Chapter 6

Phononic Crystal Resonators

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Additional information is available at the end of the chapter

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Abstract

In this chapter we present the theory of phononic crystal, classification of PnC according to its physical nature, and phononic crystal (PnC) phenomena in locally resonant materials with 2D, and 3D crystals structure. In this chapter, phononic crystal (PnC) micro-electro mechanical system (MEMS) resonators with different transduction schemes such as electrostatically, piezoresistively, piezoelectrically transduced MEMS resonators are explained. In this chapter, we employed phononic crystal strip in MEMS resonators is explained to reduce anchor loss, and analysis of eigen frequency mode of the resonators. The phononic crystal strip with supporting tethers is designed to see the formation of band gap by introducing square holes, and improvement of quality factor and harmonic response. We show that holes can help to reduce the static mass of PnC strip tether without affecting on band gaps.

Keywords: MEMS resonator, phononic crystal, piezoelectric, band gap, anchor loss

1. Introduction

Because of merits of easy fabrications and less power consumption and the better performance with high accuracy phononic crystals MEMS resonator has become hot topic in the family of flexible electronics. The concept of phononic crystal followed by a few years the analogous concept of photonic crystals [1, 2] for the propagation of electromagnetic waves.

Phononic crystals are actually the acoustic waves with periodic structures which is same as electrons crystalline structure, sometimes the acoustic waves are also refer to elastic waves. Simply we can say phononic crystals are the artificial materials are arranged in a highly ordered microscopic structure of array of particles. Phononic crystals (PnCs) have paid attention by researchers over the past two decades [3]. Phononic crystals have many potential applications, especially in the field of information and communication technologies. Propagation of waves can be control by phononic crystals. The field of phononics is progressing very quickly.

Nowadays there are many advances in the field of phononic crystals. Scientist and engineers are paying deep attention in phononic crystals (PnCs) MEMS resonator. The PnCs have significance role in the advancement of micro- and nanofields. PnCs supported tether configurations to isolate the energy leakage from resonator body into substrate [4]. A perfect PnC allows for the design of devices like waveguides and cavities to control the propagation of acoustic waves inside the band [5, 6, 34]. The PnCs can operate as coupling elements between resonators [6, 7]. Moreover the combination of PnCs and n-type doped silicon in nanostructures is a potential/promising candidate for thermoelectric applications [7].

In fact, the concept of phononic crystals is extended from one of photonic crystals for the propagation of electromagnetic waves [1–9]. The nature of phononic crystals is controlling and manipulating the propagation of elastic/acoustic waves. For example, the PnCs can prohibit the propagation of acoustic (elastic waves) inside their structures through existence of band gaps (PBG). Band gap is a frequency range in which there are no resonant guided modes or wave propagation within the structure.

2. Theory of phononic crystal

As mention above that phononics crystal is an artificial material composed by a periodic repetition of incorporation in a matrix. This periodic structure is formed by scattering inclusions located in consistent material as a lattice structure resemble with crystal lattice existed in the crystalline solid [10–12].

2.1. Lattice structure

The phononic band structure may be tailored with appropriate choices of materials, and crystal lattices. An ideal crystalline solid composed of the atoms or basis (group of atoms) are arranged by attachment of every lattice point. Let any lattice point r' can be formed from any other lattice point r in the space using translational operation [10]

$$r' = r + T \tag{1}$$

In above equation, T is the translation vector can be written as

$$T = u_1 a_1 + u_2 a_2 + u_3 a_3 \tag{2}$$

where $\boldsymbol{a_1}$, $\boldsymbol{a_2}$ and $\boldsymbol{a_3}$ three fundamental translation vectors (primitive vectors/axis) can be lie in arbitrary directions and u_1, u_2 and u_3 are three arbitrary integers.

Lattice is formed by the repetition of smallest unit cell called a primitive cell. A primitive cell (volume of space having one lattice point) is the parallelepiped defined by primitive axes $\boldsymbol{a_1}$, $\boldsymbol{a_2}$ and $\boldsymbol{a_3}$. For primitive crystal Systems with higher symmetry we use reciprocal lattice (the sum

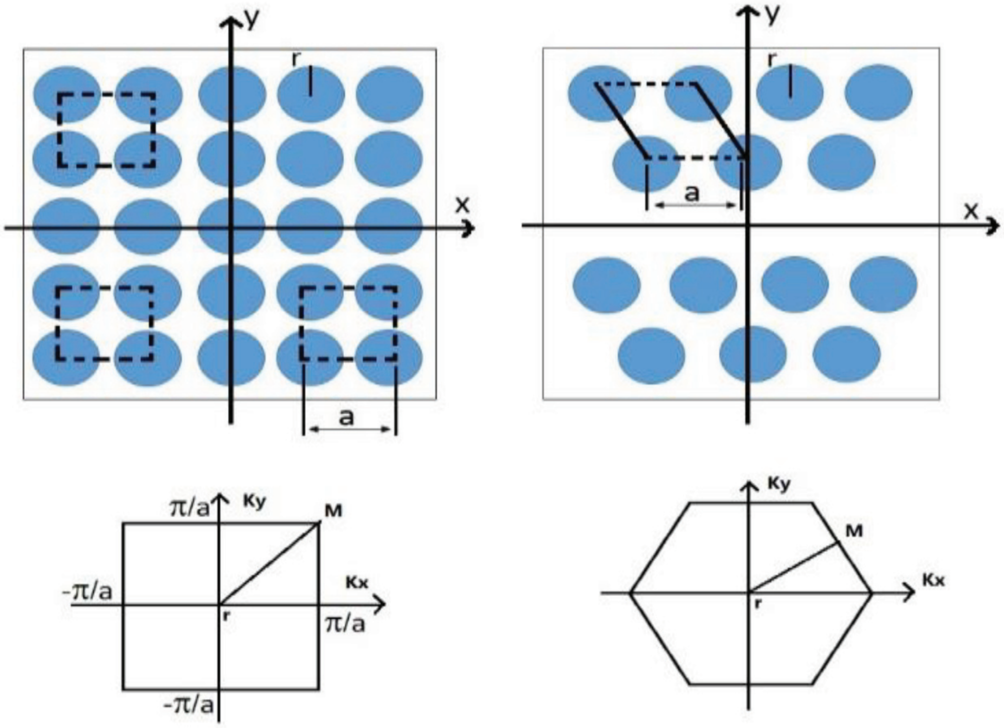

Figure 1. Brillouin zones of two-dimensional cross sections of square and hexagonal lattices with elementary unit cell of lattice parameter "a," and the radius of the inclusions "r."

