Chapter 4

Initial Condition and Behavior Patterns in Learning Dynamics: Study of Complexity and Sustainability from Time Series

Patricio Pacheco Hernández

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/intechopen.74140

Abstract

Learning is an essential part of human life. In it, our sensory organs and neural networks participate and integrate emotional behaviors, indagative and persuasive abilities, along with the ability to selectively acquire information, to mention a fraction of the media used in learning, converge to it. This study presents the results of the observational monitoring of behaviors, displayed by teams of students in learning processes, their interactions, representing them as series of time. These time series contain the dynamics of learning: weak, average, and chaotic, differentiated by the control parameter (connectivity) that is increasing respectively. The exponents of Lyapunov, the entropy of Kolmogorov, the complexity, the loss of information for each series, and the projection horizon of the processes are calculated for each series. The results, approximate, show that the chaotic dynamics propitiate the learning, given that there is an increase of connectivity within the teams breaking patterns or behavioral stereotypes. The entropic character of connectivity allows estimating the complexity of this human activity, exposing its sustainability, which brings irreversible conflicts with nature, given that the universe of nonequilibrium is a connected universe. Finally, the analysis model developed is historically contextualized, in first approximation, in some ancient civilizations.

Keywords: learnings, time series, Lyapunov coefficients, entropy, sustainability

1. Introduction

When confronting the diverse experiences of daily life, the human being concludes on the ways of acting that are adequate to get away from unpleasant situations and that place him

in conditions of greater well-being and joy. The emotions are numerous and very complex that command the different ways of reacting. Thus, for example, when faced with an event or element that bothers us or that does not give us pleasure, we can react by moving away or try to find a solution or other strategies that translate into behaviors, which will change the situation that face. Of those diverse experiences and circumstances that a person has gone through, of the dissimilar emotions that have produced him and of the multitude of decisions that a person has taken, some left in the person more profound mark than others [1]. And it is by that experience and how we have reacted before it is that it becomes the basis of reference for multiple decisions in the future, and therefore, they go on to form the baggage of a person's behavior pattern. From the above, it is clear that emotions have an enormous influence on learning [2, 3]. Emotion plays an important cognitive role [4–6]: the knowledge of life and the universe is not only intellectual, since the subtle nuances of it are provided by emotion. In effect, emotions enrich human knowledge by broadening the background, too rigorous and symmetrical, of purely intellectual concepts. Emotions are the other way of knowing about the world and themselves [6].

The learning process should give value to facts, people, and situations, shaping the initial contextualized condition, according to its influence on emotionality, given its natural impact on the learning of people. This value assignment manifests itself, neurobiologically, in attentional and perceptual selection [7, 8]; in the selection that is remembered by long-term memory and in the perception that dispositions and attitudes are "felt" as more appropriate [9, 10]. The value assignment makes learning sustainable over time. Thus arises the question that, to some extent, gives the pattern to the title of this chapter:

Is it possible to construct mathematical indicators, appropriate to be measured, that inform about the sustainability [11] of a learning process and that consider the influence of emotions in the induction of behavior patterns?

2. About the study sample

To investigate an answer to this question, we considered a sample of students who take the course of Classical Mechanics Laboratory parallel to the theoretic class of Classical Mechanics. These subjects are dictated by a higher education institution for students of the Common Plan of the Engineering area. The sample is constituted by 240 students distributed in five theoretical courses, with an average of 40 students.

The collaborative learning process that is analyzed relates to the Classical Mechanical Laboratory courses whose content base of 12 experimental activities, each of 90 minutes, programmed during a semester. A "typical" laboratory course was formed by 12 students, which grouped into four teams of three students each. The creation of teams of only women, only men, and mixed were encouraged to study, also, the relationship between gender and learning in science. The selection of the courses is made without any other a priori criteria (notes, social origin, and others). The achievement of significant learning is examined through laboratory reports developed and delivered periodically by the teams of students for each activity plus two cumulative tests with questions about alternatives and development.

The students correspond to their vast majority (90%) to the first family generation in entering higher education.

3. Initial condition: Facing stereotypes

3.1. The search for feeling good

The abandonment of chaotic behavior is at the beginning of human life (and can be translated as loss of entropy), but chaos is inherent to the environment (in life itself), which interprets as the physiopathologic loss of the adaptive possibilities in the neuronal system. This abandonment is an aspect that is not considered or considered irrelevant in a development framework, human and institutional, symmetric or homogeneous, that assumes the predictability of the processes (according to a linear approach) as a norm [12]. From the perspective of human activity, the reduction, in the short term, of anguish, anxiety, stress, and the wear and tear of unpredictable events is positive.

Modern society has turned this search into a pattern, in which it pigeonholes everything it deems necessary and too often regardless of its falsity or truth. This imperative of linearity and symmetry, which seeks to minimize the costs associated with risks, informs us and shapes the meaning of the world [12] by skewing learning. It makes us see, in a noncommutative world, commutativity in events, in the manner of algebra and linear physics. Thus, life experience and teaching associate the built order with "feeling good" [12], encouraging and stimulating behaviors. It follows then that the uncertainty bias permeates all relational forms [13, 14]. The connectivity carried by this road will have stability in relatively short periods of time, collapsing due to unresolved tensions that are incubated in its interior (truth and falsehood). The prolongation in time of relational forms will necessarily require the intervention of elements either internal or external, which can interpret as the basis of mythical, mystical conceptions [15] and of certain justifications that they misuse religion.

3.2. The history of personal experiences in the development of behavior patterns

Based on the experiences and how it reacted previously, experiences become the basis of reference for various future decisions: they constitute the stock of behavior patterns of a person. The construction and use of the contextualized initial condition seek to place these "certainties" in interdiction. Since everything life accepts and creates a bond, it will always be dependent. Two people who apparently do not need anything from each other could not form a relationship.

The way in which the human being reacts, either his way of acting, feeling, or acting, is governed by a series of external guidelines, which society accepts. Much of the behavior of human beings is learned, that is, acquired through interaction with the community in which it grows and develops [16, 17]. This means that the various groups in which he has been interacting have transmitted his guidelines and behaviors to react to the stimuli he receives from the environment.

