \omega')$ at the points. $\omega = \omega_1(\omega'_s)$, $\omega' = \omega'_s$.

2.2. Analysis of the fields' dynamics

We have arrived to very complex expressions (14). However, the field's structure (i.e., the instability behavior) may be determined by analyzing the factor

$$\exp \chi_v^{(\text{und})}(z, t) \quad (15)$$

The information, which are available from the analysis are much more detailed and complete as compared to results of well-known initial and boundary problems. The analysis gives: growth rate(s), the velocities of unstable perturbations, the character of the instability and influence of the dissipation on it, etc. The expression (15) shows that along with exponential increasing the field covers more and more space. In the absence of dissipation, the velocities of unstable perturbations range from v_0 to u . The length of the wave train increases depending on time $l \sim (u - v_0)t$. One can easily see convective character of streaming instabilities in laboratory frame, as well as in other frames moving at velocities $v < v_0$ and $v > u$. If the observer's velocity is within the range $v_0 < v < u$, the instability is absolute (see **Figure 1**, where the dependence of the SVA on z at various instants t_1, t_2 and t_3 is presented; the leading edge moves at velocity u , but the back edge moves at velocity $v_0 < u$).

The peak (and the field's properties in it) may be determined from the equation

$$\frac{\partial}{\partial z} \chi_v^{(\text{und})} = 0 \quad (16)$$

Its solution in the absence of dissipation gives $z = w_g t$, where

$$w_g = (1/3)(2u + v_0) \quad (17)$$

That is, the peak places on 1/3 of the train's length from the front and moves at the velocity w_g . Actually, w_g represents group velocity of the generated wave, with account of the beam contribution in the DR. The field's value in the peak exponentially increases and the growth

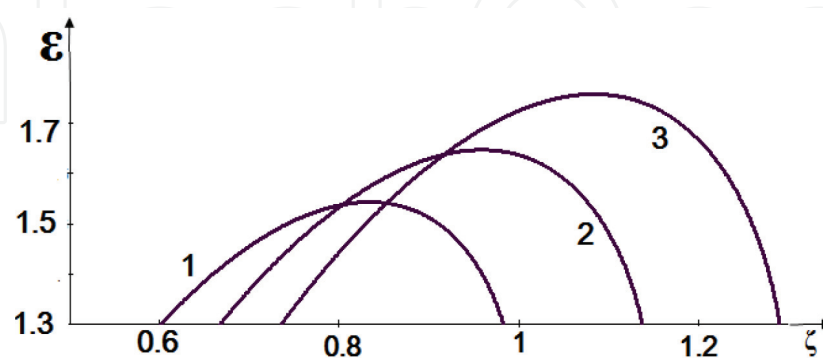


Figure 1. Asymptotic shapes of beam instability $\varepsilon = \exp \chi_0^{(\text{und})}(z, t)$ depending on longitudinal coordinate $\zeta = z\delta_0/u$ at various instants $\tau_1 = \delta_0 t_1 < \tau_2 = \delta_0 t_2 < \tau_3 = \delta_0 t_3$.

rate is equal to $\delta_m = (\sqrt{3}/2)|\delta_0|$ that is, coincides to solution of the initial problem. However, the initial problem can not specify the point, where the maximal growth occurs. The advantage of this approach is evident.

In a fixed point z , the field first increases and attains maximum at instant $t = z/w_a$ where

$$w_a = \frac{3uv_0}{u + 2v_0} \quad (18)$$

Then, the field falls off and at the time $t \geq z/v_0$ the train passes the considered point. The velocity w_a is the group velocity of the resonant wave upon amplification with account of the beam contribution in the DR. For given z , the field's maximum is

$$E_0 \sim \exp \delta_m z / (u^2 v_0)^{1/3} \quad (19)$$

The exponent $\delta_m / (u^2 v_0)^{1/3}$ coincides to solution of the boundary problem as it is the maximal spatial growth rate. The coincidence to the results of well-known initial and boundary problems testifies presented approach. It may appear that this way of instability analysis is a bit more complicate. However, it must be admitted that along with growth rates we have obtained much other information. The information obviously clarifies the picture of the instability and makes it realistic. One can easily see the merits of presented approach.

The relations between characteristic velocities are

$$v_0 < w_a < w_g < u \quad (20)$$

At fixed instant t , perturbations exist only at distances $v_0 t \leq z \leq ut$. The wave train passes given point z during the time $z/u \leq t \leq z/v_0$. In a fixed point, the amplitude attains maximum at the instant, when the peak has already passed it (see **Figure 1**). The reason is that the perturbations with smaller velocities reach considered point in longer time, and they grow more efficiently. Perturbations with velocity w_a are the most efficiently enhanced perturbations.

Generally, the dependence of the perturbations' amplitudes on their velocity v has a form $E \sim \exp \Gamma(v)t$, where

$$\Gamma(v) = \frac{3}{2^{2/3}} \frac{\delta_m}{u - v_0} \left\{ (v - v_0)^2 (u - v) \right\}^{1/3} \quad (21)$$

The character of spatial growth depending on v is

$$E \sim \exp \Gamma(v)z/v \quad (22)$$

Presented above analysis is true if we neglect dissipation. Dissipation essentially changes the instability behavior. It suppresses slow perturbations. The threshold velocity is

$$V_{thr} = \frac{\lambda u + v_0}{(1 + \lambda)} \quad ; \quad \lambda = \frac{2^{5/2}}{3^{9/4}} \left(\frac{v}{|\delta_0|} \right)^{2/3} \quad (23)$$

Only perturbations moving at higher velocities $v > V_{thr}$ develop. The wave train shortens. Dissipation decreases the field growth

$$\Gamma(v) \rightarrow \Gamma_v(v) = \Gamma(v) - v \frac{u - v}{u - v_0} \quad (24)$$

The dynamics of the field in the peak may be obtained by analyzing the Eq. (16). It takes following form

$$(z - v_0 t)(ut - z)^2 = (1/\lambda^2)(z - w_g t)^3 \quad (25)$$

If $v \ll \delta_0$, this equation leads to small corrections to the expressions (17) and (18) for characteristic velocities and for the maximal growth rate in the peak. In the opposite case of high-level dissipation, only the perturbations are unstable, whose velocity is close to the beam velocity u . In this approximation, the solution of Eq. (25) is $z = u - \Delta u$ where

$$\Delta u = 3^{-3/2} \lambda^{-1} (u - v_0), \quad (26)$$

and the expression for maximal growth rate takes the form $\Gamma_{v \rightarrow \infty} = (\delta_m^3 / v)^{1/2}$. Obviously, this case corresponds to dissipative streaming instability (DSI). The same expression for $\Gamma_{v \rightarrow \infty}$ can be obtained from Eq. (1) by direct usage of the initial problem [1]. If one specifies δ_m , he can obtain the growth rate of DSI in unbound beam-plasma system, in magnetized beam-plasma waveguide, etc.

