Min *k*-Cut for Asset Selection in Risk-Based Portfolio Strategies

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http://dx.doi.org/10.5772/intechopen.74455

Abstract

Risk-based portfolio strategies such as the equal-weighted, the minimum variance, and the risk parity portfolios vie to find portfolios that are well diversified according to their respective measures. In this chapter, we propose asset-selected risk-based portfolio strategies that aim to reduce the two known weaknesses of these strategies, namely the large portfolio size and poor diversification with respect to other measures. We formulate this task as a minimum k-cut problem through which we establish asset selection from all assets in the investable universe before the risk-based strategy is applied. Empirical results on the data sets of the S&P 500 and the KOSPI 200 indicate that our asset-selected risk-based portfolio strategies possess superior properties across extensive performance measures compared to the baseline risk-based strategies.

Keywords: alternative portfolio management, smart beta strategy, risk-based portfolio, minimum *k*-cut, portfolio optimization

1. Introduction

Portfolio selection has been a main research topic in finance for over 60 years, dating back to the seminal paper by Markowitz [1] which laid the foundation of modern portfolio theory. Markowitz's analysis has established the mean–variance-efficient portfolios which achieve optimal tradeoff between return and risk. Extensive efforts have been made into various directions since then including the development of the capital asset-pricing model (CAPM) [2]. Most recent studies on this topic of portfolio selection have focused on alternative or "smart" beta strategies which exploit risk premia other than the systematic risk, or seek a better diversification of risk. These are quantitative approaches to portfolio selection and have played an important role in the field of portfolio management recently. These lie in between an active management and the passive management manifested by the perennial market capitalization-weighted portfolio. A major category in the class of alternative beta strategies is the risk-based portfolio strategies [3–6] whose main objective is to manage risk more effectively than the market capitalization-weighted portfolio.

Currently, the existing representative risk-based portfolio strategies include the equal-weighted portfolio in which every asset has an equal weight, the minimum variance portfolio that achieves the smallest volatility, and the risk parity portfolio of [7, 8] in which every asset in the portfolio is exposed to equal risk. Clearly, diversification benefits of the three risk-based portfolios do not perfectly coincide. An important and favorable characteristic of these strategies is that they do not require the estimation of the expected returns, which is very error-prone, in their formulations. A somewhat comprehensive description of these strategies in terms of risk factors was presented in [3], and it was shown that the equal-weighted and the risk parity portfolios are special cases of the constrained minimum variance portfolio in [7]. A general framework of the quantitative asset allocation models of the three risk-based portfolio strategies has been presented in [5], and a detailed comparison of the strategies has also been provided in [6].

In this chapter, we propose to improve the characteristic and the performance of the risk-based portfolio strategies. Firstly, we address the presence of an inherent problem in the exact implementations of the equal-weighted and the risk parity portfolios that arise from the large cardinality of the respective portfolios. By construction, each of these portfolios has cardinality equal to that of the investable universe which can be too large to be implemented exactly in practice for many investors. To this end, we investigate a preselection of assets from the set of investable universe prior to implementing the risk-based portfolio strategies in order to reduce the portfolio cardinality to a more manageable size. This part relates to the improvement in the "characteristic" of the risk-based strategies.

Secondly, we address the relative weakness of a risk-based strategy with respect to some diversification measures. For example, the minimum variance portfolio produces the portfolio with the smallest variance, however, also one that is poorly diversified with respect to weight and to risk. Similarly, the equal-weighted portfolio produces the portfolio with a perfect weight diversification but one that also has a relatively high variance. To this end, as in the first case, we consider a preselection of assets and, in particular, the selection of "diversified" assets that will endow each risk-based strategy a "better" assets pool from which the portfolios are constructed. Consequently, the risk-based portfolio strategies defined on our diversified assets pool will perform superior to those defined on the original investable universe across different diversification measures and also with respect to other more popular performance measures such as the return and the Sharpe ratio as well; our results will be shown later. This part relates to the improvement in the "performance" of the risk-based strategies.

The described improvements are achieved simultaneously by formulating the problem as a minimum *k*-cut problem with assets represented as vertices in the graph. As the risk-based strategies are applied only after the assets have been selected for inclusion in the assets pool, we call our proposed strategies "asset-selected risk-based portfolio strategies." Furthermore, our asset-selected risk-based portfolio strategies require modest additional computational cost to the respective baseline risk-based ones.

We tested and compared the risk-based strategies with the proposed asset-selected risk-based strategies on the data sets of the S&P 500 from 1990 to 2016 and the KOSPI 200 from 2002 to 2016 where the latter is the set of sector-diversified largest 200 companies by market capitalization listed in the Korea Stock Exchange. Therefore, the assets that we refer to in this chapter are all stocks.

The organization of this chapter is as follows. In the next section, we present the formulation of the three risk-based portfolio strategies along with the associated diversification measures. In Section 3, we present the minimum k-cut-based asset selection method that forms the basis from which our contributions of this chapter come. It is followed by the presentation of the asset-selected risk-based portfolio strategies in Section 4. Extensive empirical results for the strategies are presented in Section 5, and the conclusion of the chapter is provided in Section 6.

