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Chapter 2

Nonlinear Calculations of the Strength of Cross-sections of Bending Reinforced Concrete Elements and Their Practical Realization

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Additional information is available at the end of the chapter

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Abstract

Calculation methodology of reinforced concrete elements based on the calculated resistance of reinforced concrete is presented. The basic depending which allows setting the strength of bending sections and elements is obtained. The reliability of the dependencies is experimentally confirmed. There are calculation examples of bending elements by the developed methodology. According to the given method, tables have been developed, which depending on the accepted parameters allow determining the resistance of the concrete, the stresses in the reinforced concrete and reinforcement, and the total relative deformation of the cross section. Using the calculated resistances of reinforced concrete allowed to reduce the calculation of reinforced concrete elements according to the nonlinear deformation model to the application of the formulas of the classical resistance of materials and to significantly simplifies the process of their calculation.

Keywords: bending, resistance, beam, deformation model, reinforced concrete

1. Introduction

Concrete is a composite material that is made of gravel, sand, cement, water, and various types of additives. Each of the components has its own characteristics, which together determine the physical and mechanical parameters of concrete. The current normative documents regulate to establish these parameters by testing the experimental samples of specified sizes—prisms or cylinders—the quality of the resulting concrete is controlled by cubes. The resistance of the concrete to the compression is determined by dividing the maximum

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compressive load into the cross-sectional area of the experimental sample of the appropriate sizes [1, 2]. Considering the fact that for different sizes of samples, this ratio will have different meanings, it can be confirming that the term "calculated resistance of concrete" is relative. However, the introduction of this term allowed to have starting points when calculating the cross sections of concrete and reinforced concrete elements under the influence of various force factors.

A similar situation with the tension. Tension is a characteristic of a stress-strain state, which is determined by multiplying the corresponding deformations into a deformation module. Thus, it is not possible to determine the tension directly by experimental way. We determine deformations and then tension by using certain assumptions. Again, without lowering the values of the accepted terms, we have rather relative parameters. Based on these considerations, the introduction of the term calculated resistance of reinforced concrete should also take place. At first sight, this term is perceived quite difficult, especially in conditions of classical reinforced concrete. But at the same time its introduction reduces the calculation of cross sections of reinforced concrete elements to formulas of resistance of materials.

2. The term calculated resistance reinforced concrete

The basic idea of accepting calculated resistance is to separate the geometric parameters from the physical and mechanical ones. When we talk about elements from a single material, this does not cause any contradictions. In the case of composite materials, there are physical, mechanical, and geometric parameters of each material. In many cases geometric parameters can be selected in general from all physical and mechanical ones, but not individually. That is why the calculated resistance of composite materials will depend on the physical and mechanical parameters of all materials of which the cross section of the element is formed. In general, this can be expressed by the following equation:

$$f_i(a_1, \ldots, a_n) = \frac{F_{Ed}}{f(b_1, \ldots, b_n)}, \tag{1}$$

where $f_i(a_1, \ldots, a_n)$ is the calculated resistance of the cross section of the element of a composite material under the condition of destruction on the i material, MPa; F_{Ed} the external calculated force factor, which corresponds to the limiting state of the element; $f(b_1, \ldots, b_n)$ the corresponding geometric characteristic; $a_1, \ldots, a_n$ the physical and mechanical parameters of material's cross section of the composite element; and $b_1, \ldots, b_n$ the geometric parameters of the cross section of the composite element.

For a single cross-section of a composite element there may be a large number of calculated resistances due to the fact that the strength of the cross section is determined by the strength characteristics of all materials from which the composite element is formed. Therefore, the total calculated resistance of a composite material is determined by the minimum value of the calculated resistances under conditions of destruction on all materials from which the cross section of the element is formed:

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$$f = \min\left(f_1(a_1, \ldots, a_n), \ldots, f_i(\ldots, a_n) \ldots, f_n(\ldots, a_n), \right. \tag{2}$$

where f is the calculated (total) resistance of the cross section of composite material.

The calculated resistance of composite materials can be obtained both theoretically and experimentally. To determine it theoretically, the necessary valid hypotheses and statics equations are adopted. The calculated resistance (obtained by this way) does not contain empirical coefficients, but is determined by generally accepted experimental and theoretically grounded hypotheses and prerequisites. In the case of the experimental setting of the calculated resistance, it is more appropriate to determine the calculated resistance separately for each condition of destruction. This allows balancing various experimental studies to the same conditions.

The feature of the use of calculated resistance is that they are determined for specific tabulated values of the corresponding classes or characteristics of the materials. In particular cases, these may be parameters that determine the characteristics of materials of a certain class. For concrete, this characteristic may be $K = 1.05(E_{cd}\varepsilon_{c1}/f_{cd})$ [3].

Introduction of the calculated resistance of composite materials allows the use of no less important term *tension in the cross-section of the composite material* σ_i. It is also a conditional hypothetical term by which it is possible to determine the parameters of a stress-strain state at different levels of load. These tensions are determined by a formula similar to expression (1):

$$\sigma_i(a_1, \ldots, a_n) = \frac{F_i}{f(b_1, \ldots, b_n)}, \tag{3}$$

where $\sigma_i(a_1,\ldots,a_n)$ is the tension in the cross section of an element of a composite material, MPa, and F_i the external force factor, which corresponds to a certain level of load.

The geometric characteristic in expressions (1) and (3) for the same type of deformation has the same meaning. To theoretically obtain these tensions it is necessary to consider systems of equations of equilibrium for a certain type of deformation and to lead them to dimensionless quantities. The parameters that are obtained this way are tabulated depending on the load level, the accepted parameters, the classes of materials, etc. Typically, the tension is determined on the condition that the material does not reach the limit values of the deformations in the operating stages of work of the cross section of the element, and therefore, unlike the calculated resistance, they will have a single value, so there is no need for the introduction of formulas of type (2).

