Chapter 7

Design Optimization of Reinforced Ordinary and High-Strength Concrete Beams with Eurocode2 (EC-2)

Fedghouche Ferhat

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/intechopen.78734

Abstract

This chapter presents a method for minimizing separately the cost and weight of reinforced ordinary and high-strength concrete (HSC) T-beams at the limit state according to Eurocode2 (EC-2). The first objective function includes the costs of concrete, steel, and formwork, and the second objective function deals with the weight of the T-beam. All the constraints functions are set to meet the design requirements of Eurocode2 and current practices rules. The optimization process is developed through the use of the generalized reduced gradient (GRG) algorithm. Two example problems are considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that this approach is economically more effective compared to conventional design methods used by designers and engineers and can be extended to deal with other sections without major alterations.

Keywords: cost and weight minimization, reinforced ordinary and high-strength concrete beams, Eurocode2 (EC-2), nonlinear optimization, algorithm

1. Introduction

Structural elements with T-shaped sections are frequently used in industrial construction. They are used for repeated and large structures because they are cost effective when using the optimum cost design model which is of great value for designers and engineers. Compression reinforcement is not often required when designing the T-beams sections. One of the great advantages of T-beams sections is the economy in the amount of steel needed for reinforcement. The objective function is usually simplified to represent the weight, disregarding the costs of shaping and the construction details. However, the economy aspects in terms of costs and gain achieved should be the area where scope exists for extending the research works [1–4].

Recent developments in the technology of materials have led to the use of the high-strength concrete (HSC); this is mainly due to its efficiency and economy. The reduction in the quantities of construction materials has enabled both a gain in weight reduction and in the foundation's cost. HSC has a high compressive strength in the range of 55–90 MPa; it not only has the advantage of reducing member size and story height, but also the volume of concrete and the area of formwork. In terms of the amount of steel reinforcement, there is a substantial difference between the normal-strength concrete structures compared to high-strength concrete structures [5, 6]. In this chapter, not only does it presents the minimum weight design but it presents a detailed objective function that considers the ratio cost not the absolute cost with sensitivity analysis of this cost ratio as well. It considers both shaping and material costs. The generalized reduced gradient (GRG) method is used to solve nonlinear programming problems. It is a very reliable and robust algorithm; also, various numerical methods have been used in engineering optimization [7–12].

This work shows a method for minimizing separately the cost and weight of reinforced ordinary and high-strength concrete (HSC) T-beams at the limit state according to Eurocode2 (EC-2). The first objective function includes the costs of concrete, steel and formwork, whereas the second objective function represents the weight of the T-beam; all the constraints functions are set to meet the ultimate strength and serviceability requirements of Eurocode2 and current practices rules. The optimization process is developed through the use of the generalized reduced gradient algorithm. Two example problems are considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that this approach is economically more effective compared to conventional design methods applied by designers and engineers and can be extended to deal with other sections without major alterations.

2. Limit state design of reinforced concrete T-section under bending

In accordance with EC-2 [13], the assumptions used at the limit state for the typical reinforced T-beam-cross section are, respectively, illustrated in **Figure 1(a)–(c)**.

In the linear strain diagram of **Figure 1b**, the symbols ε_s and ε_{cu3} designate steel strain and the ultimate strain for the rectangular stress distribution compressive concrete design stress–strain relation. The parameter α represents the relative depth of the compressive concrete zone and the plastic neutral axis is located at the distance αd from the upper fiber for the ultimate limit state design, and x is the depth of elastic neutral axis for serviceability limit state design. In the assumed uniformly distributed stress diagram of **Figure 1c**, f_{cd} is the design value of concrete compressive strength, γ_c is the partial safety factor for concrete and f_{ck} is the characteristic compressive cylinder strength of ordinary or HSC at 28 days. In accordance with EC-2, the possibility of working with rectangular stress distribution is offered. This requires the

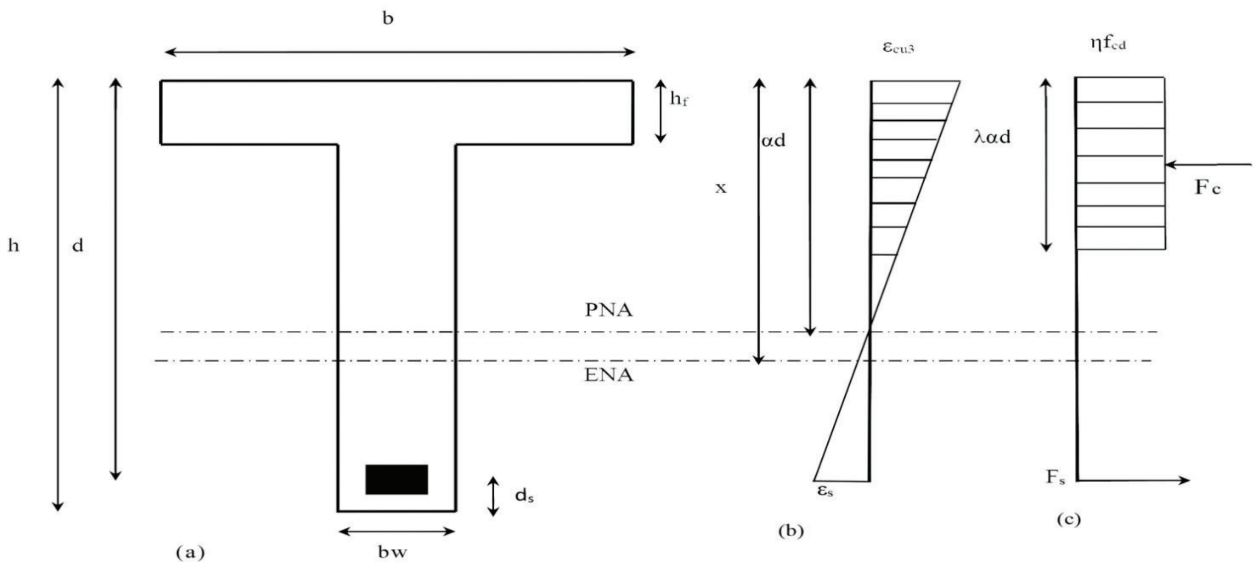

Figure 1. (a) Typical T-beam cross section; (b) strains at ultimate limit state and (c) stresses at ultimate limit state.

