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Extreme Value Analysis and Risk Communication for a Changing Climate

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Abstract

Due to climate change, the common assumption of stationarity in extreme value analysis of climate extremes has been gradually challenged. The familiar concepts such as a return period and a return level do not apply in a changing climate. To quantify and communicate risk of climate extremes in a changing climate, nonstationarity should be considered carefully. In this chapter, both the concepts and interpretations of return period, return level, failure risk, and reliability under nonstationary condition were interpreted. It was concluded that the two interpretations of the return period became divergent under nonstationary condition, while the two interpretations of failure risk were consistent irrespective of stationarity. Moreover, two examples of risk communication based on generalized extreme value (GEV) distribution for nonstationary climate extremes were presented. In the first example, climate change and its impacts on global air temperature extremes were detected. In the second example, extreme value analysis was firstly applied to precipitation extremes at two weather stations in China. Then, the fitted GEV distribution on historical records was also extrapolated for future risk communication. With these examples, the concepts those were related to risk measure and communication in a changing climate could be easily understood and applied in practice.

Keywords: extreme value theory, nonstationarity, engineering design, return level, failure risk, reliability

1. Introduction

Extreme climate events could, directly or indirectly, impact all sectors of the economy leading to severe losses of life and property [1–3]. Mitigating natural hazards caused by extreme climate events is crucial to the sustainable development of human society and economy [4]. In IPCC's report, an extreme climate event is generally defined as the occurrence of a value of a weather or

climate variable above (or below) a threshold value near the upper (or lower) ends of the range of observed values of the variable [1]. The fundamental probability theory of extreme values has been well developed for a long time and already applied in resolving the practical problems in engineering design and risk management [5–7]. For the most part, extreme value theory (EVT) assumes that extreme events are stationary, and these extremes could be successfully characterized by the probability distributions such as the generalized extreme value (GEV) and generalized Pareto (GP) distribution [8, 9]. The occurrences of extreme events are also assumed to be independent or weakly dependent, then, the return levels and return periods could be easily determined [7, 10]. Under stationary condition, there is a simple one-to-one relationship between a return period and a return level, and these two terms can be easily understood [10, 11]. Moreover, risk could be simply communicated using the probability distributions of extremes derived from EVT [11]. In this study, the term "risk" merely refers to the probability of an extreme event with substantial consequence but not the expected loss in general risk analysis.

During the past few decades, there were clear and convincing evidences for global warming and climate change [12], which raised fundamental interdisciplinary issues of risk analysis and communication [13, 14]. As climate changes, weather or climate extremes also change [4] and gradually challenge the stationarity assumption in climate and weather extreme value analysis [6, 11, 15–17]. It has been documented that in some places, climatic and hydrological extremes exhibit some type of nonstationarity in the form of trends, shifts or a combination of them [18–20]. In a nonstationary world, both the severity and frequency of climate and weather extremes will change [2, 3, 10, 16, 17]. Consequently, extreme value analysis of climate and weather extremes has to consider and account for the nonstationarity [15]. Katz et al. [21] presented a nonstationary GEV distribution by introducing time as a covariate. He further showed that both GEV and GPD distributions could be retained under nonstationarity, and maximum likelihood method was also applicable for parameter estimation [6]. Nonstationary extreme values modeling based on GEV and GPD distributions has been realized in R-package *ismev* [22] and *extRemes* [23]. The other R-package GAMLSS (generalized additive model in location, scale, and shape) also allows nonstationary modeling for block maxima, where the parameters are modeled as linear or smooth functions of covariates [24]. Another available R-package for nonstationary extreme value analysis is *GEVcdn*, in which the parameters in GEV distribution are specified as a function of covariate using a conditional density network [25]. Besides these R-packages, nonstationary extreme value analysis could also been implemented using a MATLAB toolbox *NEVA* [26]. Although the nonstationary models in these packages performed better than the stationary equivalents in fitting nonstationary climate extreme, the return period (or return level) and risk for nonstationary conditions were not explicitly presented.

In nonstationary extreme value analysis, the concepts of the return period (or return level) and risk needed to be carefully reformulated and extended, because these familiar concepts, strictly speaking, no longer apply in a nonstationary climate [27]. In stationary cases, there exist two parallel interpretations for the return period: expected waiting time to an extreme event and expected number of extreme events in a given return period [7, 11]. In addition, the return level is the same in each year under stationary conditions. In Wigley [16], the return period was defined as the expected waiting time, and the influence of nonstationarity on the risk of