Finally, it is worthwhile to note the features of using the method of calculated resistances:

1. The basics of calculation contain experimentally and theoretically grounded preconditions and hypotheses.
2. Establishment of geometric parameters allows to balance calculated systems of equations to the clear separation of geometrical, physical, and mechanical parameters of cross sections.
3. Diagrams of deformation of materials are established. It should be noted that the adopted diagrams do not play a significant role for this method. The calculated resistance for a certain type of deformation can be established for practically all existing diagrams.

4. By conducting preliminary calculations, the main calculated parameters are tabulated.

The advantages of this method should include:

1. The only methodology for calculating composite materials for nonlinear deformation of materials with classical material resistance.
2. Simplicity and convenience of the calculating process.
3. Ability to use different diagrams of deformation of materials.
4. Setting parameters of a stress-strain state at different load levels.
5. When obtaining new knowledge about the features of deformation of composite materials, it is enough to specify the value of the calculated resistance, and the method of calculation will remain unchanged, which greatly simplify the process of balance of norms [4].
6. Conducting comparative and estimating calculations of cross sections from different materials.

Regarding the disadvantages, then they primarily relate suitability of this method for some classes of materials, and some discomfort associated with using tables.

3. Calculation of bending reinforced concrete elements of a rectangular cross section

Consider the definition of the calculated resistance of reinforced concrete for bending reinforced concrete elements with single reinforcement. In order to show the universality of this method, regardless of the calculation method (force or deformation), first consider the term of the calculated resistance of the reinforced concrete for the force model laid down in the design standards SNiP 2.03.01-84* [5].

For the stress-strain state shown in **Figure 1**, the equilibrium equation is written when $\xi \le \xi_R$, taking the sum of the moments relative to the neutral line:

$$f_{yd}A_s - f_{cd}bx = 0. \tag{4}$$

$$f_{yd}A_s(d-x) + f_{cd}b\frac{x^2}{2} = M_{Ed}. \tag{5}$$

The value of x is determined from Eq. (4) and substituted by expression (5). As a result of simple transformations received:

$$f_{yd}\rho_f - \frac{f_{yd}^2\rho_f^2}{2f_c} = \frac{M_{Ed}}{bd^2}. \tag{6}$$

In formula (6) the left part is denoted by D_1; then $D_1 = M_{Ed}/bd^2$.

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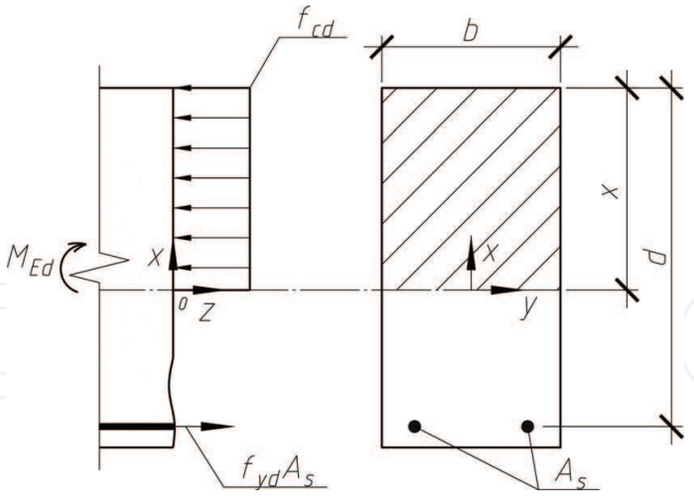


Figure 1. Scheme of forces in the cross section of bending reinforced concrete element for single reinforcement.

For the formula to take a familiar form, which is used in the resistance of materials in the calculations of metal, wooden, and stone structures, the left and right sides are multiplied by 6 and written like

$$6D_1 = \frac{6M_{Ed}}{bd^2} \quad or \quad 6D_1 = \frac{M_{Ed}}{W_c}. \tag{7}$$

In this formula W_c is the elastic moment of the resistance of the working cross section of concrete; $6D_1$ is nothing more than the calculated resistance of the reinforced concrete to the bend $f_{zM,1}$, namely:

$$f_{zM,1} = \frac{M_{Ed}}{W_c}. \tag{8}$$

Similarly, it is obtained with $\xi > \xi_R$:

$$\alpha_R f_c = \frac{M_{Ed}}{bd^2}. \tag{9}$$

The left part is denoted by D_2 and then $D_2 = M_{Ed}/bd^2$, finally receiving

$$f_{zM,2} = \frac{M_{Ed}}{W_c}. \tag{10}$$

The conditions are used (2) for the expression of the total calculated resistance:

$$f_{zM,SNiP} = \min\begin{cases} 6f_{yd}\rho_f - \dfrac{6f_{yd}^2\rho_f^2}{2f_c} \\ 6\alpha_R f_c \end{cases}. \tag{11}$$

Obtained by this way, calculated resistance of reinforced concrete for reinforcement A-400 and A-500 is shown in **Table 1**.