introduction of a factor λ for the depth of the compression zone and a factor η for the design strength. The λ and η factors are both linearly dependent on the characteristic strength f_{ck} in accordance with the following Equations [13]:

$$\lambda = 0.8 - \frac{f_{ck} - 50}{400} \tag{1}$$

$$\mu = 1.0 - \frac{f_{ck} - 50}{200} \tag{2}$$

with $50 \leq f_{ck} \leq 90$ MPa and $\lambda = 0.8, \eta = 1.0$ for $f_{ck} \leq 50$ MPa.

F_c and F_s denote the resultants of internal forces in the HSC section and reinforcing steel, respectively.

The design yield strength of steel reinforcement is $f_{yd} = f_{yk}/\gamma_s$ where f_{yk} is the characteristic elastic limit of steel and γ_s is the partial safety factor. In addition, the steel strain is considered unlimited in accordance with the Eurocode2 provisions. In this chapter, for an optimal use of steel, the strain must always be greater or equal to elastic limit strain, $\varepsilon_{yd} = f_{yd}/E_s$ where E_s represents the elasticity modulus for steel.

3. Formulation of the optimization problem

3.1. Design variables

The design variables selected for the optimization are presented in **Table 1**.

Design variables	Defined variables
b	Effective width of compressive flange
b_w	Web width
h	Total depth
d	Effective depth
h_f	Flange depth
A_s	Area of tension reinforcement
α	Relative depth of compressive concrete zone

Table 1. Definition of design variables.

3.2. Objective functions

3.2.1. Cost function

The objective function to be minimized in the optimization problems is the total cost of construction material per unit length of the beam. This function can be defined as:

$$C_0/L = C_c(b_w h + (b - b_w)h_f) + C_s A_s + C_f[b + 2h] \rightarrow \text{Minimum} \tag{3}$$

Thus, the cost function to be minimized can be written as follows:

$$C = \frac{C_O}{C_c L} = b_w h + (b - b_w)h_f + \left(\frac{C_s}{C_c}\right)A_s + \left(\frac{C_f}{C_c}\right)[b + 2h] \rightarrow \text{Minimum} \tag{4}$$

The values of the cost ratios C_s/C_c and C_f/C_c vary from one country to another and may eventually vary from one region to another for certain countries [14, 15].

3.2.2. Weight function

The weight function to be minimized can be written as follows:

$$W = (b_w h + (b - b_w)h_f)\rho \longrightarrow (\text{minimum}) \tag{5}$$

where

ρ is the density of the reinforced concrete T-beams and W is the unit weight per unit length of the reinforced concrete T- beams.

3.3. Design constraints

a. Behavior constraints:

$$M_{Ed} \leq \eta f_{cd}(b - b_w)h_f(d - 0,50h_f) + \eta\lambda f_{cd} b_w d^2\alpha(1\text{–}0,5\lambda\alpha) \tag{6}$$

http://dx.doi.org/10.5772/intechopen.78734

(external moment ≤ resisting moment of the cross-section)

$$\alpha = \left(\frac{\mathrm{f_{yd}}}{\mathrm{f_{cd}}}\right)\left(\frac{\mathrm{A_S}}{\eta\lambda \mathrm{b_w d}}\right) - \frac{(\mathrm{b} - \mathrm{b_w})\mathrm{h_f}}{\lambda \mathrm{b_w d}} \tag{7}$$

(internal force equilibrium)

$$\frac{\mathrm{A_s}}{\mathrm{b_w d}} \geq \mathrm{p_{min}} \tag{8}$$

(minimum steel percentage)

$$\frac{\mathrm{As}}{\mathrm{b_w h} + (\mathrm{b} - \mathrm{b_w})\mathrm{h_f}} \leq \mathrm{p_{max}} \tag{9}$$

(maximum steel percentage)

In Eqs. (7) and (8) above, it is assumed that the neutral axis position is under the beam flange which ensures that the section behaves as the T-beam section shown in **Figure 1a**.

Conditions on strain compatibility in steel:

$$\varepsilon_{\mathrm{cu3}}\left(\left(\frac{1}{\alpha}\right) - 1\right) \geq \frac{f_{yd}}{E_s} \tag{10}$$

(In the case of Pivot B, optimal use of steel requires that strains in steel must be limited to plastic region at the ultimate limit state (ULS).)

$$\lambda\alpha(1\text{–}0,5\lambda\alpha) \leq \mu_{limit} \tag{11}$$

(Compression reinforcement is not required.)

b. Shear strength constraint:

$$V_{Ed} \leq V_{Rd,\,max} = \nu_1 \frac{f_{cd} b_w z}{tg\,(\theta) + cotg(\theta)} \tag{12}$$

(external shear force ≤ resisting shear force)

c. Deflection constraint:

$$\frac{5wL^4}{384\ E_{cm} I_c} \leq \delta_{lim} \tag{13}$$

$$I_c = \frac{b_w h^3}{3} + \frac{(b - b_w)h^3}{3} + nA_s d^2 - A_h x^2 \tag{14}$$

$$A_h = b_w h + (b - b_w)h_f + nA_s \tag{15}$$

$$x = \frac{\frac{b_w h^2}{2} + \frac{(b-b_w)h_f^2}{2} + nA_s d}{A_h} \tag{16}$$

d. Geometric design variable constraints including rules of current practice:

$$h \geq \frac{L}{16} \tag{17}$$

$$\frac{d}{h} = 0.90 \tag{18}$$

$$0.20 \leq \frac{b_w}{d} \leq 0.50 \tag{19}$$

$$\frac{(b - b_w)}{2} \leq \frac{L}{10} \tag{20}$$

$$\frac{b}{h_f} \leq 8 \tag{21}$$

$$h_f \geq h_{fmin} \tag{22}$$

$$\frac{b}{b_w} \geq 3 \tag{23}$$

where:

μ_{limit} is the limit value of the reduced moment.