2. Risk-based portfolio strategies

We consider the following three types of risk-based portfolio strategies from which we aim to make asset selection in an effective fashion: equal-weighted, minimum variance, and risk parity. We also make comparison with the market capitalization-weighted portfolio. While this portfolio does not generate competitive returns, it has a nice property of automatic rebalancing and serves as a common benchmark against which other alternative investment strategies can assess their performances. We explore these portfolios under the long-only constraint which will guarantee a unique solution and provide more realistic investing environment for many investors. We list the four portfolio strategies subsequently where the corresponding abbreviations are shown inside the parentheses:

1. Market capitalization-weighted portfolio (M)
2. Equal-weighted portfolio (EW)
3. Minimum variance portfolio (MV)
4. Risk parity portfolio (RP)

To formulate the portfolio strategies, let us introduce some notations that will be used throughout this chapter:

N		number of assets in the investable universe
Σ		the covariance matrix
σ_i		volatility of asset i
$\boldsymbol{\sigma}$	$=$	$(\sigma_1, \cdots, \sigma_N)^t$
x_i		weight of asset i in the portfolio
$\mathbf{x}$	$=$	$(x_1, \cdots, x_N)^t$, portfolio weight vector
$\sigma(\mathbf{x})$		portfolio volatility using $\mathbf{x}$ as the portfolio weight vector

Therefore, in our notations, σ and $\mathbf{x}$ are length$-N$ vectors where t denotes the transpose operator. Note that $\sigma(\mathbf{x}) = \sqrt{\mathbf{x}^t \Sigma \mathbf{x}}$. As each portfolio strategy is completely defined by its portfolio weight vector, let us describe each strategy by its weight vector. EW is the strategy in which all assets are given equal weights. The weight vector $\mathbf{x}_{EW}$ for EW is given by the following:

$$\mathbf{x}_{EW} = \frac{1}{N}\mathbf{1} \tag{1}$$

where $\mathbf{1}$ is the all-ones vector. While the strategy looks too simple to generate consistent positive excess returns, it has shown to outperform M considerably, and no other alternative investment strategies were consistently better than EW for a wide selection of markets and holding periods [3, 4]. MV [1] is the strategy in which the portfolio volatility is minimized. The weight vector $\mathbf{x}_{MV}$ for MV is given by the following:

$$\begin{aligned} \mathbf{x}_{MV} &= \arg\min_{\mathbf{x}} \left(\mathbf{x}^t \Sigma \mathbf{x}\right) \\ \text{s.t.} &\quad \mathbf{1}^t\mathbf{x} = 1, \\ &\quad \mathbf{x} \geq \mathbf{0}. \end{aligned} \tag{2}$$

MV provides an optimal return-risk tradeoff, and in particular, it lies on the leftmost tip of the efficient frontier curve. MV is perhaps the most stable portfolio among the ones along the efficient frontier curve as most of the estimation errors come from that of the returns, and this fact adds to the appeal of MV. RP [7, 8] is the strategy in which the risk associated with each asset is the same across all assets in the portfolio. Specifically, let the risk contribution of asset i, RC_i, be defined as

$$RC_i = x_i \cdot \frac{\delta\sigma(\mathbf{x})}{\delta x_i} = \frac{x_i^2\sigma_i^2 + x_i \sum_{j\neq i} x_j \sigma_{ij}}{\sigma(\mathbf{x})} \tag{3}$$

where $\frac{\delta\sigma(\mathbf{x})}{\delta x_i}$ is the *marginal* risk contribution of asset i, and σ_{ij} is the covariance of assets i and j. RP requires that.

$$RC_i = RC_j \quad \text{for all } i, j.$$

Note that $\sum_{i=1}^{N} RC_i = \sigma(\mathbf{x})$, and thus the risk contribution from each asset adds up to the portfolio risk or volatility. We also note that in MV, the marginal risk contributions are all equal for all assets, that is, $\frac{\delta\sigma(\mathbf{x})}{\delta x_i} = \frac{\delta\sigma(\mathbf{x})}{\delta x_j}$ for all i, j. It is known that RP possesses a unique solution in long-only investment environment [9] which is our case under study. The weight vector $\mathbf{x}_{RP}$ for RP is given by the following:

$$\begin{aligned} \mathbf{x}_{RP} &= \arg\min_{\mathbf{x}} \left(\frac{1}{2}\mathbf{x}^t\Sigma\mathbf{x} - \sum_{i=1}^{n} \ln x_i\right) \\ \text{s.t.} &\quad \mathbf{1}^t\mathbf{x} = 1, \\ &\quad \mathbf{x} > \mathbf{0} \end{aligned} \tag{4}$$

which is a convex optimization formulation that can be computed efficiently [9]. One can obtain an equivalent, but computationally less efficient, optimization problem as earlier and show that RP functions as a tradeoff between EW, which maximizes weight diversification with perfect disregard for volatility or variance reduction, and MV, which minimizes variance with perfect disregard for weight diversification [7]. As a consequence, it can be deduced that.

$$\sigma(\mathbf{x}_{MV}) \leq \sigma(\mathbf{x}_{RP}) \leq \sigma(\mathbf{x}_{EW}).$$

To quantify the amount of diversification attained by the three risk-based portfolio strategies, in the following, we list the three diversification profile measures [6] for the weight vector $\mathbf{x}$ of a portfolio strategy. They are (1) weight diversification

$$\frac{1}{N\sum_{i=1}^{N} x_i^2}, \tag{5}$$

(2) volatility reduction

$$\frac{\mathbf{x}_{MV}^t \Sigma \mathbf{x}_{MV}}{\mathbf{x}^t \Sigma \mathbf{x}} \tag{6}$$

and (3) risk diversification

$$\frac{1}{N\sum_{i=1}^{N} (RC_i/\sigma(\mathbf{x}))^2}. \tag{7}$$

These equations show that the weight vector $\mathbf{x}$ for EW, MV, and RP achieves the highest weight diversification, volatility reduction, and risk diversification, respectively. Note that each of the measures assumes values between 0 and 1, which represent the lowest and the highest levels of diversification attained, respectively.