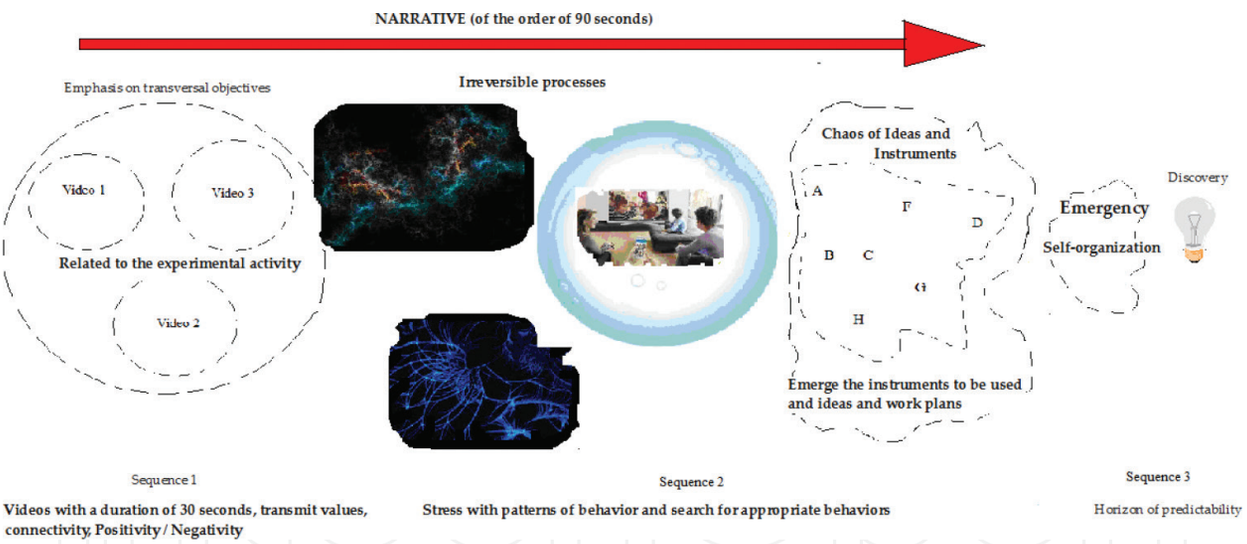
In the United States in the decade of the 1990, Chaos Theory was essentially used to solve or at least canalized racial and social conflicts that expressed in the form of school violence in schools that correspond to marginal environments, such as immigrant communities. It developed a

particular interest in the problems of educational organization and administration in these contexts. Thus, some authors [18, 19] carried out a nascent pedagogy of chaos from the theory of the same name and have used it in the resolution of determining problems of school organization and administration in unstable, violent, and conflicting educational system.

The close relationships between behavior patterns and ideas and attitudes are what have motivated anthropology to study how culture influences the development of personality. From the perspective of the chaotic model applied to the study of learning [20], the stimulus is a contextualized initial condition (CIC). The stimulus (CIC) is susceptible to construction and appeals to the history of past – present – future experience of a sample of individuals with similar life experiences. The narrative to be constructed can be in the form of a short demonstration experiment, short length videos, interaction between the teacher and the student teams through questions that emphasize unexpected relationship events, and so on. The achievement of identity with the contextualized initial condition favors the development of positive or negative emotions propitiating adaptive behaviors.

3.3. Visual narrative: A way to approach the breaking of stereotypes

The visual narrative used is explained in the outline below. This appears divided into three fundamental sequences, as shown in the diagram:



For the first phase on the left-hand side, three or four different or nonhilarious videos presented by the experimental activity will be carried out (each of no more than 10–15 seconds), which has the purpose of presenting transversal objectives such as respect for others (my actions bring consequences to other people and the environment). A question session (student–teacher) held regarding the connectivity of the events was witnessed. It seeks to stimulate curiosity but stressing the ideal of the world in which we believe we live. A predictable world manufactured in the technique to which we associate states of emotional hope.

In the middle of the picture is the real world, of irreversible processes, dominated by nonlinear dynamics, that is, chaos or fractal nature is the world of nonlinearity. Our version of life floats in that sea of irreversibility. This version means a low horizon of predictability.

Given this, our behavior patterns lead us to the illusion that the problems do not touch us or that they will resolve themselves, which is impossible. The collapse between what is learned and what is needed to face situations of high uncertainty manifested in behaviors of anger, crying, denial, impotence, stupor in the face of disaster, frustration, avoidance of error, indifference, etc., all emotions that do not contribute to solve the problem.

A convenient approach to this world is to arrange the measuring instruments being used in the experience chaotically, in such a way that through self-organization and the emergency originates an order that leads us to a possible solution in the form of discovery. Once again, the questions (student–teacher) regarding what you want to measure and how to dispose of the instruments should be the guide to the solution (in a playful way).

Finally, the final section of the video exposes a logical sequence of the disposition of the equipment to reach the objective of the activity.

The proposed sequence modifies the standard way in which the system promotes learning, given that as Albert Einstein said: "It is impossible for us to solve problems using the same procedures that generated them." The contribution of new generations, living the prevailing complexity, with a new understanding of sustainability based on connectivity is fundamental. All activity carried out by the human being entails, irremediably, error. These errors range from measuring instruments (scale of measurement, calibration, seniority, etc.), user expertise, treatment of significant figures, statistical analysis of measured values, and interpretation of measurements to the relationships between them. Therefore, this procedure is inherent to any experimental activity, and in general, we should say of any human activity. Performing activities that are keeping with the way our brain works reduces anxiety, tension, frustration, and fear.

4. Chaos and learning

The fundamental idea underlying this approach to meaningful learning is that chaos makes life and intelligence possible, since the brain is a nonlinear product of a nonlinear evolution on a nonlinear planet [21]. The brain is an unstable system that nevertheless leads and achieves the formation of new orders, as well as chaotic actions [21]. The brain has evolved to become so unstable that at the smallest stimulus, external or internal signal, it can encourage behaviors that represent a healthy break with historical behavior patterns favoring the emergence of an innovative and creative order [22].

A pattern of behavior emerges, once a narrative is recognized in a context. It allows us to optimize our human resources such as physiological, emotional, and rational, by freeing our attention and focusing on those events that burst without us having a collection of proven answers. The irruption of events in scenarios of high complexity [23, 24], created by human activity and progressively intensifies, exposes the weaker side of the human condition: the reserve of learning reactions is shown to be inoperative or too small.

Paradoxically, the built society model is fundamentally reductionist since the fundamental parameter of control is the economic one, a variable that has unleashed a form of economic "complexity" that has the planet as a great dump of the waste of human activity and technology,

garbage. This event is an irremediable "consequence" of all actions, transforming itself, its growth and treatment, and it is a process of very high complexity that very few assumed. At present, propitiate learning, regardless of how connected and globalized they are, from this petty perspective based on consumption without any responsibility to the planet, generates entropy or progressive disorder in nature, stretching it to its limits.

5. Construction of an approach to a chaotic learning model

5.1. Variables to observe

The "typical" performance matrix per student team is divided into small time intervals until the 90-minute class is completed. The observation focuses on the behaviors displayed by students in the process, for which six fields of generic behaviors were typified: Inquiry (IND), Persuasion (PER), Positivity (POS), Negativity (NEG), Internal Information (II), and External Information (IE). Each field contains a set of 13 behaviors, which assigned a numerical range from −6 to +6, as shown by the scheme N°1 for example of Positivity and Inquiry [25–27]:

This hierarchy determines a total of 78 behaviors for the six behavioral fields to observe during the 90-minute session.

To consider the interactive behavior of the team's constituent students, a time of 5 seconds was granted, which gives a total of 1080 rows of data divided into two columns. A column is the time input at intervals of 5 seconds to complete 5400 seconds, and the other is the numerical coding according to the observed behavior displayed by the team.

To construct the proportions X = IND/PER, Y = POS/NEG, and Z = II/IE, the data are divided locker by locker, for the same time, between the Inquiry and Persuasion, Positivity and Negativity, and Internal Information and External Information, using the six different value tables built (**Table 1**).