θ is the angle between concrete compression struts and the main chord

ν_1 is a nondimensional coefficient, $\nu_1 = 0.60(1\text{-}f_{ck}/250)$;

z is the lever arm, $z = 0.9d$;

h_{fmin} is the minimum depth of flange.

3.4. Optimization based on minimum cost design

The optimum cost design of reinforced concrete T-beams under the limit state can be stated as follows:

For given material properties, loading data and constant parameters, find the design variables defined in **Table 1** that minimize the cost function defined in Eq. (4) subjected to the design constraints given in Eq. (6) through Eq. (23).

3.5. Optimization based on minimum weight design

Find the design variables that minimize total weight per unit length defined in Eq. (5), subjected to the design constraints given in Eq. (6) through Eq. (23).

3.6. Solution methodology: Generalized reduced gradient method

The objective function Eq. (4), the objective function Eq. (5) and the constraints equations, Eq. (6) through Eq.(23), together form a nonlinear optimization problem. The reasons for the nonlinearity of this optimization problem are essentially due to the expressions of the cross-sectional area, bending moment capacity and other constraints equations. Both the objective function and the constraint functions are nonlinear in terms of the design variables. In order to solve this nonlinear optimization problem, the generalized reduced gradient (GRG) algorithm is used. This algorithm was first developed in late 1960 by Jean Abadie [16] as an extension of the reduced gradient method and then since has been refined by several other researchers [17, 18]. GRG nonlinear should be selected if any of the equations involving decision variables or constraints is nonlinear.

Microsoft Excel, beginning with version 3.0 in 1991, incorporates an NLP solver that operates on values and formulas of a spreadsheet model. Version 4.0 and later include LP solver and mixed-integer programming (MIP) capability for both linear and nonlinear problems. The Microsoft Office Excel Solver tool uses several algorithms to find optimal solutions. The GRG nonlinear solving method for nonlinear optimization uses the Generalized Reduced Gradient code. The Simplex LP solving method for linear programming uses the Simplex and dual Simplex method. The Evolutionary solving method for non-smooth optimization uses a variety of genetic algorithm and local search methods. The user specifies a set of cell addresses to be independently adjusted (the decision variables), a set of formulae cells whose values are to be constrained (the constraints) and a formula cell designated as the optimization objective. The solver uses the spreadsheet interpreter to evaluate the constraint and objective functions and approximates derivatives, using finite differences. The NLP solution engine for the Excel Solver is GRG.

The generalized reduced gradient method is applied as it has the following advantages: (i) the GRG method is widely recognized as an efficient method for solving a relatively wide class of nonlinear optimization problems; (ii) the program can handle up to 200 constraints, which is suitable for reinforced ordinary and HSC beam design optimization problems; and (iii) GRG transforms inequality constraints into equality constraints by introducing slack variables. Hence all the constraints are of equality form. A more detailed description of the GRG method can be found in [19].

4. Numerical results and discussion

4.1. Design example A for reinforced HSC T-beams

The numerical example A corresponds to a high-strength concrete T-beam belonging to a bridge deck, simply supported at its ends and predesigned in accordance with provisions of EC-2 design code.

The corresponding preassigned parameters are defined as follows:

$L = 25$ m; $M_{Ed} = 1.35\ M_G + 1.5\ M_Q = 9$ MNm; $V_{Ed} = 1.35\ V_G + 1.5\ V_Q = 3.1$ MN.

$w = 0.60$MN/ml (the total distribution load (dead load + live load)), $\delta_{lim} = L/250 = 0.100$ m.

Input data for HSC characteristics:

C70/85; $f_{ck} = 70$ MPa; $\gamma_c = 1.5$; $f_{cd} = 46.67$ MPa; $\rho = 0.025$ MN/m^3; $E_{cm} = 40{,}743$ MPa;

$\lambda = 0.75$; $\eta = 0.90$; $\varepsilon_{cu3}(‰) = 2.7$; $\varepsilon_{c3}(‰) = 2.4$; $h_{fmin} = 0.10$ m; $f_{ctm} = 4.6$ MPa;

$\mu_{limit} = 0.329$; $\alpha_{limit} = 0.554$ for S500 and C70/85.

Input data for steel characteristics:

S500; $f_{yk} = 500$ MPa; $\gamma_s = 1.15$; $f_{yd} = f_{yk}/\gamma_s = 435$ MPa; $n = 15$;

S400; $f_{yk} = 400$ MPa; $\gamma_s = 1.15$; $f_{yd} = f_{yk}/\gamma_s = 348$ MPa; $f_{yd}/f_{cd} = 9.32$ for classes (S500, C70/85);

$f_{yd}/f_{cd} = 7.46$ for classes (S400, C70/85); $\mu_{limit} = 0.352$; $\alpha_{limit} = 0.6081$ for S400 and C70/85;

$E_s = 2 \times 10^5$ MPa; $p_{min} = 0.26\ f_{ctm}/f_{yk} = 0.002392$; $p_{max} = 4\%$.