To get an idea of how the three risk-based strategies fare with respect to the three diversification profile measures, in **Table 1**, we list the values of the risk-based portfolio strategy-diversification profile measure pairs for the S&P 500 and the KOSPI 200 data. The values shown are the averages of the 105 and 56 periods we used in our experiments for the S&P 500 and the KOSPI 200 data, respectively. The table shows a similar pattern for the two data sets with respect to the strong and weak profile measures for each risk-based portfolio strategy, suggesting that this is a characteristic of the strategies. In particular, EW showed a relatively high-risk diversification but a relatively weak volatility reduction as intuition might suggest. MV showed a very low-weight diversification and risk diversification. RP showed a relatively high-weight diversification but relatively low volatility reduction.

Before we further proceed to examine our asset-selected portfolio strategies, let us describe our experiment setting which is as follows. For the S&P 500 data, the investing time horizon spans from February 1, 1990, to May 2, 2016, that constitutes a total of 105 quarters, and for the

	Weight diversification	Volatility reduction	Risk diversification
S&P 500			
EW	1.000	0.229	0.838
MV	0.047	1.000	0.047
RP	0.765	0.329	1.000
KOSPI 200			
EW	1.000	0.290	0.873
MV	0.095	1.000	0.095
RP	0.801	0.397	1.000

Table 1. Diversification profile measure summary.

KOSPI 200 data, it spans from May 2, 2002, to 2016, for a total of 56 quarters. Both sets of data were collected on May 2, 2016, and the stock prices have been adjusted for dividends and splits before experiment.

The daily closing prices of stocks in the formation or the look-back period of a length of 252 days were used to execute the risk-based portfolio strategies whose resulting portfolios were put into effect the following trading day. There were no missing data in the closing prices of stocks, and no preprocessing of data was made as is common in the finance literature. This portfolio is held for one quarter after which portfolio rebalancing is performed. By portfolio rebalancing, we mean an independent and new execution of the portfolio strategy using data in the formation period of the most recent 252 days to update or rebalance the portfolio. This process is iterated throughout the investing time horizon, and specifically, portfolio rebalancing was made after market close on the last trading day of January, April, July, and October of each year. To clarify the terms used in this chapter, in Ref. to the date of portfolio formation (for the first portfolio) or rebalancing (for portfolios thereafter), let us call the preceding 252 days the "formation period" and the subsequent quarter, typically 58–60 days, the "holding quarter." The sequence of consecutive holding quarters is termed the "holding period" in this chapter which is the entire investing time horizon of our experiment.

3. Min *k*-cut for asset selection

The risk-based portfolio strategies EW, MV, and RP described in the previous section provide substantial return–risk tradeoff advantage compared to M [3]. Moreover, the qualitative feature that is important about these strategies is that each achieves the maximum of the associated diversification profile measure. Building on top of these risk-based strategies, we present an improvement in two directions in this chapter. Firstly, we consider the size of the risk-based portfolio that is completely determined by the strategy that defines it. For example, strategies EW and RP, by definition, generate portfolios whose size is equal to that of the assets pool which is the universe of all investable assets, specifically, N. Therefore, to accurately implement EW and RP, the investor has to hold all assets that exist in the investable universe (normally, in the order of hundreds) in the portfolio. This may be too difficult to achieve in practice, and furthermore, holding all assets in the investable universe may not be an investor's idea of a portfolio. Thus in reality, the implementation of EW and RP is sometimes vastly approximated by various heuristic approaches created by the investor. In this chapter, we present a systematic way to reduce and control the size of the portfolios generated by these strategies. This characteristic of our proposed strategy may be very beneficial from the practical point of view. Strategy MV, on the other hand, normally generates a very concentrated set of assets whose size may be larger or, typically, smaller than an investor's preferred value. In the case of the former, our contribution will provide a systematic way to reduce the portfolio size for this strategy. In the case of the latter, our proposed strategy may have a slight adverse effect in this respect. As a side note, for the MV strategy, one may add cardinality constraint to Eq. (2) to match the size of the portfolio with investor's investment constraints; however, this will result in a mixed integer quadratic programming that is proven to be computationally hard.

Secondly, we address the possibility of performance enhancement of the risk-based portfolio strategies. Specifically, we have witnessed that risk-based strategies show, sometimes serious, weakness in some of the diversification profile measures. As this phenomenon is attributed to the disregard for one measure of diversification by a strategy that optimizes a different measure of diversification, we consider an assets pool that is well diversified on which the risk-based strategies are executed. For this matter, we present a systematic way to execute assets selection from the pool of all investable assets such that its effect will be an improved performance across all diversification profile measures and return-risk tradeoffs.