extremes was presented using some simple probability arguments. The concept of the return period was further extended to nonstationary condition in Olsen et al. [10], where the return period was defined as the expected waiting time until an exceedance as the measure of risk. The alternative interpretation of the return period (expected number of extreme events in a given return period) for nonstationary conditions was clearly explained in Parey et al. [28, 29]. Recently, Cooley [11] reviewed these two definitions of the return period suggested by Olsen et al. [10] and Parey et al. [28, 29], and proposed that the return period could be used to communicate risk in nonstationary climate. From the perspective of engineering design, Salas and Obeysekera [7] illustrated the estimation of the return period and examined the failure risk of hydrological structures in nonstationary climate. Rootzén and Katz [30] also concerned the failure risk in the design period and proposed a risk-based engineering design concept, Design Life Level, which served as the basis of risk communication in a nonstationary climate. In the above literatures, the concepts of the return period or return level have been extended and adapted to nonstationary condition; however, the interrelations between return period and risk communication, especially for engineering design purpose, was still ambiguous. The major reason causing such ambiguity is the diversified explanations of one terminology for different purposes. Therefore, a comprehensive interpretation of the return period (or return level) and failure risk (or reliability) under either stationary or nonstationary conditions simultaneously are needed.

The aim of this chapter is to present the extension process of return period, return level, and failure risk from stationary condition to nonstationary condition in a different way so that the commonness and difference could be clearly identified. Consistent with the way how a return period is defined and derived in some previous literatures, extreme value analysis will apply to the time series of annual maxima in this study. Accordingly, GEV distribution is used to illustrate the computation of the return period (return level) and failure risk (reliability) in nonstationary climate.

2. Concepts and interpretations

In some previous literatures, the interpretation of the return period usually began with the simple one-to-one relationship between a return period and a return level under stationary condition [7, 10, 28–30]. Like Cooley [11], we define random variable M_y as the annual maxima of climate or weather events for year y, and $\{M_y\}$ are assumed to be temporally independent. The cumulative probability distribution of M_y is denoted by

$$F_y(x) = P(M_y \leq x) \tag{1}$$

In this study, we try to explain the concepts of the return period and failure risk using the time series of annual maxima and the underlying stochastic process but omitting the assumption of stationarity or nonstationarity. In practice, analyzing the extremes and their probability distributions is usually considered as the basis of frequency analysis, engineering design, and risk assessment.

2.1. Waiting time-based concepts

Given a exceedance level x, let T be the waiting time (from $y = 0$) until an exceedance over this level x occurs [11], then the discrete probability density of random variable T is generally given by [7, 11]:

$$
\begin{aligned}
P(T = t) &= P\big(M_1 \leq x, M_2 \leq x, \cdots, M_{(t-1)} \leq x, M_t > x\big) \\
&= P(M_1 \leq x)P(M_2 \leq x)\cdots P\big(M_{(t-1)} \leq x\big)P(M_t > x) \\
&= \prod_{y=1}^{t-1} F_y(x)(1 - F_t(x))
\end{aligned}
\tag{2}
$$

where the second line in Eq. (2) is based on the temporal independence assumption. Then, the expectation of waiting time T is computed as

$$
\begin{aligned}
E[T] &= \sum_{t=1}^{\infty} t P(T = t) \\
&= \sum_{t=1}^{\infty} t \prod_{y=1}^{t-1} F_y(x)(1 - F_t(x)) \\
&= 1 + \sum_{i=1}^{\infty} \prod_{y=1}^{i} F_y(x)
\end{aligned}
\tag{3}
$$

The details of the derivations of Eq. (3) were shown in the appendix in [11]. The first definition of the return period is based on the expected waiting time. Specifically, a Y-year return period can be interpreted as: the expected time to the next extreme event is Y years [10].

Next, we adopt the commonly used definition of failure risk for an engineering structure, which is interpreted as the probability of the failure or the probability of exceedance over its design level in its design life period. We denote the failure risk by R and the design life period by L (in frequency analysis or engineering design, the denotations L and Y were usually not strictly distinguished). In terms of expected waiting time, the failure risk of a focal structure within its design life period is equivalent to the probability that the expected time of exceedance is less than or equals to the length of the design period, $R = P(T \leq L)$. Accordingly, the non-exceedance probability $P(T \leq L)$ can also be given by the cumulative probability of the waiting time T [7]:

$$
\begin{aligned}
R &= P(T \leq L) \\
&= \sum_{t=1}^{L} P(T = t) \\
&= \sum_{t=1}^{L} \prod_{y=1}^{t-1} F_y(x)(1 - F_t(x)) \\
&= 1 - \prod_{y=1}^{L} F_y(x)
\end{aligned}
\tag{4}
$$

Consequently, the reliability of the focal structure within its design life period is $R_\ell = 1 - R$.