Similar expressions are obtained for nonlinear calculations. Write the value of the corresponding calculated resistances for different cases of destruction:

$$f_{zM,1dm} = 6\left(\frac{\int_0^{\varepsilon_c}\sigma_c\varepsilon_c d\varepsilon_c}{\left(\int_0^{\varepsilon_c}\sigma_c d\varepsilon_c\right)^2} - \frac{\varepsilon_c}{\int_0^{\varepsilon_c}\sigma_c d\varepsilon_c}\right)\rho_f{}^2 f_{yd}^2 + 6\rho_f f_{yd}, \tag{12}$$

$$f_{zM,2dm} = 6\left(\frac{\int_0^{\varepsilon_c}\sigma_c\varepsilon_c d\varepsilon_c}{\left(\int_0^{\varepsilon_c}\sigma_c d\varepsilon_c\right)^2} - \frac{\varepsilon_c}{\int_0^{\varepsilon_c}\sigma_c d\varepsilon_c}\right)\rho_f^2 E_s^2\varepsilon_c^2 + 6\rho_f E_s\varepsilon_c. \tag{13}$$

To determine the corresponding calculated resistance, it is necessary for expressions (12) and (13) to apply an extreme criterion in the form:

$$\frac{df_{zM,idm}}{d\varepsilon_c} = 0, \quad \varepsilon_c \ \epsilon[\varepsilon_{cl}, \varepsilon_{cu}]. \tag{14}$$

The total calculated resistance to bend in calculating by the deformation model will be determined by the condition.

$$f_{zM,dm} = \min\begin{cases} f_{zM,1ДM}, \ \dfrac{df_{zM,1dm}}{d\varepsilon_c} = 0, \quad \varepsilon_c \ \epsilon[\varepsilon_{cl}, \varepsilon_{cu}]; \\ f_{zM,2ДM}, \ \dfrac{df_{zM,2dm}}{d\varepsilon_c} = 0, \quad \varepsilon_c \ \epsilon[\varepsilon_{cl}, \varepsilon_{cu}]. \end{cases} \tag{15}$$

For the further use of the expression (15), it is necessary to adopt a concrete deformation diagram. Adopted function of the deformation diagram does not have essential value, but it must satisfy the conditions for deformation of concrete. Accepting for deformation diagram for concrete Eurocode-2 [3], expression (3.14), the tabulation is executed so that the maximum fault in interpolation will not be more than 5%. The value of the calculated resistance to bending for single reinforcement for all classes of concrete and reinforcement classes A-400 and A-500 are shown in **Table 2**.

Similarly, the calculated resistance for bend for double reinforcement is obtained. For this purpose, the calculated resistance for different conditions of destruction of bending reinforced concrete elements for double reinforcement are determined:

Class of concrete	Percentage of reinforcement ρ_f								
	0.05	**0.50**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.50**	**3.00**
	f_{yd} = 375 MPa (A400C)								
C8/10	1.11	9.49	15.47	15.66	15.66	15.66	15.66	15.66	15.66
C12/15	1.11	10.01	17.54	20.37	21.73	21.73	21.73	21.73	21.73
C16/20	1.12	10.33	18.83	22.39	25.50	28.14	28.61	28.61	28.61
C20/25	1.12	10.52	19.59	23.58	27.20	30.46	33.36	35.06	35.06
C25/30	1.12	10.63	20.02	24.25	28.17	31.78	35.07	40.08	40.08
C30/35	1.12	10.71	20.34	24.74	28.88	32.75	36.35	42.73	44.77
C32/40	1.12	10.77	20.58	25.13	29.44	33.50	37.33	44.26	49.13
C35/45	1.12	10.83	20.81	25.49	29.95	34.21	38.25	45.70	52.31
C40/50	1.12	10.87	20.97	25.73	30.30	34.68	38.86	46.66	53.69
C45/55	1.12	10.90	21.09	25.93	30.59	35.07	39.38	47.46	54.84
C50/60	1.12	10.93	21.22	26.13	30.87	35.46	39.89	48.26	55.99
	f_{yd} = 450 MPa (A500C)								
C8/10	1.32	10.97	15.33	15.33	15.33	15.33	15.33	15.33	15.33
C12/15	1.33	11.71	19.85	21.22	21.22	21.22	21.22	21.22	21.22
C16/20	1.34	12.18	21.72	25.50	27.87	27.87	27.87	27.87	27.87
C20/25	1.34	12.45	22.81	27.20	31.07	34.06	34.06	34.06	34.06
C25/30	1.34	12.61	23.43	28.17	32.46	36.31	38.85	38.85	38.85
C30/35	1.34	12.72	23.88	28.88	33.49	37.71	41.54	43.31	43.31
C32/40	1.34	12.81	24.24	29.44	34.29	38.79	42.95	47.42	47.42
C35/45	1.34	12.89	24.57	29.95	35.03	39.81	44.28	51.91	51.91
C40/50	1.34	12.95	24.79	30.30	35.53	40.48	45.16	53.69	55.27
C45/55	1.34	12.99	24.98	30.59	35.94	41.05	45.90	54.84	58.28
C50/60	1.35	13.04	25.16	30.87	36.36	41.61	46.64	55.99	61.44

Note: Intermediate values are determined by straight-line interpolation.

Table 1. Calculated resistance of reinforced concrete to bend for single reinforcement $f_{zM,SNiP}$, MPa.

$$f_{zM2,1dm} = 6\frac{\frac{\int_0^{\varepsilon_c}\sigma_c\varepsilon_c d\varepsilon}{\varepsilon_c^2} + \rho_{fc}f_{yc}\left(k - nk^2\right) + \varepsilon_c(k-1)^2 k\rho_f E_s}{k^2}. \tag{16}$$

$$f_{zM2,2dm} = 6\frac{\frac{\int_0^{\varepsilon_c}\sigma_c\varepsilon_c d\varepsilon}{\varepsilon_c^2} + \rho_{fc}f_{yc}\left(k - nk^2\right) + \rho_f f_{yd}\left(k^2 - k\right)}{k^2}. \tag{17}$$

