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# Fractal Array Antennas and Applications

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## Abstract

Modern celestial and other advanced wireless communication systems require feasible array antennas with reconfigurable multibeam, broadband, high end of coverage, high gain, less side-lobe level with wider side-lobe level angles, better signal-to-noise ratio and small in size than conventionally achievable. This has initiated array antenna research in different tracks, one of which is by using fractal array antennas. The investigation on fractal-shaped antennas is basically focused on two fundamental areas such as the analysis and design of fractal antenna elements and the application of fractal geometric technology to the design of array antennas. These recursively generated antennas provide new insights into the antenna properties due to their self-similar behaviour. Owing to the feasible geometric construction and advanced properties, fractal antennas find applications in advanced wireless communications, MIMO radars, satellite communications and space observations. This work concentrated here is primarily aimed on the design of fractal array antennas using concentric elliptical ring sub-array fractal geometric design methodology and the reduction of total number of antenna elements at higher expansion factors of both conventional and proposed fractal array antennas.

**Keywords:** fractal, array antennas, array factor properties, concentric elliptical ring sub-array design methodology

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## 1. Introduction

A fractal structure is a never-finishing pattern. These structures are infinitely complex patterns that are self-similar across diverse scales [1]. Due to this self-similar performance, fractals find diverse applications in both science and engineering. The word fractal has its origin in the Latin word *fractus*, meaning an irregular surface. Coastal line of sea, mountains, sea shells, snowflakes, leaves and eye strain of a peacock are some of the naturally existed fractals [2].

**Figure 1** shows some of the naturally existed fractals in the nature. By their geometrical constructions, fractal patterns come in two main variations:

1. Random fractals
2. Deterministic or geometric fractals

All natural fractals come under random fractals because they do not have a particular deterministic way of generation and they are non-integral surfaces. These are also known as stochastic fractals. The generation of these fractals is analysed by different statistical techniques. The randomness of these fractals varies with structure to structure and way of generation. The Brownian motion of microscopic particles in fluid is also the best example for random fractal behaviour as shown in **Figure 2**. Deterministic fractals are geometry-based structures having scaled repetitive nature. These fractals have exact dimensions for the expansion unlike random fractals. Generally, all deterministic fractals are generated using iterated function system (IFS), recurrent iterated function systems (RIFS) and complex number methods. In these methods of

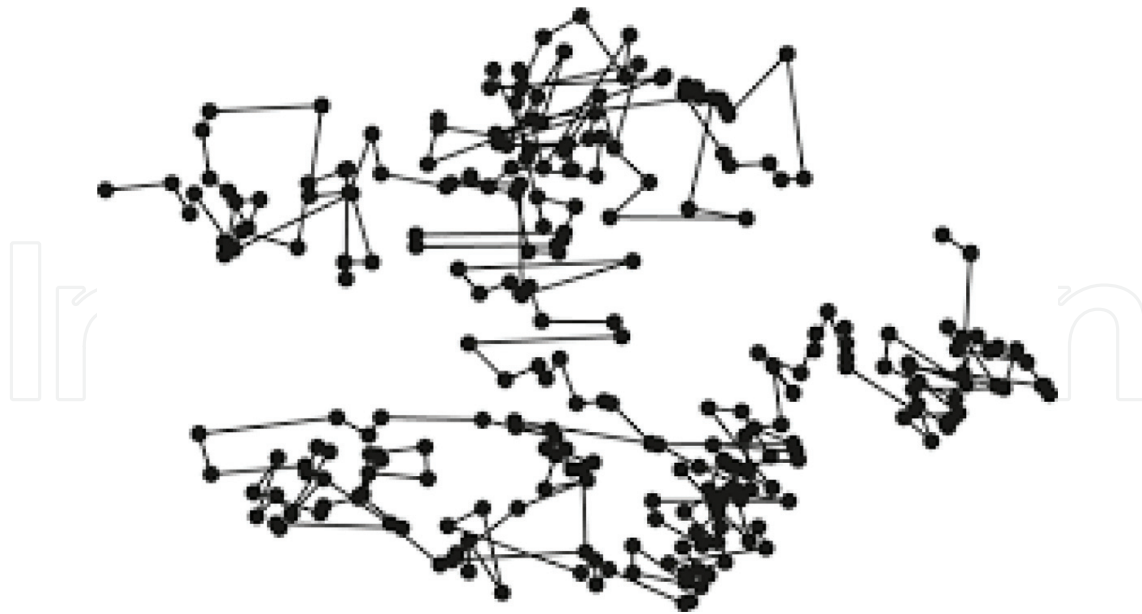


(a)



