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Exergy Flows Inside Expansion and Compression Devices Operating below and across Ambient Temperature

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Abstract

The various definitions of the coefficient of exergy efficiency (CEE), which have been proposed in the past for the thermodynamic evaluation of compression and expansion devices, operating below and across ambient temperature as well as under vacuum conditions, are examined. The shortcomings of those coefficients are illustrated. An expression for the CEE based on the concept of transiting exergy is presented. This concept permits the quantitative and non-ambiguous definition of two thermodynamic metrics: exergy produced and exergy consumed. The development of these CEEs in the cases of an expansion valve, a cryo-expander, a vortex tube, an adiabatic compressor and a monophasic ejector operating below or across ambient temperature is presented. Computation methods for the transiting exergy are outlined. The analysis based on the above metrics, combined with the traditional analysis of exergy losses, allows pinpointing the most important factors affecting the thermodynamic performance of sub-ambient compression and expansion.

Keywords: exergy efficiency, expansion, compression, sub-ambient, across ambient

1. Introduction

Cooling is part of twenty-first century life. Air conditioning, food conservation, industries such as steel, chemicals, and plastics depend on cooling. By mid-century people will use more energy for cooling than heating [1]. Almost all cold is produced by vapor-compression refrigeration and requires large amounts of electricity for its production. And since electricity is still overwhelmingly produced by burning fossil fuels, the rise in cold production will inevitably

increase both fuels consumption and power plant emissions. A climate-change irony is that cooling makes the planet hotter. Besides the development of new cooling devices using renewable energy, an important way to reduce refrigeration power consumption is through the energy efficiency improvement of vapor-compression cycles and their associated elementary processes. The processes of compression and expansion play a central role in air-conditioning, refrigeration and cryogenics. An important question still remains: How to define the efficiency of these processes by taking into account the constraints of the first and second laws of thermodynamics? The answer will be discussed in this paper.

The introduction of exergy, the thermodynamic function that takes into account the quality as well as the quantity of energy, has paved the way for a unified approach to the concept of efficiency, a subject pioneered by Grassmann [2]. Serious difficulties concerning the practical application of this concept to sub-ambient systems, however, retarded the acceptance of exergy analysis by the air-conditioning and refrigeration engineering profession. One can mention, in particular, the difficulty of formulating a coefficient of exergy efficiency (CEE) for elementary processes such as compression and expansion. The coefficient should evaluate the exergy losses, quantify the extent to which the technical purpose of an elementary process is achieved, as well as quantify the exergy consumption within the process. Finally, a uniquely determined value (not several) should be assigned to the coefficient. This paper examines some important points pertinent to these issues and presents a definition of the CEE for the thermodynamic evaluation of expansion and compression devices operating below and across ambient conditions. The definition is based on the concept of transiting exergy, introduced by Brodyansky et al. [3], that allows non-ambiguous computation of two metrics: exergy produced and exergy consumed.

2. Basic concepts of exergy analysis of sub-ambient systems

The maximum amount of work obtained from a given form of energy or a material stream, using the environment as the reference state, is called exergy [4, 5]. Three different types of exergy are important for thermodynamic analysis of the sub-ambient processes: exergy of heat flow, exergy of work (equivalent to work) and thermo-mechanical exergy, also known as physical exergy by some authors [4, 6]. Chemical exergy [7], important for some refrigeration systems based on the mixing of streams of different composition, is not considered in the present paper.

2.1. Exergy of heat flow at the sub-ambient conditions

The exergy of heat flow $\dot{Q}$ [4] is defined as:

$$\dot{E}^Q = \dot{Q} \cdot \Theta \tag{1}$$

where $\Theta = 1 - T_0/T$ is the Carnot factor determined by the temperature T of heat flow, and the ambient temperature T_0. Contrary to conditions above ambient, Θ is negative for sub-ambient

temperatures. However, according to Eq. (1), $\dot{E}^Q$ is positive due to the fact that heat is removed from a cooled object, and thus $\dot{Q}$ has a negative sign in Eq. (1). The energy and exergy balances of a reversible refrigerator (RR) are presented in **Figure 1**. One can notice that the directions of energy and exergy flows are opposite below T_0. This means that the exergy of a heat flow at $T < T_0$ is looked upon as a product of the refrigeration system rather than as feed. The exergy transfer of a RR characterizes the rate of transformation of power $\dot{W}$ to exergy of heat flow $\dot{Q}$ (exergy of produced cold). Given that the system presented in **Figure 1** is reversible, the minimum power $\dot{W}_{min}$ necessary to maintain a cooling rate $\dot{Q}$ equals $\dot{E}^Q$. Obviously it is not the case for a real (non-reversible) refrigerator, where $\dot{E}^Q$ is lower than $\dot{W}$ by the value of exergy losses $\dot{D}$.