Class of concrete	Percentage of reinforcement ρ_f								
	0.05	**0.50**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.50**	**3.00**
	f_{yd} = 375 MPa (A400C)								
C8/10	1.10	9.44	14.68	15.12	15.43	15.67	15.86	16.13	16.32
C12/15	1.11	9.97	17.38	20.09	20.85	21.27	21.60	22.10	22.45
C16/20	1.11	10.30	18.70	22.19	25.20	27.38	27.90	28.71	29.29
C20/25	1.11	10.49	19.48	23.40	26.95	30.11	32.88	34.82	35.65
C25/30	1.11	10.60	19.91	24.08	27.93	31.46	34.66	39.64	40.69
C30/35	1.12	10.68	20.24	24.59	28.66	32.45	35.96	42.12	45.45
C32/40	1.12	10.75	20.49	24.98	29.23	33.22	36.96	43.69	49.26
C35/45	1.12	10.81	20.72	25.35	29.76	33.94	37.90	45.16	51.52
C40/50	1.12	10.84	20.88	25.60	30.11	34.42	38.53	46.14	52.94
C45/55	1.12	10.87	21.01	25.80	30.40	34.82	39.05	46.95	54.10
C50/60	1.12	10.90	21.14	26.00	30.69	35.21	39.56	47.75	55.26
	f_{yd} = 450 MPa (A500C)								
C8/10	1.32	10.90	14.57	15.02	15.35	15.60	15.79	16.07	16.27
C12/15	1.33	11.66	19.40	20.15	20.70	21.13	21.48	21.99	22.35
C16/20	1.33	12.13	21.53	25.17	26.50	27.16	27.71	28.53	29.13
C20/25	1.33	12.41	22.65	26.95	30.69	32.67	33.42	34.57	35.43
C25/30	1.34	12.57	23.28	27.93	32.12	35.82	37.89	39.33	40.40
C30/35	1.34	12.69	23.74	28.66	33.18	37.28	40.93	43.80	45.10
C32/40	1.34	12.78	24.11	29.23	33.99	38.39	42.40	48.01	49.53
C35/45	1.34	12.86	24.44	29.76	34.75	39.42	43.78	51.43	54.52
C40/50	1.34	12.92	24.67	30.11	35.26	40.11	44.68	52.94	58.52
C45/55	1.34	12.96	24.86	30.40	35.68	40.69	45.43	54.10	61.40
C50/60	1.34	13.01	25.04	30.69	36.10	41.25	46.17	55.26	63.37

Note: Intermediate values are determined by straight-line interpolation.

Table 2. Calculated resistance of reinforced concrete to bend for single reinforcement $f_{zM,dm}$, MPa.

$$f_{zM2,3dm} = 6\varepsilon_c \frac{\frac{\int_0^{\varepsilon_c} \sigma_c \varepsilon_c d\varepsilon}{\varepsilon_c^3} + E_s \rho_{fc}(1-nk)^2 k + E_s \rho_f (k-1)^2 k}{k^2}. \quad (18)$$

$$f_{zM2,4dm} = 6\varepsilon_c \frac{\frac{\int_0^{\varepsilon_c} \sigma_c \varepsilon_c d\varepsilon}{\varepsilon_c^3} + E_s \rho_{fc}(1-nk)^2 k + \frac{f_{yd}}{\varepsilon_c} \rho_f (k-1) k}{k^2}. \quad (19)$$

In the given expressions, k is determined from the first equation of equilibrium under the conditions of the destruction of the element.

The total calculated resistance to bend in double reinforcement according to the deformation model will be determined by the condition:

$$f_{zM2,\,dm} = \min\begin{cases} f_{zM2,\,1dm}, \dfrac{df_{zM,\,1dm}}{d\varepsilon_c} = 0, \quad \varepsilon_c \ \epsilon[\varepsilon_{cl},\,\varepsilon_{cu}]; \\ f_{zM2,\,2dm}, \dfrac{df_{zM,\,2dm}}{d\varepsilon_c} = 0, \quad \varepsilon_c \ \epsilon[\varepsilon_{cl},\,\varepsilon_{cu}]; \\ f_{zM2,\,3dm}, \dfrac{df_{zM,\,3dm}}{d\varepsilon_c} = 0, \quad \varepsilon_c \ \epsilon[\varepsilon_{cl},\,\varepsilon_{cu}]; \\ f_{zM2,\,4dm}, \dfrac{df_{zM,\,4dm}}{d\varepsilon_c} = 0, \quad \varepsilon_c \ \epsilon[\varepsilon_{cl},\,\varepsilon_{cu}]. \end{cases} \tag{20}$$

In **Table 3** the expression of the calculated resistance to bend for double reinforcement is derived taking the diagram of deformation of concrete in the form of the function Eurocode-2 [3].

As can be seen from **Tables 2** and **3** in some cases, double reinforcement significantly (more than three times) increases the calculated resistance of reinforced concrete of bending elements and accordingly increases their bearing capacity. In this way, the reinforcement can greatly enhance the compressed area of concrete of bending reinforced concrete elements. For comparison, the data of the calculated resistance to bend for double reinforcement are presented by methodology of SNiP 2.03.01–84* [5] (**Table 4**).

Compare the calculated resistance of reinforced concrete to bending defined by the force model and deformation method for single and double reinforcement. As can be seen from **Tables 5** and **6**, the calculated resistance of reinforced concrete to the bend differs within the limits of the calculated fault. This makes it possible to say that for heavy concrete classes C8/10÷C50/60 and ordinary reinforcement classes A-400 and A-500, calculations of the strength of the cross sections of bending reinforced concrete elements with single and double reinforcement can be performed on any of the mentioned methods. By these ways, the maximum difference will be within 8% and only for certain conditions.