2.2. Expected number-based concepts

We define random variable N as the number of exceedances over a given exceedance level x occurring in Y years period beginning with the year $y = 1$ and ending with the year $y = Y$ [7, 11]. In each year, we have the following indictor function:

$$I(M_y > x) = \begin{cases} 1, & M_y > x \\ 0, & M_y \leq x \end{cases} \tag{5}$$

Then, we get

$$N = \sum_{y=1}^{Y} I(M_y > x) \tag{6}$$

The expectation of N becomes

$$\begin{aligned} E[N] &= \sum_{y=1}^{Y} E\big[I(M_y > x)\big] \\ &= \sum_{y=1}^{Y} P(M_y > x) \\ &= \sum_{y=1}^{Y} \big(1 - F_y(x)\big) \end{aligned} \tag{7}$$

Now, we say that the Y-year return period can also be interpreted in an alternative way: in Y years the expected number of exceedance events is 1 [28, 29].

Similarly, the reliability of a focal structure in its design life period L can be understood as there are no exceedance events occurring from $y = 1$ to $y = L$. Then, the reliability can be computed as

$$\begin{aligned} R_\ell &= P\big(M_1 \leq x, M_2 \leq x, \cdots, M_{(L-1)} \leq x, M_L \leq x\big) \\ &= \prod_{y=1}^{L} P(M_y \leq x) = \prod_{y=1}^{L} F_y(x) \end{aligned} \tag{8}$$

From Eqs. (4) and (8), we find that the two parallel interpretations of failure risk (or reliability) of a focal engineering structure are equivalent irrespective of $\{M_y\}$ is stationary or nonstationary.

3. Risk communication

3.1. Risk measure under stationarity

Under a stationary assumption, $\{M_y\}$ is identically distributed with a distribution function $F(x)$, where the year index y is discarded for notational simplicity. Now, the relationship between a

return period (Y) and the associated return level (x_Y, a special exceedance level) can be revealed by the following equation [11, 30]:

$$F(x_Y) = P(M \le x_Y) = 1 - 1/Y \tag{9}$$

The Y-year return level of annual extreme M is defined to be the $(1 - 1/T)$-th quantile of the distribution of climate extreme in any year. In addition, we have $P(M > x_Y) = 1/Y$. That means that the exceedance probability over the return level x_Y is $1/Y$ for each year.

It has been proved that the two interpretations of return period in the stationary case are both correct with this identical exceedance probability under stationarity assumption [11]. Substituting Eq. (9) into Eq. (3), we get the interpretation of the return period based on waiting time of exceedance:

$$E[T] = 1 + \sum_{i=1}^{Y} \prod_{y=1}^{i} (1 - 1/Y) = Y \tag{10}$$

Similarly, substituting Eq. (9) into Eq. (7), we get the alternative interpretation of return period based on expected number of exceedance events:

$$E[N] = \sum_{i=1}^{Y} 1/Y = 1 \tag{11}$$

The simple one-to-one relationship between a return period and a return level in the stationary case has been commonly utilized in frequency analysis and engineering design practice [8, 21]. For example, the frequency or expected waiting time of extreme events exceeding a given exceedance level can be easily determined using Eq. (9) in frequency analysis of climate extremes. In practice, a very important concept for an engineering structure is the design life period. Reversely, given a design life period or exceedance probability, return levels could also be determined easily. Moreover, the failure risk or reliability of a focal structure in its design life period L could also be evaluated using a simpler formulation

$$R = 1 - (F(x_D))^L \tag{12}$$

where x_D is the design level in engineering design.

3.2. Risk measure under nonstationarity

Under nonstationary condition, $\{M_y\}$ is no more identically distributed. In frequency analysis, engineering design, and risk assessment, the dependence of probability distributions $F_y(x)$ on the year index y should be considered. It is more valuable to do extreme value analysis within the design life period. We have shown the two different interpretations of a return period in Section 2. Under nonstationary condition, the relationship between the return period and the associated return level could be expressed independently using Eqs. (3) and (7). Given a return period or design life period Y (Y and L are substitutable here), the Y-year return level could be estimated by setting $E[T] = Y$ and $E[N] = 1$, respectively [10, 28, 29]. Theoretically speaking, the

Y-year return level in the nonstationary case could be estimated by solving the following two equations numerically

$$Y = 1 + \sum_{i=1}^{\infty} \prod_{y=1}^{i} F_y(x_Y) \tag{13}$$

$$1 = \sum_{i=1}^{Y} \left(1 - F_y(x_Y)\right) \tag{14}$$

To determine $F_y(x)$, fitting the historical records of annual maxima to nonstationary extreme value distribution is the first step. Moreover, to estimate the return level of extremes or assessing the failure risk of a focal structure in its rest life span, it is necessary to extrapolate the trend or shift in climate extremes. Cooley [11] showed that it was unnecessary to extrapolate $F_y(x)$ indefinitely and an accurate estimation of the return level could be obtained, when $F_y(x)$ was monotonically increasing. For computational simplicity, the definition of the return period based on the expected number of events has more advantage since the maximum extrapolation length is Y years but not indefinitely to $+\infty$.