(b)

**Figure 1.** (a) Eye strain of a peacock. (b) Fractal-shaped leaf.



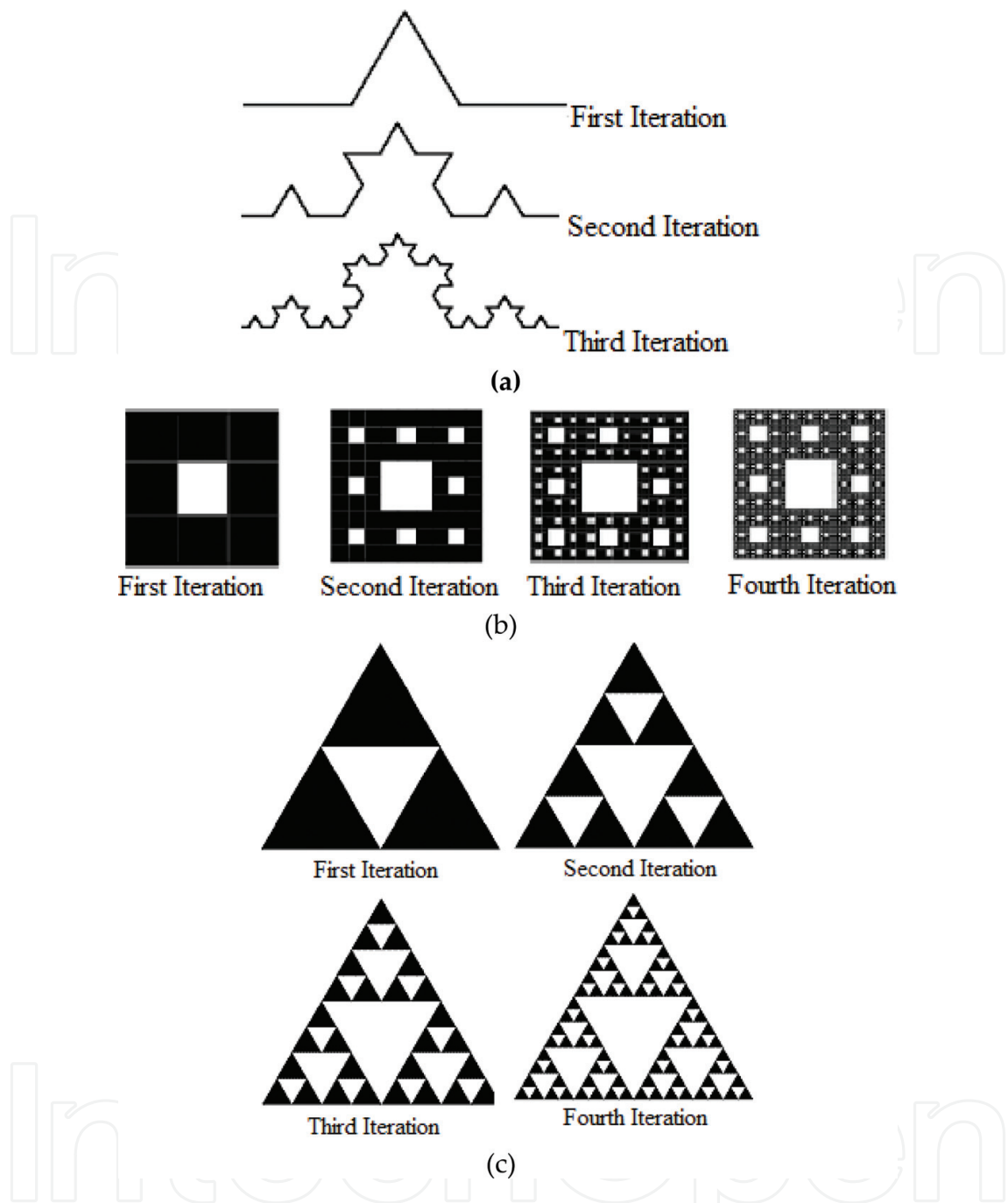
**Figure 2.** The Brownian motion of microscopic particles.

generation, fractal structures are created on the source of scaling, plan axis rotation and dislocation. The most popular IFS and complex number fractals are the Koch curve, the Sierpinski triangle, Julia sets and the Sierpinski square which are shown in **Figure 3**. In these deterministic fractal structures, the basic generator or seed is copied itself up to infinite iterations ( $p$ ) [3–5]. The design methodology proposed in this chapter for the generation of various fractal array antennas is also having a deterministic way of generation.