One of the main advantages of the deformation model in comparison with the force one is the possibility of obtaining the parameters of the stress-strain state for the operational load. Let's show how this can be done using the method of calculated resistance of reinforced concrete. For this purpose, the tensions in the bending reinforced concrete element are determined $\sigma_{zM,ДM}$ under operational loads, at which cross sections of the element can work without cracks at $M < M_W$, with cracks in the stretched zone at $M \geq M_W$, without cracks at $M \geq M_W$ (areas in the block between the cracks). In this case, it is proposed to determine the tension in the reinforced concrete for cross sections until formation of cracks.

$$\sigma_{WzM} = 6\varepsilon_{c,\,W} \times \frac{\dfrac{\int_0^{\varepsilon_{c,\,W}} \sigma_c \varepsilon_c d\varepsilon}{\varepsilon_{c,\,W}^3} + \dfrac{\int_0^{\varepsilon_{ctu}} \sigma_{ct} \varepsilon_{ct} d\varepsilon}{\varepsilon_{c,\,W}^3} + E_s \rho_{fc}(1 - nk_W)^2 k_W + E_s \rho_f (k_W - 1)^2 k_W}{k_W^2}. \tag{21}$$

Class of concrete	ρ_f = 0.01			ρ_f = 0.02			ρ_f = 0.03		
	ρ_{fc}/ρ_f								
	0.25	**0.50**	**0.75**	**0.25**	**0.50**	**0.75**	**0.25**	**0.50**	**0.75**
n = 0.06–0.1	f_{yd} = 375 MPa (A400C)								
C8/10	18.09	20.02	21.04	25.81	35.75	41.17	31.54	46.74	60.39
C12/15	19.28	20.55	21.16	31.33	38.53	41.70	37.42	52.38	61.58
C16/20	20.03	20.88	21.19	35.78	39.85	42.03	43.99	56.93	62.33
C20/25	20.46	21.07	21.23	37.52	40.63	42.22	50.11	58.67	62.76
C25/30	20.71	21.17	21.26	38.51	41.06	42.32	53.39	59.66	63.01
C30/35	20.89	21.22	21.29	39.24	41.39	42.35	55.04	60.39	63.19
C32/40	21.02	21.26	21.32	39.80	41.64	42.38	56.31	60.95	63.33
C35/45	21.14	21.31	21.36	40.33	41.87	42.41	57.50	61.48	63.44
C40/50	21.21	21.35	21.39	40.68	42.03	42.44	58.30	61.84	63.50
C45/55	21.28	21.39	21.43	40.98	42.14	42.46	58.95	62.13	63.53
C50/60	21.35	21.43	21.46	41.27	42.25	42.50	59.61	62.42	63.55
n = 0.06–0.1	f_{yd} = 450 MPa (A500C)								
C8/10	20.23	23.59	25.13	27.70	39.59	48.97	34.49	52.69	70.91
C12/15	22.45	24.35	25.25	33.11	44.76	49.73	40.27	58.17	73.21
C16/20	23.52	24.82	25.31	39.06	46.91	50.20	46.71	64.31	74.28
C20/25	24.14	25.08	25.36	43.38	48.03	50.45	52.71	68.76	74.90
C25/30	24.49	25.18	25.41	44.81	48.65	50.50	57.47	70.19	75.25
C30/35	24.74	25.26	25.45	45.87	49.11	50.55	61.98	71.25	75.50
C32/40	24.92	25.33	25.49	46.67	49.46	50.60	65.05	72.05	75.68
C35/45	25.08	25.40	25.53	47.42	49.78	50.65	66.84	72.80	75.74
C40/50	25.18	25.46	25.56	47.92	49.98	50.69	67.99	73.29	75.78
C45/55	25.27	25.50	25.60	48.32	50.13	50.73	68.92	73.70	75.82
C50/60	25.35	25.55	25.64	48.70	50.24	50.77	69.87	74.08	75.87

Note: Intermediate values are determined by straight-line interpolation.

Table 3. Calculated resistance of reinforced concrete to bend for double reinforcement $f_{zM2,dm}$, MPa.

It is noted that the tension σ_{WzM} also allows to determine the moment of formation of cracks, so depending on the tasks, it can also be called the calculated resistance of the reinforced concrete to the bend until formation of cracks.

Tension in a cross section with a crack in the stretched zone at $M \geq M_W$ is determined by the

$$\sigma_{2zM} = 6\frac{\int_0^{\varepsilon_c} \sigma_c \varepsilon_c d\varepsilon}{k^2 \varepsilon_c^2} + \frac{\int_0^{\varepsilon_{ctu}} \sigma_{ct} \varepsilon_{ct} d\varepsilon}{k^2 \varepsilon_c^2} + 6\varepsilon_c E_s \rho_{fc} \frac{(1-nk)^2}{k} + 6\varepsilon_c E_s \rho_f \frac{(k-1)^2}{k} + \frac{(k-1)}{k} \Delta\sigma_{s,x} \rho_f. \quad (22)$$

Class of concrete	ρ_f = 0.01			ρ_f = 0.02			ρ_f = 0.03		
	ρ_{fc}/ρ_f								
	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
n = 0.06–0.1	f_{yd} = 375 MPa (A400C)								
C8/10	18.20	20.06	21.04	26.23	36.61	41.20	31.51	47.37	60.48
C12/15	19.37	20.58	21.17	32.29	38.67	41.71	37.58	53.43	61.64
C16/20	20.10	20.90	21.25	36.06	39.97	42.04	44.47	57.20	62.37
C20/25	20.52	21.09	21.30	37.77	40.73	42.23	50.91	58.91	62.80
C25/30	20.76	21.20	21.32	38.73	41.15	42.33	53.91	59.87	63.04
C30/35	20.94	21.28	21.34	39.45	41.47	42.41	55.52	60.59	63.21
C32/40	21.08	21.34	21.36	40.00	41.72	42.48	56.77	61.14	63.35
C35/45	21.21	21.40	21.37	40.52	41.95	42.53	57.93	61.66	63.48
C40/50	21.30	21.43	21.38	40.87	42.10	42.57	58.71	62.00	63.57
C45/55	21.37	21.47	21.39	41.15	42.23	42.60	59.36	62.29	63.64
C50/60	21.44	21.50	21.40	41.44	42.36	42.63	60.01	62.58	63.71
n = 0.06–0.1	f_{yd} = 450 MPa (A500C)								
C8/10	20.51	23.15	24.78	26.60	37.87	47.32	32.24	49.15	66.05
C12/15	22.31	24.07	25.11	32.49	43.72	48.64	38.12	55.03	70.58
C16/20	23.44	24.64	25.32	39.14	46.02	49.47	44.78	61.69	72.44
C20/25	24.10	24.98	25.44	43.13	47.37	49.96	50.97	67.18	73.54
C25/30	24.47	25.17	25.51	44.63	48.13	50.23	55.76	68.89	74.15
C30/35	24.75	25.31	25.56	45.73	48.70	50.43	60.22	70.16	74.61
C32/40	24.97	25.42	25.60	46.59	49.14	50.59	64.33	71.15	74.97
C35/45	25.17	25.52	25.64	47.39	49.55	50.74	66.68	72.07	75.30
C40/50	25.30	25.59	25.66	47.93	49.82	50.84	67.88	72.68	75.52
C45/55	25.41	25.65	25.68	48.37	50.05	50.92	68.88	73.19	75.70
C50/60	25.52	25.70	25.70	48.82	50.27	51.00	69.89	73.70	75.89