In general, by substitution of two parameters only: growth rate and the group velocity of resonant wave in “cold” system one can obtain the behavior of specific e-beam instability.

It is not superfluous to repeat once again that the expression (14) and resulting analysis is valid for all types of e-beam instabilities: Cherenkov, cyclotron, beam instability in periodical structures, etc. Also, the analysis does not depend on specific geometry, external fields, etc.

3. The behavior of overlimiting electron beam instability

The picture described above is valid for e-beams, instability of which is due to induced radiation of the system proper waves by the beam electrons. However, it is known that with increase in beam current the physical nature of e-beam instabilities changes [6, 7, 12–14]. This is a result of influence of the beam space charge. It sets a limit for the beam current in vacuum systems. The limit may be overcome, for example, in plasma filled waveguide. The instability of over-limiting e-beams (OB) is due either to aperiodical modulation of the beam density in media with negative dielectric constant or to excitation of the NEW. In this section, we consider behavior of the first type of OB instability. It develops, for example, in uniform cross-section magnetized beam-plasma waveguide. It is clear that the change of the physical nature of the instability affects on its behavior. This instability sharply differs from the instability of conventional (underlimiting) e-beams: (1) its growth rate attains maximum at the point of exact

Cherenkov resonance, (2) it is of nonradiative type, and (3) with increase in dissipation, it turns to a new type of DSI [6, 14].

3.1. Statement of the problem: analysis of the DR

Mathematical description of OB is not so well-known as for underlimiting beams, and in order to catch the differences, we consider both cases simultaneously. Consider a cylindrical waveguide, fully filled by cold plasma. A monoenergetic relativistic electron beam penetrates it. The external longitudinal magnetic field is assumed to be strong enough to freeze transversal motion of the beam and the plasma electrons. For simplicity, we assume that the beam and plasma radii coincide to the waveguide's radius and consider only the symmetrical E -modes with nonzero components E_r , E_z , and B_φ . It is known [1] that the system under consideration is described by the following DR

$$k_\perp^2 + \left(k^2 - \frac{\omega^2}{c^2}\right) \left(1 - \frac{\omega_p^2}{\omega(\omega + iv)} - \frac{\omega_b^2}{\gamma^3(\omega - ku)^2}\right) = 0 \quad (27)$$

ω and k are the frequency and the longitudinal (along z axis) wave vector, $k_\perp = \mu_{0s}/R$. R is the waveguide's radius, μ_{0s} are the roots of Bessel function $J_0: J_0(\mu_{0s}) = 0$, $s = 1, 2, 3, \dots$, $\omega_{p,b}$ are the respective Langmuir frequencies for the beam and the plasma, u is the velocity of the beam, $\gamma = (1 - u^2/c^2)^{-1/2}$, c is speed of light. The DR (27) determines the growth rates of the beam-plasma instability. As we have mentioned earlier, the character of the beam-plasma interaction changes depending on the beam current value. This change must reveal itself in the solutions of the DR (27). In order to consider the solutions, we look them in the form $\omega = ku + \delta$, $\delta \ll ku$. The DR (27) reduces to [1, 6].

$$x^3 + i \frac{\nu}{\omega_0} \frac{\omega_p^2 v_0}{u \gamma^2 \omega_\perp^2} x^2 + \frac{\alpha v_0 u}{\gamma^2 c^2} x = \frac{\alpha}{2\gamma^4} \frac{v_0}{u} \quad (28)$$

where $x = \delta/ku$, $\alpha = \omega_b^2/k_\perp^2 u^2 \gamma^3$, $\beta = u/c$, $\omega_\perp^2 = k_\perp^2 u^2 \gamma^2$, and $v_0 = u\mu/(1 + \mu)$ is the group velocity of the resonant wave in "cold" system, $\mu = \gamma^2 \omega_\perp^2 / \omega_0^2$; $\omega_0 = (\omega_p^2 - \omega_\perp^2)^{1/2}$ is the resonant frequency of the plasma waveguide that is, ω_0 satisfies following conditions

$$D_0(\omega, k) = 0 \quad ; \quad \omega = ku \quad (29)$$

The solutions of Eq. (28) depend on the value of parameter α . This parameter actually serves as a parameter that determines the beam current value and the character of beam-plasma interaction. It corresponds (correct to the factor γ^{-2}) to the ratio of the beam current to the limiting current in vacuum waveguide [14] $I_0 = mu^3 \gamma / 4e$, that is, $\alpha = (I_b/I_0) \gamma^{-2}$ (I_b is the beam current). The values $\alpha \ll \gamma^{-2}$, correspond to underlimiting beam current $I \ll I_0$ and the instability in this case is caused by induced radiation of system proper waves by the beam electrons. Neglecting the second and third terms one can obtain the well-known growth rate of resonant beam instability in plasma waveguide

$$\delta_{\text{und}} = \frac{\sqrt{3} \omega_0}{2 \gamma} \left(\frac{\omega_b^2}{2 \omega_0^2 (1 + \mu)} \right)^{1/3} \quad (30)$$

However, if dissipation exceeds growth rate, the instability turns to DSI with the growth rate

$$\delta_{\text{und}}^{(\nu)} = \frac{\omega_b \omega_0}{2 \gamma^{3/2} \omega_p} \left(\frac{\omega_0}{\nu} \right)^{1/2} \quad (31)$$

If the beam current increases and became higher than the limiting vacuum current that is,

$$\gamma^{-2} \ll \alpha \ll 1, \quad (32)$$

the instability has the same nature as the instability in medium with negative dielectric constant. If the beam is underlimiting, this effect is slight and is not observed. But now, this effect is dominant. Its distinctive peculiarity is that this effect attains its maximum in the point of exact Cherenkov resonance. The growth rate differs from Eq. (30) and is equal [13].

$$\delta_{\text{ovl}} = \frac{\omega_b \beta}{\gamma^{1/2} (1 + \mu)^{1/2}} \quad (33)$$

The different dependence of the growth rates of Eqs. (30) and (33) on beam density should be noted.

If, along with the beam current, dissipation also increases the instability turns to DSI of overlimiting e-beam with the growth rate [6].

$$\delta_{\text{ovl}}^{(\nu)} = \frac{\beta^2 \omega_b^2 \omega_0^2}{\gamma \omega_p^2 \nu} \quad (34)$$

We emphasize new dependence on ν , that is, actually we have new type of DSI. More critical dependence on ν is due to superposition of two factors those lead to NEW excitation.

Higher values of parameter α (that is, $\alpha \gg 1$) correspond to very high currents. For example, in the case of a cylindrical waveguide this condition leads to $I_b \geq 1,4 (mc^3/e) \beta^3 \gamma^3$ and means that the beam current is more than the limiting Pierce current. Until now such high currents beams have not been used in beam-plasma interaction experiments.