The return level in Eqs. (13) and (14) are the two extensions of the return period in the stationary case; however, these two extensions are not applicable in practical engineering design [7, 30]. For engineering design purpose, Rootzén and Katz [30] presented a new concept, Design Life Level, by keeping the failure risk at a low constant level during the design life period. The relationship between Design Life Level and design life period was expressed by the following equation [30]:

$$\begin{aligned} F_{1 \sim Y}(x) &= P(M_{1 \sim Y} \leq x) \\ &= P(M_1 \leq x)P(M_2 \leq x) \cdots P\big(M_{(t-1)} \leq x\big)P(M_t \leq x) \\ &= F_1(x) * F_2(x) * \cdots * F_Y(x) \end{aligned} \tag{15}$$

where $M_{1 \sim Y} = max\{M_1, M_2, \cdots, M_Y\}$ denoted the largest annual maxima during the design life period $1 \sim Y$. Usually, the mathematical expression of $F_{1 \sim Y}(x)$ is analytically intractable, while its numerical approximation $\widehat{F}_{1 \sim Y}(x)$ is frequently used in practice. Given a failure risk, $\widehat{r}$, of a focal engineering structure during its design period $1 \sim Y$, the associated design life level could be computed by

$$DLL = \widehat{F}_{1 \sim Y}^{-1}(1 - \widehat{r}) \tag{16}$$

A variant of Design Life Level is Minimax Design Life Level [30]. The computation of Minimax Design Life Level is even simpler. During the whole design life period $1 \sim Y$, we can obtain a series of return levels: $\left\{\widehat{F}_y^{-1}(1 - \widehat{r})\right\}, y = 1, 2, \cdots, Y$, and the Minimax Design Life Level is

$$minmaxDLL = max\left\{\widehat{F}_y^{-1}(1 - \widehat{r})\right\}, y = 1, 2, \cdots, Y \tag{17}$$

Similarly, the first step to compute the Design Life Level or Minimax Design Life Level is nonstationary modeling of historical climate extremes. Then, the trends of extremes would be

extrapolated over the design life period. Moreover, the statistical uncertainty in the return period and Design Life Level can be described by computing the standard errors using the delta method [11, 30].

4. Applications

In this section, we present two examples of extreme value analysis and risk communication in a changing climate. The cumulative distribution function of the GEV is expressed as [8]:

$$F(x) = \exp\left\{-\left[1 + \varepsilon\left(-\frac{x-\mu}{\sigma}\right)\right]^{-1/\varepsilon}\right\}, 1 + \frac{\varepsilon(x-\mu)}{\sigma} > 0 \qquad (18)$$

where μ, $\sigma > 0$, and ε are the location, scale, and shape parameters, respectively. Constant parameters correspond to stationary GEV distribution, while time-varying parameters correspond to nonstationary GEV distribution. The time-varying parameters in nonstationary GEV distribution could be modeled as the function of time or other climate indictors [6]:

$$F_y(x) = \exp\left\{-\left[1 + \varepsilon_y\left(-\frac{x-\mu_y}{\sigma_y}\right)\right]^{-1/\varepsilon_y}\right\} \qquad (19)$$

where y is the year index. Commonly, the location parameter μ_y and/or the scale parameter σ_y are assumed to be time varying, while the shape parameter is assumed to be constant [6–8, 26]. In particular, the extrapolation of $F_y(x)$ into the future design life period is reasonable, only if the location and/or the scale parameters have linear or log-linear trends [7, 26]. Before extrapolation, it is needed to select a best fitting GEV distribution model, and the model selection is usually based on AIC or BIC [6].

The first example of risk communication was for global annual maximum near surface air temperature (1948–2015). The global gridded data were extracted from the reanalysis products with a spatial resolution of 2.5 * 2.5 provided by Earth System Research Laboratory, NOAA (http://www.esrl.noaa.gov). For each grid, the time series of annual maximum near surface air temperature from 1948 to 2015 was firstly constructed and the trend was detected using the Mann-Kendall (M-K) test method [32, 33]. The test result was showed in **Figure 1(a)**. Both positive and negative trends at the 5% significance level were detected during the past 68 years (1948–2015) for most part of the earth. The time series with significant trends will be fitted using nonstationary GEV distribution with time-varying parameters. Otherwise, a stationary GEV distribution with constant parameters will be applied. Like Cheng et al. [26], only the location parameter was assumed to be linearly varying with time. Nonstationary modeling was performed with the R-package *extRemes* [23]. The aim of this example was to show the changes in climate extremes caused by climate change and how this change impacted risk communication; therefore, we did not extrapolate the trends of temperature extremes but only computed the 20-year return level (the expected number-based return level during 1996–2010). Solving Eq. (14)

relied on numerical optimization techniques, and in this study, the particle swarm optimization method was applied. The result of the global 20-year return level of annual maximum near surface air temperature in 1996–2015 was shown in **Figure 1(b)**.

