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A Design of Discrete-Time Indirect Multivariable MRACS with Structural Estimation of Interactor

Wataru Kase and Yasuhiko Mutoh
Osaka Institute of Technology / Sophia University
Japan

1. Introduction

An interactor matrix introduced by Wolovich & Falb (1976) has an important role in the design of model reference adaptive control systems (MRACS) for a class of multi-input multi-output (MIMO) plants. In the early stage of the research, the interactor was supposed to be diagonal matrix and thus there were no unknown parameters (Goodwin *et al.,* 1980). But, there exist many plants which require non-diagonal interactors (Chan & Goodwin, 1982). And the design of MIMO MRACS with non-diagonal interactors had been discussed, where all elements of the interactor are assumed to be known (Elliott & Wolovich, 1982; Goodwin & Long, 1980). However, this assumption is not adequate for adaptive control systems since the parameters of the interactor depend on the *unknown* parameters of the plant, i.e., the parameter values and the relative degree of each element of the plant must be used to determine the interactor. Furthermore, even we know all of these information, the structure of the interactor is not determined uniquely.

In order to remove the assumption, the MRACS design has been proposed where the degree of diagonal elements and the upper bound of the highest degree of the lower triangular interactor matrix are assumed to be known (Elliott & Wolovich, 1984; Dugard *et al.,* 1984). Under these assumptions, off-diagonal elements of the lower triangular interactor are estimated, and the method seemed suitable for adaptive controller design. However, it is not reasonable to assume the diagonal degrees in MRACS since the determination of the degrees depends on the relative degree and parameter values of each element of a transfer matrix of a given plant. From this view point, an interactor in generic sense was considered under the assumption that the relative degrees of all elements of the transfer function matrix are known (Kase & Tamura, 1990; Mutoh & Ortega, 1993). The method covers almost of all classes of MIMO plants having the same numbers of inputs and outputs generally. But there still exist some rare plants.

By the way, there exists an idea of the certainty equivalence principle for the *indirect* MRACS design, i.e., estimate the unknown parameters of a plant first, then design the controllers on-line, using those estimated parameters. However, the design was seemed very difficult especially for MIMO plants, since large amount of calculation is needed to solve so-called Diophantine equation, beside the derivation of the interactor. In other words, there did not exist a suitable method to solve the Diophantine equation or to derive the interactor matrix. In this chapter, an indirect approach to MIMO MRACS will be shown. For this purpose, it will be presented a simple derivation of the interactor matrix and preferable solution of the Diophantine equation. Both methods are based on the state space representation for the (estimated) transfer function matrix of the plant. Unlike direct calculation of polynomial matrices, it is preferable to compute via the state space representation.

This chapter is organized as follows. In the next section, the basic controller design with *known* parameters will be shown. The derivation of the interactor matrix and the solution to the Diophantine equation will be also presented. Then, the indirect MRACS will be shown in section 3 using the results in the previous sections. Some simulation results will be presented in section 4 to confirm the validity of the proposed method. Concluding remarks will be presented in section 5.

Notations (See Wolovich, 1974)

$\boldsymbol{R}^{p\times m}[z]$: Set of $p\times m$ polynomial matrices with real coefficients.

$\partial_{ri}[D(z)]$: The *i*-th row degree of polynomial matrix $D(z)$.

$\Gamma_r[D(z)]$: The row leading coefficient matrix of $D(z)$.

$$S_I^{\nu}(z)=\begin{bmatrix} I \\ zI \\ \vdots \\ z^{\nu}I \end{bmatrix},\quad \boldsymbol{O}_{\nu}=\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{\nu} \end{bmatrix},\quad \boldsymbol{T}_{\nu}=\begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{\nu}B & CA^{\nu-1}B & \cdots & CB \end{bmatrix}$$

2. Design for Plant with Known Parameters

2.1 Problem Statement

Assume that a plant to be controlled is given by

$$y(t)=N(z)D^{-1}(z)u(t)=\widetilde{D}^{-1}(z)\widetilde{N}(z)u(t) \tag{1}$$

where $D(z),\ N(z),\ \widetilde{D}(z),\ \widetilde{N}(z)\in \boldsymbol{R}^{m\times m}[z]$ such that

$$\begin{aligned} D(z)&=D_0+zD_1+\cdots+z^{\mu}D_{\mu}=\begin{bmatrix} D_0 & D_1 & \cdots & D_{\mu}\end{bmatrix}\begin{bmatrix} I \\ zI \\ \vdots \\ z^{\mu}I \end{bmatrix}=\boldsymbol{D}S_I^{\mu}(z), \\ N(z)&=N_0+zN_1+\cdots+z^{\mu-1}N_{\mu\text{-}1}=\begin{bmatrix} N_0 & N_1 & \cdots & N_{\mu}\end{bmatrix}\begin{bmatrix} I \\ zI \\ \vdots \\ z^{\mu-1}I \end{bmatrix}=\boldsymbol{N}S_I^{\mu-1}(z), \end{aligned} \tag{2}$$

$$\widetilde{D}(z)=\widetilde{D}_0+z\widetilde{D}_1+\cdots+z^{\nu}\widetilde{D}_{\nu}=\begin{bmatrix} I & zI & \cdots & z^{\nu}I \end{bmatrix}\begin{bmatrix}\widetilde{D}_0\\ \widetilde{D}_1\\ \vdots\\ \widetilde{D}_{\nu}\end{bmatrix}=\left\{S_I^{\nu}(z)\right\}^T\widetilde{\boldsymbol{D}},$$

$$\widetilde{N}(z)=\widetilde{N}_0+z\widetilde{N}_1+\cdots+z^{\nu-1}\widetilde{N}_{\nu\text{-}1}=\begin{bmatrix} I & zI & \cdots & z^{\nu-1}I \end{bmatrix}\begin{bmatrix}\widetilde{N}_0\\ \widetilde{N}_1\\ \vdots\\ \widetilde{N}_{\nu}\end{bmatrix}=\left\{S_I^{\nu-1}(z)\right\}^T\widetilde{\boldsymbol{N}},$$

$$\boldsymbol{D}=\begin{bmatrix} D_0 & D_1 & \cdots & D_{\mu}\end{bmatrix},\ \boldsymbol{N}=\begin{bmatrix} N_0 & N_1 & \cdots & N_{\mu-1}\end{bmatrix},\ \widetilde{\boldsymbol{D}}=\begin{bmatrix}\widetilde{D}_0\\ \widetilde{D}_1\\ \vdots\\ \widetilde{D}_{\nu}\end{bmatrix},\ \widetilde{\boldsymbol{N}}=\begin{bmatrix}\widetilde{N}_0\\ \widetilde{N}_1\\ \vdots\\ \widetilde{N}_{\nu-1}\end{bmatrix}.$$

Without loss of generality, assume that $D(z)$ is column proper and $\widetilde{D}(z)$ is row proper. The purpose of control is to generate the uniformly bounded input signals $u(t)$ which cause the output signals $y(t)$ of the transfer function matrix $G(z)=N(z)D^{-1}(z)$ to follow the reference output signals $y_m(t)$ asymptotically. For this purpose, the following assumptions are made:

1. det $N(z)$ is a Hurwitz polynomial.
2. $y_m(t+1),\ y_m(t+2),\ldots,y_m(t+w)$ are available at the time instant t, where w is the degree of the interactor for $G(z)$, which will be discussed later.