3.2. Equation for SVA and its solution: transition to the new type of DSI

In order to consider the evolution of an initial perturbation in a magnetized plasma waveguide penetrated by an OB, we proceed from the DR (27). Our steps coincide to those for the case of underlimiting e-beams: expand the DR (27) in series near ω_0 and k_0 (see (29) and derive an equation for SVA. Making use the condition of OB $2\beta^2 \gamma^2 \delta / k_0 u \geq 1$ [13], one can obtain [6, 12].

$$\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial z} \right) \left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} + \nu \right) E_0(z, t) = \delta_{\text{ovl}}^2 E_0(z, t), \quad (35)$$

(the denotations coincide to those in (8)). The Eq. (35) for SVA may be solved by analogy to solution of Eq. (8). Without delving into details, we present here the results [6, 12].

$$E_0(z, t) = -\frac{J_0}{2\sqrt{\pi}} \frac{\exp \chi^{(\text{ovl})}(z, t)}{(u - v_0)^{1/2} \delta_{\text{ovl}}^{1/2} (ut - z)^{1/2}} \quad (36)$$

$$\chi^{(\text{ovl})}(z, t) = \chi_0^{(\text{ovl})}(z, t) - v \frac{ut - z}{u - v_0} \quad ; \quad \chi_0^{(\text{ovl})}(z, t) = \frac{2\delta_{\text{ovl}}}{u - v_0} \{(z - v_0 t)(ut - z)\}^{1/2}$$

The analysis of the expression (36) is similar to previous case. It again reduces to the analysis of the exponent $\chi^{(\text{ovl})}(z, t)$. The analysis shows that unstable perturbations vary through the same range from v_0 to u . The analysis of the instability character (absolute/convective) fully coincides to that for underlimiting e-beams. However, in this case, the waveform is symmetric with respect to its peak. The peak places in the middle at all instants and moves at average velocity

$$w_{g0} = 1/2(u + v_0) \quad (37)$$

The field's value in the peak exponentially increases and the growth rate is equal to maximal growth rate for OB δ_{ovl} (33) (or, the same, to solution of the initial problem).

At fixed point z the SVA attains its maximum $\sim \exp \delta_{\text{ovl}} z / (uv_0)^{1/2}$ at the instant $t = z/w_{ao}$, where

$$w_{ao} = \frac{2uv_0}{u + v_0} \quad (38)$$

The expression $\delta_{\text{ovl}}/(uv_0)^{1/2}$ is the maximal spatial growth rate at wave amplification by OB, and coincides to result of the boundary problem. The SVA depends on the perturbations' velocity v as

$$E_0(z = vt, t) \sim \exp \{\Gamma_0(v)t\} \quad ; \quad \Gamma_0(v) = 2\delta_{\text{ovl}} \frac{\sqrt{(u-v)(v-v_0)}}{u - v_0} \quad (39)$$

The character of the space growth depending on perturbations' velocity is $\sim \exp \Gamma_0(v)z / (uv_0)^{1/2}$.

Dissipation fundamentally changes this picture of the instability. For given velocity v the dependence of the SVA on the dissipation level becomes

$$\Gamma_0(v) \rightarrow \Gamma_v(v) = \Gamma_0(v) - v \frac{u - v}{u - v_0} \quad (40)$$

Dissipation suppresses slow perturbations. Only high-velocity perturbations can develop. The threshold velocity is

$$V_{th}^{(\text{ovl})} = \frac{\lambda u + v_0}{1 + \lambda} \quad ; \quad \lambda = v^2 / 4\delta_{\text{ovl}}^2 \quad (41)$$

The dynamics of the peak in the presence of dissipation may be obtained by analyzing the equation

$$(z - w_{go}t)^2 - \lambda(ut - z)(z - v_0t) = 0 \quad (42)$$

The solution of Eq. (42) presents the peak's coordinate z_m

$$z_m = w_{go}t \left\{ 1 + \sqrt{\frac{\lambda}{1 + \lambda} \left(1 - \frac{uv_0}{w_{go}^2} \right)} \right\} \quad (43)$$

Substitution of z_m into $\chi^{(ovl)}$ gives the maximal growth rate under arbitrary v/δ_{ovl}

$$E_0(z = z_m, t) \sim \exp(\delta_{ovl}t \cdot f(\lambda)) \quad ; \quad f(x) = \sqrt{1 + x^2} - x \quad (44)$$

In limit of high-level dissipation, we have

$$E \sim \exp \delta_{ovl}^{(v)} t \quad (45)$$

where $\delta_{ovl}^{(v)}$ is given by Eq. (34). That is, with increase in level of dissipation the instability of OB transforms to the new type of DSI. The shapes of the waveform for OB instability for various level of dissipation are plotted in **Figure 2**. **Figure 3** presents the curve $f(x)$.

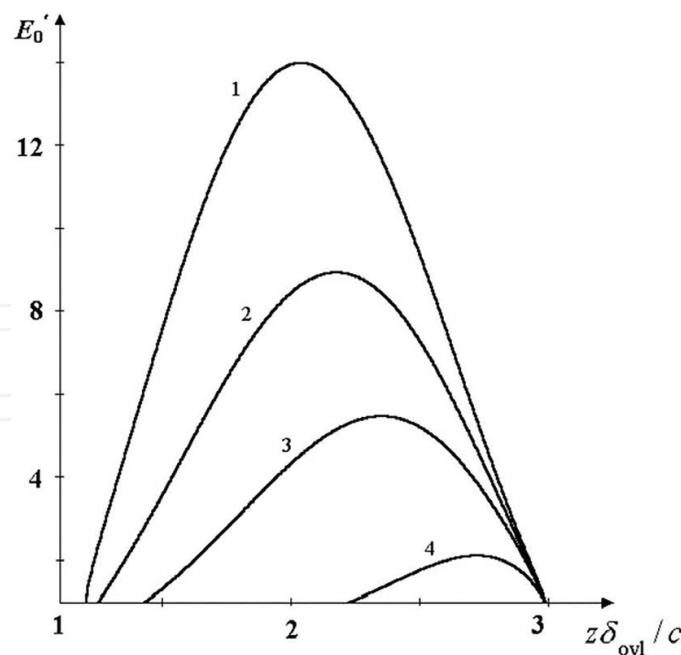


Figure 2. Shapes of the waveform versus longitudinal coordinate at fixed instant $t = 3/\delta_{ovl}$ for various values of parameter $k = v/\delta_{ovl}$ $k_1 = 0$, $k_2 = 1$, $k_3 = 2$, $k_4 = 4$.