In the second example, we used two time series of annual maximum precipitation (AMP) to illustrate the risk measure and communication under both stationary and nonstationary conditions. The two AMP time series were extracted from observation dataset of daily precipitation,
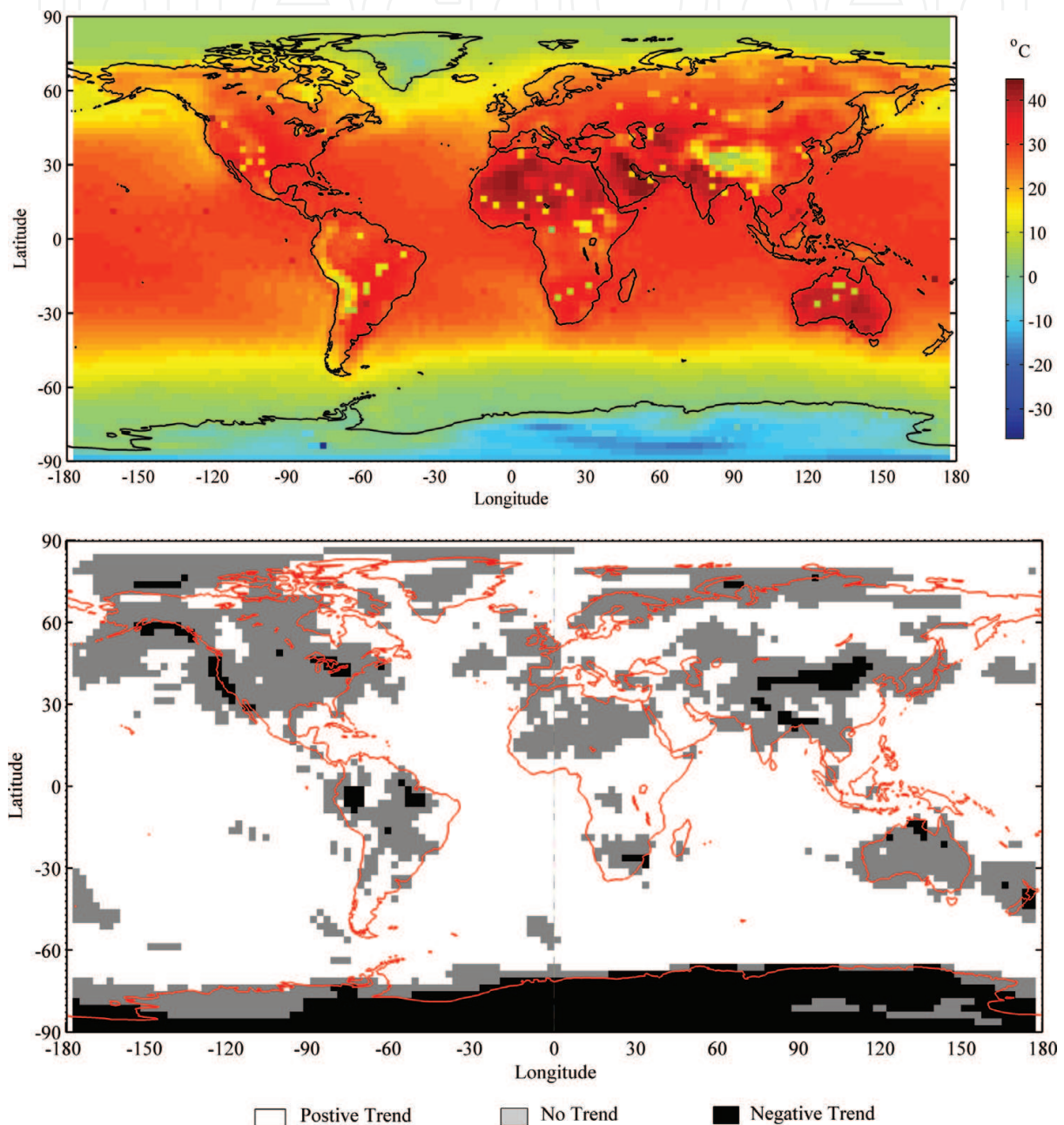