	Codification
6	Interprets
5	Group in tables of values
4	To size
3	Calibrate a measuring instrument
2	Characterize measurements variables
1	Explore
0	Neutro
-1	Do not explore
-2	Not characterize measurement variables
-3	Not calibrate a measuring instrument
-4	Do not measure
-5	It does not group in tables of values
-6	Does not interpret

t (min)	Inquiry behavior
0	
9	
18	
27	
36	
45	
54	
63	
72	
81	
90	

Table 1. Illustration of a table with the inquiry behavior and its coding taxonomy.

5.2. Experimental procedure

Two laboratory classrooms were used to record the observations, with two video cameras located in each one. Each camera records sound and image by storing the information on external hard drives. The cameras were positioned at the height of 1.8 m fixed to the wall and in such a way as to completely cover the student teams. Previously, a document was created that requested the student's authorization for the filming, which had to be signed by each one of them. Noted that during the filming process, the behavior of the students did not alter the presence of the cameras. In one room, the experimental group to which the initial condition was applied, while in another room, the control group develops its activities with no initial condition. At the end of the lecture session, the collected information is saved indicating the date, time, actively developed, and course.

5.3. Team to collect the information recorded in the videos

Any measurement or data collection instrument must collect two essential requirements: validity and reliability. In general terms, validity refers to the degree to which an instrument measures the variable it intends to measure. Validity is a concept from which different types of evidence have related [28, 29]. This evidence is the content (the degree to which an instrument reflects a specific implicit domain that is measured), criterion (validity of an instrument of measurement compared with an external judgment), and construct (quality of measurement related consistently with other measurements according to hypotheses derived theoretically and that concern the measured concept).

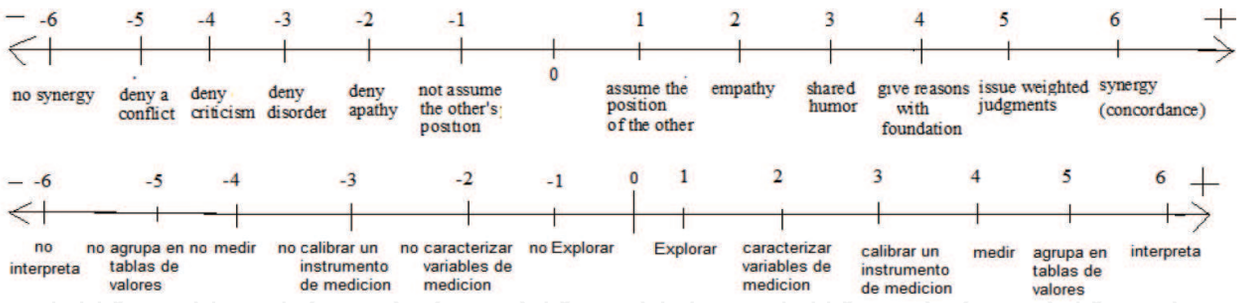
The reliability of the instrument is the degree to which its repeated application to the same subject or object produces the same results. If the correlation between the results of the different appliances is highly positive, the instrument is considered reliable (in psychometrics, Cronbach's alpha [30] is a coefficient used to measure the reliability of a measurement scale).

Applying the Hypothesis Testing (or dokimasia) to the tables of measurements of each one of the variables under observation, we could know if the measurement instrument calibrates from the accuracy.

The procedure for extracting information from the videos of the sessions was divided into two parts: Theory and Praxis.

From the theoretical point of view, the observers trained in the graduation of behaviors for the fields defined in this research: Inquiry, Persuasion, Positivity, Negativity, Internal Information, and External Information. This graduation of the behaviors for Positivity and Inquiry is illustrated in **Scheme 1**. Is it possible to do a finer gradation? Obviously, it is. Similarly, it noted that a specific exploration of facial gestures was not performed (frowning, opening or closing the eyes too much, changing the line of the mouth, and so on and so forth) [31–33]. It is a subject that opens a future work.

From practice, each component of the team in charge of registration, consisting of four people, gives independently the same scene (image and sound) of some of the videos captured of the activity carried out by the students. This brief scene, of the order of 10 minutes, is numerically

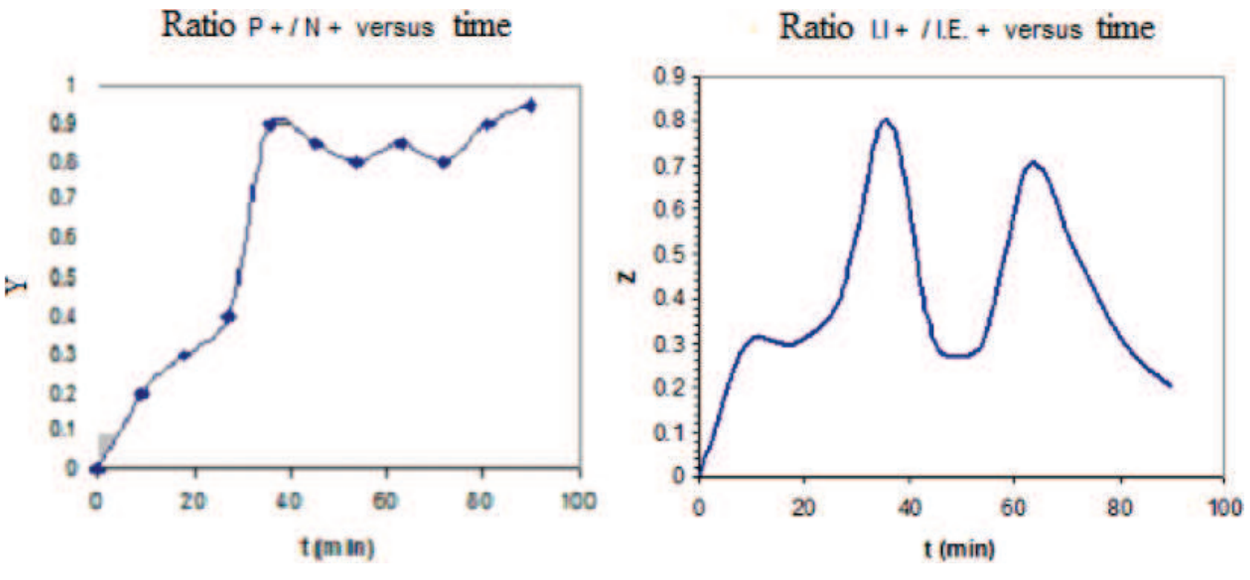


Scheme 1. It indicates the domain of double polarity and the coding scale, in its dual polarity, of Positivity and Inquiry, with its 13 associated behaviors.

coded according to the behaviors observed in the same field. In general, the coding tables have 1080 rows, separated by intervals of 5 seconds. Once this process is finished, the four tables generated are analyzed by the psychologist and a statistician who study the convergences and divergences between the measures: the guideline, in the form of feedback, given to the observers is to identify the actions that promote relationships within the team. Gradually, the times of study of scenes are extended, until covering the session of complete learning about 90 minutes. The comparative analysis of the measures is the one that indicates if the instruments (the observers) present similarity in their registers [27]. After a work period of about 1 month, it was possible to certify that the observers were calibrated and reliable "instruments" (results illustrated in Section 5.5 between the control and experimental team).