of the components in the *k*-space). Therefore, the axis vectors of the reciprocal lattice can be constructed from three fundamental translation vectors $\boldsymbol{a_1}$, $\boldsymbol{a_2}$ and $\boldsymbol{a_3}$ [10] (**Figure 1**).

$$b_1 = \frac{2\pi(a_2 \times a_3)}{a_1.a_2 \times a_3}, b_2 = \frac{2\pi(a_3 \times a_1)}{a_1.a_2 \times a_3}, b_3 = \frac{2\pi(a_1 \times a_2)}{a_1.a_2 \times a_3} \tag{3}$$

Any periodic structure, the propagation of acoustic waves in a phononic crystal is determined by the Bloch [12] from which the band structure can be derive in the Brillouin zone. The Brillouin zone is a unit cell in the reciprocal lattice. It should be noted that Brillouin zone can be in one (1D), two (2D), or three dimensions (3D). For desiring the possibility of absolute band gaps phononic crystals has been studied in One Dimension (1D) phononic crystals [13] on the basis of literature, Two Dimension (2D) [14, 15], and Three Dimension 3D [14, 15].

2.2. Band gap

Band gaps are used to explain electronic band structures of materials. Bloch theorem tells us that waves of a certain frequency can propagate without scattering through periodic media. But the propagation of waves is stopped at other frequencies. The frequencies range where the propagation is allowed is called bands and where the propagation is stopped is called band gaps. Phononic band gap in the periodic structure can cause the reflection of mechanical wave when incident on phononic crystals. So the propagation is stopped by generating the mechanical wave inside the phononic crystal. The propagation of mechanical wave with audible frequency range is not permitted in phononic crystals of periodicities ranging from meters to centimeters.

To find the band gap in a phononic crystals, we need to understand the energy band structure of a solid for electrons in a crystalline solid by using following Schrödinger equation [16]:

$$E\psi(r) = -\left[\frac{\hbar^2}{2m}\nabla^2 + V(r)\right]\psi(r) \tag{4}$$

where E is the total energy, Ψ is the wave function, $\hbar$ is the Planck's constant, m is the effective mass and V is the potential, r is the position vector, and ∇^2 is the differential operator.

The above single nonrelativistic particle Eq. (4) shows the total energy is the sum of kinetic energy, and potential energy. Bloch proved the solutions of the wave function in the Schrödinger equation for a periodic potential with periodic function u analogous with crystal as

$$\psi_k(r) = u_k(r)e^{ikr} \tag{5}$$

where ψ is the Bloch wave, k is the crystal wave vector, r is the position, e is Euler's number with imaginary unit i. Actually it consist of product of a plane wave, and a periodic function u_k. The band structure is usually in the form of a dispersion relation between the angular frequency ω and the wave vector k. And k should be in the primitive cell of the primitive lattice vectors of the reciprocal lattice (the first Brillouin zone). Let "a" is the periodicity of one dimensional system, then primitive reciprocal lattice vector is $P = (2\pi/a)$. So the region $[(-\pi/a), (\pi/a)]$ is the first Brillouin zone (**Figure 2**) [10].

Note that gaps width depends upon the difference of wave velocities in the two materials. It means that more difference in periodic medium gives wider band gap. Now comes to phononic band gap. As we are much familiar that in a solid medium (material) atoms cannot move independently since they are connected by chemical bonds and also they move around their equilibrium positions and exert a force on their neighboring atoms to displace, and this displacement cause the phonons creation. The phononic crystals' band structure depend upon the propagation of the elastic/ acoustic waves with suitable materials, shape, crystal lattices, and inclusions with background material [17–19] based on Bragg scattering [17] or by local

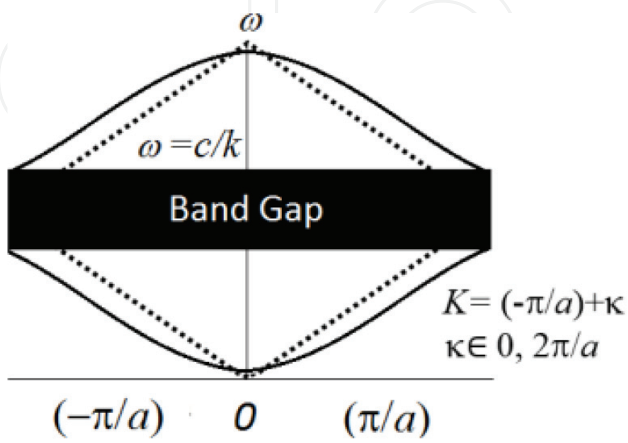

Figure 2. Frequency vs. wave vector for one dimensional linear homogeneous medium (dotted lines), and two dimensional periodic medium (solid lines).

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resonance (LR) mechanism [17, 20] in which band gap formed by the internal resonances of the individual inclusions.

As we are familiar with propagation of wave with the motion of atom (say n) with wave number k and angular frequency ω which satisfy the following equation

$$\psi_n = Ae^{ikna}e^{i\omega t} \tag{6}$$

With dispersion relation ω, and upper bound limit of angular frequency ω_0

$$\omega = \omega_0 \left| \sin k\frac{a}{2} \right| \tag{7}$$

So the dispersion relation is in symmetric interval wave vector k $\in(-\pi/2, \pi/2)$.

2.3. Dispersions in phononic crystal

The dispersion relations are expressed in terms of the angular frequency $\omega(k)$ and wavenumber (wave vector) k. Dispersion represent the band structure.

$$\omega(k) = V(k)k \tag{8}$$

where $V(k)$ is the wave speed (V is the function of k), and k is the wave vector can be written as $k = 2\pi/\lambda$. In term of phase velocity k should be $V_p=\omega/k$. So the rate of change of angular frequency with respect to time is

$$V_g = \frac{\partial\omega}{\partial k} \tag{9}$$

Eq. (8) shows that dispersion curves are dependent of materials characteristics like elastic constant, and phononic crystal structure. The band gap can be calculated as frequency range between two continuous dispersion curves associated with wave vector k. The propagation of acoustic wave in phononic crystal can be more due to large gap. **Figure 14** describes the band structure with dispersion curves in phononic crystal.