Input data for units cost ratios of construction materials:

$C_s/C_c = 40$ for HSC concrete;

$C_f/C_c = 0.01$ for wood formwork;

$C_f/C_c = 0.10$ for metal formwork;

$C_f/C_c = 0.00$ in the case of the cost of the formwork is negligible.

4.1.1. Comparison between the minimum cost design and the minimum weight design of HSC T-beams

The vector of design variables including the geometric dimensions of the T-beam cross-section and the area of tension reinforcement as obtained from the standard design approach solution and the optimal cost design solution using the proposed approach is shown in **Table 2**.

Design variables vector.	**Initial design**	**Optimal solution with minimum cost (S500, C70/85), $C_s/C_c = 40$, $C_f/C_c = 0.01$ wood formwork**	**Optimal solution with minimum weight**
b(m)	1.20	0.86	0.52
b_w(m)	0.40	0.28	0.28
h(m)	1.40	1.58	1.56
d(m)	1.26	1.42	1.40
h_f(m)	0.15	0.11	0.10
A_S(m^2)	185x10^{-4}	161x10^{-4}	181 x10^{-4}
α	0.554	0.342	0.554
Gain		22%	47%

Table 2. Comparison of the optimal solutions with minimum weight and minimum cost design for HSC.

http://dx.doi.org/10.5772/intechopen.78734

The optimal solutions using the minimum cost design and the minimum weight design are shown in **Table 2**.

It is shown from **Table 2** that the gain and optimum values for minimum cost design and for minimum weight design are different.

From the above results, it is clearly shown that significant cost saving of the order of 47% can be obtained using the proposed minimum weight design formulation and 22% through the use of minimum cost-design approach.

4.1.2. Parametric study

In this section, the optimal solution is obtained according to practical consideration: (i) the total depth is imposed, $h = h_{imposed;}$ (ii) the effective width of compressive flange is imposed, $b = b_{imposed;}$ (iii) the reinforcing steel is imposed, $A_s = A_{simposed;}$ and (4i) the flange depth is imposed, $h_f = h_{fimposed}$.

The gain depends on the type of formwork used. We distinguish the wood formwork: $C_f/C_c = 0.01$ and the steel formwork $C_f/C_c = 0.10$.

Further practical requirements can also be implemented, such as esthetic, architectural and limited authorized templates. The optimal solutions obtained using the particular conditions imposed are shown in **Table 3**.

From the above results, it is clearly seen that a significant cost saving between 08% and 23% can be obtained by using this parametric study.

4.1.3. Sensitivity analysis

The relative gains can be determined for various values of unit-cost ratios: $C_s/C_c = 10; 20; 30; 40; 50; 60; 70; 80; 90; 100$ for a given unit cost ratio $C_f/C_c = 0.01$

The corresponding results are reported in **Table 4** and represented in **Figure 2**.

Optimal solution with.	**Gain (%)**
Classes(S500, C70/85); $C_s/C_c = 40$; $C_f/C_c = 0.01$wood formwork	22
Classes(S500, C70/85); $C_s/C_c = 40$; $C_f/C_c = 0.10$ steel formwork	19
Classes(S500,C70/85) and $C_f/C_c = 0$ the cost of the formwork is negligible	23
Classes(S400,C70/85); $C_s/C_c = 40$; $C_f/C_c = 0.01$wood formwork	08
Imposed height h = 1.70 m; S500 and C70/85	21
Imposed width b = 1.00 m; S500 and C70/85	22
Imposed reinforcement $A_s \leq 0.0150\ m^2$; S500 and C70/85	22
Imposed flange depth $h_f = 0.10$ m; S500 andC70/85	22

Table 3. The variation of relative gain with particular conditions imposed such as the HSC T-beam dimensions and reinforcing steel.

(S500; C70/85) $C_f/C_c = 0.01$	Gain (%)
10	33
20	27
30	24
40	22
50	22
60	22
70	23
80	24
90	26
100	27

Table 4. Variation of relative gain in percentage (%) versus unit cost ratio C_s/C_c for a given cost ratio $C_f/C_c = 0.01$.

It is shown in **Table 4** and **Figure 2** that the relative gain decreases for increasing values of the unit cost ratio C_s/C_c, stabilizes around an average value for $40 \le C_s/C_c \le 60$ and then increases significantly beyond this average value for a given cost ratio $C_f/C_c = 0.01$.

The relative gains can be determined for various values of unit cost ratios: $C_f/C_c = 0.01$; 0.02; 0.03; 0.04; 0.05; 0.06; 0.07; 0.08; 0.08; 0.09; 0.10 for a given unit cost ratio $C_s/C_c = 40$.

The corresponding results are reported in **Table 5** and presented in **Figure 3**.

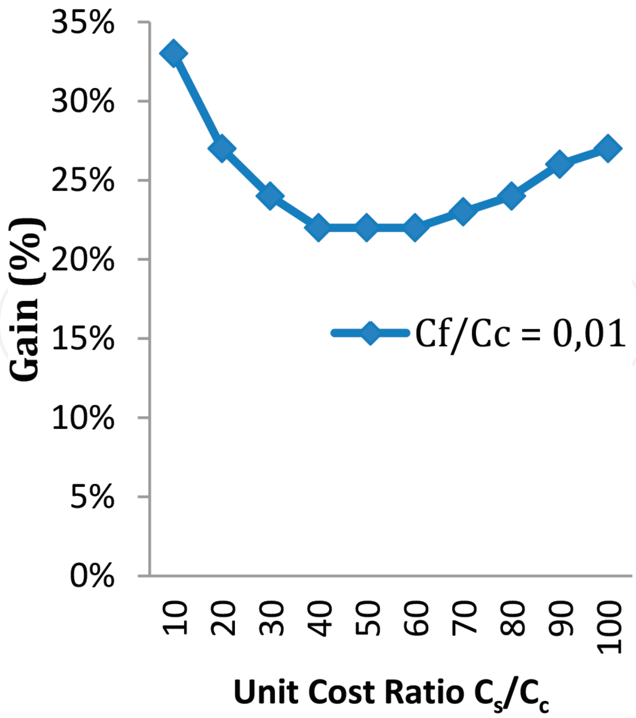

Figure 2. Variation of relative gain in percentage (%) versus unit cost ratio C_s/C_c for a given cost ratio $C_f/C_c = 0.01$.