Figure 1. (a) M-K test for global annual maximum near surface air temperature (1948–2015) (positive trend in white: no significant trend in gray; negative trend in black). (b) Nonstationary 20-year return level of global annual maximum near surface air temperature based on the expected number of events during 1996–2015.

which was provided by the National Meteorological Information Center (NMIC) of the China Meteorological Administration (CMA). The two AMP time series were selected, because either positive or negative trends were detected. The corresponding weather stations are Qionghai (Station ID: 59855) and Zunhua (Station ID: 54429), which are located at N19°14′E110°28′ and N40°12′E117°57′, respectively. The valid observation periods were 1953–2013 (Qionghai) and 1956–2013 (Zunhua). Both stationary and nonstationary GEV distributions were used to fit the AMP time series denoted by the following four candidate models:

$$\begin{cases} M0: & \{\mu, \sigma, \varepsilon\} \\ M1: & \{\mu_0 + \mu_1 y, \sigma, \varepsilon\} \\ M2: & \{\mu, \sigma_0 + \sigma_1 y, \varepsilon\} \\ M3: & \{\mu_0 + \mu_1 y, \sigma_0 + \sigma_1 y, \varepsilon\} \end{cases} \tag{20}$$

Akaike information criterion (AIC) was also computed for model fitting evaluation [31]. The model that was preferred was having the minimum value of AIC. For AMP time series at station Qionghai, the best fitting model was $M1$. For AMP time series at station Zunhua, the best fitting model was $M3$. The observed values of AMP, the estimated median, and the 5th and 95th percentiles were shown in **Figure 2(a)**. With the best fitting models, trends in precipitation extremes were extrapolated to the next 50 years (2014–2063). In other words, the design life period was assumed to be 50 years starting from 2014 to 2063. The scale parameter σ_y was constrained to be positive by $max\{0, \sigma_0 + \sigma_1 y\}$ in the design life period. With the extrapolated $F_y(x)$ in 2014–2063, we computed the return levels (or Design Life Level and Minimax Design Life Level) along with their standard errors, expected waiting time (or the return period that has been given in advance), failure risk, and reliability in the design life period. The return levels for nonstationary conditions presented in [10, 28, 29] were computed using Eqs. (13) and (14), while Design Life Level and Minimax Design Life Level are computed using Eqs. (16) and (17), respectively. The corresponding standard errors were computed using the delta method [11, 30]. Expected waiting time was computed based on Eq. (3), and the failure risk and reliability were computed using Eqs. (4) and (8).