The control input to achieve the above objective is given by

$$u(t)=\frac{X(z)}{z^{\nu-1}}u(t)+\frac{Y(z)}{z^{\nu-1}}y(t)+L(z)y_m(t) \tag{3}$$

where $X(z),\ Y(z)\in \boldsymbol{R}^{m\times m}[z]$ satisfy the following Diophantine equation:

$$X(z)D(z)+Y(z)N(z)=z^{\nu-1}\{D(z)-L(z)N(z)\} \tag{4}$$

and $L(z)\in \boldsymbol{R}^{m\times m}[z]$ satisfies the following relation:

$$\lim_{z\to\infty}L(z)G(z)=I. \tag{5}$$

$L(z)y_m(t)$ in eqn.(3) is available from assumption 2. $L(z)$ is known as an interactor matrix. Substituting eqn.(3) to eqn.(1),

$$\begin{aligned} y(t)&=N(z)\left\{z^{\nu-1}D(z)-X(z)D(z)-Y(z)N(z)\right\}^{-1}z^{\nu-1}L(z)y_m(t)\\ &=N(z)\left\{z^{\nu-1}L(z)N(z)\right\}^{-1}z^{\nu-1}L(z)y_m(t)\end{aligned} \tag{6}$$

Thus from assumption 1, the purpose will be achieved if

1. det $L(z)$ is a Hurwitz polynomial, and
2. The degrees of $X(z)$ and $Y(z)$ are, at most, $\nu-2$ and $\nu-1$ respectively.

In the following subsections, simple calculation methods of $L(z)$, $X(z)$ and $Y(z)$ satisfying the above constraints will be shown.

2.2 A Simple Derivation of Interactor (Kase & Mutoh, 2008)

Consider the problem to find an interactor $L(z)$ for a given $m\times m$ non-singular transfer function matrix $G(z)$. In the direct MRACS, an interactor with lower triangular structure is useful to insure the global stability of the overall system. But, in this chapter, the interactor is not assumed to have any special form.

Let (A, B, C) denote a realization of $G(z)$. Then, using the Markov parameters, $G(z)$ can be expressed by

$$G(z) = z^{-1}CB + z^{-2}CAB + z^{-3}CA^2B + \cdots \tag{7}$$

If we set $L(z)$ by

$$L(z) = zL_1 + z^2L_2 + \cdots + z^wL_w = \begin{bmatrix} L_1 & L_2 & \cdots & L_w \end{bmatrix} \begin{bmatrix} zI \\ z^2I \\ \vdots \\ z^wI \end{bmatrix} = \boldsymbol{L}zS_I^{w-1}(z). \tag{8}$$

Then,

$$\begin{aligned} L(z)G(z) &= \begin{bmatrix} L_1 & L_2 & \cdots & L_w \end{bmatrix} \begin{bmatrix} zI \\ z^2I \\ \vdots \\ z^wI \end{bmatrix} (z^{-1}CB + z^{-2}CAB + z^{-3}CA^2B + \cdots) \\ &= \boldsymbol{L} \begin{bmatrix} CB + z^{-1}CAB + z^{-2}CA^2B + \cdots \\ zCB + CAB + z^{-1}CA^2B + \cdots \\ \vdots \\ z^{w-1}CB + z^{w-2}CAB + \cdots + CA^{w-1}B + \cdots \end{bmatrix} \\ &= \boldsymbol{L} \begin{bmatrix} \cdots & CAB & CB & 0 & \cdots & 0 \\ \cdots & CA^2B & CAB & CB & \cdots & 0 \\ & \vdots & \vdots & & \ddots & \vdots \\ \cdots & CA^wB & CA^{w-1}B & \cdots & CAB & CB \end{bmatrix} \begin{bmatrix} \vdots \\ z^{-1}I \\ I \\ zI \\ \vdots \\ z^{w-1}I \end{bmatrix} \end{aligned} \tag{9}$$

If eqn.(5) holds, then

$$\boldsymbol{L}\boldsymbol{T}_{w-1} = \boldsymbol{J}_{w-1}, \quad \boldsymbol{J}_{w-1} := \begin{bmatrix} I & 0_{m\times m(w-1)} \end{bmatrix} \tag{10}$$

must hold. Conversely, the identity interactor $L(z)$ can be obtained by solving this equation and the solvability of this is guaranteed if and only if

$$\operatorname{rank}\begin{bmatrix} \boldsymbol{T}_{w-1} \\ \boldsymbol{J}_{w-1} \end{bmatrix} = \operatorname{rank} \boldsymbol{T}_{w-1}, \tag{11}$$

and define the integer w by the least integer satisfying the above equation. Thus, using Moore-Penrose pseudo-inverse $\boldsymbol{T}_{w-1}^{+}$ of $\boldsymbol{T}_{w-1}$, $\boldsymbol{L}$ can be calculated if and only if

$$\boldsymbol{L} = \boldsymbol{J}_{w-1}\boldsymbol{T}_{w-1}^{+}. \tag{12}$$

All of $\boldsymbol{L}$ that satisfy eqn.(11) are coefficient matrices of the identity interactor, which form a subset of coefficient matrices of the interactor defined by Mutoh & Ortega (1993). In the paper, a certain calculating algorithm was used to obtain $\boldsymbol{L}$ and to assign stable zeros of the interactor as well. But, for the identity interactor, as shown above, it is quite natural to solve eqn.(11) using Moore-Penrose pseudo-inverse, because $\boldsymbol{J}_{w-1}$ is a fixed matrix and the pseudo-inverse can be calculated easily using some standard softwares in these days. Then, since we need a stable interactor in control design problems, the remaining problem is to check the location of zeros of the identity interactor given by eqn.(12). For $\boldsymbol{T}_{w-1}^{+}$, the following Lemma holds.

Lemma 1. For the integer $k \geq w-1$, the following equation holds:

$$\boldsymbol{T}_k^{+} = \begin{bmatrix} \boldsymbol{M}_k \\ \boldsymbol{Z}_k - \boldsymbol{T}_{k-1}^{+}\boldsymbol{O}_{k-1}AB\boldsymbol{M}_k \end{bmatrix} \tag{13}$$

where

$$\boldsymbol{M}_k = \begin{bmatrix} \boldsymbol{L} & \boldsymbol{0}_k \end{bmatrix}, \ \ \boldsymbol{Z}_k = \begin{bmatrix} 0_{km\times m} & \boldsymbol{T}_{k-1}^{+} \end{bmatrix}, \ \ \boldsymbol{0}_k = 0_{m\times m(k-w+1)}. \tag{14}$$

Proof. Let

$$\boldsymbol{T}_k^{+} = \begin{bmatrix} \boldsymbol{M}_k \\ \boldsymbol{P}_k \end{bmatrix}, \quad \boldsymbol{P}_k \in \boldsymbol{R}^{km\times(k+1)m}. \tag{15}$$

Since $\boldsymbol{T}_k^{+}$ is the pseudo-inverse of $\boldsymbol{T}_k$,

$$\boldsymbol{T}_k\boldsymbol{T}_k^{+} = (\boldsymbol{T}_k\boldsymbol{T}_k^{+})^T.$$