(S500; C70/85) $C_s/C_c = 40$ C_f/C_c	Gain (%)
0.01	22
0.02	21
0.03	21
0.04	20
0.05	20
0.06	19
0.07	19
0.08	19
0.09	19
0.10	18

Table 5. Variation of relative gain in percentage (%) versus unit cost ratio C_f/C_c for a given cost ratio $C_s/C_c = 40$.

From **Table 5** and **Figure 3**, the gain decreases monotonically with the increase of unit cost ratio C_f/C_c for a given cost ratio $C_s/C_c = 40$.

4.2. Design example B for reinforced ordinary concrete T-beams

The numerical example B corresponds to a concrete T-beam belonging to a pedestrian deck, simply supported at its ends and predesigned in accordance with the provisions of EC-2 design code.

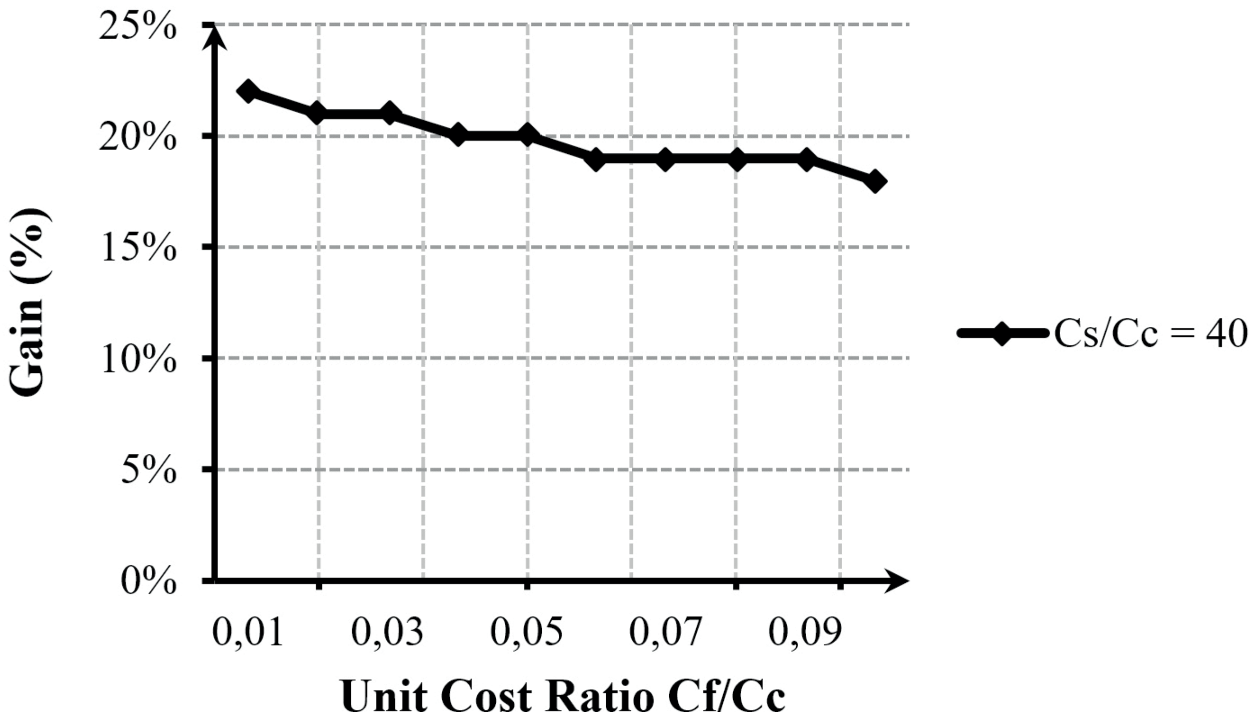

Figure 3. Variation of relative gain in percentage (%) versus unit cost ratio C_f/C_c for a given cost ratio $C_s/C_c = 40$.

The preassigned parameters are defined as follows:

$L = 20$ m; $M_{Ed} = 5$MNm; $V_{Ed} = 1.1$MN; $w = 0.043$MN/ml; $\delta_{lim} = L/250 = 0.080$ m.

Input data for ordinary concrete characteristics:

C20/25; $f_{ck} = 20$ MPa; $\gamma_c = 1.5$;$f_{cd} = 11.33$ MPa; $\rho = 0.025\text{MN/m}^3$; $E_{cm} = 30{,}000$ MPa;

$\lambda = 0.80$; $\eta = 1.00$; $\varepsilon_{cu3}(‰) = 2$; $\varepsilon_{c3}(‰) = 3.5$; $h_{fmin} = 0.15$ m; $f_{ctm} = 2.20$ MPa; $n = 15$;

$\mu_{limit} = 0.372$; $\alpha_{limit} = 0.6167$ for S500 and C20/25.

$\mu_{limit} = 0.392$; $\alpha_{limit} = 0.6680$ for S400; and C20/25.