## 2. Applications of fractals

Fractal geometric technology has permeated numerous areas of science and engineering, such as astrophysics, image processing, biological sciences, bioinformatics, antenna engineering, computer graphics and medical applications:

- Image compression using fractal image coding has led to a major fall in memory requirements and processing time than conventional techniques [6]. **Figure 4** exemplifies the process of fractal image compression. The output images of shape 'A' unite to the Sierpinski triangle. This last image is called 'attractor' for this photocopying mechanism. Any original image will be transformed to the attractor if the mechanism runs repetitively. This characteristic is the advantage to the fractal image compression.
- The fractal structures inspired from the human blood vessels of fractal nature offer an easy low-pressure network to achieve a silicon chip to allow a cooling fluid to uniformly flow across the surface of the chip, and this keeps the computer cool.
- The human body is also having fractal nature. The DNA, retina, blood vessels and lobes of the lungs are self-similar structures. Euclidean geometry is powerless to study and

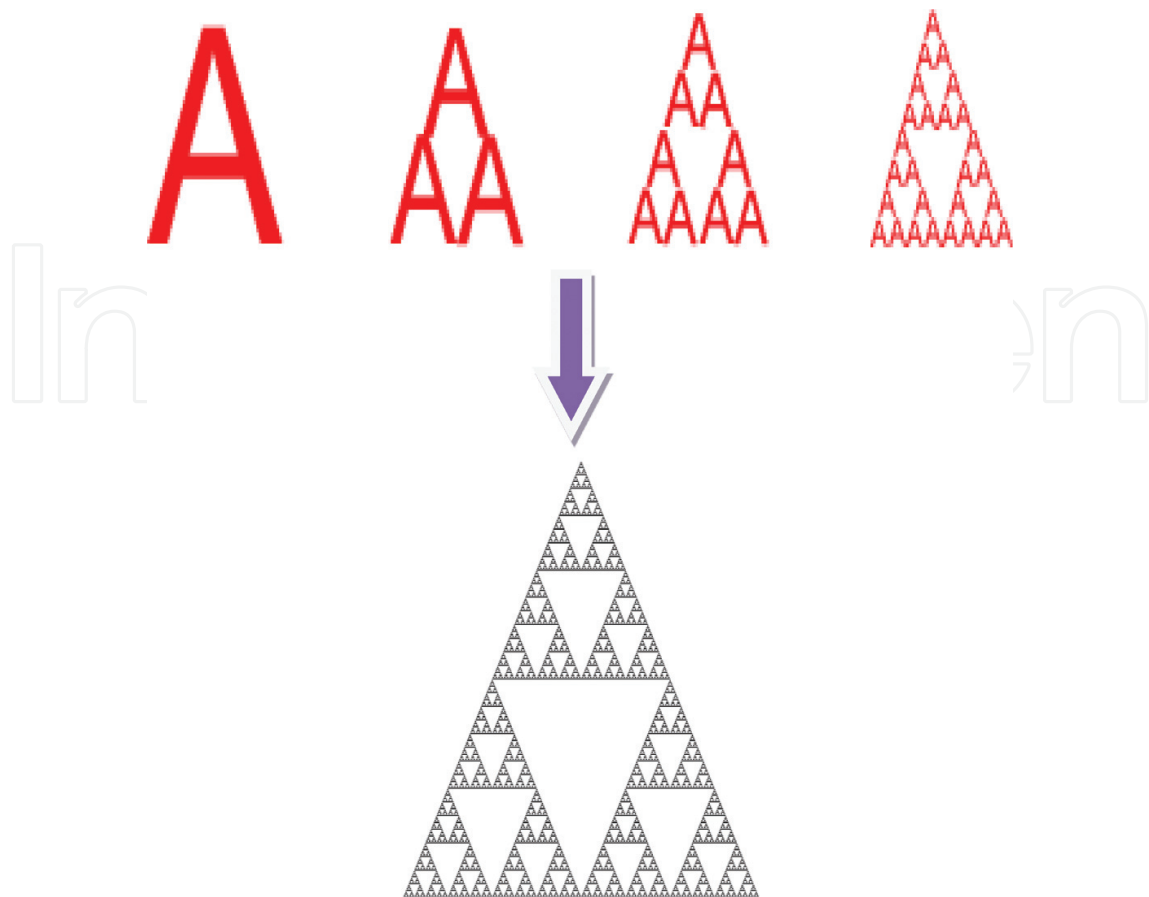


**Figure 3.** (a) Koch curve up to three iterations. (b) Sierpinski square or carpet up to four iterations. (c) Sierpinski triangle up to four iterations.