Substituting the above equation into eqn.(15),

$$\begin{bmatrix} CB \\ \boldsymbol{O}_k AB \end{bmatrix}\boldsymbol{M}_k + \begin{bmatrix} 0_{m\times km} \\ \boldsymbol{T}_{k-1} \end{bmatrix}\boldsymbol{P}_k = \begin{bmatrix} \boldsymbol{M}_k^T & \boldsymbol{P}_k^T \end{bmatrix}\boldsymbol{T}_k^T. \tag{16}$$

By post-multiplying the above equation by $\boldsymbol{M}_k^T$ and then using eqn.(10), it follows that

$$\begin{bmatrix} CB \\ \boldsymbol{O}_k AB \end{bmatrix}\boldsymbol{M}_k\boldsymbol{M}_k^T + \begin{bmatrix} 0_{m\times km} \\ \boldsymbol{T}_{k-1} \end{bmatrix}\boldsymbol{P}_k\boldsymbol{M}_k^T = \boldsymbol{M}_k^T. \tag{17}$$

From the existence of $\boldsymbol{P}_k$,

$$(I_{mk} - \boldsymbol{T}_{k-1}\boldsymbol{T}_{k-1}^T)\left(\begin{bmatrix} L_2^T \\ L_3^T \\ \vdots \\ L_w^T \\ \boldsymbol{0}_k^T \end{bmatrix} - \boldsymbol{O}_k \boldsymbol{L}\boldsymbol{L}^T\right) = 0 \tag{18}$$

holds. Using the singular value decomposition, $\boldsymbol{T}_{k-1}$ can be written as

$$\boldsymbol{T}_{k-1} = U_{k-1}\begin{bmatrix} \Sigma_{k-1} & 0 \\ 0 & 0 \end{bmatrix} V_{k-1}^T \tag{19}$$

for some unitary matrices U_{k-1} and V_{k-1}. Since eqn.(10) implies

$$\begin{bmatrix} L_2 & L_3 & \cdots & L_w & \boldsymbol{0}_k \end{bmatrix} \boldsymbol{T}_{k-1} = 0, \tag{20}$$

post-multiplying the above equation by $V_{k-1}\begin{bmatrix} \Sigma^{-2} & 0 \\ 0 & 0 \end{bmatrix} V_{k-1}^T$ gives

$$\begin{bmatrix} L_2 & L_3 & \cdots & L_w & \boldsymbol{0}_k \end{bmatrix} \left(\boldsymbol{T}_{k-1}^T\right)^T = 0. \tag{21}$$

Thus,

$$\begin{bmatrix} L_2^T \\ L_3^T \\ \vdots \\ L_w^T \\ \boldsymbol{0}_k^T \end{bmatrix} = \left(I_{mk} - \boldsymbol{T}_{k-1}\boldsymbol{T}_{k-1}^+\right)\boldsymbol{O}_k AB\boldsymbol{L}\boldsymbol{L}^T \tag{22}$$

is obtained from eqns.(18) and (19). Using a free parameter matrix $\boldsymbol{Z}_k \in \boldsymbol{R}^{mk\times m(k+1)}$, the general solution of eqn.(17) is given by

$$\boldsymbol{P}_k = \boldsymbol{Z}_k - \boldsymbol{T}_{k-1}^+\boldsymbol{T}_{k-1}\boldsymbol{Z}_k\boldsymbol{M}_k^T\left(\boldsymbol{M}_k^T\right)^+ - \boldsymbol{T}_{k-1}^+\boldsymbol{O}_k\boldsymbol{M}_k. \tag{23}$$

Finally, by choosing $\boldsymbol{Z}_k$ as in eqn.(14), eqn.(13) is obtained from eqns.(21) and (23). It is easy to verify that the above $\boldsymbol{T}_k^+$ satisfies the rest of conditions for the pseudo-inverse, i.e.

$$\left(\boldsymbol{T}_k^+\boldsymbol{T}_k\right)^T = \boldsymbol{T}_k^+\boldsymbol{T}_k, \quad \boldsymbol{T}_k\boldsymbol{T}_k^+\boldsymbol{T}_k = \boldsymbol{T}_k, \quad \boldsymbol{T}_k^+\boldsymbol{T}_k\boldsymbol{T}_k^+ = \boldsymbol{T}_k^+. \tag{24}$$

Therefore the Lemma has been proved. ∇∇∇

In MRACS case, it can not be assumed to know the exact value of w in eqn.(11). Lemma 1 shows that non-zero parameters in the interactor are not changed, if the upper bound of w is known. This is a nice property of the proposed method for MRACS. The following Theorem shows that all zeros of the interactor by proposed method lie at the origin. So, the stability of the interactor is clear although it does not have a lower triangular form. Moreover, the proposed interactor is optimal for the LQ cost with singular weightings. See Kase *et al*. (2004) for the proof.

Theorem 1. If the interactor is given by

$$L(z) = \boldsymbol{J}_{w-1}\boldsymbol{T}_{w-1}^{+} z S_I^{w-1}(z), \tag{25}$$

then the following properties hold:

$$\begin{aligned} &\mathbf{P1} \qquad L(z)L^{\sim}(z) = \boldsymbol{L}\boldsymbol{L}^T, \\ &\mathbf{P2} \qquad \begin{bmatrix} CB \\ CA_F B \\ \vdots \\ CA_F{}^{w-1} B \end{bmatrix} = \boldsymbol{L}^{+}, \\ &\mathbf{P3} \qquad CA_F^w = 0 \end{aligned} \tag{26}$$

where $\boldsymbol{L}^{+}$ is the pseudo-inverse of $\boldsymbol{L}$, and

$$L^{\sim}(z) = L^T(z^{-1}) = z^{-1}L_1^T + z^{-2}L_2^T + \cdots + z^{-w}L_w^T, \quad A_F = A - B\boldsymbol{L}\boldsymbol{O}_{w-1}A.$$

2.3 A Solution of Diophantine Equation (Kase, 1999; 2008)

There are many methods to solve eqn.(4). In this subsection, a method using state space parameters is presented. First, the following lemma holds.

Lemma 2. Let $P(z),\ N(z),\ D(z) \in \boldsymbol{R}^{m\times m}[z]$, where $D(z)$ is non-singular. Then, there exist polynomial matrices $Q(z),\ R(z) \in \boldsymbol{R}^{m\times m}[z]$ such that

$$\begin{aligned} &P(z)N(z) = Q(z)D(z) + R(z), \\ &R(z)D^{-1}(z) \text{ is strictly proper.} \end{aligned} \tag{27}$$

Furthermore, let (A, B, C) denote any realization of $N(z)D^{-1}(z)$. Then, $Q(z)$ and $R(z)D^{-1}(z)$ are given by the following equations:

$$\begin{aligned} &Q(z) = \boldsymbol{P}\begin{bmatrix} 0_{m\times fm} \\ \boldsymbol{T}_{f-1} \end{bmatrix} S_I^{f-1}(z) \\ &R(z)D^{-1}(z) = \boldsymbol{P}\boldsymbol{O}_f(zI - A)^{-1}B \end{aligned} \tag{28}$$

where f denotes the degree of $P(z) = P_0 + zP_1 + \cdots + z^f P_f$, and $\boldsymbol{P}$ is defined by $\boldsymbol{P} = \begin{bmatrix} P_0 & P_1 & \cdots & P_f \end{bmatrix}$.