5.4. Time series

The proportions X = IND/PER, Y = POS/NEG, and Z = II/IE constructed constitute numerical series of 1080 data that are called time series. After applying the initial condition with emphasis on emotions at 30 minutes, some of the graphs of the moving averages in time are below (**Graph 1**):



Graph 1. Represents the variation over time of the Time Series of the Quantities Positivity/Negativity (= Y) and of the ratio Internal Information/External Information (= Z).

http://dx.doi.org/10.5772/intechopen.74140

5.5. Reliability and validity of measuring instruments

Illustration with the Dimension Y = POS/NEG in time using moving averages.

A STUDENT TEAM (COURSE 1)

t (min)	Y_1	Y_1 -0.63	$(Y_1$- 0.63 $)^2$
0	0	-0.63	0.3969
9	0.2	-0.43	0.1849
18	0.3	-0.33	0.1089
27	0.4	-0.23	0.0529
36	0.9	0.27	0.0729
45	0.85	0.22	0.0484
54	0.8	0.17	0.0289
63	0.85	0.22	0.0484
72	0.8	0.17	0.0289
81	0.9	0.27	0.0729
90	0.95	0.32	0.1024
N =11 Sum =	6.95	Sum =	1.1464
Sum / 11 = Average	6.95 / 11 = 0.63	VAR1 =	1.1464 / 11 = 0.104

A STUDENT TEAM (COURSE 2)

T(min)	Y_2	Y_2 – 0..75	$(Y_2$ - 0.75$)^2$
0	-1	-1.75	3.0625
9	-0.5	-1.25	1.5625
18	1.5	0.75	0.5625
27	0.8	0.05	0.0025
36	1	0.25	0.0625
45	1	0.25	0.0625
54	1	0.25	0.0625
63	0.8	0.05	0.0025
72	1.25	0.5	0.25
81	1.2	0.45	0.2025
90	1.2	0.45	0.2025
N =11 Sum =	8.25	Sum =	6.035
Sum / 11 = Average	8.25 / 11 = 0.75	VAR2 =	6.035 / 11 = 0.5486

where:

$(Y_1$-0.55)*$(Y_2$-0.66)
1.1025
0.5375
-0.2475
0.0115
0.0675
0.055
0.0425
0.011
0.085
0.1215
0.144
1.9305

Pearson's Sample Correlation Coefficient [34] for the fundamental dimension, Y, referring to two items from two different courses:

$$r = \cos(\alpha) = \frac{\sum_{i=1}^{N}(y_{1i}-\bar{y}_1)(y_{2i}-\bar{y}_2)}{\sqrt{\sum_{i=1}^{N}(y_{1i}-\bar{y}_{1i})^2}\sqrt{\sum_{i=1}^{N}(y_{2i}-\bar{y}_2)^2}} \approx \frac{1.9305}{\sqrt{1.07}\sqrt{2.456}} \approx 0.75 \approx p \tag{1}$$

Cronbach's alpha coefficient [30]:

Confidence interval

$$\bar{Y} \pm t_{n-1;1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \tag{2}$$

with $\bar{Y} = 0.63$, $S = 0.322$, α at level 0.05, $r = n - 1 = 11 - 1 = 10$ = degrees of freedom where n is the number of measurements.

$$t_{n-1;1\,1-\frac{\alpha}{2}} = t_{10-1;1-\frac{0.05}{2}} = t_{9;0.975} = 2.228 \text{ (for } \textit{Distribution } t\text{-Student).}$$
$$\text{confidence interval} = 0.63 \pm 2.228 * \frac{0.322}{\sqrt{11}} = \begin{cases} 0.41 & \text{(minimum)} \\ 0.85 & \text{(maximum)} \end{cases} \tag{3}$$

Finally, we want to know if the measuring instrument calibrated for accuracy.

Hypothesis test: defined H_0 = Null Hypothesis and H_1 = Alternative hypothesis [35].

Process:

H0: μ = 0.56, the instrument is calibrated for accuracy.

H1: μ ≠ 0.56, the instrument is not calibrated. There is a systematic error.

$$t = \frac{\bar{Y} - \mu}{\frac{S}{\sqrt{n}}} = \frac{0.63 - 0.56}{\frac{0.322}{\sqrt{11}}} \approx 0.72 \tag{4}$$

where $\bar{Y} = 0.63$, μ = 0.56 is a control team (calibrated with valid procedures).

Different observational monitoring teams were employed to perform the measurements and obtain μ

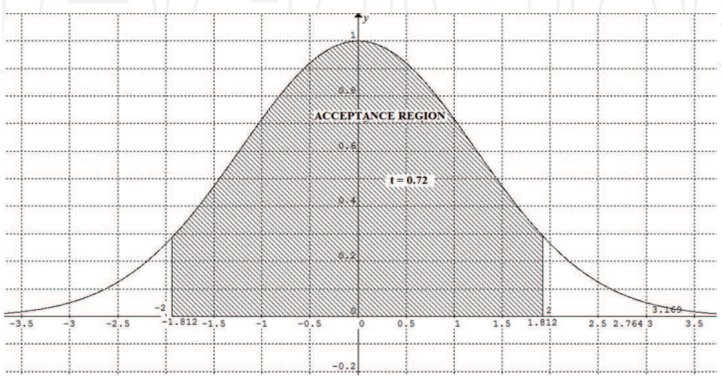
r = n − 1 = 11–1 = 10

$$t_{n-1;;1-\frac{\alpha}{2}} = \left\{ t_{10;0.95} = 1.812; \quad t_{10;0.99} = 2.764; \quad t_{10;0.999} = 3.169 \right.$$

The numerical values of t extracted from [34].

Graph 2 represents the Normal Distribution of the indicators t.

The accuracy calibrated instrument hypothesis is accepted.

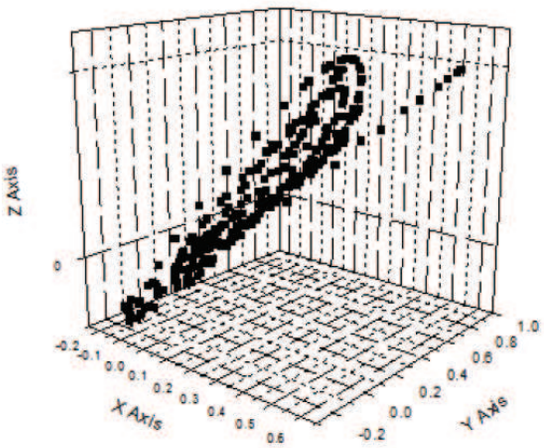


Graph 2. Normal distribution of the indicators t.