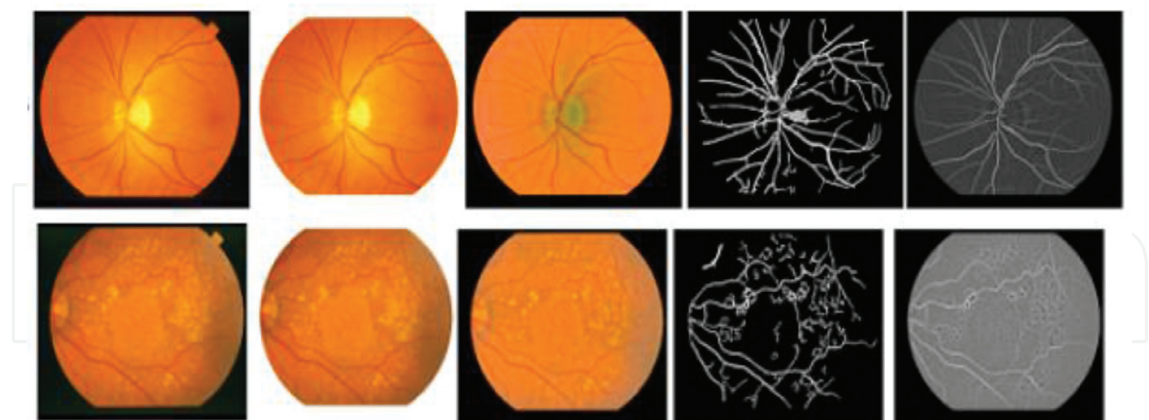
analyse abnormalities in these structures. Fractal geometric technology and fractal analysis tools are more useful to diagnose irregularities in the human body [7], and **Figure 5** shows the retina network of fractal nature.

- Fractal mesh invention has divulged to diminish memory requisites and CPU time for finite element analysis of quivering problems [8].





**Figure 4.** Fractal image compression using Sierpinski triangle.



**Figure 5.** The retina network of fractal nature.

- Bridges, cables and different zones in cities such as industrial, commercial and residential places are designed using this repetitive fractal geometric technology as shown in **Figure 6** [9].
- The largest use of fractals exists in computer graphics. Many image processing schemes use fractal algorithms to create natural and artificial fractal structures digitally [10].



**Figure 6.** Triangular and circular landscape of a city zone.

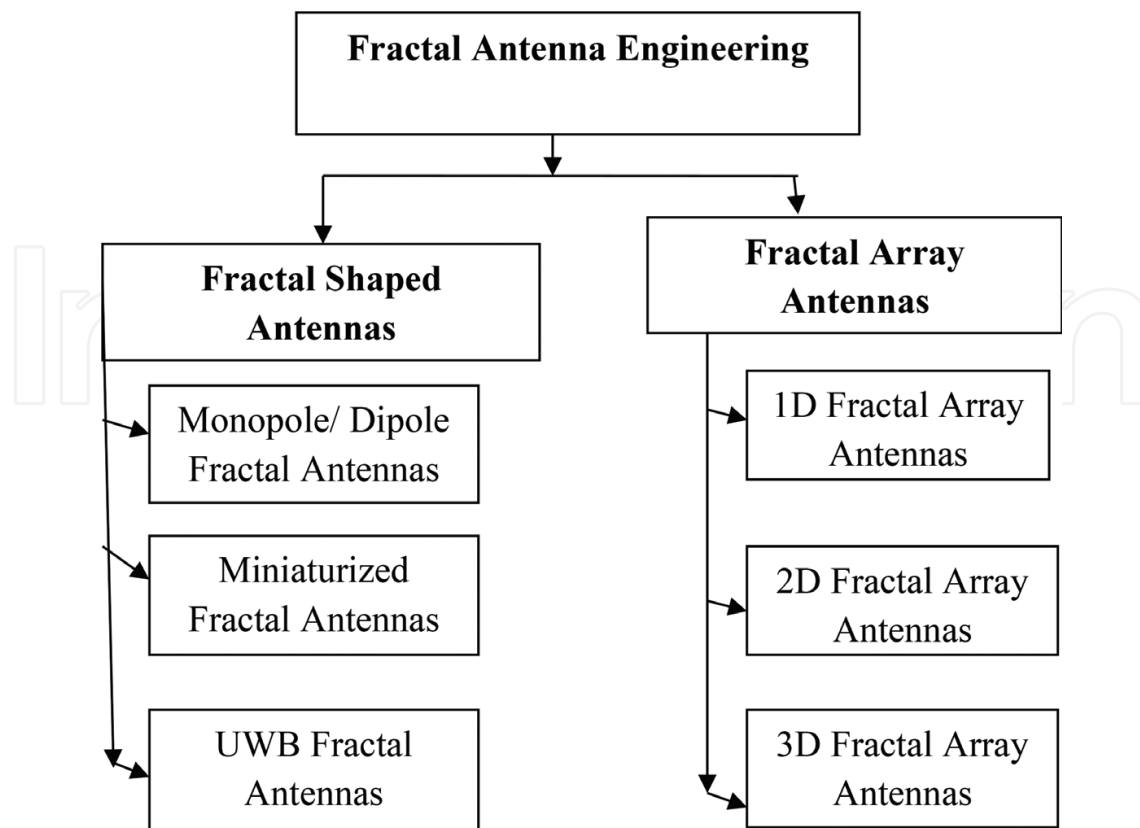
### 3. Fractal geometric technology in antenna engineering