Input data for steel characteristics:

S400; $f_{yk} = 400$ MPa; $\gamma_s = 1.15$; $f_{yd} = f_{yk}/\gamma_s = 348$ MPa;

$E_s = 2 \times 10^5$ MPa; $p_{min} = 0.26\ f_{ctm}/f_{yk} = 0.00143$; $p_{max} = 4\%$;

$f_{yd}/f_{cd} = 30.71$ for classes (S400, C20/25);

$f_{yd}/f_{cd} = 38.39$ for classes (S500, C20/25).

Input data for units cost ratios of construction materials:

$C_s/C_c = 30$ for ordinary concrete.

$C_f/C_c = 0.10$ for metal formwork.

$C_f/C_c = 0.01$ for wood formwork.

4.2.1. Comparison between the minimum cost design and the minimum weight design of ordinary concrete T-beams

The optimal solutions using the minimum weight design and the minimum cost design are shown in **Table 6**.

It is shown in **Table 6** that the gain and the optimum values for minimum weight design and for minimum cost design are different.

From the above results, it is clearly shown that a significant cost saving of the order of 23% can be obtained using the proposed minimum weight design formulation and 14% through the use of the minimum cost design approach.

4.2.2. Parametric study

In this section, the optimal solution is obtained through the considerations: (i) one of the dimensions of HSC T-section is imposed, $h = 1.50$ m; (ii) the imposed reinforcing steel $A_s = 120 \times 10^{-4}\ \text{m}^2$; (iii) imposed web width $b_W = 0.30$ m; and (iv) imposed relative depth of compressive concrete zone $\alpha = 0.6000$

http://dx.doi.org/10.5772/intechopen.78734

Design variables vector	Initial design, C20/25 & S400	Optimal solution with minimum weight, C20/25 & S400	Optimal solution with minimum cost, C20/25 & S400
b(m)	1.20	1.30	1.25
b_w(m)	0.40	0.28	0.29
h(m)	1.60	1.57	1.60
d(m)	1.44	1.41	1.44
h_f(m)	0.14	0.17	0.16
$A_S(m^2)$	125×10^{-4}	123×10^{-4}	122×10^{-4}
α	0.668	0.668	0.668
C	1.171		1.0281
Gain		23%	14%

Table 6. Comparison of the optimal solutions with minimum weight and minimum cost design.

Optimal solution with	Gain (%)
$f_{yd}/f_{cd} = 30.71$; $C_s/C_c = 30$; $C_f/C_c = 0.01$ wood formwork, C20/25 & S400	14
$f_{yd}/f_{cd} = 38.39$; $C_s/C_c = 30$; $C_f/C_c = 0.01$wood formwork, C20/25 & S500	09
$f_{yd}/f_{cd} = 30.71$; $C_s/C_c = 30$; $C_f/C_c = 0.00$; C20/25 & S400	15
Imposed web with $b_w = 0.30$ m; $f_{yd}/f_{cd} = 30.71$; $C_s/C_c = 30$; $C_f/C_c = 0.01$; C20/25 & S400	13
Imposed reinforcement$A_s \leq 0.0120$ m^2; $f_{yd}/f_{cd} = 30.71$; $C_s/C_c = 30$; $C_f/C_c = 0.01$; C20/25 & S400	14
Imposed height h = 1.50 m; $f_{yd}/f_{cd} = 30.71$; $C_s/C_c = 30$; $C_f/C_c = 0.01$; C20/25 & S400	11
Imposed relative depth$\alpha = 0.600$; $f_{yd}/f_{cd} = 30.71$; $C_s/C_c = 30$; $C_f/C_c = 0.01$; C20/25 & S400	14

Table 7. Variation of relative gain with particular conditions imposed such as the T-beam dimensions, reinforcing steel and weight.

(S400; C20/25) $C_f/C_c = 0.01$	Gain (%)
10	18
20	16
30	14
40	13
50	12
60	12
70	12
80	11
90	11
100	11

Table 8. Variation of relative gain in percentage (%) versus unit cost ratio C_s/C_c for a given cost ratio $C_f/C_c = 0.01$.

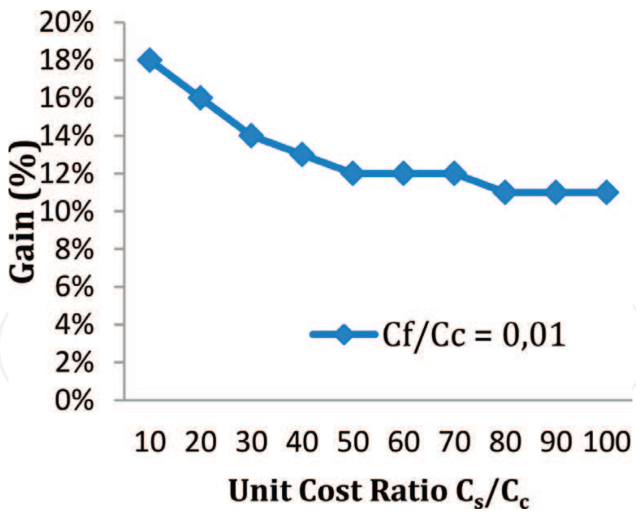

Figure 4. Variation of relative gain in percentage (%) versus unit cost ratio C_s/C_c for a given cost ratio $C_f/C_c = 0.01$.

(S400; C20/25) $C_s/C_c = 30$ C_f/C_c	Gain (%)
0.01	14
0.02	14
0.03	13
0.04	13
0.05	13
0.06	12
0.07	12
0.08	12
0.09	12
0.1	12

Table 9. Variation of relative gain in percentage (%) versus unit cost ratio C_f/C_c for $C_s/C_c = 30$.