The results were shown in **Tables 1** and **2**, respectively. The first three rows in **Tables 1** and **2** mainly illustrated the relationship between the return period and the return level under stationary and nonstationary conditions. The last two rows in **Tables 1** and **2** showed the two concepts, Design Life Level and Minimax Design Life Level, for the purpose of engineering design under nonstationary conditions. For AMP time series at Qionghai station, there was a significant positive trend in precipitation extremes. Given a 50-year return period, the associated return level under stationary assumption was much lower than those under nonstationary assumption. From the perspective of engineering design, the return levels shown in the first three rows were unacceptable, because the failure risks in the following 50 years (design life period) were all larger than 0.55. To ensure a low failure risk, a higher design level is needed. For AMP time series at Zunhua station, there was a significant negative trend in precipitation extremes. When nonstationarity was considered, the return level became lower due to the decreasing trend in precipitation extremes. Furthermore, due to the same reason, the Design Life Level and Minimax Design Life Level in 50-year design life period (2014–2063) were lower than the 50-year return level under stationary assumption.
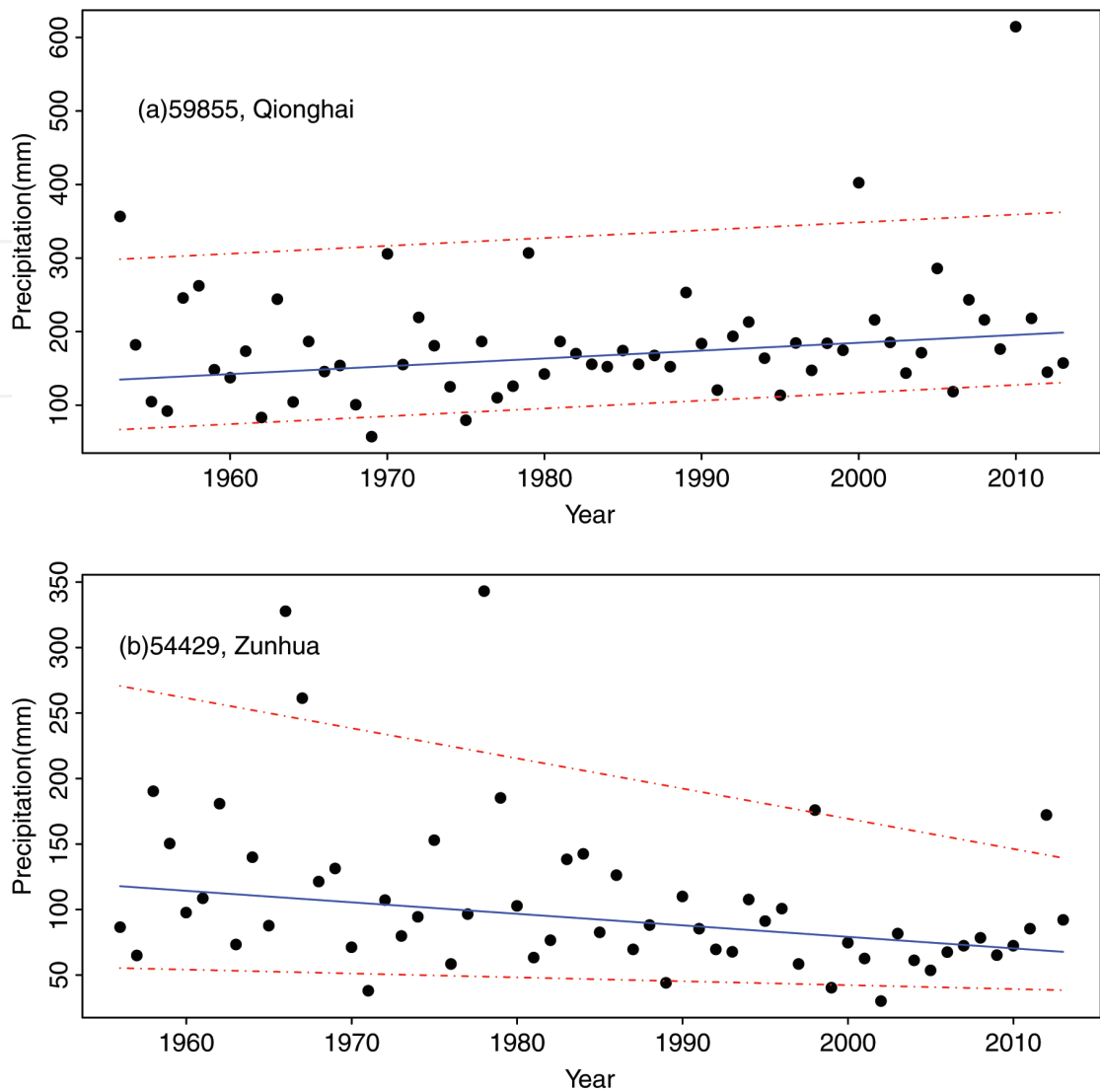
Figure 2. Summary of the nonstationary modeling of annual time series of precipitation extremes using GEV distribution models. (a) Station Qionghai (ID:59855); observation period: 1953–2013; the best fitting nonstationary GEV distribution: $M1$. (b) Station Zunhua (ID:54429); observation period: 1956–2013; the best fitting nonstationary GEV distribution: $M3$. Symbols: observed values (dots), the estimates of the median (solid lines), and the 5th and 95th percentiles (dashed lines).

		Risk communication				
Model	**Equation**	**Return level**	**Standard error**	**Return period (or EWT)**[a]	**Risk**	**Reliability**
$M0$	Eq. (9)	398.31	46.15	50	0.6358	0.3642
	Eq. (13)	485.00	65.67	50	0.5594	0.4406
$M1$	Eq. (14)	466.61	61.77	50	0.6359	0.3641
	Eq. (16)	818.87	13.4	177.03	0.0407	0.9593
	Eq. (17)	790.58	15.0	171.96	0.05	0.95

[a]EWT stands for expected waiting time.

Table 1. Results of risk communication for precipitation extremes with positive trend at station Qionghai (ID: 59855).

		Risk communication				
Model	Equation	Return level	Standard error	Return period (or EWT)[a]	Risk	Reliability
M0	Eq. (9)	282.3	50.59	50	0.6358	0.3642
	Eq. (13)	81.44	20.07	50	0.7647	02353
M3	Eq. (14)	83.72	18.75	50	0.6548	0.3452
	Eq. (16)	105.62	11	199.78	0.001	0.9989
	Eq. (17)	98.41	19.59	189.98	0.05	0.95

[a]EWT stands for expected waiting time.

Table 2. Results of risk communication for precipitation extremes with negative trend at station Zunhua (ID: 54429).