Proof. Since

$$P(z)N(z)D^{-1}(z) = \boldsymbol{P}\begin{bmatrix} I \\ zI \\ \vdots \\ z^f I \end{bmatrix}\left(z^{-1}CB + z^{-2}CAB + z^{-3}CAB + \cdots\right)$$

$$= \boldsymbol{P}\begin{bmatrix} z^{-1}CB + z^{-2}CAB + z^{-3}CAB + \cdots \\ CB + z^{-1}CAB + z^{-2}CAB + \cdots \\ \vdots \\ z^{f-1}CB + z^{f-2}CAB + \cdots + CA^{f-1}B + z^{-1}CA^f B + \cdots \end{bmatrix}$$

$$= \boldsymbol{P}\begin{bmatrix} 0 & 0 & \cdots & 0 \\ CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{f-1}B & CA^{f-2}B & \cdots & CB \end{bmatrix}\begin{bmatrix} I \\ zI \\ \vdots \\ z^{f-1}I \end{bmatrix} + \boldsymbol{P}\begin{bmatrix} z^{-1}CB + z^{-2}CAB + \cdots \\ z^{-1}CAB + z^{-2}CA^2B + \cdots \\ z^{-1}CA^2B + z^{-2}CA^3B + \cdots \\ \vdots \\ z^{-1}CA^f B + z^{-2}CA^{f+1}B + \cdots \end{bmatrix}$$

$$= \boldsymbol{P}\begin{bmatrix} 0_{m\times fm} \\ \boldsymbol{T}_{f-1} \end{bmatrix} S_I^{f-1}(z) + z^{-1}\boldsymbol{PO}_f B + z^{-2}\boldsymbol{PO}_f AB + z^{-3}\boldsymbol{PO}_f A^2B + \cdots$$

the results can be obtained. ∇∇∇

Theorem 2. Let (A, B, C) denote any realization of $N(z)D^{-1}(z)$. Then, there exist $X(z)$ and $Y(z)$ which satisfy

1) Diophantine equation (4).
2) $X(z)/z^{\nu-1}$ is strictly proper.
3) $Y(z)/z^{\nu-1}$ is proper.

If and only if the following relation holds:

$$\boldsymbol{YO}_{\nu-1} = -\boldsymbol{LO}_{w-1}A^{\nu} \tag{29}$$

where $\boldsymbol{Y}$ is defined by $\boldsymbol{Y} = [Y_0 \quad Y_1 \quad \cdots \quad Y_{\nu-1}]$ for $Y(z) = Y_0 + zY_1 + \cdots + z^{\nu-1}Y_{\nu-1}$.

Proof. If eqn.(4) holds, then from Lemma 2,

$$\boldsymbol{YO}_{\nu-1} = -[0_{m\times m\nu} \quad \boldsymbol{L}]\boldsymbol{O}_{\nu+w-1} = -\boldsymbol{LO}_{w-1}A^{\nu}.$$

Conversely, if eqn.(29) holds, multiply the both sides of eqn.(29) by $\sum_{i=1}^{\infty} z^{-i}A^{i-1}B$, it follows that

$$\boldsymbol{YO}_{\nu-1}\sum_{i=1}^{\infty} z^{-i}A^{i-1}B = \boldsymbol{Y}\left(\begin{bmatrix} N(z)D^{-1}(z) \\ zN(z)D^{-1}(z) \\ z^2N(z)D^{-1}(z) \\ \vdots \\ z^{\nu-1}N(z)D^{-1}(z) \end{bmatrix} - \begin{bmatrix} 0 \\ CB \\ CAB+zCB \\ \vdots \\ CA^{\nu-2}B+zCA^{\nu-3}B+\cdots+z^{\nu-1}CB \end{bmatrix}\right)$$

$$= Y(z)N(z)D^{-1}(z) - \boldsymbol{Y}\begin{bmatrix} 0_{m\times m(\nu-1)} \\ \boldsymbol{T}_{\nu-2} \end{bmatrix} S_I^{\nu-2}(z)$$

$$= -[0_{m\times m\nu} \quad \boldsymbol{L}]\boldsymbol{O}_{\nu+w-1}\sum_{i=1}^{\infty} z^{-i}A^{i-1}B$$

$$= -[0_{m\times m\nu} \quad \boldsymbol{L}]\left(\begin{bmatrix} N(z)D^{-1}(z) \\ zN(z)D^{-1}(z) \\ z^2N(z)D^{-1}(z) \\ \vdots \\ z^{\nu+w-1}N(z)D^{-1}(z) \end{bmatrix} - \begin{bmatrix} 0 \\ CB \\ CAB+zCB \\ \vdots \\ CA^{\nu-2}B+zCA^{\nu-3}B+\cdots+z^{\nu+w-1}CB \end{bmatrix}\right)$$

$$= -z^{\nu-1}L(z)N(z)D^{-1}(z) + z^{\nu-1}I + \boldsymbol{L}\begin{bmatrix} CA^{\nu-1}B & \cdots & CAB \\ \vdots & & \vdots \\ CA^{\nu+w-2}B & \cdots & CA^{w}B \end{bmatrix} S_I^{\nu-2}(z).$$

Defining $X(z)$ by

$$X(z) = -\boldsymbol{L}\begin{bmatrix} CA^{\nu-1}B & \cdots & CAB \\ \vdots & & \vdots \\ CA^{\nu+w-2}B & \cdots & CA^{w}B \end{bmatrix} S_I^{\nu-2}(z) - \boldsymbol{Y}\begin{bmatrix} 0_{m\times m(\nu-1)} \\ \boldsymbol{T}_{\nu-2} \end{bmatrix} S_I^{\nu-2}(z),$$

then the Diophantine equation (4) can be obtained. It is clear that $X(z)/z^{\nu-1}$ and $Y(z)/z^{\nu-1}$ are proper from the above discussion. ∇∇∇

It is worth noting that Theorem 2 holds for any realization of $N(z)D^{-1}(z)$. So, this method is easy to apply for the indirect adaptive control.

3. Indirect Adaptive Controller Design

Using some suitable parameter estimation algorithm, such as the least squares algorithm, obtain the estimated values of $\tilde{\boldsymbol{D}}$ and $\tilde{\boldsymbol{N}}$. Then, obtain the observability canonical realization $\hat{A}(t)$, $\hat{B}(t)$ and $\hat{C}(t)$ from these estimated values. After that, the control input is generated by calculating $\hat{L}(z,t)$, $\hat{X}(z,t)$ and $\hat{Y}(z,t)$ recursively. Before the discussions of the adaptive controller design, the following assumptions are imposed:

1. The upper bound degree $\overline{w}$ of the interactor is known.
2. $y_m(t+1),\ y_m(t+2), \ldots, y_m(t+\overline{w})$ are available at the time instant t.

3. det $\widetilde{N}(z)$ is a Hurwitz polynomial.
4. $\partial_{ri}[\widetilde{D}(z)] = \nu_i$ is known and $\Gamma_r[\widetilde{D}(z)]$ is a lower triangular with ones in the diagonal positions.
5. $\hat{A}(t)$, $\hat{B}(t)$ and $\hat{C}(t)$ converge to their true values.
6. A lower bound of the minimum singular decomposition value $\varepsilon_{\min}$ of $\boldsymbol{T}_{\overline{w}-1}$ is known.