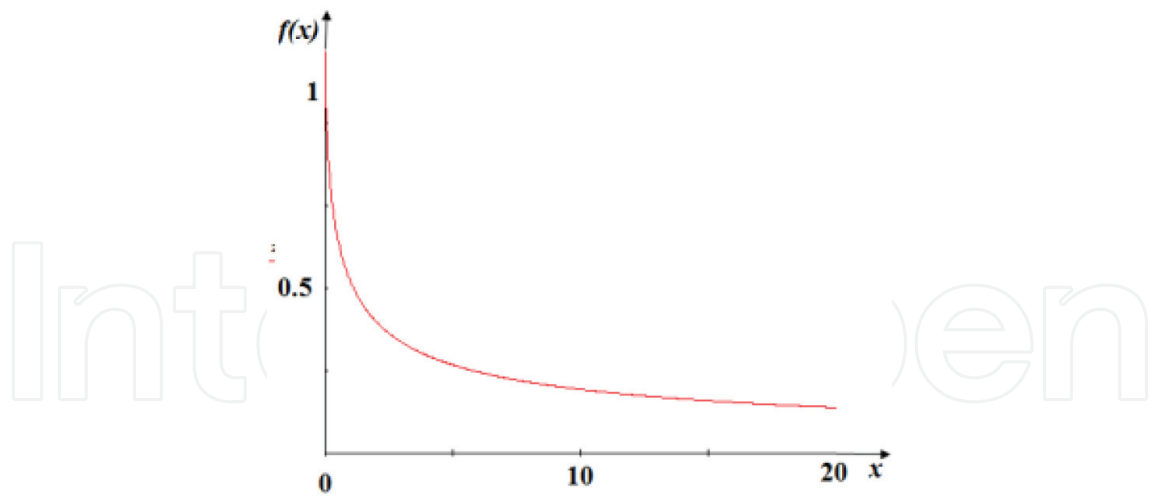


Figure 3. The function $f(x)$ gives the dependence of maximal growth rate on dissipation level.

4. The behavior of the instability in spatially separated beam-plasma system

4.1. Statement of the problem: the dispersion relation

There is a factor which significantly influences on the physics of beam-plasma interaction. The factor is the level of overlap of the beam and the plasma fields. The well-known beam-plasma instability corresponds to full overlap of the beam and the plasma fields (strong beam-plasma coupling). In this case, physical nature of developing instability is due to induced radiation of the system's normal mode oscillations by the beam electrons. The oscillations are determined by plasma alone, as its density is assumed much higher than the beam density. The beam oscillations are actually suppressed and do not reveal themselves. Excited fields are actually detached from the beam in that they exist in beam absence.

The opposite case when the beam and plasma fields are overlapped slightly is the case of weak beam-plasma coupling. It may be realized, for instance, if the beam and the plasma are spatially separated in transverse direction. This transverse geometry provides conditions for increasing the role of the beam's normal mode oscillations. In this case, the beam-plasma interaction has other physical nature. Electron beam is actually left to its own. Its oscillations come into play. Account of the beam's normal mode oscillation leads to substantially new effects. Moreover, there is NEW among beam proper waves. Its growth causes instability due to the sign of energy. The growth rate of this instability attains maximum in resonance of plasma wave with NEW. Resonance of this (wave-wave) type comes instead of wave-particle resonance (conventional Cherenkov Effect) and was named "Collective Cherenkov Effect" [14, 15].

Consider weak interaction of monoenergetic electron beam and plasma in waveguide in general form [8, 14]. The only assumption is following. The beam and plasma are separated spatially, which implies weak coupling of the beam and the plasma fields. For a start, we do not particularize the cross sections. The beam current is assumed to be less than the limiting

vacuum current. Dissipation in the system is taken into account by introducing collisions in plasma. We restrict ourselves by the case of strong external longitudinal magnetic field that prevents transversal motion of beam and plasma particles.

In strong external magnetic field, perturbations in plasma and beam have longitudinal components only. In such system, it is expedient to describe perturbations by using polarization potential ψ [14]. This actually is a single nonzero component of well-known Hertz vector.

We proceed from equations for ψ and for the beam and the plasma currents $j_{p,b}$.

$$\begin{aligned} \frac{\partial}{\partial t} \left(\Delta_{\perp} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \psi &= -4\pi (j_{bz} + j_{pz}) \quad ; \quad E_z = \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} \\ \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial z} \right)^2 j_b &= \frac{\omega_b^2 \gamma^{-3}}{4\pi} \frac{\partial}{\partial t} E_z \quad ; \quad \left(\frac{\partial}{\partial t} + \nu \right) j_p = \frac{\omega_p^2}{4\pi} E_z. \end{aligned} \quad (46)$$

Here $j_{bz}(\mathbf{r}_{\perp}, z, t) = p_b(\mathbf{r}_{\perp}) j_b(z, t)$ and $j_{pz}(\mathbf{r}_{\perp}, z, t) = p_p(\mathbf{r}_{\perp}) j_p(z, t)$ are perturbations of the longitudinal current densities of the beam and the plasma. Functions $p_{b,p}(\mathbf{r}_{\perp})$ describe transverse density profiles for beam and plasma. For homogeneous beam/plasma $p_{b,p} \equiv 1$, for infinitesimal thin beam/plasma $p_{b,p} \sim \delta(r - r_{b,p})$ (δ is Dirac function). Δ_{\perp} is the Laplace operator over transverse coordinates, z is longitudinal coordinate, t is the time, c is speed of light, $\omega_{p,b}$ are the Langmuir frequencies for plasma and beam respectively, ν – is the collision frequency in plasma, γ is the relativistic factors of the beam electrons, u is the beam velocity.

In general, the analytical treatment of the problem may be developed in different ways. The traditional way is to consider a multilayer structure of given geometry. With increase in number of layers this way leads to a very cumbersome DR. However, in the case of weak coupling (namely when the integral describing the overlap of the beam and the plasma fields (see below) is small), the interaction may be considered by another approach. The approach is perturbation theory over wave coupling [14]. Parameter of weak beam-plasma coupling serves as a small parameter that underlies this approach. This way leads to a DR of much simpler form, which, in addition, clearly shows the interaction of the beam and the plasma waves. Also, the procedure is not associated with a specific shape/geometry; that is, obtained results may be easily adapted to systems of any cross-section.

The set of Eq. (46) reduces to following eigenvalue problem

$$\Delta_{\perp} \psi - \kappa^2 \left[1 - p_p(\mathbf{r}_{\perp}) \delta \varepsilon_p - p_b(\mathbf{r}_{\perp}) \delta \varepsilon_b \right] \psi = 0 \quad ; \quad \psi|_{\Sigma} = 0 \quad (47)$$

where ψ is the proper function of the problem, Σ means the surface of the waveguide (it is not specified yet).

$$\kappa^2 = k^2 - \frac{\omega^2}{c^2} \quad ; \quad \delta \varepsilon_p = \frac{\omega_p^2}{\omega(\omega + i\nu)} \quad ; \quad \delta \varepsilon_b = \frac{\omega_b^2}{\gamma^3(\omega - ku)}. \quad (48)$$

ω and k are the frequency and longitudinal wave vector, ν is the frequency of plasma collisions. As we have mentioned earlier, direct solution of the problem (47) presents considerable

difficulties. However, in case of spatially separated beam and plasma that is, when $p_b(\mathbf{r}_\perp)p_p(\mathbf{r}_\perp) = 0$ and the integral describing the overlap of the fields (see below) is small, it is possible to apply perturbation theory. It assumes that in zero order approximation the beam and the plasma are independent and they may be described by two independent eigenvalue problems for plasma and beam respectively [14].