3. Physical nature of phononic crystal

Nature of materials (solid or fluid), and physical characteristics (density and elastic constants) of the inclusions plays an important role in the gaps bandwidth. So, PnC can be define into three classification according to its physical nature.

3.1. Solid–solid phononic crystals

The band gap in these structures is formed by the low and high contrast [20] between different materials. This type of PnC can be square, triangle, and honeycomb [21] which shows its band gap impact. Moreover for two dimensional solid-solid PnC the elastic displacement is perpendicular to the cylindrical axis in-plane propagation, and parallel to the cylindrical axis out-of-plane propagation [22].

3.2. Fluid–fluid phononic crystals

Only longitudinal modes can exist in these PnCs. These PnCs made up of two different fluids. Large band gap for this PnC can be found by arrangement of Soft polymer hollow cylinders in a water background at low frequencies [23].

3.3. Solid–fluid (mixed composite) phononic crystals

These PnCs can be constructed by solid inclusions in a fluid (condensed liquid [24, 25, 33] or a gas [26, 27]) matrix and vice versa. Only complex modes of vibration occur from longitudinal in the fluid to longitudinal and transverse in the solid region. So that is why the mixed composite PnCs' acoustic band structures cannot be predicted accurately by using plane wave expansion (PWE) method [23].

Moreover shape of the inclusions play an important role in the formation of band gap. According to geometry PnCs can be in one (1D), two (2D), or three dimensions (3D). Absolute phononic gap should be appear at frequency below the Bragg limit, so this phenomena can happen in locally Resonant materials [28] and can obtained in 2D, and 3D phononic crystals.

4. Phononic crystal composition

Phononic crystals consist of different dimensional periodicity structure having their own characteristics.

4.1. One dimensional phononic crystal

One dimensional phononic crystals (PnCs strip) [4, 18, 20, 29] are composed of two or many layers repetition of geometrical space in a certain direction. The one -dimensional PnCs are also called super lattices (SLs) [30]. The combination of solid–solid or solid–fluid-layered formed each cell of super lattices. The only one direction is responsible for the propagation of an elastic (acoustic) waves in these models.

The periodic band gap structures of SLs consist of crystalline, amorphous semi-conductors. One-dimensional PnC is made up of N cells which show two types of confined states [12]:

i. N–1 states in the allowed bands

ii. One and only one state corresponding to each band gap and do not depend on the width of the crystal N [31].

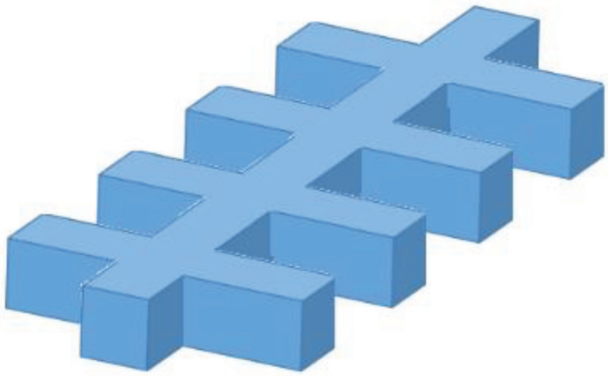


Figure 3. 1D phononic crystal structure.

From 1D systems we conclude that their design is more suitable and based on very simple analytical and numerical calculations to understand different physical properties relevant with band gaps. One dimensional PnCs mainly focus on exploiting the properties of stubs like the shape of the stubs, locations of stubs on the background material, types of waves and creation of defect of background to widen or lower band gaps [18]. In the range of low-frequency there is a wave speed for propagation perpendicular/parallel to layering [31, 32], the one-wave speed for propagation perpendicular to the layering, and two-wave speeds for propagation parallel to the layering (**Figure 3**).

4.2. Two dimensional phononic crystal

As compare to one dimension (1D) PnC the two dimension (2D) PnC has better ability to trap the elastic energy. Repetition of the periodicity in two directions of the space formed a 2D PnCs structure. Its structural arrangement is like the pattern of air holes on silicon or piezoelectric materials [5, 6]. This type of PnC slab can be constructed in square, triangle, hexagonal lattices, or folded structure [34] stepped pillars and holes [3], honeycomb lattice [33], square pillar [19], chessboard-patterned bi-component array, square lattice with cylinder pillars [35], and polygonal graphene like lattice [36]. Following is the schematic of square pillars PnC plate (**Figure 4**) [18].

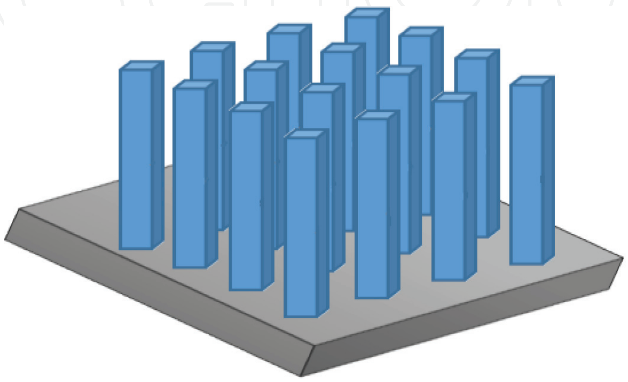
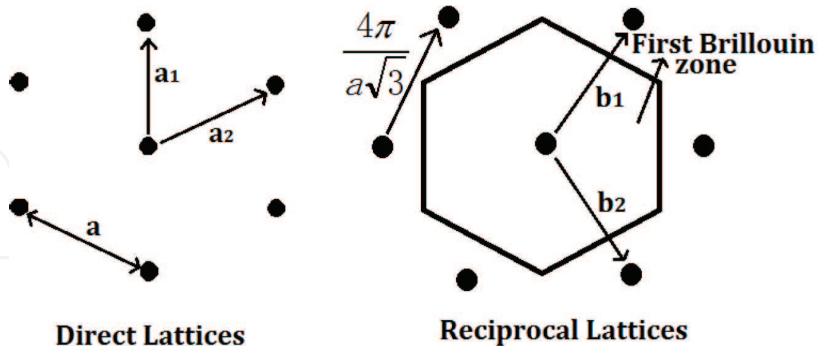


Figure 4. 2D phononic crystal structure.