Relating the above two directions of improvement, in this chapter, we present an assets selection method that realizes the two improvements simultaneously. To describe our method, consider the correlation matrix $R = \left(\rho_{ij}\right)_{i,j=1}^{N}$ of the set of all investable assets, where $\rho_{ij}\left(\overset{\Delta}{=}\frac{\sigma_{ij}}{\sigma_i\sigma_j}\right)$ is the correlation coefficient between assets i and j. To make the matrix to have nonnegative entries, let $\widetilde{\rho_{ij}}\overset{\Delta}{=}e^{\rho_{ij}}$ for all $i, j = 1, \cdots, N$, and let us consider the new matrix $\widetilde{R} = \left(\widetilde{\rho_{ij}}\right)_{i,j=1}^{N}$. Now, form the weighted graph G whose adjacency matrix is $\widetilde{R}$. This complete graph G has the property that more (less) correlated pair of assets has higher (lower) weighted edge. Therefore, to obtain a set of k ($\ll N$) assets that are least correlated with each other, one may partition the graph G into k-connected subgraphs so that the edges removed to obtain the partition has a minimum weight and then pick an asset in each partition according to some rule. This approach to obtaining a set of k assets with the described property complements the main objective of some of the risk-based portfolios. Specifically, recall that the main objectives of EW and RP are the maximizations of weight and risk diversifications, respectively, with perfect disregard for other measures. This suggests the implication of the presence of, possibly highly, correlated assets in the obtained portfolios. Thus, for these strategies, the preselection of assets seems beneficial. On the other hand, the main objective of MV is the maximization of volatility reduction in which lesser correlated assets are selected to some degree. Therefore for this strategy, the benefit of the preselection of assets with the described property seems not to be as large as in the other cases.

In all cases, by reducing the assets pool from the universe of all investable assets of size N to a set of k-diversified assets with respect to correlation with other assets, we have effectively executed a *diversified assets selection.* Therefore, it remains to describe (1) how to partition the graph into k-connected subgraphs, satisfying the constraints mentioned above, and (2) how to pick an asset in each partition.

The first part of this is precisely the minimum k-cut problem of which finding for the exact solution is well known to be NP-hard [10]. To this end, we use an efficient approximation algorithm to this problem that finds a minimum k-cut within a factor of $2\left(1-\frac{1}{k}\right)$ of the optimal due to [11], which is as follows:

Min k-cut approximation algorithm [11]:

1. For each edge $\widetilde{\rho_{ij}}$, pick a minimum weight cut that separates the end points of $\widetilde{\rho_{ij}}$.
2. Sort these cuts by increasing weight, obtaining the list $\rho'_1, \rho'_2, \cdots, \rho'_{N(N-1)/2}$.
3. Greedily, pick cuts from this list until their union is a k-cut; cut ρ'_i is picked only if it is not contained in $\rho'_1 \cup \cdots \cup \rho'_{i-1}$.

We note that the factor of $2\left(1-\frac{1}{k}\right)$ of the optimal is still known as the best approximation factor for tractable algorithms for the minimum k-cut problem [12]. The complexity of this algorithm is dominated by that of finding the cuts $\rho'_1, \rho'_2, \cdots, \rho'_{N(N-1)/2}$ which can efficiently be calculated through the use of Gomory-Hu tree representation. Specifically, $N-1$ max flow computations suffice to implement the above Min k-Cut Approximation Algorithm. Moreover, using the Gomory-Hu tree representation, all partitions but one in the k-cut contain exactly one vertex each with the remaining $N-k+1$ vertices being contained in the last partition. This characteristic of the algorithm when used with Gomory-Hu tree representation almost eliminates the need for the second part of our diversified assets selection as we need to pick only one vertex in the only partition that contains more than one vertex. Nevertheless, it remains to describe how to pick an asset in this last partition with more than one asset. To this end, we define the affinity of asset i, $a(i)$, as

$$a(i) = \sum_{j \neq i} \widetilde{\rho_{ij}} \tag{8}$$

from the matrix $\tilde{R}$. Therefore, the affinity of asset i gives a measure of how the asset i is correlated with all other assets. Eq. (8) shows that the larger the value of $a(i)$, the more correlated the asset i is with other assets. To pick the one vertex in the last partition with $N-k+1$ vertices, we picked the vertex i with the highest $a(i)$ in the partition as this vertex would appropriately serve as the "representative" of this partition. To see the effect of using affinity as the criterion for asset selection, we also tried picking the one vertex with the smallest $a(i)$ in the partition. We labeled the former asset selection method as "Max," and the latter one as "Min." We note that this second part adds negligible computational burden on the overall diversified assets selection process as the correlation matrix R and thereby $\tilde{R}$ is easily obtained from the covariance matrix Σ.

In the execution of the overall asset-selected portfolio selection, this part of asset selection is executed first and then the risk-based portfolio strategy is conducted. We have presented an "efficient" method to obtain a reduction in the size of the assets pool to any number of choices in this section. In the next two sections, we empirically demonstrate that this method also proves to be very "effective," characterized by superior return–risk tradeoff performances compared to the baseline risk-based portfolio strategies.

4. Asset-selected risk-based portfolio strategies

In this section, we formally present our proposed asset-selected risk-based portfolio strategies. As mentioned before, the purpose of our proposed asset selection-based strategy is twofold. The first is endowing the investor the option to choose the exact size of the portfolio when the risk-based strategy is either EW or RP. The second purpose is obtaining a superior return-risk tradeoff through effective subset selection of the assets prior to applying the risk-based portfolio strategy. We empirically demonstrate that our proposed strategies can generate returns that are sufficiently higher than the pure, or the baseline, risk-based strategies.