Note: Intermediate values are determined by straight-line interpolation.

Table 4. Calculated resistance of reinforced concrete to bend for double reinforcement $f_{zM2,SNiP}$, MPa.

The tensions between the cracks are determined by

$$\sigma_{mzM} = 6\frac{E_s^2(k-1)^2\int_0^{\frac{\sigma_{s,m}}{E_s(k-1)}}\sigma_c\varepsilon d\varepsilon}{k^2\sigma_{s,m}^2} + 6\frac{(k-1)^2\int_0^{\varepsilon_{ctu}}\sigma_{ct}\varepsilon_{ct}d\varepsilon}{k^2\varepsilon_{ctu}^2} + 6\frac{k-1}{k}\sigma_{s,m}\rho_f + 6\frac{(1-kn)^2}{k(k-1)}\rho_{fc}\sigma_{s,m}. \quad (23)$$

All of the above tensions in concrete are determined by expression:

Class of concrete	Percentage of reinforcement ρ_f								
	0.05	**0.50**	**1.00**	**1.25**	**1.50**	**1.75**	**2.00**	**2.50**	**3.00**
	f_{yd} = 375 MPa (A400C)								
C8/10	0.997	0.995	0.949	0.965	0.985	1.001	1.013	1.030	1.042
C12/15	0.997	0.996	0.991	0.986	0.960	0.979	0.994	1.017	1.033
C16/20	0.996	0.997	0.993	0.991	0.988	0.973	0.975	1.003	1.024
C20/25	0.996	0.997	0.994	0.992	0.991	0.989	0.986	0.993	1.017
C25/30	0.996	0.998	0.995	0.993	0.992	0.990	0.988	0.989	1.015
C30/35	0.996	0.998	0.995	0.994	0.992	0.991	0.989	0.986	1.015
C32/40	0.996	0.998	0.996	0.994	0.993	0.992	0.990	0.987	1.003
C35/45	0.996	0.998	0.996	0.995	0.994	0.992	0.991	0.988	0.985
C40/50	0.996	0.998	0.996	0.995	0.994	0.993	0.991	0.989	0.986
C45/55	0.996	0.998	0.996	0.995	0.994	0.993	0.992	0.989	0.986
C50/60	0.996	0.997	0.996	0.995	0.994	0.993	0.992	0.990	0.987
	f_{yd} = 450 MPa (A500C)								
C8/10	0.998	0.993	0.951	0.980	1.001	1.018	1.030	1.049	1.061
C12/15	0.997	0.995	0.977	0.950	0.976	0.996	1.012	1.037	1.054
C16/20	0.997	0.996	0.991	0.987	0.951	0.975	0.994	1.024	1.045
C20/25	0.996	0.997	0.993	0.991	0.988	0.959	0.981	1.015	1.040
C25/30	0.996	0.997	0.994	0.992	0.990	0.987	0.975	1.012	1.040
C30/35	0.996	0.997	0.994	0.992	0.991	0.989	0.985	1.012	1.042
C32/40	0.996	0.997	0.995	0.993	0.991	0.990	0.987	1.012	1.044
C35/45	0.996	0.998	0.995	0.993	0.992	0.990	0.989	0.991	1.050
C40/50	0.996	0.998	0.995	0.994	0.992	0.991	0.989	0.986	1.059
C45/55	0.996	0.998	0.995	0.994	0.993	0.991	0.990	0.986	1.053
C50/60	0.996	0.998	0.995	0.994	0.993	0.991	0.990	0.987	1.031

Table 5. Comparison of the calculated resistance of reinforced concrete to bend for single reinforcement $f_{zM,dm}/f_{zM,SNiP}$.

$$\sigma_{izM} = \frac{M}{W_c}. \tag{24}$$

Parameters of the stress-strain state at the operating load levels are necessary for determining the deflection and width of the crack opening. Therefore, the basic parameters that are necessary for this will be: tension in the reinforcement and curvature.

The tension in the reinforcement until formation of cracks is determined by expression

$$\sigma_{s,W} = (k_W - 1)\varepsilon_{c,W}, \tag{25}$$

under certain values k_W, $\varepsilon_{c,W}$.