A triangular Bravais lattice crystal [37] having cylinders shaped assembled structure with vertices of the equilateral triangles with vectors $\boldsymbol{a_1}$, $\boldsymbol{a_2}$ considered as direct lattice, and b_1, b_2 correspond to reciprocal lattice as shown in structure below.



$\vec{a_i} \cdot \vec{b_j} = 2\pi\delta_{ij}$, and $|\vec{b_1}| = |\vec{b_2}| = \frac{4\pi}{a\sqrt{3}}$, δij is known as Kronecker delta.

4.3. Three dimensional phononic crystal

Crystals with scattering units (rod, sphere) [25] that are simply air void cylinder which gives rise to Bragg reflections of the acoustic (elastic) waves. So constructive or destructive interference creates in crystal and these constructive and destructive interference creates frequency range at which wave propagate or block. The propagation, and blocking of waves refer to pass bands, and stop bands respectively. Structure of crystal plays an important role in the creation of band gap. It means that contrast between the materials can be produced with the large band gaps. Like changing from water to epoxy (liquid matrix to the solid) gives larger band gaps [38]. The fabrication process of the 3D phononic crystals requires high accuracy of structural patterns (**Figure 5**) [38, 51].

Structure of 3D phononic crystal is made up of a face-centered cubic (FCC) crystal having spheres shaped assembled structure obtained from one sphere which is added to the center of each face of the simple cubic unit cell.

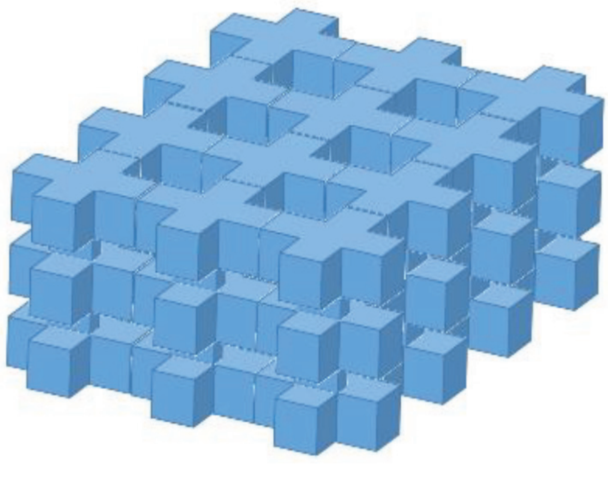
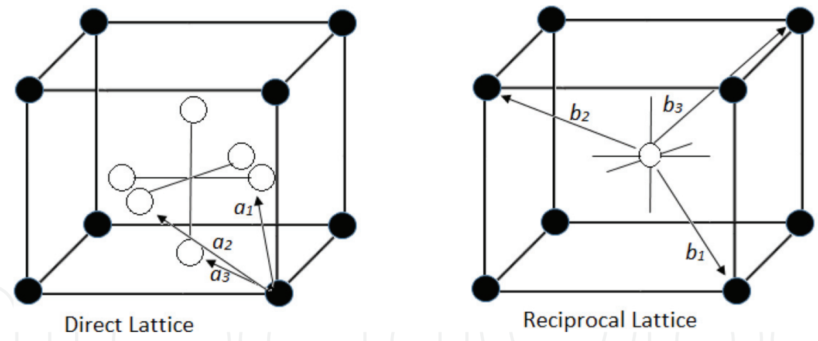


Figure 5. 3D phononic crystal structure.



where $\vec{a_1}$, $\vec{a_2}$, $\vec{a_3}$, $\vec{b_1}$, $\vec{b_2}$, $\vec{b_3}$ are the primitive vectors, and a is the cube edge length.

$\left|\vec{a_1}\right| = |a_2| = |a_3| = \frac{a}{\sqrt{2}}$, and $\left|\vec{b_1}\right| = \left|\vec{b_2}\right| = \left|\vec{b_3}\right| = \frac{2\sqrt{3}\pi}{a}$.

So artificially complex structure of 3D phononic crystal fascinating the researcher to develop new kind of phononic crystal with more precise attenuation band in the range of acoustic frequency with better performance used in an engineering field.

5. Phononic crystal based MEMS resonator

Now a days MEMS technologies and the applications of MEMS resonators in communication systems are widely used. MEMS technologies covers many of devices like micro-sensors, actuators, accelerometers, variable capacitors, switching filters, oscillators, couplers, and the main is resonators, and other sensing devices. These operation of PnC MEMS devices is based on the energy conversion between the mechanical and electrical domains [9]. There are many MEMS component which are used in electronic application systems, Telematics, Medical Electronics, etc., but PnC MEMS resonator play an important role in such kind of application systems and improve the performance of devices by resonant frequency stability, quality factor, motional resistance, nonlinearity, and power handling. Insertion loss in the filters, and phase noise in the oscillators can be reduce by PnC MEMS resonators. It can also help to avoid signal distortion and stabilize the operating frequency. Air scattering inclusion on solid background and two-dimensional structures are common employing in PnCs MEMS resonators. The micro-mechanical structures of MEMS resonators operate on an electromechanical transduction mechanism. This mechanism is the conversion of reversible process between electrical and mechanical energy [39]. Electrostatic, piezoresistive, and piezoelectric are the three main

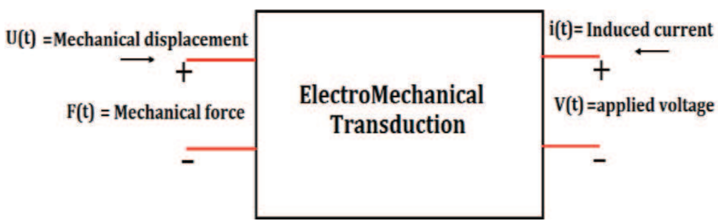


Figure 6. Transduction mechanism.

Transduction methods which are used in MEMS resonators. Following diagram shows the basis transduction mechanism in MEMS resonators (**Figure 6**).