2.2. Thermo-mechanical exergy

The thermo-mechanical exergy equals the maximum amount of work obtainable when the stream of substance is brought from its initial state to the environmental state, defined by pressure P_0 and temperature T_0, by physical processes involving only thermal interaction with the environment [3, 4]. The specific thermo-mechanical exergy $e_{P,T}$ is calculated according to:

$$e_{P,T} = [h(P, T) - h(P_0, T_0)] - T_0 \cdot [s(P, T) - s(P_0, T_0)] \tag{2}$$

The value of $e_{P,T}$ may be divided by two components: thermal exergy e_T due to the temperature difference between T and T_0, and mechanical exergy e_P due to the pressure difference between P and P_0. It is important to emphasize that this division is not unique, because e_T depends on pressure conditions and e_P in its turn depends on temperature conditions. As a result, the division has no fundamental meaning and leads, as will be illustrated further, to ambiguities for the exergy efficiency definition. By conventional agreement [4], e_T and e_P are defined as:

$$e_T = [h(P, T) - h(P, T_0)] - T_0 \cdot [s(P, T) - s(P, T_0)] \tag{3}$$

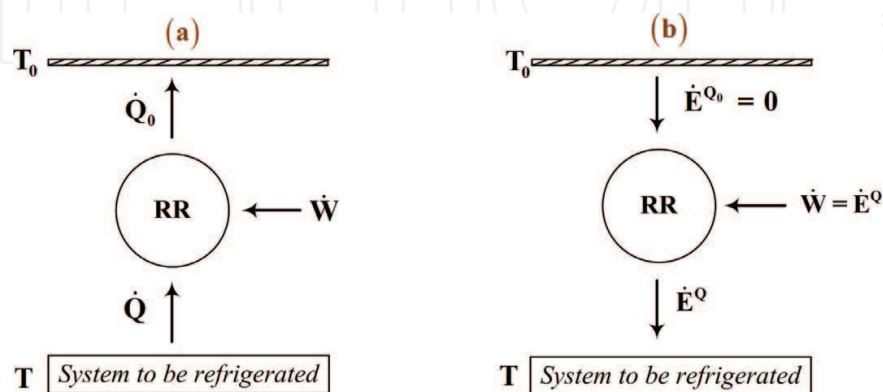$$e_P = [h(P, T_0) - h(P_0, T_0)] - T_0 \cdot [s(P, T_0) - s(P_0, T_0)] \tag{4}$$

Figure 1. Energy (a) and exergy (b) balances of a reversible refrigerator (RR).

The contribution of e_T and e_P to the value of $e_{P,T}$ can be clearly visualized on the exergy-enthalpy diagram presented in **Figure 2**. For instance, the thermal exergy for point 1 is illustrated as the segment $(e_T)_1$ defined by the intersections of two isotherms T_1 and T_0 with the isobar P_1. The mechanical exergy for point 1 is illustrated as the segment $(e_P)_1$ defined by the intersections of two isobars P_1 and P_0 with the isotherm T_0. Whatever the temperature conditions are ($T < T_0$ or $T > T_0$), the thermal exergy is always positive [3], as clearly presented on the e-h diagram. In this sense the e_T behavior is similar to that of the exergy of heat flow, that is always positive, as has been discussed. Meanwhile, e_P is only positive for conditions $P > P_0$ (see for example point 1), but it is negative for $P < P_0$, as illustrated by point 2 in **Figure 2**.