Class of concrete	ρ_f **= 0.01**			ρ_f **= 0.02**			ρ_f **= 0.03**		
	ρ_{fc}/ρ_f								
	0.25	**0.50**	**0.75**	**0.25**	**0.50**	**0.75**	**0.25**	**0.50**	**0.75**
1	2	3	4	5	6	7	8	9	10
n = 0.06–0.1	f_{yd} = 375 MPa (A400C)								
C8/10	0.994	0.998	1.000	0.984	0.977	0.999	1.001	0.987	0.999
C12/15	0.996	0.998	1.000	0.970	0.996	1.000	0.996	0.980	0.999
C16/20	0.997	0.999	0.997	0.992	0.997	1.000	0.989	0.995	0.999
C20/25	0.997	0.999	0.997	0.993	0.998	1.000	0.984	0.996	0.999
C25/30	0.997	0.999	0.997	0.994	0.998	1.000	0.990	0.996	1.000
C30/35	0.998	0.997	0.998	0.995	0.998	0.999	0.991	0.997	1.000
C32/40	0.997	0.996	0.998	0.995	0.998	0.998	0.992	0.997	1.000
C35/45	0.997	0.996	0.999	0.995	0.998	0.997	0.993	0.997	0.999
C40/50	0.996	0.996	1.001	0.996	0.998	0.997	0.993	0.997	0.999
C45/55	0.996	0.996	1.002	0.996	0.998	0.997	0.993	0.997	0.998
C50/60	0.996	0.997	1.003	0.996	0.997	0.997	0.993	0.997	0.998
n = 0.06–0.1	f_{yd} = 450 MPa (A500C)								
C8/10	0.986	1.019	1.014	1.041	1.045	1.035	1.070	1.072	1.074
C12/15	1.006	1.012	1.005	1.019	1.024	1.022	1.056	1.057	1.037
C16/20	1.003	1.007	0.999	0.998	1.019	1.015	1.043	1.042	1.025
C20/25	1.002	1.004	0.997	1.006	1.014	1.010	1.034	1.024	1.019
C25/30	1.000	1.000	0.996	1.004	1.011	1.005	1.031	1.019	1.015
C30/35	0.999	0.998	0.996	1.003	1.009	1.002	1.029	1.015	1.012
C32/40	0.998	0.996	0.995	1.002	1.007	1.000	1.011	1.013	1.010
C35/45	0.996	0.995	0.996	1.001	1.005	0.998	1.002	1.010	1.006
C40/50	0.995	0.995	0.996	1.000	1.003	0.997	1.002	1.008	1.004
C45/55	0.994	0.994	0.997	0.999	1.002	0.996	1.000	1.007	1.002
C50/60	0.993	0.994	0.997	0.998	0.999	0.995	1.000	1.005	1.000

Table 6. Comparison of the calculated resistance of reinforced concrete to bend for double reinforcement $f_{zM2,dm}/f_{zM2,SNiP}$.

The tension in the reinforcement in the cross section with the crack in the stretched zone are calculated at known values k, ε_c, $\Delta\sigma_{s,x}$. Average tensions in the reinforcement on the section in the block between the cracks $\sigma_{s,m}$ are defined as the arithmetic mean of the tensions that are determined by expressions (22) and (23).

The curvature of the cross sections of reinforced concrete elements, taking into account the hypothesis of plane cross sections, is determined by the expression

$$1/r = \sum \varepsilon/d, \tag{26}$$

where $\Sigma\varepsilon$ is the total deformation of fibrous concrete fibers and stretched reinforcement.

The total deformation of fibrous concrete fibers and stretched reinforcement must be determined by the following formulas:

- For cross sections without cracks at $M < M_W$:

$$\sum \varepsilon = \varepsilon_{c,W} + \varepsilon_{s,W} = \varepsilon_{c,W} + \sigma_{s,W}/E_s. \tag{27}$$

- For cross sections with a crack in the stretched zone:

$$\sum \varepsilon = \varepsilon_{c,2} + \varepsilon_{s,fic} = \varepsilon_{c,2} + (k_{W,2} - 1)\varepsilon_{c,2}. \tag{28}$$

- For cross sections without cracks at $M \geq M_W$

$$\sum \varepsilon = \varepsilon_{c,m} + \varepsilon_{s,m} = \frac{\sigma_{s,m}}{E_s(k-1)} + \frac{\sigma_{s,m}}{E_s}. \tag{29}$$

Deflections are determined by curvature by using numerical methods.

According to the given method, tables have been developed, which depending on the accepted parameters allow to determining the resistance of the concrete, the stresses in the reinforced concrete and reinforcement, and the total relative deformation of the cross section. For this purpose, the deformation diagram was adopted in the form of Eurocode-2 function [3]. These tables are given in [6].

The calculation of the strength of the cross sections of bending reinforced concrete elements and crack resistance is recommended to be performed according to the formula:

$$\frac{M_{Ed}}{W_c} \leq f_{zM}\left(f_{WzM}\right). \tag{30}$$

Calculation of tension limitation in the reinforcement is carried out as follows:

$$\sigma_{zM} = \frac{M_e}{W_c} \rightarrow \varepsilon_{si} \rightarrow \sigma_s \leq \sigma_{s,Table}, \tag{31}$$

where $\sigma_{s,Table}$ is the tension in the reinforcement, in which there is no need to determine the width of the cracks opening, which are determined by Table 2.5 [7].

The calculation of the width of the cracks opening is carried out in same scheme:

$$\sigma_{zM} = M_e/W_c \rightarrow \varepsilon_{si} \rightarrow \sigma_{si} \rightarrow S_r \rightarrow W_k. \tag{32}$$

Calculation of deflections is performed in the following order:

$$\sigma_{zM} = M_e/W_c \rightarrow \Sigma\varepsilon \rightarrow 1/r = \Sigma\varepsilon/d \rightarrow f. \tag{33}$$

A separate important issue in the theory of reinforced concrete is the consideration of regime loads and influences: long-term, quasi-constant, low cycle, temperature, humidity and others. Thus, taking into account the long-term load can be realized by introducing the creep coefficient to the curvature or by introducing into the calculation of the deformation diagrams with the corresponding parameters. The calculation of regime load under the first condition can be carried out according to the given method by using tables for short-term load. When performing calculations under the second condition, it is necessary to use the tables obtained for the corresponding parameters of the diagrams. Similar tables can be made for virtually all regime loads and influences, which greatly simplify the calculations of strength, crack resistance, stiffness and width of crack opening. This is an issue that needs to be studied in detail, but the use of the calculated resistance of reinforced concrete gives confidence in the successful solution of this problem.