$$\Delta_\perp \psi_\alpha - \kappa^2 [1 - p_\alpha(\mathbf{r}_\perp) \delta \varepsilon_\alpha] \psi_\alpha = 0 \quad ; \quad \psi_\alpha|_\Sigma = 0 \quad ; \quad \alpha = p, b \quad (49)$$

Proper functions ψ_p and ψ_b of these zero-order problems as well as the zero-order DR for the beam and the plasma are assumed to be known. If one applies perturbation theory to the zero-order problems those are described by the DR

$$\left\{ D_p(\omega, k) \right\}_{\substack{\omega = \omega_0 \\ k = k_0}} = 0 \quad ; \quad \left\{ D_b(\omega, k) \right\}_{\substack{\omega = \omega_0 \\ k = k_0}} = 0 \quad (50)$$

(the point $\{\omega_0, k_0\}$ is the intersection point of the plasma and the beam curves) and search the solution of Eq. (47) in the form $\psi = A\psi_p + B\psi_b$, $A, B = \text{const}$, he can obtain in first order approximation the following DR

$$D_p(\omega, k) D_b(\omega, k) = G (\kappa^4 \delta \varepsilon_p \delta \varepsilon_b)_{\substack{\omega = \omega_0 \\ k = k_0}} \quad (51)$$

where

$$D_{p,b}(\omega, k) = k_{\perp p,b}^2 - \kappa^2 \delta \varepsilon_{p,b} = 0. \quad (52)$$

G is the coupling coefficient. It shows the efficiency of beam-plasma interaction, $k_{\perp p,b}$ are the actual transverse wavenumbers for the beam and the plasma respectively (see also [8])

$$G = \frac{\left(\iint_{S_w} p_p \psi_p \psi_b d\mathbf{r}_\perp \right) \left(\iint_{S_w} p_b \psi_p \psi_b d\mathbf{r}_\perp \right)}{\left(\iint_{S_w} p_p \psi_p^2 d\mathbf{r}_\perp \right) \left(\iint_{S_w} p_b \psi_b^2 d\mathbf{r}_\perp \right)} > 0 \quad (53)$$

$$k_{\perp p,b}^2 = \left(\iint_{S_w} \left((\nabla_\perp \psi_{p,b})^2 + \kappa^2 \psi_{p,b}^2 \right) d\mathbf{r}_\perp \right) \left(\iint_{S_w} p_{p,b}(\mathbf{r}_\perp) \psi_{p,b}^2 d\mathbf{r}_\perp \right)^{-1}$$

Mathematically, G is expressed in terms of integrals those represent the overlap of the beam and the plasma fields. Physically, it determines as far the field of plasma wave penetrates into beam and vice versa. According to our consideration, G is small $G \ll 1$. One more condition of validity of presented consideration is homogeneity of the beam and the plasma inside the cross sections.

4.2. The growth rates

The spectra of the beam waves are given by D_b Eq. (52) and have following form

$$\omega_{\pm} = ku(1 + x_{\pm}) \quad ; \quad x_{\pm} = \frac{\sqrt{\alpha}}{\gamma} \left(\pm \sqrt{\beta^4 \gamma^2 \alpha + 1} - \beta^2 \gamma \sqrt{\alpha} \right) \quad (54)$$

where $\alpha = \omega_b^2/k_{\perp b}^2 u^2 \gamma^3$ is the parameter that determines the beam current value (see previous section) $\beta = u/c$. The beam-plasma interaction in the absence of dissipation leads to conventional beam instability that is caused by excitation of the system normal mode waves by the beam electrons. Its maximal growth rate depends on beam density as $n_b^{1/3}$. With increase in level of dissipation the conventional beam instability is gradually converted to that of dissipative type. Its maximal growth rate depends on dissipation as $\sim 1/\sqrt{\nu}$. For these instabilities the normal mode oscillations of the beam are neglected. The concept of the NEW is invoked only to explain the physical meaning of DSI. These results are valid only for the case of strong beam-plasma coupling. The decrease in beam-plasma coupling leads to exhibition of the beam's normal mode oscillation. In this case, the instability is caused by the excitation of the NEW. Specific features of weak beam-plasma interaction should appear themselves in solutions of Eq. (51). If one looks them in the form $\omega = ku(1 + x)$, then Eq. (51) becomes

$$(x + q + iv/ku)(x - x_+) (x - x_-) = G\alpha/2\gamma^4 \quad (55)$$

where $q = (1/2\gamma^2) (k_{\perp p}^2 u^2 \gamma^2 / \omega_p^2 - 1)$. The usual Cherenkov resonance of the beam electrons with plasma wave corresponds to the condition $q = 0$; however, the resonance between the beam slow wave and plasma wave (collective Cherenkov effect) corresponds to $q = -x_-$. The interaction of the beam and plasma waves leads to instability. Mathematically, it is due to corrections to the expression for NEW. Using the condition of collective Cherenkov resonance one can obtain

$$\left(x' + i \frac{\nu}{2\gamma^2 ku} \right) x' = - \frac{G\sqrt{\alpha}}{4\gamma^3}. \quad (56)$$

where $x' = x - x_-$. In the absence of dissipation the growth rate of instability caused by NEW growth is

$$\delta_{\text{new}}^{(\nu=0)} = \frac{ku}{2\gamma} \sqrt{\frac{G\sqrt{\alpha}}{\gamma}}. \quad (57)$$

It depends on beam density as $n_b^{1/4}$. Under conventional Cherenkov resonance the system is stable. Dissipation exhibits itself as additional factor that intensifies growth of the NEW. Eq. (56) gives following expression for the growth rate upon arbitrary level of the dissipation [8].

$$\delta(\lambda) = \delta_{\text{new}}^{(\nu=0)} \left\{ \sqrt{1 + \lambda^2/4} - \lambda/2 \right\} \quad (58)$$

where $\lambda = \nu / (2\delta_{\text{new}}^{(\nu=0)} \gamma^2)$. The expression (58) shows gradual transition of no dissipative instability to that of dissipative type with increase in level of dissipation. This dependence on

dissipation coincides to that depicted in **Figure 3**. In the limit of strong dissipation $\lambda \gg 1$, Eq. (58) becomes

$$\delta(v \gg \delta_{\text{NEW}}^{(\nu=0)}) = \delta_{\text{NEW}}^{(\nu \rightarrow \infty)} \left[1 - \frac{G\gamma\sqrt{\alpha}(ku)^2}{\nu^2} \right], \quad (59)$$

where

$$\delta_{\text{NEW}}^{(\nu \rightarrow \infty)} = \frac{2\gamma^2 \left(\delta_{\text{NEW}}^{(\nu=0)} \right)^2}{\nu} = \frac{G\sqrt{\alpha}}{2\gamma} \frac{(ku)^2}{\nu} \quad (60)$$

$\delta_{\text{NEW}}^{(\nu \rightarrow \infty)}$ presents the maximal growth rate of the new type of dissipative instability, shown up in [8]. It also follows from Eq. (56) by neglecting first term in parentheses. The new type of dissipative beam-plasma instability is now substantiated for beam and plasma layers in waveguide. The cross-sections of the layers and the waveguide are arbitrary. The instability of new type results from the superposition of dissipation on the instability that is already caused by the growth of the NEW. The instability comes instead of the conventional DSI (with growth rate $\sim 1/\sqrt{\nu}$) when beam-plasma coupling becomes small. The dependence on dissipation becomes more critical. The same instability can be substantiated in finite external magnetic field also [18].