5.1. Electrostatically-transduced MEMS resonators

Electrostatically-transduced MEMS Resonators are also known as capacitive MEMS resonators. The basic principle of this kind of resonator is variation between electrodes and resonating body when the resonant structure vibrates in its mode shape so that the capacitance change. The current is change at output due to change in capacitance by following relation [11].

$$i_m(\omega) = \frac{\partial C_d}{\partial x} V_{DC} \dot{x}(\omega) \tag{10}$$

where $i_m\ (\omega)$ is the motional current, ω is the angular frequency, C_d is the capacitance between the gap and resonant body with bias voltage V_{DC}, and $\dot{x}(\omega)$ is the vibration amplitude of the resonator. A direct current (DC) (which is bias voltage) is applied to the resonator body, and an alternating current (AC) signal to input electrodes. So the capacitance takes place between the output electrode and resonant body. Due to this phenomena an electrostatic force between the input/drive electrodes and resonant body generates from the combination of the AC and DC voltages. So the structure is set into its resonant mode (frequency of the drive signal is same as the resonant frequency of the resonator) (**Figure 7**).

The above structural mechanism is quite very simple it consist of two parallel-plates called electrodes one is input (excite) and other is output (sense) electrode placed at two sides of the resonator. Applied the DC voltage to the resonant body through anchor/support tether. The output signal is taken from the sense electrode by giving AC signal to excite electrode. Although a very high Q is the great advantage of electrostatic transduction through capacitive MEMS resonators. But the main drawback of such resonator are high impedance and low transduction efficiency at the high frequency [40].

5.2. Piezoresistively-Transduced MEMS resonators

In 1856 L. Kelvin discovered the piezoresistive effect, and later this effect is applied on MEMS resonators as a transduction which is studied by some researchers. [41–44]. These kind of

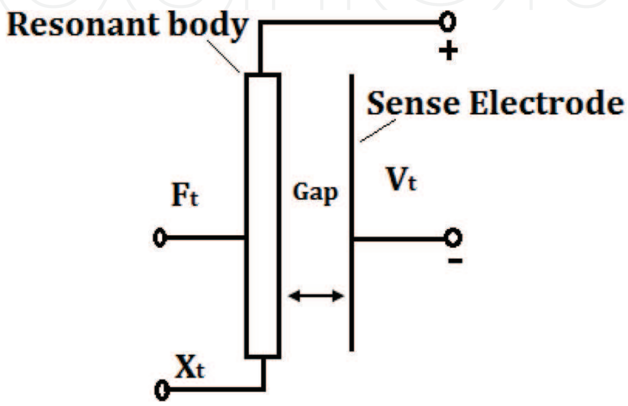


Figure 7. Electrostatic transduction scheme.

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MEMS resonators operate based on the change of electrical resistance of material caused by applied mechanical stress or material deformation by using transduction scheme with silicon piezoresistive. Such phenomena can be seen in some crystalline (non-amorphous) materials. Piezoresistive MEMS resonator show low effective impedance. Moreover piezoresistivity depend upon electrical resistivity by following Eq. (11)

$$\frac{\Delta\rho}{\rho} = (GF)\varepsilon \tag{11}$$

where $\Delta\rho/\rho$ is the relative change in specific resistivity with piezoresistor strain ε, and GF is the strain factor (Gauge factor). Some times GF can be expressed as πE. Here E is the Young's modulus and π is the piezoresistivity matrix expressed below.

$$\pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{11} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_{44} \end{bmatrix}$$

The principle of piezoresistively-transduced MEMS resonator is not so complicated. The electrodes of resonator is applied by both voltage sources AC and DC then vibration is generated by the electrostatic force and the resistance of resonator changes due to this effect, also an AC current is induce by this vibration [44, 45] (**Figure 8**).

V_{DC} is connected through the resistors while V_{AC} is applied through the capacitor, and from the supporting tether the output current has been taken.

5.3. Piezoelectrically-transduced MEMS resonators

In 1880 French physicist P. Curie was first found the piezoelectric phenomena [46] on crystals of quartz, tourmaline, cane sugar, topaz. Piezoelectrically transduced MEMS resonators have been developed in recent years based upon piezoelectric effects and early studies on quartz

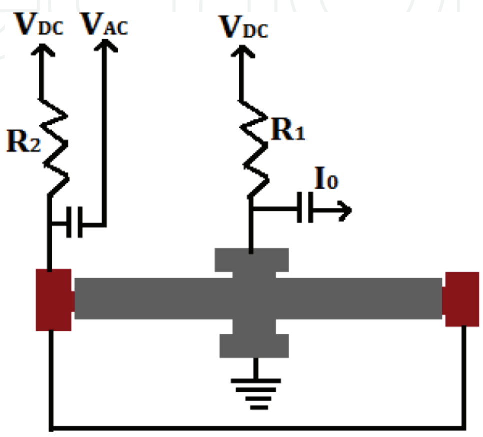

Figure 8. Schematic of piezoresistive resonator.

piezoelectric resonators. The basic principle of the piezoelectric MEMS resonators is that piezoelectric effect happening in piezoelectric materials to induce electric charges on surface of output electrodes. DC voltage must applied to resonator for operation. The impedance of the resonators can be reduced by increasing the DC voltage, so it can improve the performance of resonator. Electromechanical coupling of the resonators is effected by the gap between electrodes and resonant body. Coupling can be high if the gap is narrow. In Electrostatically-Transduced, and piezoresistively-Transduced MEMS Resonators there is a problem of high motional impedance, so this sort of problem can be reduce by piezoelectrically-Transduced scheme because piezoelectric operates on vibration mode then induces charges on the surface of output electrodes when AC is applied to electrodes.

Moreover the performance of resonator depends upon quality factor, resonant frequency, motional resistance, power handling, nonlinearity and frequency stability. Quality factor and operating frequency are the two main parameters that can improve the performance of MEMS devices such as electrostatically-transduced MEMS devices and piezoresistively-transduced MEMS devices which is known as silicon-on-insulator (SOI) technology [42, 49]. But the electrostatically transduced based designs is almost limited at high frequencies due to their inherently small coupling coefficients.

5.4. Quality factor and band width

Quality factor is the ability of energy storage under damping mechanisms at their resonant frequency. When the quality factor is higher, the better the performance of resonator will be better. An ideal resonator can have an infinite quality factor value. Attenuation of quality factor may cause the damping sources. Damping can be generated by temperature, and the nature of materials. If Q factor is higher, then energy loss is low.