In the adaptive controller design, it will be carried out by the recursive calculation of the previous section. That is, based on the $\hat{A}(t)$, $\hat{B}(t)$ and $\hat{C}(t)$, set

$$\hat{\boldsymbol{O}}_{\overline{w}-1}(t) = \begin{bmatrix} \hat{C}(t) \\ \hat{C}(t)\hat{A}(t) \\ \vdots \\ \hat{C}(t)\hat{A}^{\overline{w}-1}(t) \end{bmatrix}, \quad \hat{\boldsymbol{T}}_{\overline{w}-2}(t) = \begin{bmatrix} \hat{C}(t)\hat{B}(t) & 0 & \cdots & 0 \\ \hat{C}(t)\hat{A}(t)\hat{B}(t) & \hat{C}(t)\hat{B}(t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{C}(t)\hat{A}^{\overline{w}-2}\hat{B}(t) & \hat{C}(t)\hat{A}^{\overline{w}-3}\hat{B}(t) & \cdots & \hat{C}(t)\hat{B}(t) \end{bmatrix}.$$

Then, solve

$$\hat{\boldsymbol{L}}(t)\hat{\boldsymbol{T}}_{\overline{w}-1}(t) = \boldsymbol{J}_{\overline{w}-1} \tag{30}$$

using pseudo-inverse of $\hat{\boldsymbol{T}}_{\overline{w}-1}(t)$. Assume that $\hat{T}_{\overline{w}-1}(t)$ is given by

$$\begin{aligned} &\hat{\boldsymbol{T}}_{\overline{w}-1}(t) = \hat{U}_{\overline{w}-1}(t)\begin{bmatrix} \hat{\boldsymbol{\Sigma}}_{\overline{w}-1}(t) & 0 \\ 0 & 0 \end{bmatrix}\hat{V}^T_{\overline{w}-1}(t), \\ &\hat{\boldsymbol{\Sigma}}_{\overline{w}-1}(t) = \mathrm{diag}\left\{\hat{\lambda}_1(t), \quad \hat{\lambda}_2(t), \quad \ldots, \quad \hat{\lambda}_r(t)\right\}, \ \hat{\lambda}_i(t) > 0 \end{aligned} \tag{31}$$

for some unitary matrices $\hat{U}_{\overline{w}-1}(t)$ and $\hat{V}_{\overline{w}-1}(t)$. If

$$\hat{\lambda}_i(t) < \varepsilon_{\min} \tag{32}$$

for some integers i, then modify

$$\hat{\lambda}_i(t) = \varepsilon_{\min} \tag{33}$$

and calculate $\hat{\boldsymbol{T}}^{+}_{\overline{w}-1}(t)$ by

$$\hat{\boldsymbol{T}}^{+}_{\overline{w}-1}(t) = \hat{V}_{\overline{w}-1}(t)\begin{bmatrix} \hat{\boldsymbol{\Sigma}}^{-1}_{\overline{w}-1} & 0 \\ 0 & 0 \end{bmatrix}\hat{U}^T_{\overline{w}-1}(t). \tag{34}$$

Next, solve

$$\hat{\boldsymbol{Y}}(t)\hat{\boldsymbol{O}}_{\nu-1}(t) = -\hat{\boldsymbol{L}}(t)\hat{\boldsymbol{O}}_{\overline{w}-1}(t)\hat{A}^{\nu}(t). \tag{35}$$

Note that $\hat{\boldsymbol{O}}_{\nu-1}(t)$ always has full column rank since $(\hat{C}(t), \hat{A}(t))$ is in the observability canonical form. So eqn.(35) is easy to solve. In general, $\hat{\boldsymbol{O}}_{\nu-1}(t)$ is a tall matrix. For the solution to eqn.(35), the method employing the pseudo-inverse is effective for the plant with measurement noise (Kase & Mutoh, 2000). It may be also useful for the improvement of the transit response of MRACS.

Finally, calculate $\hat{\boldsymbol{X}}(t)$ by

$$\hat{\boldsymbol{X}}(t) = -\hat{\boldsymbol{Y}}(t)\begin{bmatrix} 0_{m\times m(\nu-1)} \\ \hat{\boldsymbol{T}}_{\nu-2}(t) \end{bmatrix} - \hat{\boldsymbol{L}}(t)\hat{\boldsymbol{V}}(t) \tag{36}$$

where

$$\hat{\boldsymbol{V}}(t) = \begin{bmatrix} \hat{C}(t)\hat{A}^{\nu}(t)\hat{B}(t) & \cdots & \hat{C}(t)\hat{A}(t)\hat{B}(t) \\ \vdots & & \vdots \\ \hat{C}(t)\hat{A}^{2\nu+\overline{w}-2}(t)\hat{B}(t) & \cdots & \hat{C}(t)\hat{A}^{\nu+\overline{w}-2}(t)\hat{B}(t) \end{bmatrix}. \tag{37}$$

Then, using $\hat{\boldsymbol{L}}(t)$, $\hat{\boldsymbol{X}}(t)$ and $\hat{\boldsymbol{Y}}(t)$, the adaptive control input is given by

$$u(t) = \hat{\boldsymbol{X}}(t)\begin{bmatrix} u(t-1) \\ \vdots \\ u(t-\nu+1) \end{bmatrix} + \hat{\boldsymbol{Y}}(t)\begin{bmatrix} y(t) \\ \vdots \\ y(t-\nu+1) \end{bmatrix} + \hat{\boldsymbol{L}}\begin{bmatrix} y_m(t+1) \\ \vdots \\ y_m(t+\overline{w}) \end{bmatrix}. \tag{38}$$

The global stability of the over-all system may be proved under the assumption 5. However, the details are under studying.

4. Numerical Examples

Consider the following plant:

$$G(z) = \begin{bmatrix} \dfrac{a}{z+0.1} & \dfrac{1}{z+0.2} \\ \dfrac{1}{z+b} & \dfrac{1}{z+1.2} \end{bmatrix}$$
$$= \begin{bmatrix} z^2+0.3z+0.02 & 0 \\ 0 & z^2+(b+1.2)z+1.2b \end{bmatrix}^{-1}\begin{bmatrix} a(z+0.2) & z+0.1 \\ z+1.2 & z+b \end{bmatrix}$$

Although the plant seems simple enough, there exist three variations of the interactor depend on the values of a and b.

[Case 1] $a = 1.5,\ b = 0.9$

In this case, the interactor of the plant is

$$L(z) = z\begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}.$$

[Case 2] $a = 1,\ b = 0.9$

In this case, the interactor of the plant is

$$L(z) = z\begin{bmatrix} -10z+5 & 10z+5 \\ 10z-4 & -10z-4 \end{bmatrix}.$$

[Case 3] $a=1,\ b=1.1$

In this case, the interactor of the plant is

$$L(z)=z\begin{bmatrix} 10z^2-3.6z+2.8 & -10z^2-6.4z+2.8 \\ -10z^2+3.8z-2.4 & 10z^2+6.2z-2.4 \end{bmatrix}.$$

For the above three cases, the interactor can be obtained by solving $\hat{\boldsymbol{L}}(t)\hat{\boldsymbol{T}}_2(t)=\hat{\boldsymbol{J}}_2$. Using the above $\hat{\boldsymbol{L}}(t)$, $\hat{\boldsymbol{Y}}(t)$ can be obtained by solving

$$\hat{\boldsymbol{Y}}(t)\begin{bmatrix} C \\ C\hat{A}(t) \end{bmatrix}=-\hat{\boldsymbol{L}}(t)\begin{bmatrix} C\hat{A}^2(t) \\ C\hat{A}^3(t) \\ C\hat{A}^4(t) \end{bmatrix}.$$

Since C -matrix does not depend on t in this example and (C,A) is observable independent on t, $\hat{\boldsymbol{O}}_1(t)$ always has column-full rank and thus the above equation has a solution.