4.3. The space–time dynamics of the instability in spatially separated beam and plasma

We have already obtained some properties of the instability in system with spatially separated beam and plasma. Consider now the behavior of this instability in detail. In so doing, we consider the evolution of an initial perturbation in system with spatially separated e-beam and plasma. We proceed from the DR (51). The successive steps are known: to derive the equation for SVA, solve it and analyze the solution. As a result, we have following equation for SVA:

$$\left(\frac{\partial}{\partial t} + v_b \frac{\partial}{\partial z} \right) \left(\frac{\partial}{\partial t} + v_p \frac{\partial}{\partial z} + \nu^* \right) E_0(z, t) = \delta_0^2 E_0(z, t). \quad (61)$$

where $\delta_0 \equiv \delta_{\text{NEW}}^{(\nu=0)}$ (57), $v_{p,b}$ are group velocities of the plasma wave and the NEW of the beam, respectively, and $\nu^* = \text{Im } D_p (\partial D_p / \partial \omega)^{-1}$ is proportional to collision frequency $\nu^* = \text{const} \cdot \nu$.

The Eq. (61) is actually the same Eq. (35). This implies that the fields' space–time evolution at the instability development in spatially separated beam-plasma system qualitatively coincides to that of over-limiting e-beam instability. It remains to repeat briefly the milestones of the analysis above for behavior of OB instability in new terms (assuming $v_b > v_p$) and, where it is needed, to interpret results according new denotations. For this, we first rewrite the analyzing expression in new denotations

$$\chi_v^{(\text{ovl})} \rightarrow \chi_v^{(\text{ss})} = \frac{2\delta_0}{v_b - v_p} \sqrt{(z - v_p t)(v_b t - z)} - \nu^* \frac{v_b t - z}{v_b - v_p}. \quad (62)$$

For the instability under weak beam-plasma coupling the velocities of unstable perturbation vary through the range $v_p \leq v \leq v_b$. The character of the instability is determined by group velocities of plasma wave and the NEW. The statements on the character of the instability (convective or absolute) remain valid with account of replacements $v_0 \rightarrow v_p$ and $u \rightarrow v_b$. The place and the velocity of the peak of the wave train can be obtained, as earlier, by solving the equation

$$\frac{\partial}{\partial z} \exp \chi_v^{(ss)} = 0. \quad (63)$$

In the absence of dissipation, the peak places in the middle of the train at all instants that is, it moves at the average velocity $w_{gs} = (1/2)(v_b + v_p)$. The field value in the peak exponentially increases and the growth rate is equal to $\delta_{NEW}^{(v=0)}$ (57). In the absence of dissipation, the waveform is symmetric with respect to its peak at all instants.

Dissipation suppresses slow perturbations. The threshold velocity is (compare to previous subsection)

$$V_{th}^{(ss)} = \frac{\lambda' v_b + v_p}{1 + \lambda'} \quad ; \quad \lambda' = \left(\frac{v^*}{2\delta_0} \right)^2 \quad (64)$$

The wave train shortens. Only high velocity perturbations (at velocities in the range $V_{th} < v < v_b$) develop. Herewith the behavior of the fields in the peak (and the place/velocity of the peak) may be obtained by analyzing Eq. (63). If one takes into account the dissipation, the solution of (63) yields $z = w_0 t$, where

$$w_0 = \frac{1}{2} \left\{ v_b + v_p + \sqrt{\frac{\lambda'}{1 + \lambda'}} (v_b - v_p) \right\} > w_{gs} \quad (65)$$

The peak shifts to the front of wave train. For high-level dissipation, we have $w_0, V_{th}^{(ss)} \rightarrow v_b$ that is, one can conclude: the group velocity of perturbation of the new DSI is equal to the group velocity of the NEW. This distinguishes the DSI under weak coupling from the DSI of OB (where the velocity of perturbations was equal to the beam velocity).

Substitution of Eq. (65) into $\chi_v^{(ss)}$ gives us the dependence of the growth rate on dissipation of arbitrary level. The field value in the peak depends on dissipation as

$$E_0 \sim \exp \delta_0 t \left(\sqrt{1 + \lambda'^2/4} - \lambda'/2 \right) \quad (66)$$

This result agrees to Eq. (58). This coincidence actually serves as an additional proof of the correctness of the approach based on analysis of developing wave train (i.e., correctness of the initial assumptions, derived equation for SVA, its solution etc.). Analogous coincidence exists in case of underlimiting e-beams (see Section 2), but very cumbersome expressions (solutions of third-order algebraic equation) prevent showing it obviously.

In conclusion to present section, we can state that two various types of e-beam instabilities: (1) the OB instability and (2) the instability under weak beam-plasma coupling have similar behavior. Both these instabilities transform to dissipative instabilities with the maximal growth rate $\sim 1/\nu$. In spite of their different physical nature, these instabilities have similar mathematical description. The contribution of the OB in the DR is given by expression having first order pole. The DR of the systems with spatially separated beam and plasma also may be reduced to analogous form. For comparison: the contribution of underlimiting e-beam is given by an expression with second-order pole for all types beam instabilities (Cherenkov, cyclotron etc.). This leads to their similar behavior. However, a difference between these two DSI also exists. In system with OB dissipation shifts the velocities of unstable modes to the beam velocity u . In the second case, the velocities are approximately equal to group velocity of NEW.

5. The behavior of the Buneman instability in dissipative plasma

5.1. Statement of the problem: the equation for SVA

The physical essence of the Buneman instability (BI) [4] is in the fact that the proper space charge oscillations of moving electrons due to the Doppler Effect experience red shift, and this greatly reduced frequency becomes close to the proper frequency of ions. Actually, the BI is due to resonance of the negative energy wave with the ion oscillations. For future interpretations and comparisons, we present the well-known [1, 4] DR and the maximal growth rate for the simplest case of the BI (cold e-stream, heavy ions, and accounting for collisions)

$$1 - \frac{\omega_{Le}^2}{(\omega - \mathbf{k}\mathbf{u})(\omega + i\nu_{Bn} - \mathbf{k}\mathbf{u})} - \frac{\omega_{Li}^2}{\omega^2} = 0 \quad ; \quad \delta_{Bn}^{(m)} = \frac{\sqrt{3}}{2} \omega_{Le} \left(\frac{m}{2M} \right)^{1/3} \quad (67)$$

(\mathbf{u} is the velocity of streaming electrons, ω_{Le} and ω_{Li} are Langmuir frequencies for electrons and ions respectively, ν_{Bn} is the frequency of collisions). The BI develops if $\omega_{Le} \geq \mathbf{k}\mathbf{u}$, and the growth rate attains its maximum under $\omega_{Le} \approx \mathbf{k}\mathbf{u}$.