Now, to formally describe the strategies, let us denote the baseline risk-based strategy which does not employ asset selection by S and the asset-selected risk-based strategy by

$$\mathrm{S\_}\mu$$

where S is the name of the risk-based portfolio strategy, that is,

$$\mathrm{S} \in \{\mathrm{EW}, \mathrm{MV}, \mathrm{RP}\},$$

and μ is the name of the asset selection method. For our asset selection-based strategies, we picked the number of partitions k equal to approximately 25% of the total number of assets N in all of the experiments conducted in this chapter. In addition to the two methods of μ described in the previous section, we also tried selecting the asset with the largest market capitalization in the last partition with $N - k + 1$ assets. We denote this method as "MC," that is,

$$\mu \in \{\mathrm{Max}, \mathrm{Min}, \mathrm{MC}\}.$$

To illustrate the performance of these risk-based portfolio strategies, the next three figures, **Figures 1–3**, exhibit the annualized return versus the annualized volatility plots of the three strategies, respectively, for the S&P 500 (marked by $\circ$) and the KOSPI 200 (marked by Δ) data. **Figure 1** shows the plots for the EW-based strategies for the two data sets. Any point to the left of and/or above the point of the baseline strategy can be interpreted as improvement over the baseline strategy. We observe that EW_Max, EW_Min, and EW_MC all lie above and/or to the left of EW for both data sets. The defining favorable characteristic of the asset selection-based

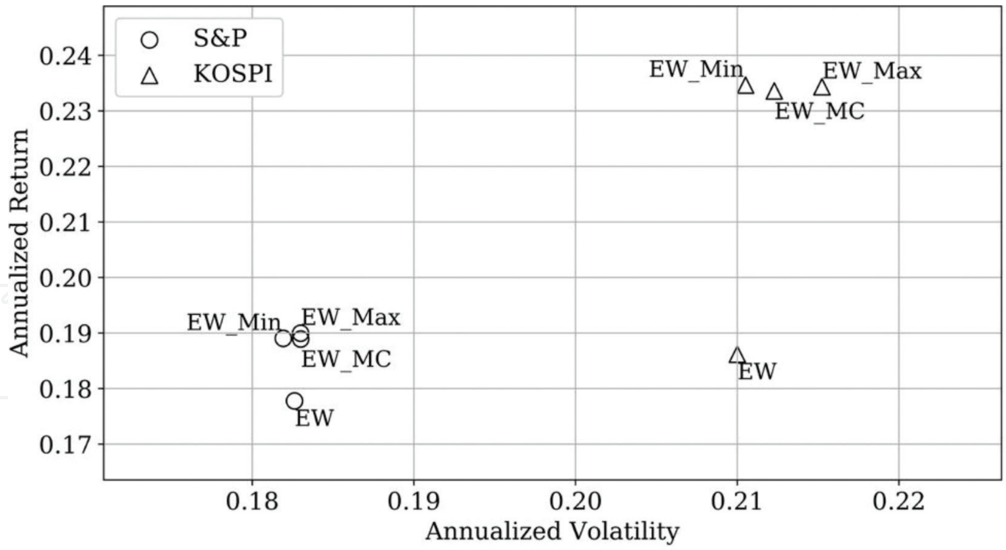

Figure 1. Annualized return versus volatility for EW-based strategies.

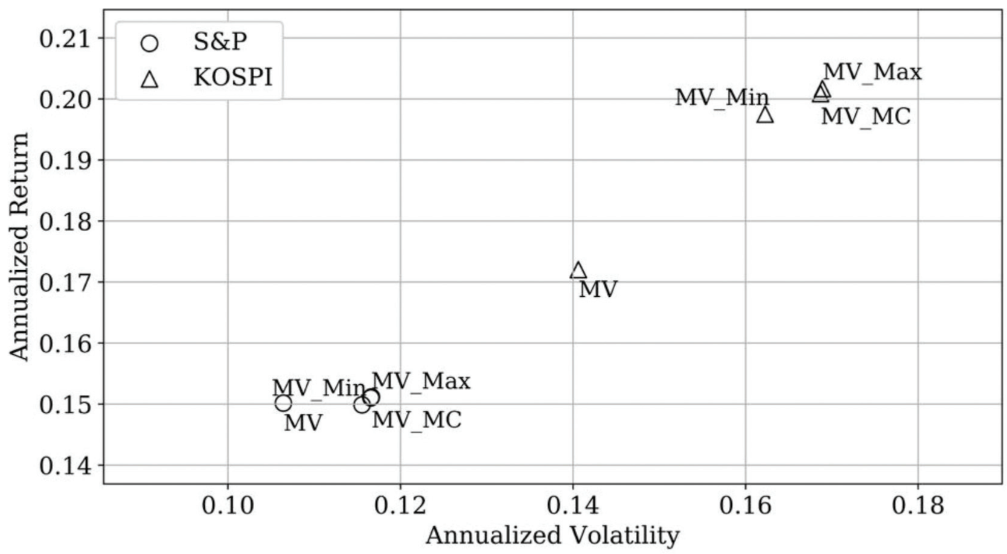

Figure 2. Annualized return versus volatility for MV-based strategies.

strategies that can be deduced from the performances of EW_Max, EW_Min, and EW_MC is that the number of assets in the portfolio can be reduced significantly to any number of investor's choice, facilitating easier portfolio management while, at the same time, improving the return–risk performance!