The concept of fractal geometric technology to the antenna engineering was pioneered by Kim and D. L. Jaggard [11]. They introduced random fractal array antennas for less side-lobe levels. Conventional methods to the design and analysis of antennas have their base in Euclidean geometric methodology. There has been a substantial amount of current interest, however, in the option of developing antennas and array antennas that utilize fractal geometric technology in their design methodologies. Actually, designing of antennas using Euclidean geometry is based on a certain formula and analytical equations, but in this fractal geometry, designing of antennas depends on iterative functions and their recursive algorithms.

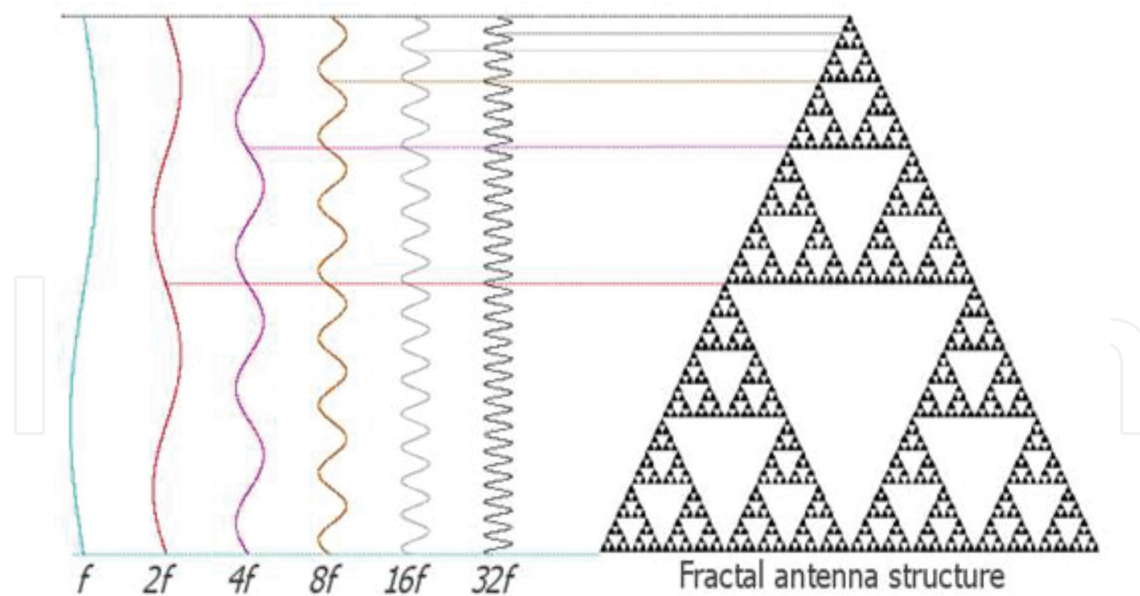
Fractal antenna engineering is having two main branches of antenna design methods to fulfil the requirements of wireless-based communication systems. **Figure 7** shows the two main branches of 'fractal antenna engineering'. Depending on their properties and designing parameters, both fractal-shaped radiators and fractal array antennas are again classified into various types. Both types are playing a significant role in the advanced communication systems owing to their magnificent radiation characteristics and miniaturized design techniques.