Further practical requirements can also be implemented, such as esthetic, architectural and limited authorized template.

The optimal solutions obtained using the particular conditions imposed are shown in **Table 7**.

From the above results, it is clearly seen that a significant cost saving between 09 and 15% can be obtained by using this parametric study.

4.2.3. Sensitivity analysis

The relative gains can be determined for various values of the unit cost ratios: $C_s/C_c = 10$; 20; 30; 40; 50; 60; 70; 80; 90; 100 for a given unit cost ratio $C_f/C_c = 0.01$

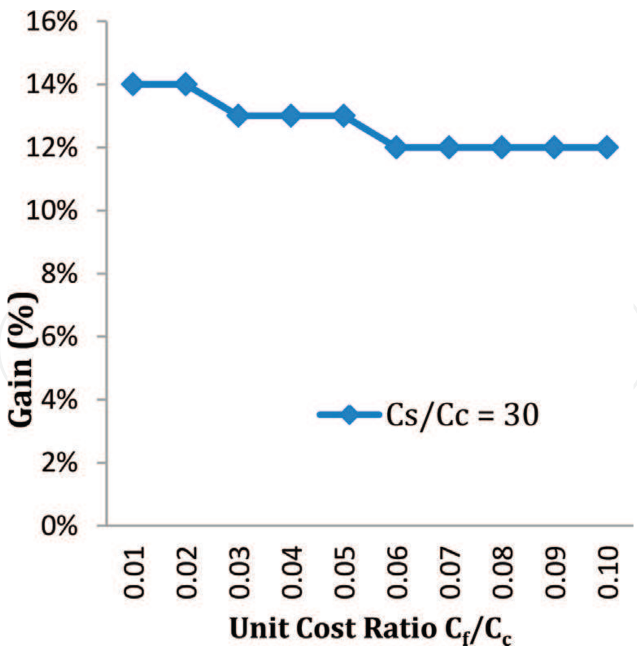

Figure 5. Variation of relative gain in percentage (%) versus unit cost ratio C_f/C_c for a given cost ratio $C_s/C_c = 30$.

The corresponding results are reported in **Table 8** and presented graphically in **Figure 4**.

It is shown in **Table 8** and **Figure 4** that the relative gain decreases for increasing values of the unit cost ratio C_s/C_c for a given value of $C_f/C_c = 0.01$.

The relative gains can be determined for various values of the unit cost ratios: $C_f/C_c = 0.01$; 0.02; 0.03; 0.04; 0.05; 0.06; 0.07; 0.08; 0.08; 0.09; 0.10 for a given unit cost ratio $C_s/C_c = 30$.

The corresponding results are reported in **Table 9** and illustrated graphically in **Figure 5**.

From **Table 9** and **Figure 5**, the gain decreases monotonically with the increase of unit cost ratio C_f/C_c for a given value of $C_s/C_c = 30$.

5. Conclusions

The following important conclusions are drawn on the basis of this chapter:

- The problem formulation of the optimal cost design of reinforced concrete T-beams can be cast into a nonlinear programming problem; the numerical solution is efficiently determined using the GRG method in a space of only a few variables representing the concrete cross-section dimensions.
- The space of feasible design solutions and the optimal solutions can be obtained from a reduced number of independent design variables.
- The optimal values of the design variables are only affected by the relative cost values of the objective function and not by the absolute cost values.

- The optimal solutions are found to be insensitive to changes in the shear constraint. Shear constraint is not usually critical in the optimal design of reinforced concrete T-beams under bending and thus can be excluded from problem formulation.
- The observations of optimal solution results reveal that the use of optimization based on the optimum cost design concept may lead to substantial savings in the amount of construction materials to be used in comparison to classical design solutions of reinforced concrete T-beams.
- The objective function and the constraints considered in this chapter are illustrative in nature. This approach based on nonlinear mathematical programming can be easily extended to other sections commonly used in structural design. More sophisticated objectives and considerations can be readily accommodated by suitable modifications of the optimal cost design model.
- In this chapter, we have included the additional cost of formwork which makes a significant contribution to the total costs. This integration is important for an economical approach to design and manufacture.
- The suggested methodology for optimum cost design is effective and more economical compared to the classical methods. The results of the analysis show that the optimization process presented herein is effective and its application appears feasible.
- The comparison of optimal solutions for minimum cost and minimum weight shows that the construction cost affects significantly the optimal sizes. Not only do we use the mass but the cost as objective function as well which contains the material and construction provision costs. The difference is caused by construction details costs.

Appendix

List of symbols

The following symbols are used in this chapter:

C20/25	Class of ordinary concrete
C70/85	Class of HSC
S400	Grade of steel
S500	Grade of steel
f_{ck}	Characteristic compressive cylinder strength of ordinary or HSC at 28 days
f_{ctm}	Tensile strength of concrete
f_{cd}	Design value of concrete compressive strength
γ_c	Partial safety factor for concrete