2.3. Exergy efficiency of processes operating below and across the ambient temperature

The exergy balance around any process under steady state conditions and without external irreversibilities (the case considered in this paper) may be written as [4]:

$$\dot{E}_{in} = \dot{E}_{out} + \dot{D}_{int} \tag{5}$$

Here $\dot{E}_{in}$ and $\dot{E}_{out}$ are the inlet and outlet exergy flows; $\dot{D}_{int}$ is the rate of internal exergy losses. There is abundant scientific literature [3–9] on the subject of the exergy performance criteria definition based on Eq. (5). However, there are only a few definitions of this criteria applied to the processes of expansion and compression operating below and across ambient temperature.
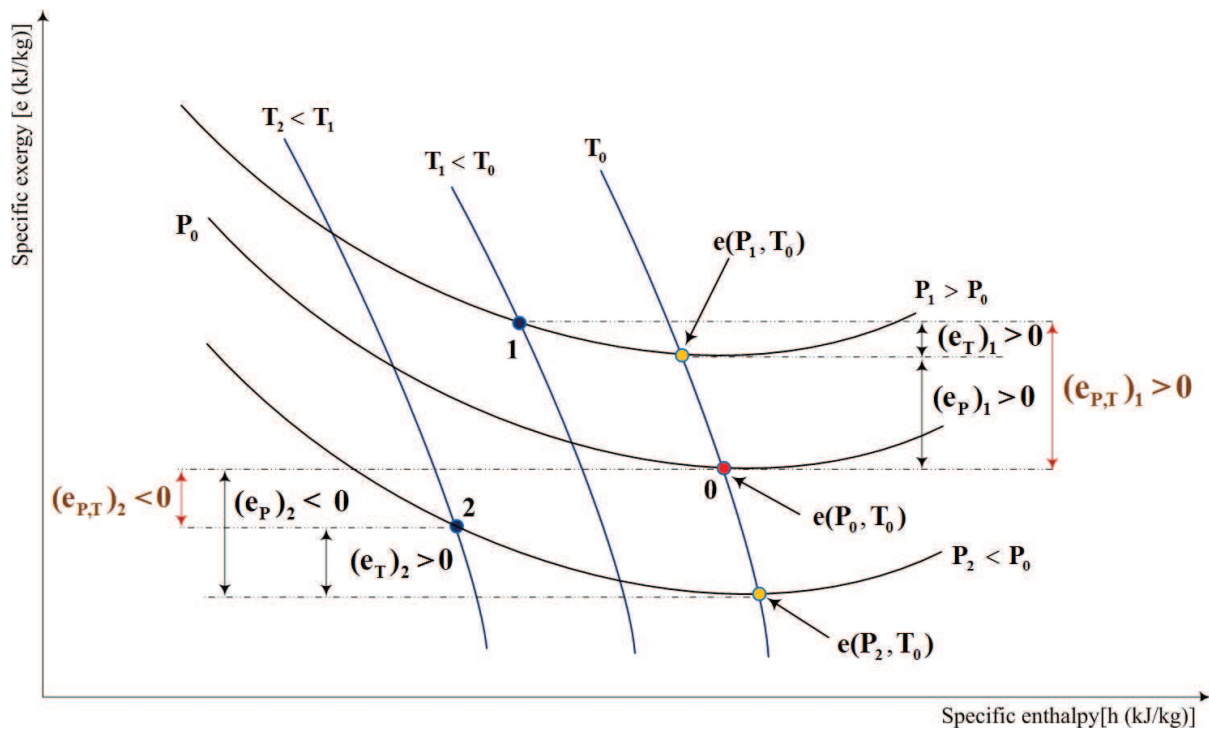

Figure 2. Thermal e_T and mechanical e_P exergy components on an exergy-enthalpy diagram.

Among them, three exergy efficiency definitions may be distinguished: input-output efficiency, products-fuel efficiency, and efficiency that accounts for the transiting exergy.

The input-output efficiency $\eta_{in\text{-}out}$, first proposed by Grassmann [2], is computed according to Eq. (6):

$$\eta_{in\text{-}out} = \frac{\dot{E}_{out}}{\dot{E}_{in}} \tag{6}$$

The shortcomings of this definition, particularly for its application to sub-ambient problems, are well documented [3–9]. The main one is the fact that often $\eta_{in\text{-}out}$ does not evaluate the degree to which the technical purpose of a process is realized; the subject will be illustrated in Section 3. The products-fuel efficiency $\eta_{pr\text{-}f}$ proposed by Tsatsaronis [10] and Bejan et al. [6] in the context of expansion and compression processes is computed as:

$$\eta_{pr\text{-}f} = \frac{\text{Exergy of Products}}{\text{Exergy of Fuel}} \tag{7}$$

Under the terms "products" and "fuel" the authors meant either the differences in exergies of the streams at the inlet and outlet of a process, or the exergies of streams themselves. For example, while evaluating the efficiency of an adiabatic compression operating above ambient conditions, the "fuel" is the supplied work, and the "product" is the increment of thermo-mechanical exergy. The problem with this approach is that it is possible to obtain different values of $\eta_{pr\text{-}f}$ of the sub-ambient expansion and compression processes due to the fact that different things can be understood under the notions "products" and "fuel". It should be also mentioned that some authors used a different terminology to express the numerator and denominator of Eq. (7). For example, Kotas [4] used "desired output" vs. "necessary input"; Szargut et al. [5] used "exergy of useful products" vs. "feeding exergy".