Finally, $\hat{\boldsymbol{X}}(t)$ can be calculated by

$$\hat{\boldsymbol{X}}(t)=-\hat{\boldsymbol{Y}}\begin{bmatrix} 0 \\ C\hat{B}(t) \end{bmatrix}-\hat{\boldsymbol{L}}(t)\begin{bmatrix} C\hat{A}(t)\hat{B}(t) \\ C\hat{A}^2(t)\hat{B}(t) \\ C\hat{A}^3(t)\hat{B}(t) \end{bmatrix}.$$

The reference signal vector was given by

$$y_m(t)=G_m(z)r(t)=\begin{bmatrix} \sin \pi t/15 \\ \sin 2\pi t/25 \end{bmatrix}.$$

The least square algorithm with constant trace was used to estimate the parameters where the initial values of the covariance matrix is 10^6, and $\varepsilon_{\min}=0.02$. Fig.1, 3 and 5 show the tracking errors of the proposed indirect MRACS for Case 1-3 respectively. Fig.2, 4 and 6 show the tracking errors of the proposed indirect MRACS for Case 1-3 *without modification* respectively. The results show the effectiveness of the proposed method.

5. Conclusion

In this chapter, an indirect MIMO MRACS with structural estimation of the interactor was proposed. By using indirect method, unreasonable assumptions such as assuming the diagonal degrees of interactor can be avoided. Since the controller parameters are calculated based on the observability canonical realization of the estimated values, the proposed method is suitable for on-line calculations. In the proposed method, the degree of the controllers do not depend on the estimated structure of the interactor. The global stability of the overall system is under studying.

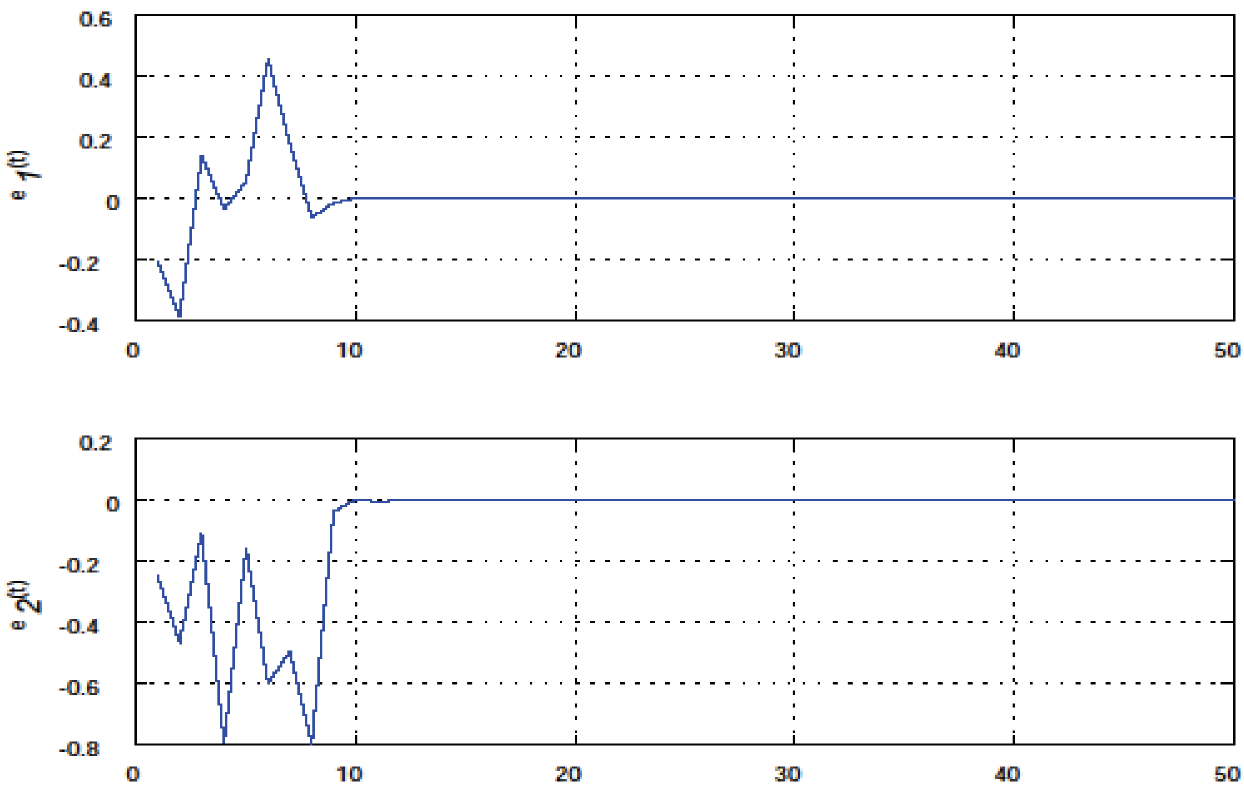

Figure 1. Output tracking error of the proposed MRACS (Case 1; with modifications)