Figure 9 above shows the resonance width (band width) Δf, and f (refer to f_r) is the resonant frequency. So $Q = f/\Delta f = 2\pi\, E_{stored}/E_{dissipate\ per\ cycle}$, where E refers to energy.

Figure indicates the high, low, and intermediate Q factors are said to be an underdamped ($Q > 1/2$), over damped ($Q < 1/2$), and critically damped ($Q = 1/2$) respectively. The parameters like bandwidth, spurious signals, ringing also dependent on Q. When the value of Q factor increase

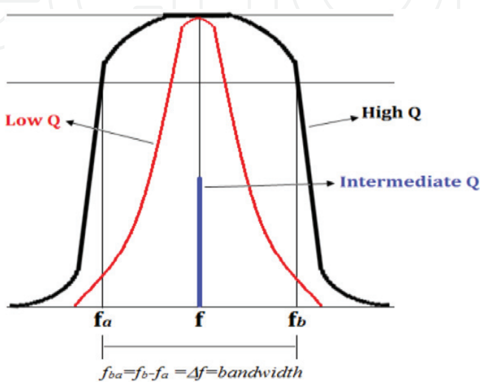

Figure 9. Different frequency responses in MEMS resonators.

the band width of circuit must decrease, so energy storage is better and response of circuit can increase. Moreover when the value of Q increase the spurious signals can be removed by the circuit (losses decrease) and circuit will be able to ring more.

5.5. Motional resistance

Motional resistance is an impedance of resonator can be expressed by the following formula having angular frequency ω, resonator's equivalent mass m_{eq}, and quality factor Q.

$$R_m = \frac{\omega m_{eq}}{\eta_1 \eta_2 Q} \tag{12}$$

where

$$m_{eq} =^{eq} \frac{\rho T}{U_m(x_m, y_m)^2} \int\int U(x,y)^2 dxdy \tag{13}$$

In above equation η is the mechanical coupling coefficient which depends upon piezoelectric transduction mechanism (ratio of the current passing through the resonator to the maximum velocity) can be expressed as

$$\eta = \frac{i}{v_{\max}} = \frac{Q_T}{U_{\max}} \tag{14}$$

where Q_T is the induced electric charge can be expressed as

$$Q_T = \int\int D_i dxdy \tag{15}$$

The motional resistance and quality factor of MEMS resonators are inversely proportional to each other.

6. Phononic crystal strips in MEMS resonator

The phononic crystals are presented as main theories for designing the support tethers in thin film aluminum nitride on diamond contour mode MEMS resonators.

6.1. Support tether configuration

Here we introduce our work [47] on MEMS Resonators with supporting tether configurations which is based on reflector and phononic crystal strip by using thin films piezoelectric material (Aluminum Nitride) on diamond. Diamond is used as a substrate material. **Figure 10** shows the PnC strips support tethers of MEMS resonators.

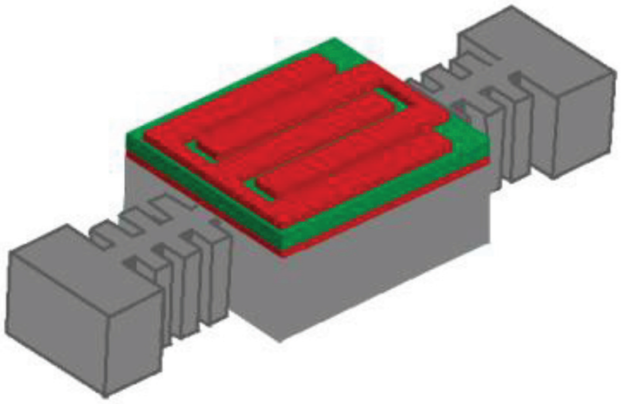

Figure 10. PnC MEMS resonator with supporting tethers.

In this work the quality factor has been improved. Because the tether structures improve the quality factor of MEMS resonators. PnCs also support elimination of anchor loss in the resonators. PnCs can also be designed for sensor applications.

From **Figure 11** we see that each resonator consists of a thin-film aluminum nitride piezoelectric layer sandwiched between two gold (Au) metallic electrode layers which is located on thick diamond substrate layer and operate at 115 and 156 MHz, respectively. Gold has a very high electrical conductivity and very low resistivity [47] so it can reduce the energy dissipation. When thin film aluminum nitride is applied by an electric field from gold electrodes then strain field is created in the thin film and mechanical vibration of resonators increase. Following Eq. (16), and Eq. (17) represent the resonant frequency of WG and WS resonator with effective mass density ρ_{eff} respectively.

$$f_{WG} = \frac{1}{2L}\sqrt{\frac{E_{eff}}{(1+V_{eff})\rho_{eff}}} \tag{16}$$

$$f_{WS} = \frac{1}{2W}\sqrt{\frac{E_{eff}}{\rho_{eff}}} \tag{17}$$

where L is the side length, and W is the side width of WG resonator, and WS resonator respectively, E_{eff}, and v_{eff} is the effective Young's modulus, and the effective Poisson's ratio respectively. We can calculate the values of E_{eff}, ν_{eff}, and ρ_{eff} by using following formula

$$P_{eff} = \frac{t_{AIN}p_{AIN} + 2t_{AU}p_{AU} + t_{Di}p_{Di}}{t_{AIN} + 2t_{AU} + t_{Di}} \tag{18}$$

where t_{AIN}, t_{Au} and t_{Di} are the thickness of aluminum nitride.

Clamping of tether with MEMS resonators at corners is obtained by COMSOL through FE simulation as shown in **Figure 11** above. Now come to the PnC strip with supporting tether [47], as we discussed above that PnC is a highly periodic structure of unit cell which is the basic block. In this work we take the PnC strip of five unit cells as shown below (**Figure 12**).

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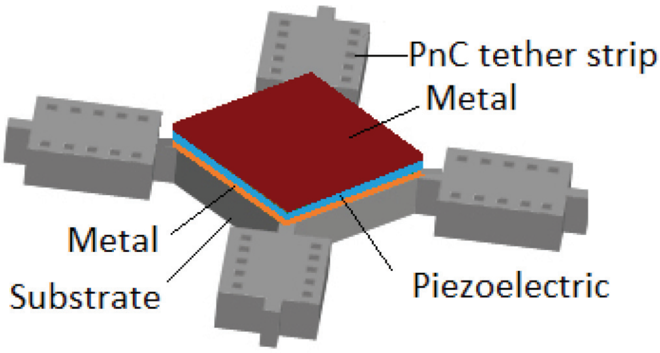

Figure 11. Resonators with Eigen mode shapes.