Brodyansky et al. [3] proposed a definition of efficiency based on the subtraction of the exergy that has not undergone transformation within an analyzed process. The latter was named "transiting exergy", $\dot{E}_{tr}$, and the exergy efficiency is defined as:

$$\eta_{tr} = \frac{\dot{E}_{out} - \dot{E}_{tr}}{\dot{E}_{in} - \dot{E}_{tr}} = \frac{\Delta\dot{E}}{\nabla\dot{E}} \tag{8}$$

where $\Delta\dot{E}$ and $\nabla\dot{E}$ are the exergy produced and exergy consumed in the process. It is clear that the difference between the denominator and the numerator in Eq. (8) equals the exergy losses within the process. It will be illustrated that the unambiguous definition of $\dot{E}_{tr}$ paves the way for the uniquely determined thermodynamic metrics $\Delta\dot{E}$ and $\nabla\dot{E}$ in the case of thermo-mechanical exergy transformation for sub-ambient processes. The evaluation of η_{tr} to assess the rate of exergy transfer of the mechanical exergy component to the thermal exergy component for these processes will be based on these metrics.

3. Transiting thermo-mechanical exergy and its link to the thermal and mechanical components

Following the equations proposed by Brodyansky et al. [3], the specific transiting thermo-mechanical exergy e_{tr} of an analyzed system is defined as the minimum exergy value that can be assigned to a material stream, considering the pressure P and temperature T at the inlet and outlet, as well as the ambient temperature T_0. With this definition, there are three possible combinations of P_{in}, T_{in}, P_{out}, T_{out} and T_0 that determine the value of e_{tr}:

$$\text{If } (T_{in} > T_0 \text{ and } T_{out} > T_0): \quad \dot{E}_{tr} = \dot{m}.e_{tr}(P_{min}, \mathbf{T_{min}}) \tag{9a}$$

$$\text{If } (T_{in} < T_0 \text{ and } T_{out} < T_0): \quad \dot{E}_{tr} = \dot{m} \cdot e_{tr}(P_{min}, \mathbf{T_{max}}) \tag{9b}$$

$$\text{If } (T_{in} > T_0 \text{ and } T_{out} < T_0) \, \text{OR} \, (T_{in} < T_0 \text{ and } T_{out} > T_0): \quad \dot{E}_{tr} = \dot{m} \cdot e_{tr}(P_{min}, \mathbf{T_0}) \tag{9c}$$

Inspection of these equations shows that for all three cases e_{tr} is determined by using the lowest pressure P_{min} among the inlet and outlet values. The situation is different for temperature, where the minimum exergy value is determined using the lowest temperature T_{min} for processes above ambient, the highest temperature T_{max} for sub-ambient processes, and by using T_0 for the case of processes operating across ambient temperature. In order to understand the physical meaning of the transiting exergy, let us analyze the throttling process of a real gas taking place under these three different temperature conditions.

3.1. Adiabatic throttling process

The case of the throttling process operating above T_0 is presented on an e-h diagram (see **Figure 3a**). According to Eq. (9a), the value of e_{tr} is:

$$e_{tr} = e(P_{out}, T_{out}) \tag{10}$$

This value coincides with the value e_2 as illustrated in **Figure 3a**. The specific exergy losses (d) are also presented on the diagram. Following Eq. (8), the values $\Delta\dot{E}$ and $\nabla\dot{E}$ are:

$$\Delta\dot{E} = \dot{m} \cdot (e_{out} - e_{tr}) = 0 \tag{11}$$

$$\nabla\dot{E} = \dot{m} \cdot (e_{in} - e_{tr}) = \dot{m} \cdot d \tag{12}$$

where $\dot{m}$ is the gas mass flow rate.