### 4. Fractal-shaped radiators

The multiband behaviour of fractal-shaped antennas was introduced by C. P. Baliarda. In that study, Sierpinski and Koch monopole antennas were initiated, and these fractal antennas have multiband performance over different frequency bands as shown in **Figure 8**. Such performance



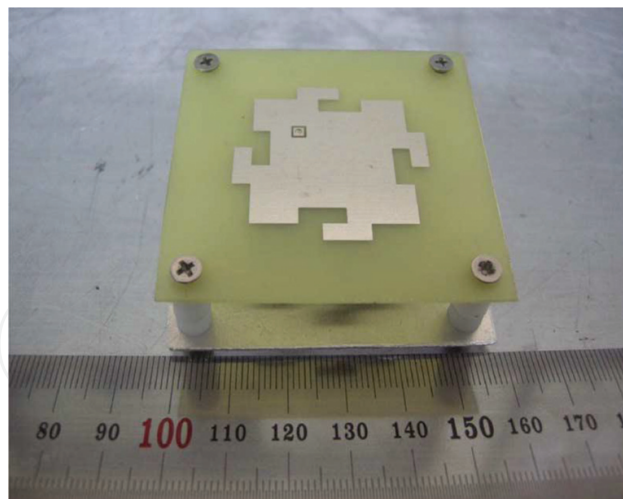
**Figure 7.** Basic classifications of fractal antennas.



**Figure 8.** Logarithmic frequency response of from Sierpinski fractal structure.

is based on the repetitive nature of the fractal structures, bends and corners [12–15], and some more fractal antennas such as modified Sierpinski monopole, modified half-Sierpinski gasket and Mod-P Sierpinski fractal antennas were introduced for multiband applications in [16–18].





**Figure 9.** Miniaturized Giuseppe Peano microstrip patch antenna [32].

Like Sierpinski fractal gasket antennas, Sierpinski fractal carpet structures have also been used in the designing of antenna elements [19]. One of the serious setbacks with a small loop antenna is that the input resistance is very small, building it hard to couple power to the antenna. By using a fractal loop, the input impedance of the antenna increases. Koch Island, Minkowski and hexagonal geometry with triangular loop are the best examples for the fractal loop antennas [20–24].

Fractal geometric technology was also introduced in microstrip patch antennas instead of conventional rectangular, circular and square geometries, and this leads to improved gain of those antennas with multiband and ultrawideband behaviour [25]. The analogous insight of raising the electrical length of a radiator can be applied to a patch antenna [26]. The patch antenna can be analysed as a ‘microstrip transmission line’. So, if the current will be forced to pass through the convoluted path of a fractal structure rather than a conventional Euclidean pathway, the area needed to engage the resonant transmission line will be reduced. This method has been applied to patch antennas in a range of forms [27–29]. Recently, novel patterns of fractal antennas are projected for miniaturization applications, and miniaturized Giuseppe Peano microstrip patch is shown **Figure 9** [30–32].

## 5. Fractal array antennas

An array antenna is one of the best solutions for the long-range communication systems rather than aperture antennas. The multiplicity of antenna elements allows more particular control of the radiation pattern, thus resulting in lower side lobe level and high directive scanned beams. Due to these fundamental properties, array antennas play a vital role in military, defence and other space applications. Owing to novel insights into array antenna parameters like low side lobe level with narrow beams and wider side lobe level angles, ultrawideband, multibeam,

feasible and simple design methodologies and algorithms of fractal array antennas, usage of these arrays increases quite commonly in the antenna literature from the past two to three decades. Due to these properties, fractal array antennas find applications in celestial and other advanced communication systems.

Random and deterministic fractal array antennas are the two basic types of fractal arrays based on their geometric construction. Again, deterministic fractal array antennas are also divided into three types based on their geometric patterns [33–36]:

1. Linear (1D) fractal array antennas
2. Planar (2D) fractal array antennas
3. Conformal (3D) fractal array antennas

This chapter focused on the design methodology of linear and planar deterministic fractal array antennas using concentric elliptical ring sub-array design methodology. In this process of design, the behaviour of fractal nature should apply to the regular concentric elliptical antenna array. This recursive process will produce self-similar concentric elliptical geometry as depicted in **Figure 10**. It is clear from the definition of fractal that the same shape repeats again and again; in this manner geometric structure considered here also repeats again and again. The array antennas proposed in this work can be defined as arrays of arrays, which means that the original counterpart of the array antennas repeats again and again. The general array factor of fractal nature is defined in Eq. (1), which is based on the definition of self-similar nature. The equation for the fractal array factor is the product of generating sub-array factor [37–38]:

$$A.F_p(A.F(\theta, \varphi)) = \prod_{p=1}^P \text{GSA}(S^{p-1}(A.F(\theta, \varphi))) \quad (1)$$

where GSA and A.F stand for generating sub-array and array factors, respectively. The array factor of concentric elliptical ring sub-array geometric generator for the design of linear and planar deterministic fractal array antennas is given in Eq. (2):