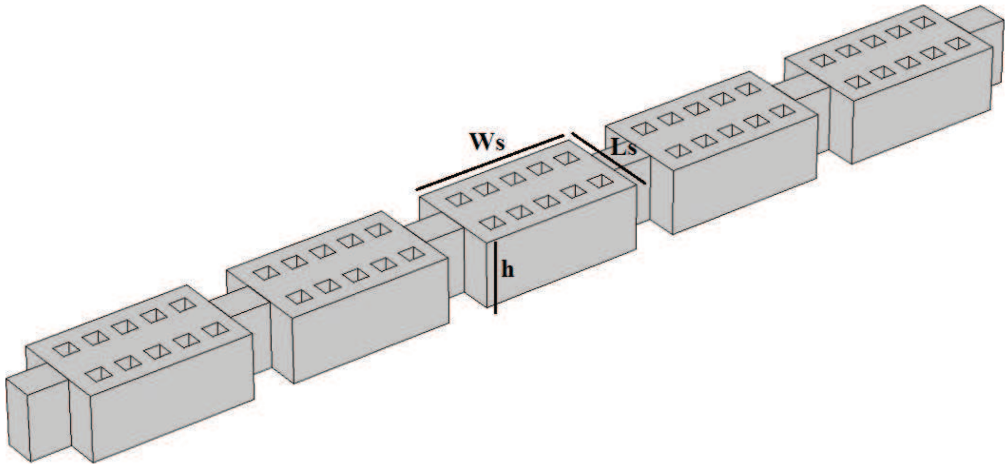

Figure 12. Phononic crystal strip.

Moreover this work only investigates the band gap variation with W_s (stub width), L_s (stub length), and W_h (side hole) of unit cells of the PnC strip.

The parameters of the WG and WS mode resonators and a unit cell of the PnC strip are given in **Table 1**. Following **Figure 13** shows the analysis of eigen frequency mode of the resonators for positioning of anchor tether placement location.

As we discussed above that band gaps is used to explain electronic band structures of materials and can cause the reflection of mechanical wave in the periodic structure when incident on phononic crystals. So here is the band structure with simulated dispersion curve represented as blue dotted lines, and band gaps represented in yellow area having stub width: Ws = 28 μm, stub length: Ls = 30 μm, and hole width: Wh = 2.5 μm (**Figure 14**).

From these results we arrived at this point that.

The role of Stub is important in the formation of band gaps, particularly in its length. If L_s is large the band gap is wide.

i. Holes can help to reduce the static mass of the PnC strip tether without effecting on band gaps.

	WG				WS					
	Resonator	**Electrode**	**Substrate**	**Piezoelectric**	**Resonator**	**Electrode**	**Substrate**	**Piezoelectric**	**Unit cell**	**Stub**
Thickness (μm)		0.1	3	0.4		0.1	3	0.2	3	
Width (μm)	60	60			48	23			6	28
Length (μm)	60				80					30

Table 1. Parameters of PnC strip WG and WS mode resonators.

http://dx.doi.org/10.5772/intechopen.78584

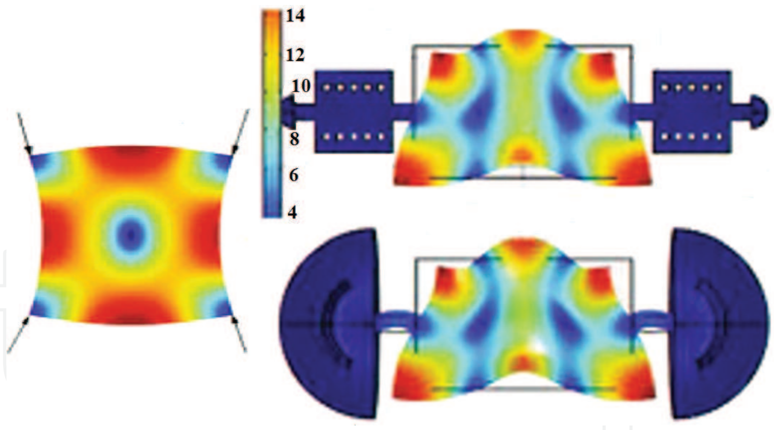

Figure 13. Resonators with Eigen mode shapes.

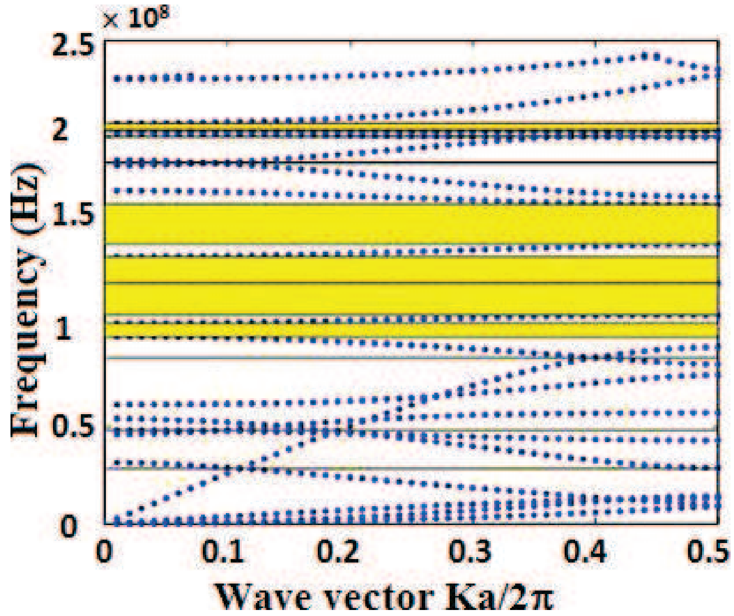

Figure 14. Band structure with dispersion curves in PnC.