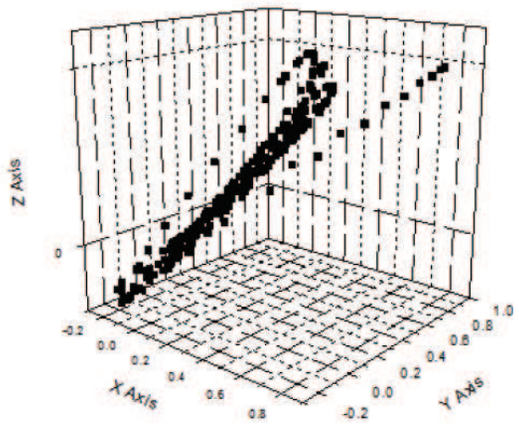
6. Dynamics

From the time series of X (t), Y (t), and Z (t), the discretized column vectors are constructed (observed that although vectors with a minimum of 1000 elements allow making good estimations, the ideal is that contain over 5000 components for the stability of the Lyapunov coefficients). According to the significant learning of the team of students, the graphs of the time series in the phased space acquire such forms:

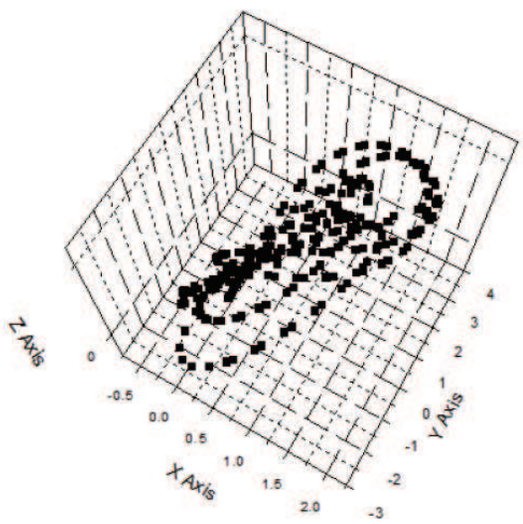
Weak attractor

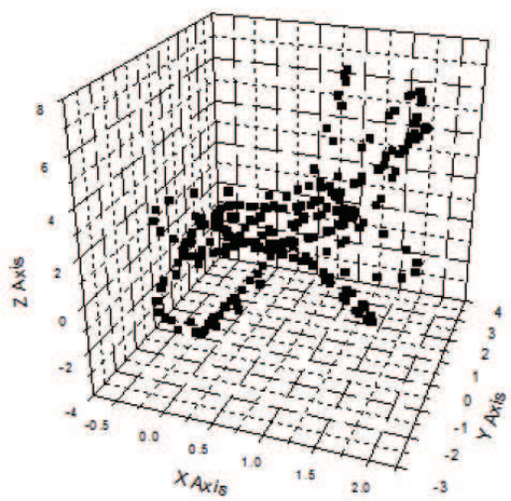


Middle attractor



Chaotic Attractor

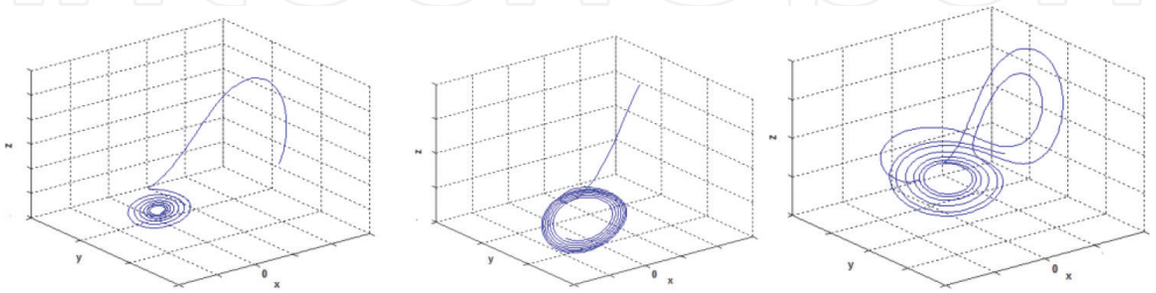




The time series and the graph obtained are those that allow incorporating elements of chaos theory in their study. They satisfy two fundamental conditions of this theory: sensitivity to initial conditions and the existence of Lyapunov exponents greater than zero. Applying to the experimental data, the Lorenz equations [36, 37] modified according to the fourth order Runge Kutta numerical method, the dynamics classified from the control parameter

(also called connectivity). Its values deliver the performance of the teams that make up the Experimental Group: Low (r = 16.5, weak attractor), Medium (r = 20.5, medium attractor), and High (r = 28.7, chaotic attractor).

These values compared with those that arise from theoretical iterative cycles (using, for example, adjustment by Fourier Time Series) for X (t), Y (t), and Z (t), based on the range of their experimental domains. Programming in MatLab (software for numerical calculation and scientific analysis) the modified Lorenz equations [28, 33] (also possible by Neural Networks [38] or Cellular Automata [39]), the graphs are obtained as shown:



r = 18 r = 21 r = 31

These graphics are classified according to the values of the theoretic control parameter, r, which roughly matches with the values of r for weak, medium and chaotic attractor, respectively, emerging from the experimental Time Series.

It was observed that the contrast between the performance of the experimental groups (selecting a team with chaotic dynamics) and the control groups (traditional courses without initial condition: choosing a good performance team) is carried out through cross-correlation. The cross-correlations by group according to the influence exerted by the variable of emotions Y (= Positivity/Negativity) on the variable X (= Inquiry/Persuasion) [20–25] is observed in **Table 2**.

The experimental group treated with contextualized initial conditions, which promote high connectivity within each team, shows that the balanced presence of positivity/negativity in their relationships exerts an influence 1.7 ~ 2, approximately, on the variable X, which is inquiry/persuasion (the most rational part of the team's work). Thus, the team leads more efficiently and safely toward the achievement of meaningful learning. This influence translated into connectivity and emotional field evolution reflected in the value that students give to learning and in its achievements. These achievements range from the experience of collaborative work, each component is determined in the learning process, to the formal evaluation procedures applied ranging from the weekly reports, entrance test at the beginning of the teaching session, tests, oral interrogation of any component of the team whose performance is extended to the whole team, etc.

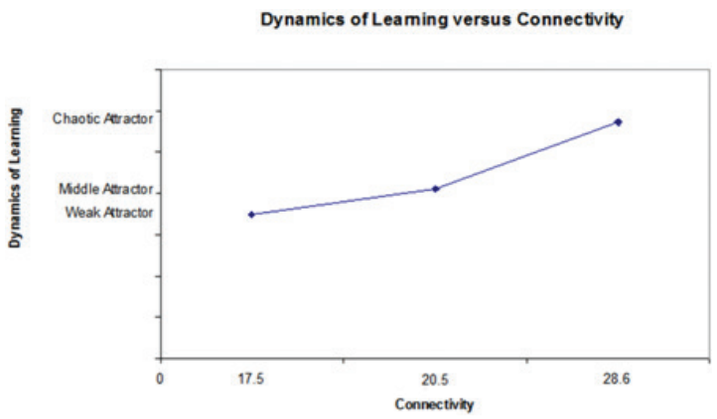
Group	Control	Experimental	Comparison: experimental/control
Cross-correlation	0.3	0.5	1.7

Table 2. Comparison between cross-correlation according to the Control and Experimental Teams.