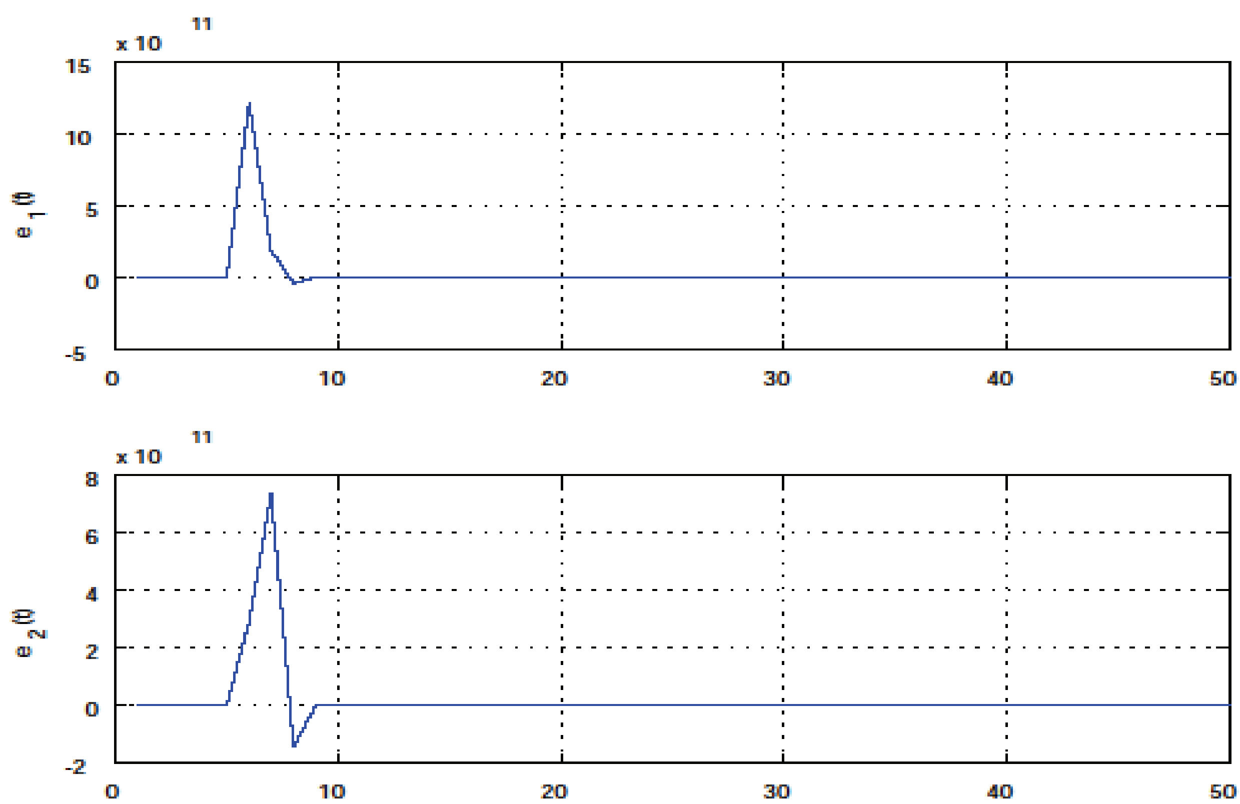

Figure 2. Output tracking error of the proposed MRACS (Case 1; *without modifications*)

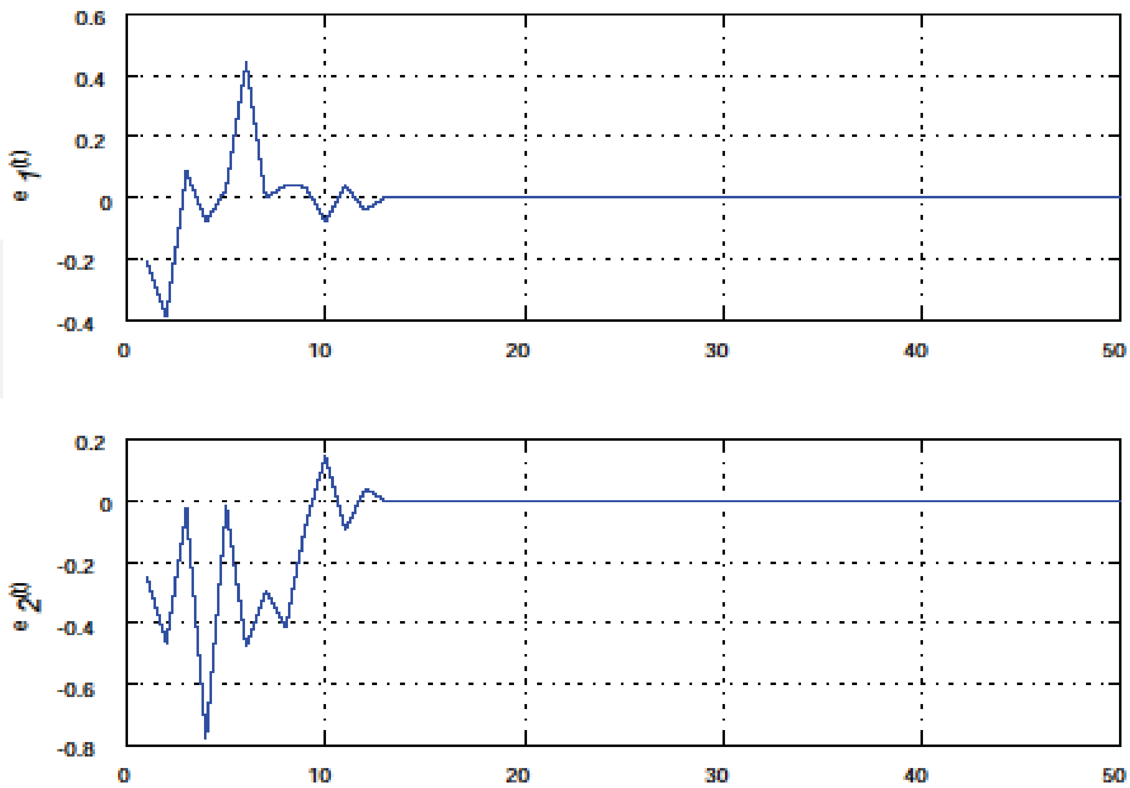

Figure 3. Output tracking error of the proposed MRACS (Case 2; with modifications)

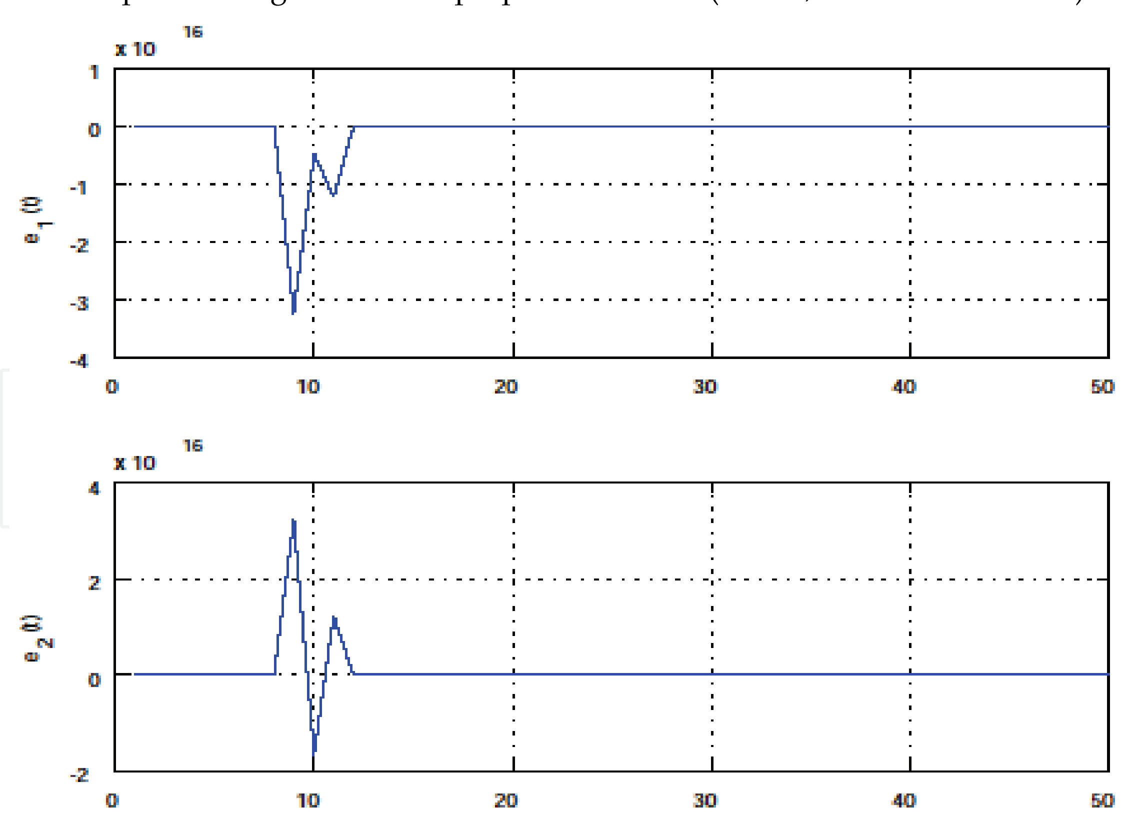

Figure 4. Output tracking error of the proposed MRACS (Case 2; *without modifications*)