As a result, the efficiency $\eta_{tr} = 0$, meaning that the exergy consumed is completely lost during the process and there is no produced exergy. It should be mentioned that the input-output efficiency calculated according to Eq. (6) has a negative value in this particular case and has no physical meaning. This is due to the fact that $\dot{E}_{out} < 0$, because the throttling ended at the vacuum conditions. For this particular case the products-fuel efficiency according to Eq. (7) gives the same value as η_{tr}.
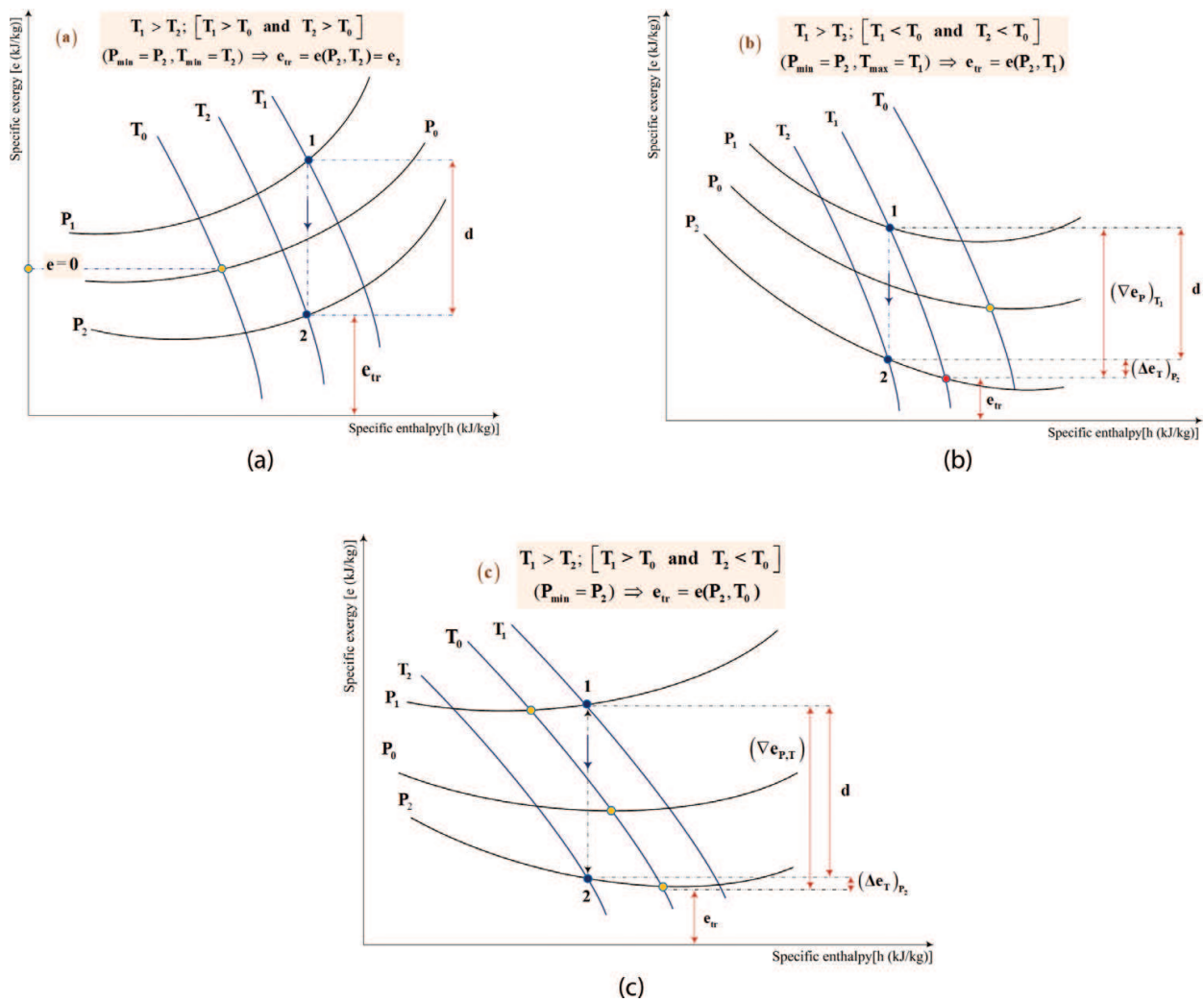
Figure 3. Transiting thermo-mechanical exergy presentation for a throttling process of a real gas operating above (a), below (b), and across (c) ambient temperature (T_0).