7. Connectivity

The control parameter r (connectivity [40]) gives the transition between the different dynamics that favor meaningful learning. Connectivity defined as the capacity shown by the components of a system to expand the actions of others by their actions and to expand their actions from the actions of others [41, 42]. This definition is a glimpse into an underlying referential framework, inherent in all things, sustained by the complex intervariable interferences that characterize them, in a first approximation.

These interferences induce clutter dynamics that create an intelligent collective order, but temporary, which makes it imperative to incorporate them in learning. Teams with high connectivity and high POS/NEG quotients (greater than or equal to: 2.5 [43], 4.3 [44], and 5 [17]) are sustained over time and achieve the objectives of the activity [24, 45]. When observing **Graph 3**, we can see a growth in connectivity, as we approach the chaotic or complex dynamics:



Graph 3. Dynamics of learning versus connectivity.

What does this increasing behavior of connectivity (entropic connectivity) mean for learning? Is it possible to calculate it? How is it related to the complexity of the learning process under study?

Answering these questions, different numerical procedures were applied to the time series [46], which allow determining the Lyapunov coefficients [47], the Kolmogorov entropy (S_K) [48, 49], the complexity [50], and finally, the uncertainty in information [51].

8. Irreversibility and the sustainability of learning

The position of Ilya Prigogine [52, 53] on irreversibility and entropy varies that of traditional physics. In his lecture The Birth of Time (Rome, 1987), Prigogine argued:

"Entropy always contains two dialectical elements: a creator element of disorder, but also a creative element of order. (…) We see, then, that instability, fluctuations, and irreversibility

play a role at all levels of nature: chemical, ecological, climatological, biological - with the formation of biomolecules - and finally cosmological".

In this way, it was observed that the phenomenon of irreversibility for Prigogine is constructive, highlighting the "creative role of time," which, at least at a macroscopic level, supposes a kind of antientropy: "*the universe of non-equilibrium is a connected universe.*".

According to Wackernagel et al. [11]:

"Sustainability requires that life is within the regeneration capacity of the biosphere. In an attempt to measure the degree to which humanity satisfies this requirement, existing data have been used to translate human demand on the environment in the area required for the production of food and other goods, as well as in the absorption of waste. Numerical estimates indicate that human demand may well have outgrown the regenerative capacities of the biosphere since the 1980s. According to this preliminary and exploratory evaluation, the carrying capacity of humanity corresponds to 70% of that of the world biosphere in 1961, growing up to 120% in 1999".

All processes are irreversible because they are connected entropically making the complexity increases. Human activity is not exempt from this principle.

Chaotic systems consume considerable energy and information to maintain their level of complexity while being very sensitive to environmental fluctuations [54].

9. Some general mathematical concepts

9.1. The coefficient of Lyapunov

The standard procedure of determining whether or not a system is chaotic is through the exponents of Lyapunov represented by λ. When considering two nearest points in a stage n, x_n and $x_n + d\,x_n$, in the next temporal stage, they will diverge, particularly at x_{n+1} and $x_{n+1} + d\,x_{n+1}$. It is this average ratio of divergence (or convergence) that the exponents of Lyapunov capture. Another way of thinking about the exponents of Lyapunov is as a proportion in which the information about the initial conditions lashes. There are so many exponents of Lyapunov as a dimension of phase space.

The signs of the Lyapunov exponents, λ, provide a qualitative picture of a system's dynamics. One-dimensional maps are characterized by a single Lyapunov exponent.

If the exponent of Lyapunov is positive, $\lambda > 0$, then the system is chaotic and unstable [55, 56]. Next points will diverge regardless of how close they are. Although there is no order, the system is still deterministic. The magnitude of Lyapunov exponents is a measure of sensitivity to initial conditions, the primary characteristic of a chaotic system.

If $\lambda < 0$, then the system is attracted to a fixed point or stable periodic orbit [55]. The absolute value of the exponents indicates the degree of stability.

If $\lambda = 0$, the system is in a marginally stable orbit [55].

In a three-dimensional continuous dissipative dynamical system, the only possible spectra, and the attractors they describe, are as follows: (+,0,−), a strange attractor; (0,0,−), a two-torus; (0,−,−), a limit cycle; and (−,−,−), a fixed point [55].

The equation that allows calculating the coefficient of Lyapunov is given by:

$$\lambda = \lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{L} \ln \left| \frac{d\,x_{n+1}}{d\,x_n} \right| \tag{5}$$

9.2. The exponent of Hurst, H

H is used as a measure of long-term memory of time series. It refers to the autocorrelations of the time series, and the speed at which they decrease as the gap between pairs of values increases. The inverse of the Hurst exponent is equal to the fractal dimension of a time series. H takes values between 0 and 1:

$H \approx 0.5$ indicates the absence of long-term dependence [57].

$0.5 < H < 1$, it is a persistent series, graphically presents a smooth appearance. $H \approx 1$ indicates that the degree of persistence or long-term dependence holds.

$H < 0.5$, corresponds to anti-persistence, contrary to long-range dependency (LRD), indicates a strong negative correlation of the process that fluctuates violently.

$0 < H < 0.5$ indicates that the time series, in the long term, change high and low values of adjacent pairs of data; this tendency remains to fluctuate for a long time [57–59].

H is an index for the categorization of complexity, quantifies the chaotic dynamics, and is directly related to the fractal dimension, D, where $1 < D < 2$, such that $D = 2–H$.

9.3. Embedding dimension

The Embedding Theorem serves to remake from the observed or measured time series, the evolution of the states in the phase space, where the exponents of Lyapunov and the fractal dimension can be calculated (for example). It uses the method of delayed coordinates (reconstruction with delays). If we have the data series $x_1, x_2, x_3, x_4 \ldots x_n$, we can form the set of points $(x_1, x_2 \ldots, x_p)$, $(x_2, x_3 \ldots, x_{p+1})$ …, and $(x_i, x_{i+1} \ldots, x_{p+i})$. These points determine a trajectory in the space R^p. The dynamics of the empirical system represented by the "minimum" dynamics (in a dimensional sense) of this set of points:

If the system is random, the fractal dimension grows, as the dimension of the embedding space increases, that is, p.

If the system is periodic, the fractal dimension grows to a value k and then remains constant and whole (it is not fractal).

If the system is chaotic, the fractal dimension stabilizes for a certain embedding dimension p. Also, at least one exponent of Lyapunov will be positive.

9.4. Matrix and correlation dimension

The decomposition of time series into main analysis components gives rise to the correlation matrix. This matrix is two-dimensional (M × M) constructed by placing the values of the correlation function for τ = 0 along the main diagonal. Then, the values for τ = 1 put to the right and left of the diagonal, following with τ = 2 and so on until completing the matrix, which can be as large as 16 × 16 [37, 60].

$$\begin{pmatrix} f_{11}(\tau_0) & f_{12}(\tau_1) & & \\ f_{21}(\tau_1) & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \ddots \end{pmatrix}_{M \times M}$$

The number of significant eigenvalues of the correlation matrix, of the order of the correlation dimension, is a measure of the complexity of the system [37]. A procedure of these characteristics was applied the time series of X (t), Y (t), and Z (t).