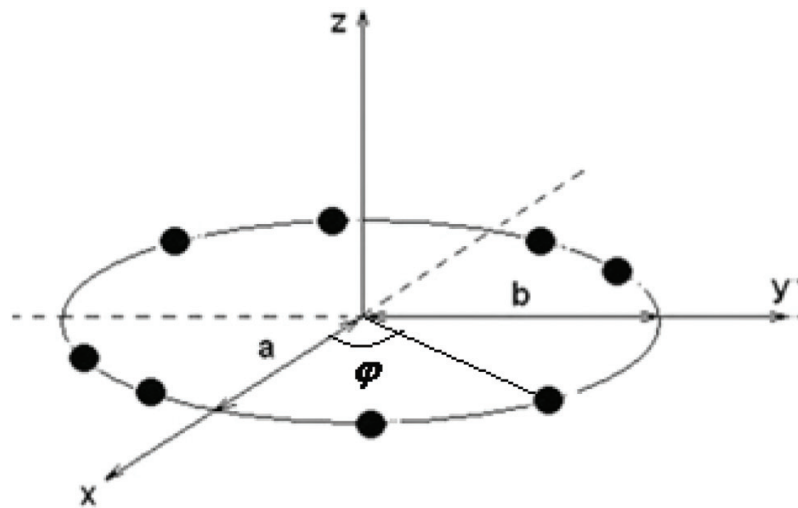
$$A.F_p(\theta, \varphi) = \prod_{p=1}^P \left[ \sum_{m=1}^M \sum_{n=1}^N I_{mn} e^{jkS^{p-1}\Psi_{mn}} \right] \quad (2)$$

$$\begin{aligned} \Psi_{mn} = & (a \cos \varphi_{mn} \cos \varphi + b \sin \varphi_{mn} \sin \varphi) \sin \theta \\ & - (a \cos \varphi_{mn} \cos \varphi_0 + b \sin \varphi_{mn} \sin \varphi_0) \sin \theta_0 \end{aligned} \quad (3)$$

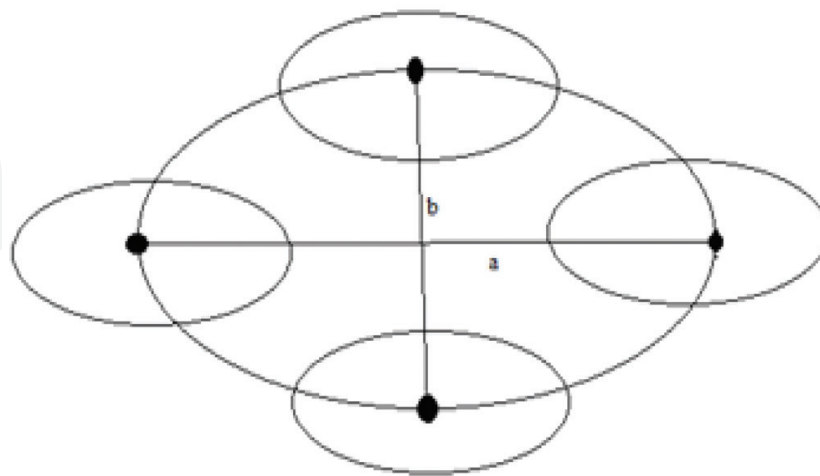
$$\varphi_{mn} = \frac{2\pi}{N} (mn - 1) \quad (4)$$

where S is the scaling factor and two is the scaling factor of the considered sub-array; P is the iterations and four successive iterations have considered in this work, and it can be extended up to infinite iterations; M is the number of concentric rings and here only one concentric has been considered; N is the number of antenna elements and a number of antenna elements are

depending on the iterations;  $k$  is the wave equation,  $I_{mn}$  uniform current amplitudes;  $\varphi_{mn}$  is the position of the antenna element in x-y region; and  $\theta_0$  and  $\varphi_0$  are steering angles. This geometric technique replicates the concentric circular ring sub-array design methodology, but in this case, the circular generator is replaced with the elliptical generator as depicted in **Figure 10**. The proposed methodology permits choice in broadening the shape of a radiation beam or for designing multiple beams for any deterministic 1D and 2D fractal array antennas without entailing any amplitude variation and with less power constrain. The triangular fractal array antenna of expansion factor of 2 and four successive iterations have been designed by concentric elliptical ring sub-array design methodology which is observed in **Figure 11**, and