Figure 2 shows the similar plots for the MV-based strategies. As one can expect, since the baseline strategy achieves the minimum volatility among all strategy types, improvement cannot be and was not made with respect to annualized volatility from the asset selection-based strategies. On the other hand, for the KOSPI 200 data set in particular, asset selection-based strategies were able to produce higher returns than the baseline strategy which may be attributable to the diversified assets selection. **Figure 3** shows the plots for the RP-based

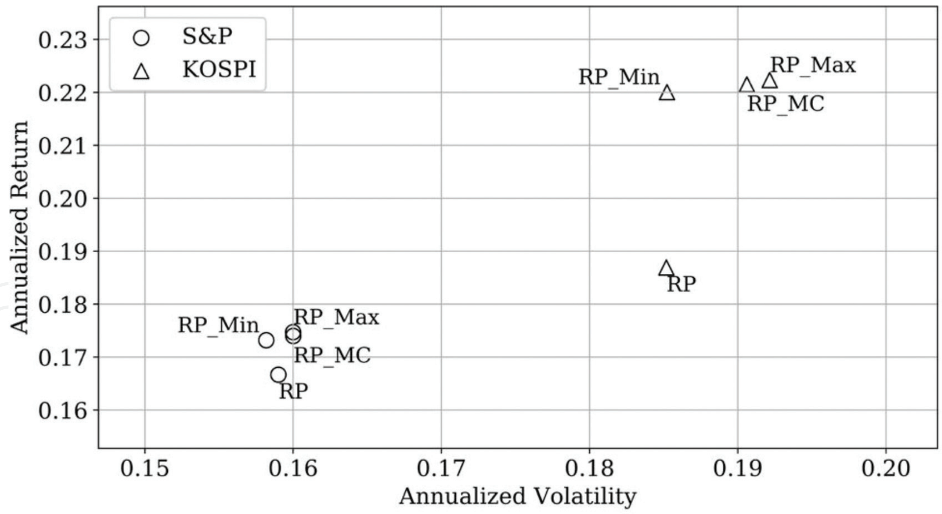

Figure 3. Annualized return versus volatility for RP-based strategies.

	Weight diversification	Volatility reduction	Risk diversification
S&P 500			
EW_Max/Min/MC	1.000	0.349/0.344/0.350	0.835/0.829/0.835
MV_Max/Min/MC	0.131/0.136/0.131	1.000	0.131/0.137/0.131
RP_Max/Min/MC	0.842/0.826/0.843	0.479/0.484/0.480	1.000
KOSPI 200			
EW_Max/Min/MC	1.000	0.456/0.440/0.470	0.863/0.857/0.869
MV_Max/Min/MC	0.253/0.256/0.257	1.000	0.253/0.257/0.257
RP_Max/Min/MC	0.859/0.835/0.864	0.603/0.595/0.612	1.000

Table 2. Diversification profile measure summary for asset selection-based strategies.

strategies, and in particular, the plots resemble those of the EW-based strategies, that is, RP_Max, RP_Min, and RP_MC all lie above and/or to the left of RP. As RP produces the portfolio only with equal-risk exposure to all assets, it is clearly benefitted by making diversified assets selection.

In **Table 2**, we list the values of the three diversification profile measures of S_μ, $S \in \{$ EW, MV, RP $\}$ and $\mu \in \{$ Max, Min, MC $\}$ for the S&P 500 and the KOSPI 200 data analogous to **Table 1**. The diversification profile measures were calculated so that S_μ, $S \in \{$ EW, MV, RP $\}$ achieves 1 for weight diversification, volatility reduction, and risk diversification, respectively, for every $\mu \in \{$ Max, Min, MC $\}$. **Table 2** shows a similar trend with respect to the stronger and weaker profile measures for each of the asset-selected risk-based portfolio strategies as in **Table 1**. However, a noteworthy finding from **Table 2** is that, compared to **Table 1**, the values of the profile measures have increased significantly across all profile measures for all portfolio strategies. Furthermore, such an improvement is consistent across all asset selection methods and both data sets. This result serves as good evidence that effective subset selection of assets prior to applying the risk-based portfolio strategy improves the "quality" of the risk-based strategies in all of the diversification profile measures considered. In summary, **Figures 1–3** and **Tables 1** and **2** indicate that our proposed asset-selected risk-based portfolio strategies provide clear superior return-risk tradeoff and diversification profile measure performances to the baseline risk-based portfolio strategies.

5. Results

In this section, we present a broader set of empirical results for the asset-selected risk-based portfolio strategies presented in this chapter. This allows us to better understand the advantages and the disadvantages of the asset-selected portfolio strategies. For this matter, we examine the following set of performance measures: (A) cumulative return, (B) annual return average, (C) annual return standard deviation, (D) annualized return, (E) annualized volatility, (F) Sharpe ratio, (G) beta, (H) portfolio size, (I) maximum drawdown, and (J) one-way turnover. In this set of performance measures, all return measures are simple returns except the "cumulative return (A)" whose value is set to 1 on the first day of the holding period to explicitly show the increase in the value of the initial asset throughout the entire investing time horizon. The "portfolio size (H)" is the ratio of the size of the portfolio to that of the investable universe which is equal to N. We recall that the performances of all strategies considered in this chapter except the market capitalization-weighted portfolio M reflect survivorship bias of the same degree as all data pertaining to assets were collected on the same day. For the M strategy, we used the index to calculate for the performance measures.

5.1. S&P 500 data

Table 3 shows the results of M, EW-based, MV-based, and RP-based strategies for the S&P 500 data. For the EW-based and the RP-based strategies, our asset-selected strategies showed a clear superior performance in terms of all types of returns considered, a comparable or slightly worse maximum drawdown performance, and an inferior one-way turnover performance to the respective performances of the baseline strategies. It seems that this single drawback of higher one-way turnover for the asset-selected strategies is an intrinsic characteristic of the strategies that stems from the construction and can be viewed as an implementation cost for the improved return-risk tradeoff performances gain. For the MV-based strategies, essentially no improvement was gained through asset selection. This behavior can be attributed to the fact that the baseline MV portfolio is already a somewhat diversified portfolio with respect to covariances between assets, so that adding the asset selection phase in portfolio construction does not help in terms of the performance measures. In fact, for the S&P 500 data, asset selection only contributed to adverse effect in terms of portfolio size, maximum drawdown, and one-way turnover as indicated by the table. Therefore, to improve the MV strategy in a similar order of magnitude as the EW and RP strategies, it seems that different asset selection