Now, let us analyze the case of throttling at sub-ambient conditions presented in **Figure 3b**. According to Eq. (9b):

$$e_{tr} = e(P_{out}, T_{in}) \tag{13}$$

Thus $\Delta \dot{E}$ and $\nabla \dot{E}$ are calculated as:

$$\Delta \dot{E} = \dot{m} \cdot (e_{out} - e_{tr}) = \dot{m} \cdot [e(P_{out}, T_{out}) - e(P_{out}, T_{in})] = \dot{m} \cdot (\Delta e_T)_{P_{out}} \tag{14}$$

$$\nabla \dot{E} = \dot{m} \cdot (e_{in} - e_{tr}) = \dot{m} \cdot [e(P_{in}, T_{in}) - e(P_{out}, T_{in})] = \dot{m} \cdot (\nabla e_P)_{T_{in}} \tag{15}$$

The term $(\Delta e_T)_{P_{out}}$ in Eq. (14) is the increase of the specific thermal exergy due to an isobaric temperature drop under **sub-ambient** conditions at constant pressure P_{out}. The term $(\nabla e_P)_{T_{in}}$ in Eq. (15) is the decrease of the specific mechanical exergy due to an isothermal pressure drop

at constant temperature T_{in}. Finally, the case presented in **Figure 3c** illustrates a throttling process started above ambient and ended at sub-ambient conditions. According to Eq. (9c):

$$e_{tr} = e(P_{out}, T_0) \tag{16}$$

The values $\Delta\dot{E}$ and $\nabla\dot{E}$ are calculated as:

$$\Delta\dot{E} = \dot{m} \cdot (e_{out} - e_{tr}) = \dot{m} \cdot [e(P_{out}, T_{out}) - e(P_{out}, T_0)] = \dot{m} \cdot (\Delta e_T)_{P_{out}} \tag{17}$$

$$\nabla\dot{E} = \dot{m} \cdot (e_{in} - e_{tr}) = \dot{m} \cdot [e(P_{in}, T_{in}) - e(P_{out}, T_0)] = \dot{m} \cdot (\nabla e_{P,T}) \tag{18}$$

Again, the input-output efficiency is not suitable for the evaluation of the processes presented in **Figure 3b** and **c**, given that the outlet exergy $e_2 = e(P_{out}, T_{out})$ is negative. Another difficulty is linked to the application of the products-fuel efficiency for these cases. The exergy transfer of the throttling process at sub-ambient conditions consists in the partial transformation of mechanical exergy ("fuel") into thermal exergy ("product"). The problem stems from the fact that there are multiple possibilities to define "fuel" and "product" in this case; as a result multiple values of η_{pr-f} may be formulated, leading to the ambiguity in the products-fuel efficiency application. Indeed, the different increments of thermal exergy may be considered as a "product" for the case in **Figure 3b**, for example, the increase in thermal exergy following the isobar P_1 or the isobar P_2. In the same way, different decrements of mechanical exergy may be considered as a "fuel" in the same figure, for example, the decrease of mechanical exergy following the isotherms T_1 or T_2.

Contrary to "products-fuel", the transiting exergy approach does not attempt to individually compute the thermal and mechanical exergy component variations. It relies, rather, on the unaffected part of the thermo-mechanical exergy entering and leaving the system.

As illustrated in **Figure 3a** and **c**, the transiting exergy may be considered as the introduction of a new reference state to evaluate exergy consumed and produced. Instead of the reference point $e = 0$ (the intersection of the isobar P_0 and the isotherm T_0), the new reference point is presented by e_{tr}: the intersection of the isobar P_2 and the isotherm T_2 for the case 3a; of the isobar P_2 and the isotherm T_1 for the case 3b; and of the isobar P_2 and the isotherm T_0 for the case 3c. Finally, the transiting exergy approach provides the foundation for the non-ambiguous definition of the terms $\Delta\dot{E}$ and $\nabla\dot{E}$, and thus of η_{tr}.

Example 1

The initial parameters of air at the inlet of a throttling valve are: $\dot{m} = 1$ kg/s, $P_1 = 3$ MPa, $T_1 = 140$ K. The ambient temperature $T_0 = 283$ K. Calculate the variation of $\dot{E}_{tr}$, $\Delta\dot{E}$ and $\nabla\dot{E}$ and η_{tr} as a function of the outlet pressure P_2 in the range 0.1–1 MPa.