η	Design strength factor
λ	Compressive zone depth factor
ε_{c3}	Strain at the maximum stress for the rectangular stress distribution compressive concrete
ε_{cu3}	Ultimate strain for the rectangular stress distribution compressive concrete design stress–strain relation
f_{yk}	Characteristic elastic limit for steel reinforcement
γ_s	Partial safety factor for steel
f_{yd}	Design yield strength of steel reinforcement
ε_{yd}	Elastic limit strain
E_s	Young's elastic modulus of steel
E_{cm}	Modulus of elasticity of concrete
p_{min}	Minimum steel percentage
p_{max}	Maximum steel percentage
α_{limit}	Limit value of relative depth of compressive concrete zone
μ_{limit}	Limit value of reduced moment
L	Beam span
w	The total distribution load (dead load+ live load)
V_G	Maximum design shears under dead loads
V_Q	Maximum design shears under live loads
$V_{Rd,max}$	Maximum resistant shear force
V_{Ed}	Ultimate shear force
$M_{Rd, max}$	Maximum resisting moment
M_{Ed}	Ultimate bending moment
M_G	Maximum design moments under dead loads
M_Q	Maximum design moments under live loads
F_s	Resultant tensile internal force for steel
F_c	Resultant compressive internal force for HSC
n	Ratio of the modulus of elasticity of steel to that of concrete
b	Effective width of compressive flange

b_w	Web width
h	Total depth
h_f	Flange depth
d	Effective depth
d_s	Effective cover of reinforcement
As	Area of reinforcing steel
h_{fmin}	Minimum depth of flange
δ_w	The mid-span deflection of simply supported beam under distribution load w
δ_{lim}	Limit deflection
θ	Angle between concrete compression struts and the main chord
ν_1	A nondimensional coefficient; $\nu_1 = 0.60(1\text{-}f_{ck}/250)$
z	Lever arm, z = 0.9d
ρ	Density of the reinforced concrete T-beams
W	Unit weight per unit length of the reinforced concrete T- beams
C_0/L	Total cost per unit length of T-beam
C_s	Unit cost of reinforcing steel
C_c	Unit cost of concrete
C_f	Unit cost of formwork

Author details

Fedghouche Ferhat

Address all correspondence to: ferfed2002@yahoo.fr

École Nationale Supérieure des Travaux Publics (ENSTP), Département Infrastructures de Base (DIB), Laboratoire des Travaux Publics ingénierie de Transport et Environment (LTPiTE), Algiers, Algeria

References

[1] Babu Narayan KS, Venkataramana K. Shape optimization of steel reinforced concrete beams. Computers and Concrete. 2007;**4**:317-330. DOI: 10.12989/cac.2007.4.4.317

[2] Ozturk HT, Durmus A. Optimum design of a reinforced concrete beam using artificial bee colony algorithm. Computers and Concrete. 2012;**10**:295-306. DOI: 10.12989/cac.2012.10.3.295

[3] Bordignon R, Kripka M. Optimum design of reinforced concrete columns subjected to uniaxial flexural compression. Computers and Concrete. 2012;**9**:345-358. DOI: 12989/cac.2012.9.5.327

[4] Medeiros GF, Kripka M. Structural optimization and proposition of pre-sizing parameters for beams in reinforced concrete buildings. Computers and Concrete. 2013;**11**:253-270. DOI: 10.12989/cac.2013.11.3.253

[5] Rashid MA, Mansur MA. Reinforced high strength concrete beams in flexure. ACI Structural Journal. 2005;**03**:462-471

[6] Ozbay E, Oztas A, Baykasoglu A. Cost optimization of high strength concretes by soft computing techniques. Computers and Concrete. 2010;**7**:221-237. DOI: 10.12989/cac.2010.7.3.221

[7] Nocedal J, Wright SJ. Numerical optimization. Second ed. New York: Springer Verlag; 2006

[8] Jiang P, Zhu M, XLJ. An optimization algorithm for minimizing weight of the composite beam. Advances in Intellegent and soft.Computing. 2012;**135**:769-775. DOI: 10.1007/978-3-642-27708-5_106

[9] Kaveh A, Kalateh-Ahani M, Fahimi-Farzam M. Constructability optimal design of reinforced concrete retaining walls using a multi-objective optimization of foundation using global-local gravitational search algorithm. Structural Engineering and Mechanics. 2013;**50**:257-273. DOI: 10.12989/sem.2014.50.3.257

[10] Fedghouche F, Tiliouine B. Minimum cost design of reinforced concrete T-beams at ultimate loads using Eurocode 2. Engineering Structures. 2012;**42**:43-50. DOI: 10.1016/j.engstruct.2012.04.008

[11] Abbasnia R, Shayanfar M, Khodam A. Reliability-based design optimization of structural systems using a hybrid genetic algorithm. Structural Engineering and Mechanics. 2014;**52**:1099-1120. DOI: 10.12989/sem.2014.52.6.1099

[12] Fedorik F, Kala J, Haapala A, Malaska M. Use of design optimization techniques in solving typical structural engineering related design optimization problems. Structural Engineering and Mechanics. 2015;**55**:1121-1137. DOI: 10.12989/sem.2015.55.6.1121

[13] Eurocode 2. Design of concrete structures-part 1–1: general rules for buildings.EN 1992–1-1:2004, European Committee for Standardization (CEN), Brussels; 2004

[14] Pratt DJ. Fundamentals of Construction Estimating. 3rd ed. Australia: Delmar cengage Learning; 2011

[15] Davis L. Spon's Architects' and Builders' Price Book 2011. 136th ed. Taylor & Francis Ltd.; 2010

[16] Abadie J, Carpentier J. In: Fletcher R, editor. Generalization of the Wolfe Reduced Gradient Method to the Case of Nonlinear Constraints. Optimization (Sympos., Univ. Keele, Keele, 1968). London: Academic Press; 1969

[17] Lasdon S, Warren AD, Jain A, Ratner M. Design and testing of a generalized reduced gradient code for nonlinear programming. ACM Trans. Math. Software. 1978;**4**(1):34-50

[18] Lasdon LS, Waren AD. Generalized reduced gradient method for linearly and nonlinearly constrained programming. In: Greenberg HJ, editor. Design and Implementation of Optimization Software. the Netherlands: Sijthoff and Noordhoff; 1978. pp. 363-397

[19] Yeniay O. A comparative study on optimization methods for the constrained nonlinear programming problems. Mathematical Problems in Engineering. 2005;(2):165-173. DOI: 10.1155/MPE.2005.165