	A	B	C	D	E	F	G	H	I	J
M	6.282	0.085	0.159	0.073	0.180	0.243	1.000	—	2.200	—
EW	73.249	0.190	0.171	0.178	0.183	0.816	0.977	1.000	12.885	0.019
EW_Max	95.972	0.203	0.180	0.190	0.183	0.881	0.965	0.252	16.067	0.370
EW_Min	94.036	0.202	0.179	0.189	0.182	0.881	0.957	0.252	15.730	0.368
EW_MC	93.786	0.202	0.179	0.189	0.183	0.875	0.965	0.252	15.786	0.366
MV	39.299	0.156	0.136	0.150	0.106	1.141	0.453	0.106	4.064	0.379
MV_Max	40.084	0.158	0.130	0.151	0.117	1.049	0.489	0.068	5.505	0.573
MV_Min	39.043	0.156	0.130	0.150	0.116	1.049	0.483	0.070	5.575	0.562
MV_MC	40.218	0.158	0.130	0.151	0.117	1.050	0.489	0.068	5.531	0.574
RP	57.126	0.176	0.154	0.167	0.159	0.868	0.840	1.000	9.487	0.072
RP_Max	68.401	0.185	0.159	0.175	0.160	0.912	0.836	0.252	11.168	0.398
RP_Min	66.100	0.183	0.157	0.173	0.158	0.913	0.823	0.252	10.798	0.392
RP_MC	67.305	0.184	0.158	0.174	0.160	0.908	0.836	0.252	11.040	0.394

Table 3. Performance summary for S&P 500 data.

methods need to be explored. We note that in all of the asset-selected risk-based strategies, as $k - 1$ assets have already been selected before selecting the last asset using the asset selection method μ, the choice of the asset selection method seemed not to matter too significantly in terms of the performances as shown in the table.

Next, in **Figure 4**, we show the curves representing the cumulative returns over the entire investing time horizon for the S&P 500 data. For the asset-selected strategies, only the $\mu = \text{Max}$

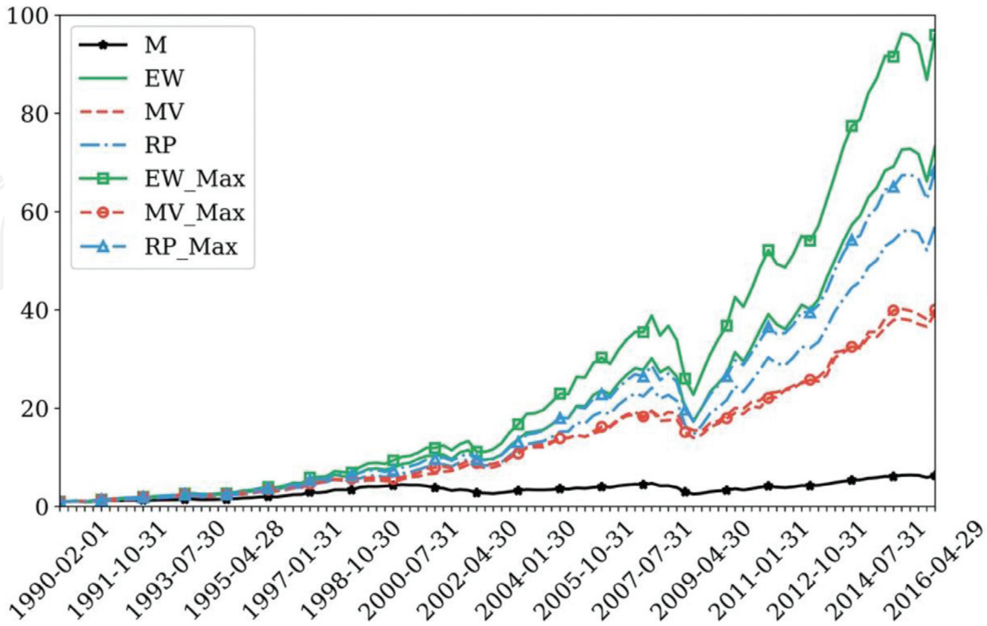

Figure 4. Cumulative return curves for S&P 500 data.

method is shown in the figure as this method was generally the best, albeit marginally, among the three methods. All curves start at 1, and we used the same color to represent the same type of risk-based portfolio strategy. **Figure 4** shows that EW_Max performs the best, and in particular, all asset-selected risk-based strategies outperform their respective baseline strategies, sometimes significantly.

5.2. KOSPI 200 data

Table 4 shows the results of M, EW-based, MV-based, and RP-based strategies for the KOSPI 200 data. Similar to **Table 3**, for the EW-based and the RP-based strategies, our asset-selected strategies showed a clear superior performance in terms of all types of returns considered, in this case including a maximum drawdown as well, and an inferior one-way turnover performance to the respective performances of the baseline strategies. As in **Table 3**, the portfolio sizes are approximately 25% of N which facilitates easy portfolio management in contrast to the baseline strategies. Even for the MV strategy, our asset-selected strategies produced performance improvement across all measures but the one-way turnover. The magnitude of the improvement was not as large as the asset-selected strategies of the EW and RP cases; however, even in this MV case, adding the asset selection phase in portfolio construction facilitated a more comprehensive assets diversification than that obtainable only through variance minimization. As before, the choice of the asset selection method seemed not to matter too significantly while $\mu = $ Max method generally outperformed the others. So for the KOSPI 200 data, our proposed asset-selected strategy produced a clear superior performance to the baseline strategy for all strategy types considered in this chapter.