Solution

The outlet temperature of the air is calculated by using the software Engineering Equation Solver (EES) [11]. Given that the expansion of air takes place at sub-ambient conditions, Eqs. (13)–(15) are used to evaluate $\dot{E}_{tr}$, $\Delta\dot{E}$ and $\nabla\dot{E}$. The results are presented in **Table 1**. One observation is obvious, that the rise in exergy losses as well as the decrease in η_{tr} go along with

P_2 (MPa)	T_2 (K)	$\nabla\dot{E}$ (kW)	$\Delta\dot{E}$ (kW)	$\dot{D}$ (kW)	$\dot{E}_{tr}$ (kW)	η_{tr} (%)
1.0	118.6	101.8	30.8	71.0	245.9	30.3
0.9	117.3	110.8	32.4	78.4	236.9	29.2
0.7	114.7	132.0	35.6	96.4	215.7	26.9
0.5	111.9	160.1	38.8	121.3	187.6	24.2
0.3	109.1	202.3	42.1	160.2	145.4	20.8
0.1	106.0	292.3	45.5	246.8	55.5	15.6

Table 1. Variation in temperature and exergy metrics with the outlet pressure for a throttling process of air.

the decreasing outlet pressure P_2. Less obvious is that the exergy produced $\Delta\dot{E}$ rises with decreasing P_2, reflecting the production of a more important cooling effect. It can also be noticed that the transiting exergy $\dot{E}_{tr}$ decreases with decreasing outlet pressure P_2.

Example 2

The expansion valve of a refrigeration mechanical vapor compression cycle is supplied with the subcooled working fluid R152a at the rate $\dot{m}$ = 0.15 kg/s at P_1 = 615.1 kPa. The fluid is expanded to a pressure of P_2 = 142.9 kPa. The ambient temperature T_0 = 278 K. Calculate the variation of $\dot{E}_{tr}$, $\Delta\dot{E}$ and $\nabla\dot{E}$ and η_{tr} as a function of the subcooling ΔT_{subC} in the range 275–281 K.

Solution

A vapor compression cycle is presented on a Ts-diagram in **Figure 4**. The subcooling process is represented by the line 3f-3. Given that the expansion of R152a takes place across ambient temperature, Eqs. (16)–(18) are used to evaluate $\dot{E}_{tr}$, $\Delta\dot{E}$ and $\nabla\dot{E}$. The results are shown in **Table 2**.

The transiting exergy does not change with the subcooling, because it is the function of constant parameters T_0 and P_2, meanwhile the exergy produced increases and exergy consumed decreases. The new result is that η_{tr} is rising with the subcooling. It should be mentioned that increasing the amount of subcooling is well documented as a way to increase the COP (coefficient of performance) of vapor compression cycles [4]. Thus, the rise in η_{tr} of an expansion device guarantees the COP improvement of the overall cycle, a conclusion that may lead to practical recommendations for optimization of refrigeration cycles.