5. Discussion and conclusions

Due to the climate change, the stationary assumption that was commonly used in statistical analysis of climate extremes gradually became unacceptable [6, 15, 17]. How to quantify and communicate risk of climate extremes in nonstationary climate is essential for engineering design and risk assessment [7, 11, 30]. There were many attempts to quantify and communicate risk in a changing climate such as extending the concepts of the return period from stationary condition to nonstationary condition [10, 28, 29] or developing a new concept of the return level [30]. In stationary climate, frequency analysis, engineering design, and risk assessment were all based on the stationary extreme value distribution model [8]. It was assumed that the fitted extreme value distribution model on historical records also applied for future observations. Also due to stationarity, the concepts of risk measure for different purposes had not been strictly distinguished. Unlike the simple one-to-one relationship between a return level and a return period under stationary condition, risk measure and communication were more complicated under nonstationary condition, especially due to the time-varying essence of climate extremes. Therefore, a clear interpretation and illustration of the methods for risk measure and communication in a changing climate are of great importance.

In this study, climate extremes were presented in the form of annual maxima of extreme climate events. This chapter began with the two parallel interpretations of the return period, in which, the implicit relationship between a return level and a return period was included, but the stationary or nonstationary assumptions were omitted. This implicit relationship was also considered as the basis for frequency analysis and engineering design. In the stationary case, the two interpretations of the return period were equivalent. Although they were no more equivalent in the nonstationary case, they both provided independent methods for determining the associated return level for a given return period. Risk assessment usually aims to a focal engineering structure with a given design level. We showed that the concept of failure risk (or reliability) also had two parallel interpretations, and these two interpretations were consistent irrespective of stationary or nonstationary assumptions. In order to illustrate how risk was quantified and communicated in a changing climate, two examples of nonstationary climate extremes were used. Totally, we have reviewed two methods for estimating the return

level for a given return period under nonstationary condition [10, 28, 29] and two newly refined concepts of the return level in a given design life period for engineering design purpose [30]. In the first example, we detected the trend of annual maximum of global near surface air temperature during 1948–2015. Nonstationary GEV distribution with a time-varying location parameter was used to fit near surface air temperature extremes with significant trends, and the expected number-based return levels in 1996–2015 were computed. In the second example, time series of observed annual precipitation extremes at two weather stations in China with significant trends was analyzed. Both stationary and nonstationary GEV distribution models were used to fit the precipitation extremes. For each station, one best fitting GEV distribution was identified. Then, the linear trends in the parameters of GEV distribution were extrapolated into the following 50 years (also considered as the design life period). Return level, Design Live Level, and Minimax Design Live Level were all computed for the design life period (2014–2063), respectively. It was concluded that the communication of risk in a changing climate was obviously different from that in a stationary climate. For frequency analysis purpose, general return level/return period might be quite capable of communicating risk of climate extremes. While for the engineering design purpose, Design Live Level or Minimax Design Live Level were recommended, because the failure risk of a focal structure would be very low if Design Live Level or Minimax Design Live Level were chosen as the design level.

A reliable statistical modeling on long-term data was the basis of risk communication in a changing climate [26]. In nonstationary extreme value modeling, there are usually many candidate models. The choice of extreme value distribution models might influence the risk measure substantially in nonstationary and changing climate, because the trend captured by the extreme value distribution should be extrapolated into the future design life period. In this chapter, we only chose time as the covariate in the nonstationary extreme value analysis, and the parameters in GEV distribution model were expressed as the linear function of time. Perhaps, there might be more suitable trends such as quadratic or exponential trends leading to more candidate models. Evaluating all these candidate models was not an easy task. In practice, only a few of commonly used models was evaluated and compared. Moreover, the model selection process is not simply by using tools such as AIC or BIC [11]. Sometimes, additional expert knowledge is needed. In nonstationary extreme value modeling, besides time, some other climate indictors representing the variability of the climate system were also chosen as covariates. Although the historical data could be successfully fitted with these additional climate indictors, it was difficult to extrapolate the historical climate variability. That is because the climate variability itself is difficult to predict due to the complicity of the climate system. To reduce the uncertainty in statistical extrapolation, the output of numerical climate models was also used. However, the reasonability of simulation results was constrained by the parameters setting and initial values [30]. Additionally, the standard error of return levels and Design Live Level was all estimated using the delta method. Although standard error is a simple measure to quantify the uncertainty of nonstationary extreme value modeling, the uncertainty cannot be properly reflected using the symmetric confidence interval. Lastly, as pointed in [30], neither Design Life Level nor Minimax Design Life Level could be used as the criteria for realistic engineering design, because more economic and political factors should be considered besides the failure risk and reliability. All the above mentioned things are outside the scope of this chapter; here, our primary objective was to discriminate the concepts and their interpretations of the return period/return level, failure risk/

reliability under stationary and nonstationary conditions, and to illustrate the computations using realistic climate extremes. With these examples, we believed that the concepts those were related to risk measure and communication in a changing climate could be easily understood and applied.

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