	A	B	C	D	E	F	G	H	I	J
M	2.267	0.084	0.230	0.061	0.229	0.113	1.000	—	1.044	—
EW	10.495	0.216	0.299	0.186	0.210	0.718	0.840	1.000	2.232	0.021
EW_Max	18.156	0.256	0.283	0.234	0.215	0.924	0.840	0.254	2.439	0.396
EW_Min	18.216	0.257	0.283	0.235	0.211	0.946	0.815	0.254	2.463	0.400
EW_MC	18.003	0.254	0.278	0.234	0.212	0.933	0.834	0.254	2.358	0.390
MV	8.897	0.191	0.237	0.172	0.141	0.971	0.399	0.201	1.257	0.354
MV_Max	12.553	0.217	0.211	0.202	0.169	0.984	0.488	0.120	1.039	0.503
MV_Min	11.964	0.216	0.224	0.198	0.162	0.999	0.439	0.122	1.437	0.495
MV_MC	12.437	0.215	0.208	0.201	0.169	0.981	0.490	0.121	1.022	0.502
RP	10.585	0.214	0.280	0.187	0.185	0.818	0.729	1.000	1.891	0.079
RP_Max	15.868	0.241	0.255	0.222	0.192	0.972	0.734	0.254	1.863	0.417
RP_Min	15.466	0.239	0.256	0.220	0.185	0.997	0.694	0.254	1.767	0.419
RP_MC	15.723	0.239	0.251	0.222	0.191	0.976	0.734	0.254	1.790	0.409

Table 4. Performance summary for KOSPI 200 data.

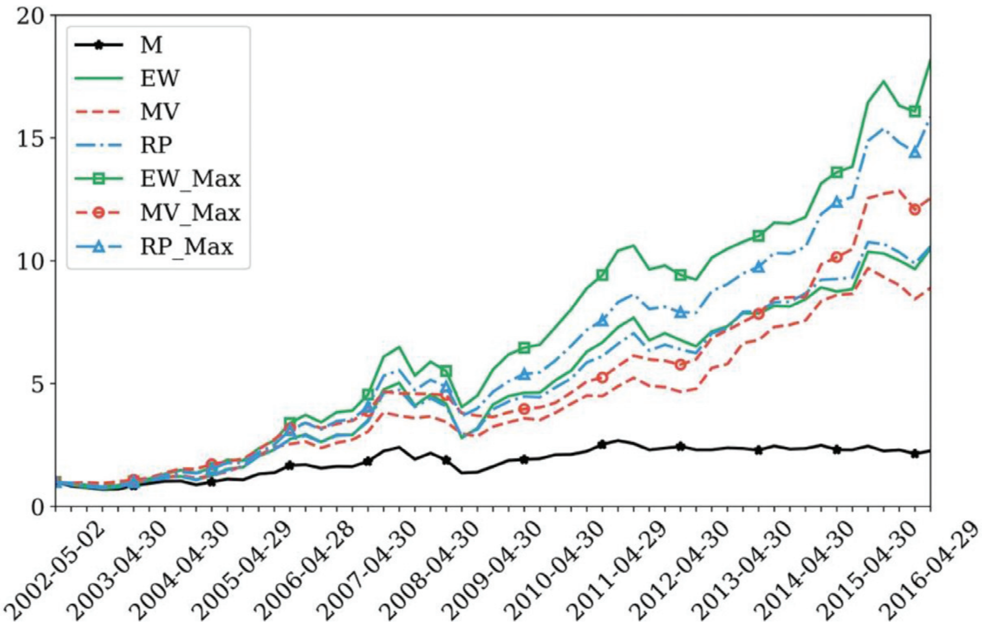

Figure 5. Cumulative return curves for KOSPI 200 data.

Next, in **Figure 5**, we show the curves representing the cumulative returns over the entire investing time horizon for the KOSPI 200 data. As in **Figure 4**, only the μ = Max method is shown for the asset-selected strategies as this method was the best among the three methods. As before, all curves start at 1, and we used the same color to represent the same type of risk-based portfolio strategy. **Figure 5** shows that EW_Max performs the best followed by RP_Max and then MV_Max. Consequently, all asset-selected risk-based strategies outperform their respective baseline strategies as in **Figure 4**. Summing up, **Figures 4** and **5** describe that our proposed strategy's performance improvement is robust across both data sets which serves as evidence that diversified assets selection contributes to superior portfolio returns.

6. Conclusions

In this chapter, we considered the three types of risk-based portfolio strategies that have played an important role recently in the area of smart beta strategies. They are the equal-weighted, the minimum variance, and the risk parity portfolios. By establishing an efficient and effective asset selection from assets in the investable universe before the risk-based portfolio strategies are applied, improvements in the characteristic and in the performance of the risk-based portfolio strategies were obtained. The improvement in the characteristic part allows the investor to pick the exact size of the portfolio for the equal-weighted and the risk parity portfolios. The improvement in the performance part is related to the performance improvement in all three risk-based portfolio strategies for various performance measures such as the returns, the Sharpe ratio, and the diversification measures. Empirical results on the data sets of the S&P 500 and the KOSPI 200 have indicated that our asset-selected risk-based portfolio strategies show, sometimes significant, advantages across various performance measures compared to the baseline risk-based strategies.

Acknowledgements

This research was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) and funded by the Ministry of Education (NRF-2017R1D1A1B03032722).

Conflict of interest

The authors declare no conflict of interest.

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