6.2. Quality factor and harmonic response

Several MEMS resonators has been fabricated [48] with high quality factor, temperature stability with high frequency. Several techniques have been used to minimize the anchor loss, and improve quality factor in resonator such as impedance mismatching between resonator body and support tethers, quarter-wavelength tethers, narrowed-width tethers, geometrical shape-based tethers, acoustic wave reflection based tethers. And one of the sound technique is phononic crystal (PnC) based tethers [47] which is highly effective in reduction of anchor loss, and improve the quality factor in resonator. High quality factor reduces motional resistance. Phononic crystals boosting the anchor quality factor and present the ability of acoustic/elastic wave propagation isolation as well as reflection. **Figure 15** shows the Q factor, and anchor Q factor for PnC strip tethers.

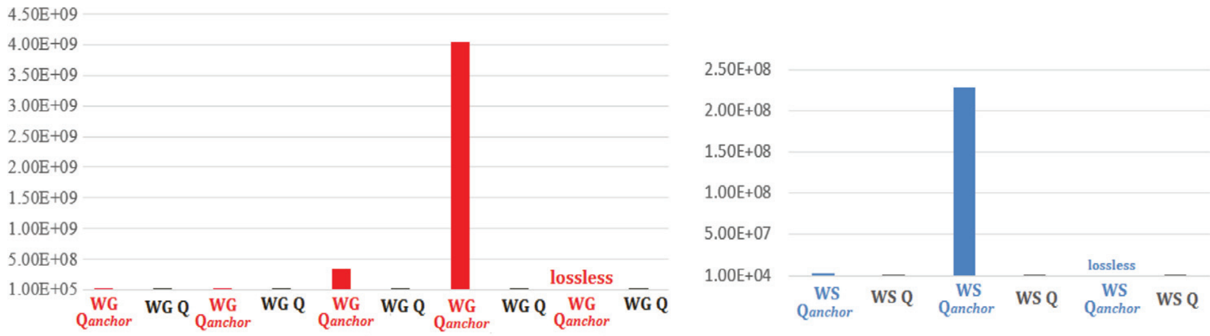

Figure 15. WG and WS modes for PnC strip supporting tethers.

The maximum value of Q obtained from WG mode resonator with five-unit cell PnC strip tethers is 398.5% and from WS mode resonator with three-unit cell PnC strip tethers is 591.1%. The values of Q and Qanchor for WS WG mode resonator, and WS mode resonator for their corresponding unit cells are shown in **Table 2**.

To see the harmonic response of resonators voltage is applied by two sources 1 V and -1 V. **Figure 15** depicted the curve between frequency and displacement. Narrow curve indicate that the quality factor is much higher, but this is fact that the quality factor is always limited by energy losses. In MEMS resonator the harmonic response is represented by electric charge and admittance (**Figure 16**).

6.3. Anchor loss

Anchor is basically the attachment of supporting frame mechanical connection between the resonators In all micromechanical resonator there must be the energy loss called an anchor damping or anchor loss due to radiation of acoustic wave energy from the resonant structure via supporting tether [10, 48, 50, 51], so the energy entered in the substrate when resonator vibrate.

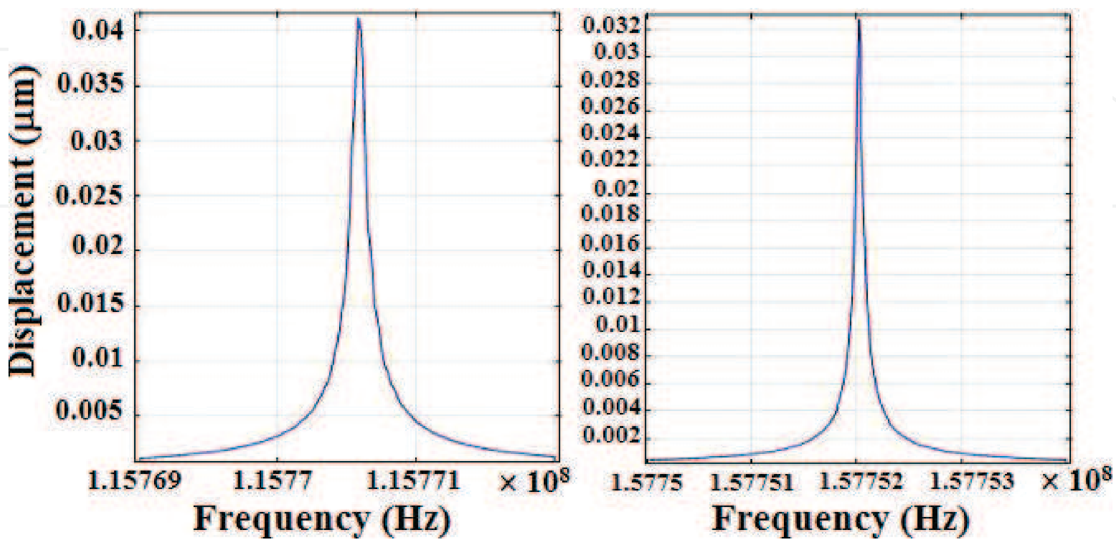

Figure 16. Harmonic response: WG and WS mode PnC strip resonators with supporting tethers.

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Unit Cell	WG mode		WS mode	
	Q_{anchor}	Q	Q_{anchor}	Q
1	1.2954E6	4.5638E5	3.0251E6	3.1128E5
2	2.7839E7	6.9205E5	2.2752E8	3.8383E5
3	3.3660E8	7.1349E5	lossless	4.1347E5
4	4.0541E9	7.2244E5	—	
5	lossless	7.2406E5	—	

Table 2. Q, and Q_{anchor} of PnC strip tether resonators.

In other words we can say that in the resonator, elastic waves are trapped at resonance. This phenomena may cause the loss of energy. The anchor size is responsible for the loss of energy (anchor loss). One way of reduce the anchor loss is to increase the number of tethers and slightly reduce the size of tethers [52].

7. Summary

This chapter has employed the theory of phononic crystal, classification of PnC according to its physical nature, and PnC phenomena in locally resonant materials with 2D, and 3D crystals structure. In this chapter PnC MEMS resonators with different transduction schemes such as electrostatically, piezoresistively, piezoelectrically-transduced MEMS resonators are explained. In this chapter phononic crystal strip in MEMS resonators is explained to reduce anchor loss, so phononic crystal strip with supporting tethers is designed to see the formation of band gap by introducing square holes, and improvement of quality factor. Moreover few simulation tools like COMSOL Multi-physics for designing, MATLAB for extracting parameters and EXCEL for representation of graphs are used.

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