4. Examples of calculation of bending reinforced concrete elements

Example 1. Reinforced concrete beam with working cross section $b \times d = 20 \times 45$ sm is made of concrete of class *C*25/30 and reinforced 4∅25 of steel of class A500C. Determine the carrying capacity of the beam.

Solution. The percentage of beam reinforcement with stretched reinforcement is calculated:

$$\rho_f = \frac{A_s}{bd} \times 100\% = \frac{19.63}{20 \times 45} = 2.181\%.$$

According to the tables the calculated resistance of the reinforced concrete to the bend is determined:

$$f_{zM} = 37.12 \text{ MPa}.$$

The carrying capacity of the beam is calculated by the formula:

$$M_{Ed} = W_c f_{zM} = \frac{bd^2}{6} f_{zM} = \frac{20 \times 45^2}{6} 37.12 \times 10^{-3} = 250.56 \text{ kNm}.$$

Example 2. Reinforced concrete beam with working cross section $b \times d = 30 \times 45$ sm is made of concrete of class C25/30 and steel of class A400C and should take an external moment $M_{Ed} = 266.46$ kNm. Determine element reinforcement.

Solution. The moment of resistance of the concrete cross section is determined:

$$W_c = \frac{bd^2}{6} = \frac{30 \times 45^2}{6} = 10125 \text{ sm}^3.$$

The required calculated resistance of the reinforced concrete to the bending is calculated:

$$f_{zM} = \frac{M_{Ed}}{W_c} = \frac{266.46 \times 10^3}{10125} = 26.32 \text{ MPa}.$$

According to the tables the required percentage of reinforcement is determined:

$$\rho_f = 1.453\%.$$

The area of the cross section of the working reinforcement is equal:

$$A_s = \rho_f \times b \times d = 0.01453 \times 30 \times 45 = 19.62 \text{ sm}^2.$$

By gage 4∅25, A_s = 19.63 sm^2 is accepted.

Example 3. Determine the cross-sectional dimensions of the beam of concrete class C16/20 and the area of the cross section of the working reinforcement of steel of class A400C, if the beam perceives the bending moment M_{Ed} = *136* kNm, and the contents of the working armature are ρ_f = 1.25%.

Solution. According to the tables, the calculated resistance of the reinforced concrete to the bend is found: f_M = 21.60 MPa. The moment of resistance is determined:

$$W_c = \frac{bd^2}{6} = \frac{M_{Ed}}{f_{zM}} = \frac{136 \times 10^3}{21.60} = 6296 \text{ sm}^3.$$

Accepting the ratio $b = 0.5d$, calculate

$$d = \sqrt[3]{12 \times W_c} = \sqrt[3]{12 \times 6296} = 42.27 \text{ sm}.$$

Accepting $b \times d$ = 20 × 42 sm, then the area of cross section of the working reinforcement will be $A_s = \rho_f \times b \times d$ = 0.0125 × 20 × 42 = 10.5 sm^2. By gage 2∅20 + 2∅18, A_s = 11.37 sm^2.

Example 4. Reinforced concrete beam with working cross section $b \times d$ = 30 × 55 sm has to perceive the bending moment M_{Ed} = 486 kNm. To define the conditions under which the bearing capacity of the beam will be provided and accept the reinforcement.

Solution. The required calculated resistance of the reinforced concrete to the bend is calculated:

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$$f_{zM} = \frac{M_{Ed}}{W_c} = \frac{6M_{Ed}}{bd^2} = \frac{6 \times 486 \times 10^3}{30 \times 55^2} = 32.12 \text{ MPa}.$$

From the tables it is clear that such a calculated resistance can be provided for different classes of concrete, reinforcement, and the percentage of reinforcement, starting with the concrete of class C20/25 and percentage of reinforcement 1.5 and more. Following physical, economic, and technological considerations, the designer takes the option that best suits the customer, for example, concrete class C25/30 and reinforcement of class A500C with percentage of reinforcement $\rho_f = 1.42\%$. Then the area of the cross section of the working reinforcement will be $A_s = \rho_f \times b \times d = 0.0142 \times 30 \times 55 = 23.43$ sm^2. By gage 3∅28 + 2∅20, $A_s = 24.75$ sm^2 is accepted.

5. Conclusions

Obtained parameters of the stress-strain state of bending reinforced concrete elements: the calculated resistance of reinforced concrete to bend and tension in the cross section of the bending reinforced concrete element, the above dependencies allow to solve a number of problems, namely:

1. Calculation of the strength of the cross section of the bending reinforced concrete element with known cross-sectional dimensions of the concrete and reinforcement area.
2. Determination of the required cross-sectional area of the reinforcement for a given load with known cross-sectional dimensions of concrete.
3. Foundation the dimensions of the cross section of concrete and reinforcement for a certain percentage of reinforcement and the given load.
4. Checking strength with known cross-sectional area of the reinforcement and given cross-sectional dimensions of the concrete.
5. Verification of the conditions for ensuring the strength of the cross section of reinforced concrete element.
6. Calculation of the moment of formation of cracks.
7. Calculation the width of the crack opening under operating load.
8. Determination of the deflections of the elements under the operational load.

Using the calculated resistances of reinforced concrete allowed to reduce the calculation of reinforced concrete elements according to the nonlinear deformation model to the application of the formulas of the classical resistance of materials and to significantly simplify the process of their calculation.

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