Now consider a plasma system, the DR of which may be written as

$$D_0(\omega, \mathbf{k}) + \Delta D = 0 \quad (68)$$

where $\Delta D = -\omega_{Li}^2/\omega^2$ describes the contribution of ions in the DR, while $D_0(\omega, \mathbf{k})$ describes contribution of moving electrons as well as collisions/dissipation in the system. In following consideration, we do not specify the form of $D_0(\omega, \mathbf{k})$. As $\omega_{Li} \ll \omega_{Le}$ we have $|\Delta D| \ll |D_0|$ and the ions in Eq. (68) play a role under small ω that is, $\omega_{Li} \gg \omega \rightarrow 0$. One can at once see imaginary roots of the Eq. (68). The system becomes unstable (low frequency instability) and the growth rate may be obtained from

$$|\omega(k)|^3 = \omega_{Li}^2 \left[\left(\frac{\partial D_0(\omega, k)}{\partial \omega} \right)_{\omega \rightarrow 0} \right]^{-1} \quad (69)$$

An initial perturbation arises and the instability begins to develop in point $z = 0$ (electron stream propagates in the direction $z > 0$) at instant $t = 0$. Our aim is to obtain the shape of the perturbation and investigate in detail the behavior of the BI. The procedure for obtaining the equation for SVA is known. Applying this procedure, we arrive to following Eq. [16]

$$\frac{\partial^2}{\partial t^2} \left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z} + \nu \right) E_0(z, t) = i |\delta_{Bn}|^3 E_0(z, t) \quad (70)$$

$$|\delta_{Bn}|^3 = \left(\frac{\omega_{Li}^2}{\partial D_0 / \partial \omega} \right)_{\omega \rightarrow 0, k=k_0} ; \quad v_0 = - \left(\frac{\partial D_0 / \partial k}{\partial D_0 / \partial \omega} \right)_{\omega \rightarrow 0, k=k_0} ; \quad \nu = \left(\frac{\text{Im} D_0}{\partial D_0 / \partial \omega} \right)_{\omega \rightarrow 0, k=k_0} + ik_0 v_0 \equiv \nu' + ik_0 v_0$$

$\text{Im} \delta_{Bn}$ is the general form of the resonant growth rate of the low-frequency BI [1, 4] (compare to Eq. (67)); v_0 is the group velocity of the resonant wave in the system. Here, it is equal to velocity of streaming electrons; ν' actually presents dissipation. In unbound plasma, the main cause of dissipation is collisions of plasma particles. Equality of the ν' in this form to collision frequency is not obligatory.

Eq. (70) may be solved in known manner: that is, by using the Fourier and Laplace transformations. The problem reduces to integration in the inverse transformation. All these steps are known. So as not to repeat, we at once present resulting expression for the SVA [16]

$$E_0(z, t) = \frac{J_0}{\sqrt{2\pi}} \frac{\exp \left\{ \chi_{Bn}(z, t) - \nu' \frac{z}{v_0} \right\} e^{i \left(\frac{\chi_{Bn} + \sqrt{3} k_0 z}{\sqrt{3}} - \frac{\pi}{6} \right)}}{(6 v_0 z |\delta_{Bn}|^3)^{1/2}} \quad (71)$$

$$\chi_{Bn}(z, t) = \frac{3\sqrt{3}}{4} |\delta_{Bn}| \left\{ \frac{2z\tau^2}{v_0} \right\}^{1/3} ; \quad \tau = t - z/v_0,$$

5.2. Analysis of the Buneman instability behavior

As earlier, the structure of the fields is basically determined by the factor [16].

$$\exp \left\{ \frac{3\sqrt{3}}{4} |\delta_{Bn}| \left\{ \frac{2z\tau^2}{v_0} \right\}^{1/3} - \nu' \frac{z}{v_0} \right\}. \quad (72)$$

In the absence of dissipation the velocities of unstable perturbations range from 0 to the group velocity v_0 . The length of the induced wave train increases as $l \approx v_0 t$. The condition

$$\frac{\partial}{\partial z} \left(\chi_{Bn} - \nu' \frac{z}{v_0} \right) = 0 \quad (73)$$

(compare to Eq. (16)) determines the peak's movement. In the absence of dissipation the peak disposes on $2/3$ of the train's length from its front and moves at velocity $v_0/3$. Substitution of $z = v_0 t/3$ into Eq. (72) gives the field's behavior in the peak. It grows exponentially

$E_0 \sim \exp[(\sqrt{3}/2)|\delta_{Bn}|t]$ and the growth rate is equal to the maximal growth rate of the BI obtained earlier as a result of initial problem (e.g., see [1, 4] and Eq. (67)). However, in contrary to this approach, the initial problem does not give the point of the maximal growth. This approach gives the point. In addition, it gives the rates of the field growth in every point of the wave train (in the presence of dissipation also).

Dissipation changes the fields' dynamics and mode structure. It is easily seen from Eq. (72) that dissipation suppresses fast perturbations. The threshold velocity v_{th} can be obtained from the equation $\chi_{Bn}(z, t) = vz/v_0$ and is equal

$$v_{th} = \frac{v_0}{1 + \lambda_0^{3/2}} \quad ; \quad \lambda_0 = \frac{2^{5/3}}{3^{-3/2}} \frac{v'}{|\delta_{Bn}|} \quad (74)$$

The wave train shortens. Actually the pulse slows down. Dissipation influences on the peak location/movement. Its place $z = z_{max}$ can be obtained from the equation

$$(v_0 t - 3z)^3 = (3\lambda_0)^3 z^2 (v_0 t - z) \quad (75)$$

The solution of this third-order algebraic equation gives location and velocity of the peak under arbitrary ratio v'/δ_{Bn} . To avoid cumbersome expressions, we present here the solution only in the most interesting limit of high dissipation $\lambda_0 \rightarrow \infty$.

$$z = z_{max} = \left(\frac{3^{3/4}}{2^{5/2}} \right) \left(\frac{|\delta_{Bn}|}{v'} \right)^3 v_0 t \quad (76)$$