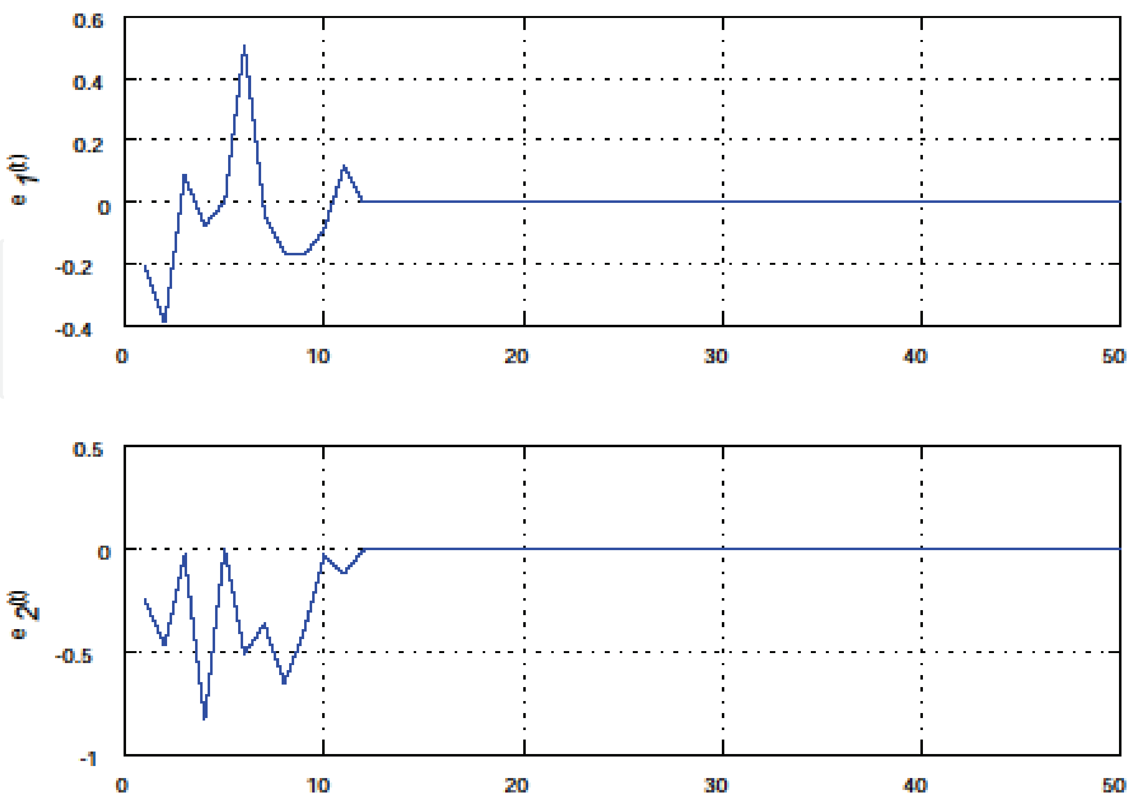

Figure 5. Output tracking error of the proposed MRACS (Case 3; with modifications)

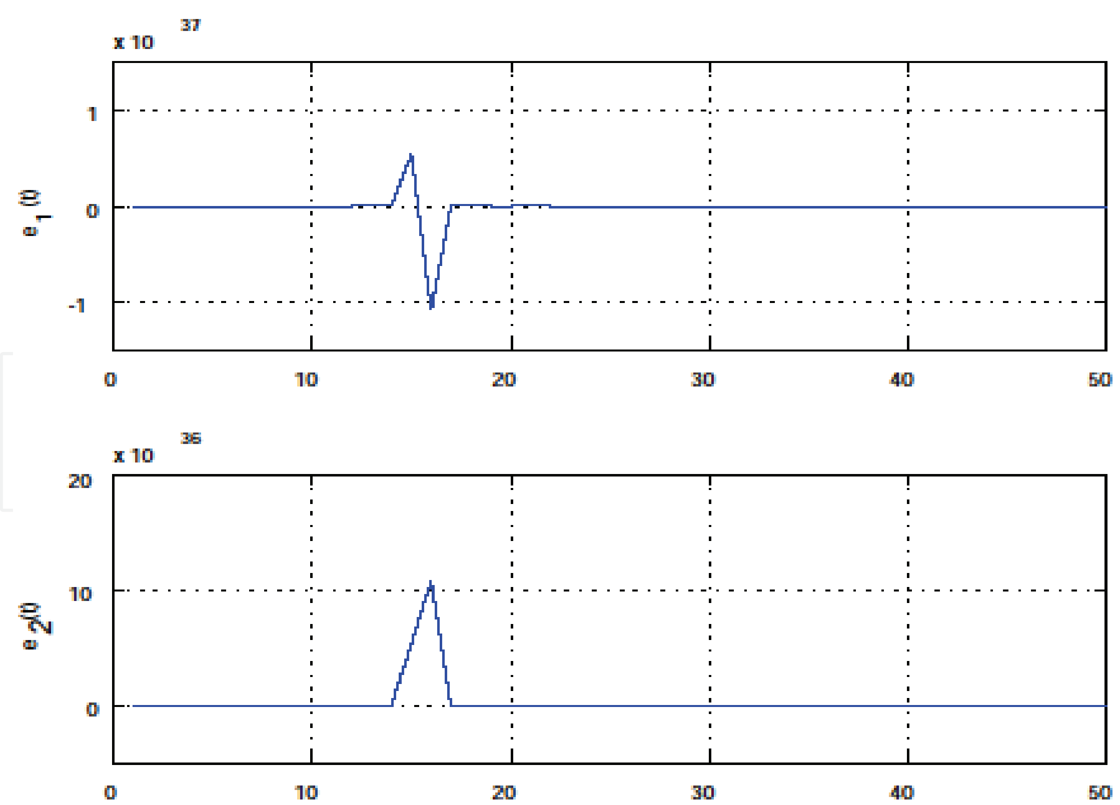

Figure 6. Output tracking error of the proposed MRACS (Case 3; *without modifications*)

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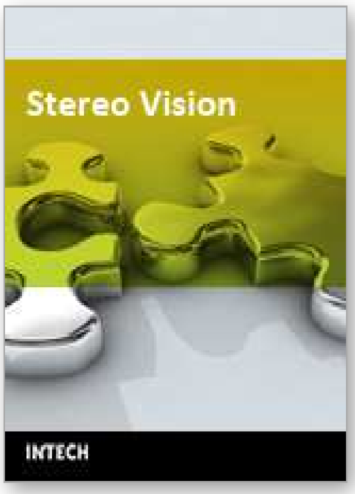

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The objective of this book is to provide an up-to-date and state-of-the-art coverage of diverse aspects related to adaptive control theory, methodologies and applications. These include various robust techniques, performance enhancement techniques, techniques with less a-priori knowledge, nonlinear adaptive control techniques and intelligent adaptive techniques. There are several themes in this book which instance both the maturity and the novelty of the general adaptive control. Each chapter is introduced by a brief preamble providing the background and objectives of subject matter. The experiment results are presented in considerable detail in order to facilitate the comprehension of the theoretical development, as well as to increase sensitivity of applications in practical problems