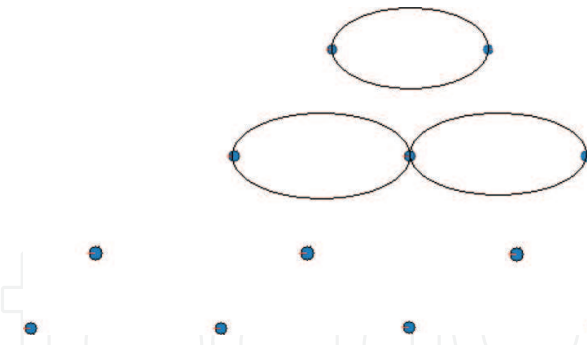


(a)

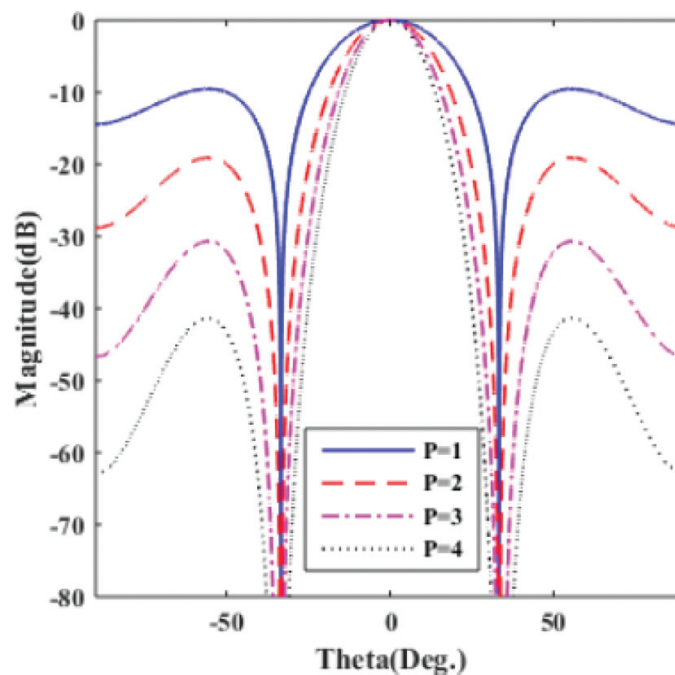


(b)

**Figure 10.** Concentric elliptical sub-array geometric generators for (a) stage 1 and (b) stage 2 [38].



**Figure 11.** The first four iterations of linear fractal array antenna for an expansion factor of 1 [37].



**Figure 12.** Array factors of triangular fractal planar array antennas generated by concentric elliptical sub-array methodology for  $S = 1$  and up to four iterations [37].

the corresponding design equation is described in Eq. (4). The basic triangular array starts with three elements and expands up to four iterations in this chapter. Same distance ( $d = \lambda/2$ ) between the antenna elements maintained for both expansion factors ( $S$ ) of 1 and 2. Nearly one third of the antenna elements can be thinned in the first iterations due to recursive nature of the proposed methodology. **Figure 12** depicts array factor of proposed triangular fractal array antenna. Wide side lobe level angles of  $53.1^\circ$ ,  $55.4^\circ$ ,  $54.1^\circ$  and  $55.8^\circ$  are observed in four successive iterations with a proper balance between the remaining array factor properties:

$$A.F_p(\theta, \varphi) = \prod_{p=1}^4 \left[ \sum_{m=1}^1 \sum_{n=1}^3 I_{mn} e^{jkS^{p-1}\Psi_{mn}} \right] \quad (5)$$



## 6. Conclusions

Fractals are self-similar structures. Various fields of science and technology have inspired by these self-similar structures to develop easy and reliable systems. Application of fractal concepts to the antenna engineering leads to new insights into the antenna parameters. Any polygon-shaped fractal array can be constructed using concentric elliptical ring sub-array design methodology. This design methodology and fractal array antennas generated by this methodology can be helpful for the generation of multiple beams with different array factor properties using a single fractal array antenna without any hardware complexity.

## Author details

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