Substitution of this expression into $\chi_{Bn}(z, t)$ gives the field's behavior in the peak under high-level dissipation. The field's value increases exponentially

$$E_0 \sim \exp\{\delta_v t\} \quad (77)$$

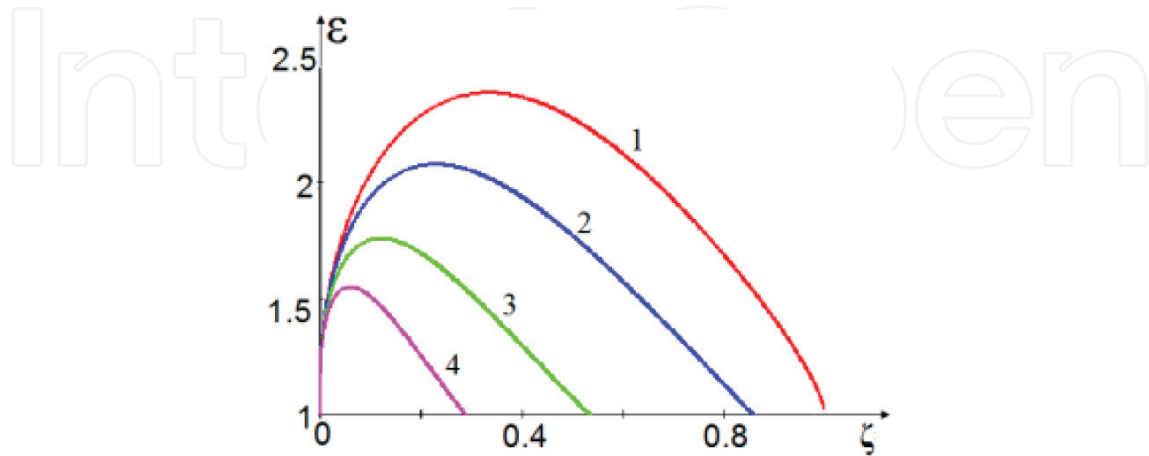


Figure 4. The shapes of initial perturbation for various level of dissipation. The dimensionless distance $\zeta = z\delta_{Bn}/v_0$, and the dimensionless field $\varepsilon = E_0/(J_0/(v_0\delta_{Bn}))$ are marked along the axes. Curve 1 corresponds to $\lambda' = v/|\delta_{Bn}| = 0$; curve 2 – To $\lambda' = 0.5$; curve 3 – To $\lambda' = 1.5$; curve 4 – To $\lambda' = 3$.

where the growth rate $\delta_\nu = \sqrt{|\delta_{Bn}|^3/2\nu}$ is nothing else, as the growth rate of DSI of conventional type [1, 16, 17]. This once again justifies that high-level dissipation transforms the BI to DSI.

In addition, the expression for $\chi_{Bn}(z, t)$ gives much other information on the character of BI development. For example, by substituting $z = vt$ one can investigate the behavior of the perturbation, moving at given velocity v and determine the rate of their growth

$$E_0(z = vt, t) \sim \exp G(v)t \quad ; \quad G(v) = \frac{3\sqrt{3}}{\nu_0} |\delta_{Bn}| \left\{ 2v(\nu_0 - v)^2 \right\}^{1/3} - \nu' \frac{v}{\nu_0} \quad (78)$$

Figure 4 presents shapes of induced wave train for various levels of dissipation.

6. Conclusion

Now, we can generalize the properties of the SI. Originated perturbations form a wave train, carrier frequency and wave vector of which are determined by resonant conditions. The expression for space–time distribution of the fields gives much information on the behavior of the instability in limit of comparatively large times. The solutions of conventional initial and boundary problems follow from the expression by itself. The growth rate in the peak is equal to maximal growth rate of resonant instability δ , which usually describes given instability. The initial value problem gives the same growth rate without specifying where the growth takes place. That is, the approach gives realistic picture of the SI development. Dissipation leads to shortening of the wave train. With increase in level of dissipation the SI gradually turns to dissipative type. In the limit $\nu \gg \delta$ (ν is the collision frequency) the growth of the fields takes place according to dissipative instability. The approach gives also information on the growth rate for arbitrary δ/ν . Obvious expression may be obtained by solving algebraic equation of second/third order.

The approach justifies existence of two new, previously unknown types of DSI. For these DSI, the role of the beam's space charge and/or proper oscillation becomes decisive. For both DSI, the growth rates have more critical dependence on dissipation as compared to conventional. Presented approach obviously shows the transition to the new types of DSI.

Actually the approach presents solution of the well-known problem of time evolution of initial perturbation in systems those undergo the instabilities of streaming type. The importance of the problem is doubtless. Its traditional solution is restricted by mathematical difficulties. Presented methods allows without any difficulties obtain result for various SI in spite of their different mathematical description (e.g., the description of Buneman instability differs from the instability in spatially separated beam-plasma system and from beam-plasma instabilities; herewith, the description various types of beam-plasma instabilities (Cherenkov, cyclotron, and other) also differs from each other). The approach by itself unified the differences. For beam-plasma instabilities results of the approach are unified even more and their usage is not

more difficult than usage of the result of the initial and boundary problems (in spite of presented approach gives incomparably more data). In this sense, the approach can be used instead of the problems. It could seem that the procedure is a bit more difficult. However, this difficulty only seems.

The general character of presented approach should be emphasized once more. It is based on very general assumptions and does not refer on any particular model. The approach transforms the general form of the DR to an equation for SVA of the developing wave train. For a large class of beam-plasma instabilities (Cherenkov, cyclotron, etc.), the equation for SVA is actually the same. Its solution gives analytical expression describing evolution of initial perturbation. Various SI evolve in similar manner. This emphasizes identity of their physical nature (induced radiation of the system proper waves by the beam electrons). For given instability, one should specify two parameters only: the resonant growth rate and the group velocity of the resonant wave. Obtained expression gives detailed information on the instability. The information is: the shape of developing wave train (envelope), velocities of unstable perturbations, the type of given instability (absolute or convective), location of the peak and the character of its movement, the rate of field's growth in the peak, temporal and spatial growth rates, the rate of growth for perturbation moving at given velocity. Most of these data are unavailable by other methods.

Validity limitations also should be mentioned. Obtained results may not be applied to the systems where beam instability is caused by finite longitudinal dimension, for example, Pierce instability.

Presented approach has neither inner contradictions, no contradictions to previous results of the beam-plasma interaction theory. Its results fully coincide to those obtained by direct analysis of the DR. In some cases, (e.g., for overlimiting e-beam instability and the instability in spatially separated beam-plasma system) obvious analysis is possible due to comparatively simple contribution of the beam in the DR (namely when the contribution has first (but not second) order pole).

The results of presented approach actually are continuation and further development of the results of the initial and boundary problems. In its turn, the results of the problems have been repeatedly tested and rechecked experimentally. This actually can serve as confirmation of validity of the approach.

In [19, 20] the nonlinear dynamics of the beam-plasma instability was investigated numerically at no stationary beam injection into plasma-filled systems. The results show that at the initial stage of instability development the field has a shape matching reasonably to presented results.

Obtained results on SI evolution help to understand how the instability transforms given equilibrium of background plasma, estimate the level and/or scale of originated irregularities clear up how the nonlinear stage arises and predict saturation mechanisms. The systems, to which this may be applied are numerous, as the SI are the most common instabilities: from the Earth ionosphere to current carrying plasma (where the Buneman instability plays important role). Not to mention relativistic microwave electronics etc.

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