9.5. The entropy of Kolmogorov and its relation to the loss of information

Following Farmer [61, 62], one of the essential differences between chaotic and predictable behavior is that chaotic trajectories continuously generate new information, while predictable trajectories do not. Metric entropy makes this notion more rigorous. In addition to providing a good definition of "chaos," metric entropy provides a quantitative way to describe "how chaotic" a dynamic system is.

The entropy of Kolmogorov [48, 49] is the average information loss [51, 63], when "l" (cell side in units of information) and τ (time) become infinitesimal:

$$S_K = -\lim_{\tau \to 0} \lim_{l \to 0} \lim_{n \to \infty} \frac{l}{n\tau} \sum_{0 \ldots n} P_{0 \ldots n} \log P_{0 \ldots n} \quad \text{(Kolmogorov entropy)} \tag{6}$$

Expressed in information bits/sec or bits/orbits for a continuous system and bits/iteration for a discrete system.

The entropy difference of Kolmogorov (ΔS_K) between one cell and another ($S_{K\,n+1}$ - $S_{K\,n}$) represents the additional information needed to know, in which cell (i_{n+1}) system will be found in the future. Therefore, the difference ($S_{K\,n+1}$ - $S_{K\,n}$) measures the loss of system information over time.

In conclusion, the calculation of the entropy of Kolmogorov:

1. Check if the entropy of Kolmogorov is between zero and infinity ($0 < S_K < \infty$), which allows verifying the presence of chaotic behavior. If the Kolmogorov entropy is equal to 0, no information loss, and the system is regular and predictable. If $S_K = \infty$, the system is entirely random and it is impossible to make any prediction.
2. Determine the amount of information needed to predict the future behavior of a system, in this case, a learning process.
3. Calculate the speed with which the system loses (or downgrade information over time).
4. To establish the maximum horizon of temporal predictability of the system, from which no prediction can make, nor elaboration of scenarios.

9.6. Determination of information loss

There is a relationship between the entropy of Kolmogorov and the characteristic parameter of chaos, the exponent of Lyapunov, λ, which shows that it is proportional to the loss of information, $< \Delta I >$ [51]:

$$\langle \Delta I \rangle_i \log 2 = -\lambda_i \Rightarrow \langle \Delta I \rangle_i = \frac{-\lambda_i}{\log 2}, \quad i = X, Y, Z \tag{7}$$

The coefficients of Lyapunov λ_x, λ_y, and λ_z are associated with the time series of the Inquiry/Persuasion (X (t)), Positivity/Negativity (Y (t)), and Internal Information/External Information (Z (t)) coefficients obtained according to learning dynamics.

The expressions for $<\Delta I>_i$ are in bits/time and show the relationship between the entropy of Kolmogorov and the exponent of Lyapunov, λ.

If $\lambda < 0$, the movement is not chaotic, information does not lose, because the prediction is accurate. (It is the main idea of the current educational paradigm).

If $\lambda > 0$, the movement is chaotic, the prediction is less accurate and, therefore, the loss of information is greater [51, 64].

10. Experimental results: Application of the chaos data analyzer software (CDA) to the experimental time series

In chaos theory, the calculation of the Lyapunov coefficients is fundamental because it allows studying the effect of the initial condition, the irreversibility of the processes, the entropy, the time of predictability, the complexity, and, based on these parameters, to characterize the sustainability of a learning process.

In the cases of weak attractor dynamics and middle attractor dynamics, chaos theory does not apply, since they are predictable or deterministic systems.

10.1. Chaotic attractor dynamics

For the analysis of the time series, the CDA Software, Chaos Data Analyzer Programs [37, 60], and the Golden Surfer Software were used to fill incomplete time series [65].

Notation:

λ: Exponent of Lyapunov (bits/units of time) [37, 47, 66].

D: Encrusting dimension [37].

n: is the number of sample intervals over which each pair of points followed before a new pair is selected.

A: is the relative accuracy of the data before the expected noise begins to dominate.

H: Exponent of Hurst is related to the smoothness of the curve and the dimension fractal, according to Mandelbrot [67–69], $0 \leq H \leq 1$. To $0.5 \leq H \leq 1.0$ indicates persistence (the past tends to persist in the future).

S = Correlation entropy [37–60] for each variable (measured in bits/units of time) (**Table 3**):

Variable	Λ	D	N	A	H	Correlation dimension	S_K
X	1.150 ± 0.099	1	2	0.0001	0.925	0.719 ± 0.247	1.027
Y	0.723 ± 0.084	1	2	0.0001	0.919	0.728 ± 0.245	0.477
Z	1.469 ± 0.105	1	2	0.0001	0.89	0.737 ± 0.251	0.728

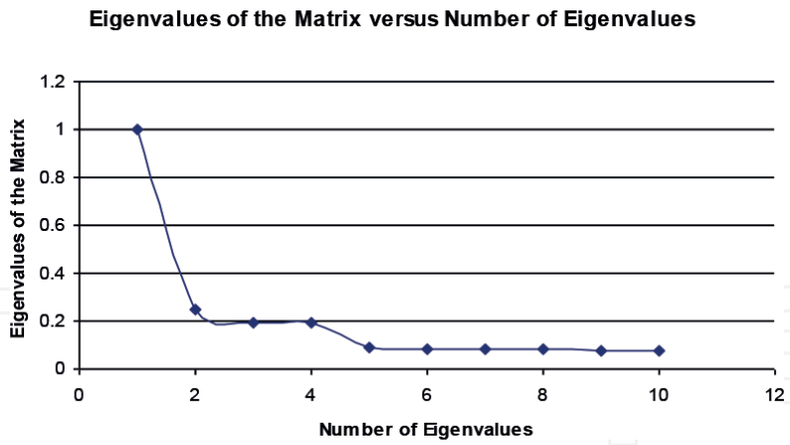
Table 3. The correlation entropy, S_K, is the entropy of Kolmogorov, and its reciprocal delivers the time for which the prediction is significant.

Applying the calculation of the correlation matrix to the time series X (t) of the chaotic attractor, the number of eigenvalues, of the order of the correlation dimension, is a measure of the complexity of the system [37]. In this case, two significant eigenvalues were determined (not zero) (**Graph 4**):

The time series of Y (t) and Z (t) treated in a similar way present the same number of eigenvalues. So, we conclude that given the number of eigenvalues, the series represent a complex system.

According to the López-Corona approach [50] for complexity, with normalized S_k (**Table 3**) $\rightarrow S_{K_{obs}}$:

$$C_i(t) = aS_{K_{obs,i}}\left(1 - S_{K_{obs,i}}\right) > 0, \quad i = X, Y, Z \tag{8}$$



Graph 4. Eigenvalues of the matrix versus number eigenvalues.

The values of C(t) for time series give results greater than zero for the three time series (X(t), Y (t), and Z (t)), which would characterize a complex system and is coincident with the result of the matrix of correlation.

In the learning process studied, applying Chaos Theory to time series arising from characteristic variables that are common to all learning processes, three particularities revealed entropic connectivity, irreversibility, and complexity. In the same way, the sustainability of the process is due to the quotient of the POS/NEG emotions, when positive influence exists (POS) in the case of the Chaotic Attractor Dynamics, which is the one that presents the highest achievement in meaningful learning (rudely indicated as performance). These learnings lead to the consequent formation of patterns of "desired" behavior. Increasing connectivity is increasing entropy, which to maintain the "beloved" order, entropy (negentropy) must be transferred to the environment (to the planet), is quantifiable evidence of such process, the increase of garbage and pollution. This corollary demonstrates the irreversibility of the process in the current narrative. Is it possible to modify this plot? Within the current narrative, it seems unlikely. A new form of relationship between ourselves and with nature must build. (Chaos theory does not admit complexity for the weak and middle attractor, i.e., it does not apply).

11. The future in the past? Ancient civilizations

The considerable cognitive requirements of life in complex societies have resulted in many primate species having larger and more expensive brains [70], with all that this implies in connectivity. The human immersed in evolution has historically transferred the cost of learning the complexity of nature, and there is ample evidence.

11.1. Mesopotamia

In the interior of ancient Mesopotamia, agriculture and livestock farming were imposed as the primary economic activity between 6000 and 5000 BC. Due to unfavorable natural conditions

for this practice in large part of this territory, men built and used canals to transport water from distant sources and thus obtain good harvests. Because of these facilities, they were able to achieve very high performances. On the absence of excavations in rural areas, the knowledge of ancient Mesopotamian agriculture is based mainly on old texts, including the numerous records of the practice of field sales. Exploitation contracts or loans for farmers, as well as the abundant documentation, were found in the administrative buildings of the palaces and temples of the cities of Mesopotamia.

The irrigation technology in the fields implemented implied the risk of soil salinization. The evaporation of water causes the minerals it contains to rise, and if the soil salt content is too high, the field can no longer be cultivated and the water must be drained off the field to replenish the soil. This problem affected many lands in southern Mesopotamia, which became uncultivable and abandoned after intensive exploitation. In contrast, palm trees grow very well in salinized grounds, which explains their growth in the oldest Mesopotamia.

11.2. The Mayans

The collapse of the Mayan civilization was because of the destruction of the environment caused by it due to the mismanagement of resources, indicated the American archeologist Richard D. Hansen [71, 72], one of the principal researchers of that old culture. "The Mayans themselves damaged their environment. They destroyed it. The impact of the damage (to the environment) was so strong that they caused the collapse of civilization," says Hansen.

In the Cuenca Mirador, the expert explained, the Mayans developed "the first economic state in the Americas." "In the pre-Classic period (in the year 1500 BC), they formed the first political State, almost an empire, where there was a development with strong economic management and large populations," but due to a strategic error "of government," the same Mayas caused its collapse.

Starting in 150 AC, "due to multifactors" associated with the environment such as diseases, drought, and deforestation, "people started to leave the area." "But it was not a case of abandonment in which people leave, but come back. Here they left and did not return. The collapse of the Mayas was a total abandonment "due to the lack of resources, Hansen stressed. The Mayans "were human" and as such "made mistakes," "abused the resources they had at their disposal." They fell into "conspicuous consumption." Preferring to build great palaces "without thinking about the needs of the people, without feeding them, until they finished everything, "he said.

11.3. Roman Empire

In the year 100 BC, the Roman Empire was spread along the Mediterranean. The Romans could have stayed in this area, near the sea, but the explorations gave good results, and they were encouraged to continue their territorial expansion by increasing connectivity. However, transportation by land was slow and expensive, unlike maritime transport, so the increase in the connectivity became expensive.

According to Joseph Tainter [73, 74], professor of environment and society at Utah State University, one of the most important lessons of the fall of Rome is that complexity comes at a cost. In the third century, Rome added more and more new elements: a considerable

army, cavalry, and subdivided provinces (each with its bureaucracies, courts, and defenses), all components necessary to maintain the cohesion of its almost 60 million inhabitants of the more varied races. Eventually, it could not, to the eaves of knowledge and technique of the time that already left their trace of disorder in the environment, continue to sustain that growing complexity entering a long collapse and fragmentation.

12. Conclusion

The mathematical theory of chaos when applied to experimental time series of learning processes is shown to be efficient and rigorous in revealing properties that underlie the interior of those, such as irreversibility and entropic connectivity. Similarly, when categorizing the performance of learning according to the dynamics (weak, medium, chaotic), it exhibits the behavioral patterns produced by each dynamic. Learning, as a human activity perennial in time, induces behavior patterns, an order associated with emotions, exporting entropy to the biosphere: "order wins" wanted "exporting disorder" not wanted. This learning reveals an overestimation, historical, and cultural, of the regenerative capacities of the environment and planet Earth, especially concerning one of the forms of the disorder ("unwanted"), more characteristic of modern human activity: garbage and pollution, which we leave in charge of the planet. At present, we can perceive these regenerative limits in the form of depletion of croplands and productive demands on agricultural land through increasingly powerful fertilizers with unexpected consequences, consumption of drinking water, acidification and desertification of the oceans, atmospheric pollution, climate change, and so on. Would we expect another result? As we say about the time series, we face the increasing, and the frenetic entropy connectivity of the learning (much of trial and error) overstimulated by a POS/NEG ratio exacerbated toward positivity. On the other hand, an economic-technological system that essentially seeks to optimize profit as a synonym of well-being and, by extension, an illusion of happiness. Given the current state of the biosphere [75–77], would we expect another result? The complexity of this disjunctive is that nothing is more proper to human nature than its willingness to learn. Civilization, from its inception, has shown that an essential evolutionary characteristic of its learning is to adapt the environment to its utilities and needs, which will always be influenced by the certainties and uncertainties of knowledge and its transforming polarity. Innovating patterns of behavior, which have given us confidence in our possibilities by feeling exclusive of species on a planet with supposedly infinite resources, are complex. It means accepting limits. It recognizes in human activities not only its load of positivity, but also negativity, which broadens its meaning. There is no more provocative word for freewill than the idea of limits to what we want and can do. It is necessary to face that in all the processes that they want to carry out, the existence of limits will play an essential role. A predicament that is not new, and always proclaimed—and most of the time, interestingly overlooked—by the most varied forms and themes of reflection: complex systems, entropy, science, emotions, the human brain, religion, and more.

It is probable that the revenge for the untied man (restless Nietzsche would say), of today, is the irruption of the "contemplative man" that Nietzsche would point us out of tomorrow. The challenge is greater, as indicated, because it requires us to explore new forms of

relationship between ourselves and with nature [58], of which we are a part, and of the planet Earth in particular. Our viability of species is in interdiction, even expanding our environment of space, and this is a medium, even extremely hostile to human life [59].

Acknowledgements

Public acknowledgment to Ms. Marisol Fuentes Olmos, Master of Education, in the designing, construction, and linguistic help in the revision of this handwriting.

Author details

Patricio Pacheco Hernández

Address all correspondence to: patricio.pacheco@utem.cl

Technological Metropolitan University, Santiago, Chile

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