3.2. Expansion in low temperature systems with work production and heat transfer

The primary purpose of expansion processes in the sub-ambient region is the production of cooling effect. The power that may be produced can be considered as a useful by-product. This type of expansion takes place in cryo-expanders. There are two types of these devices: adiabatic and non-adiabatic gas expansion machines. The energy and exergy balances around a non-adiabatic expander are presented in **Figure 5**. It should be emphasized that the directions
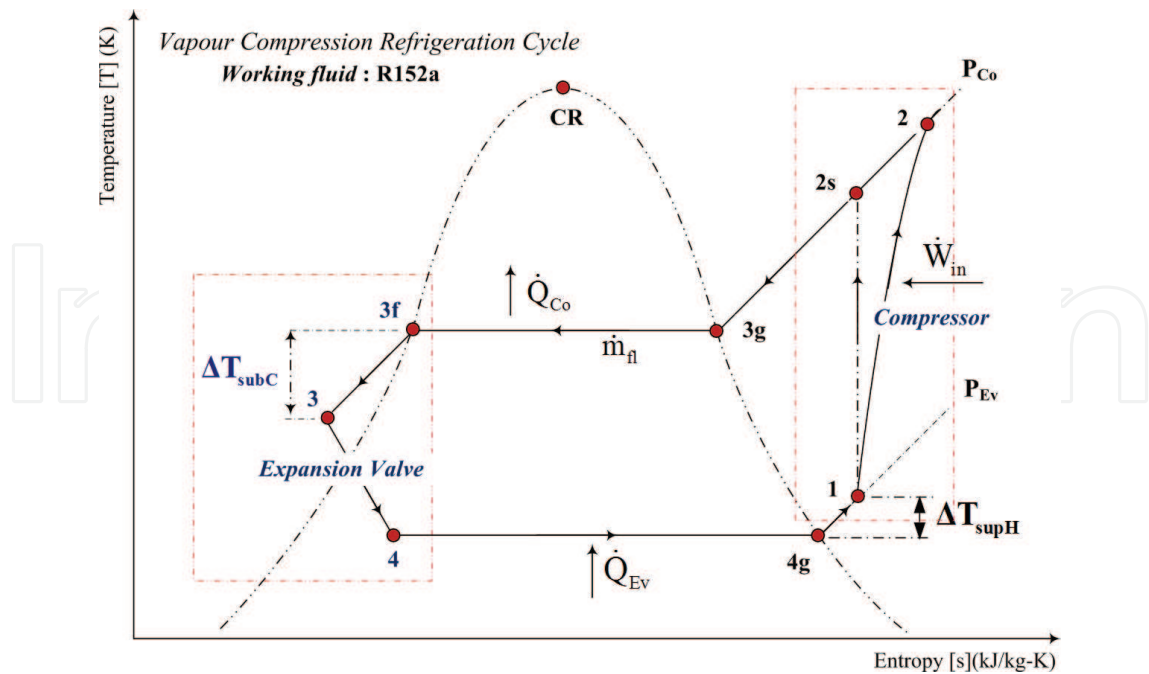
Figure 4. A vapor compression cycle presentation on a Ts-diagram.

ΔT_{subC} (K)	$V\dot{E}$ (kW)	$\Delta\dot{E}$ (kW)	$\dot{D}$ (kW)	$\dot{E}_{tr}$ (kW)	η_{tr} (%)
275	4.083	3.208	0.875	1.744	78.6
276	4.066	3.230	0.836	1.744	79.4
278	4.034	3.274	0.760	1.744	81.2
279	4.020	3.295	0.725	1.744	82.0
281	3.994	3.339	0.655	1.744	83.6

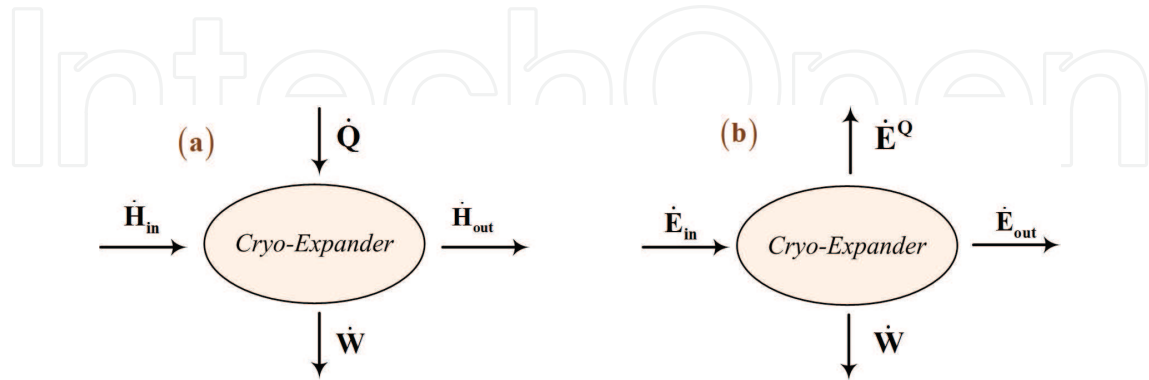**Table 2.** Variation in exergy metrics with subcooling for the expansion process of R152a.

Figure 5. Energy (a) and exergy (b) balances around a non-adiabatic cryo-expander.

of heat flow $\dot{Q}$ and exergy $\dot{E}^Q$ are opposite. This is due to the fact that heat transfer from a cooled object to the expanding fluid occurs at $T < T_0$. As a result $\dot{E}^Q$ calculated according to Eq. (1) is presented as the outlet flow in **Figure 5b**.

The process of gas expansion in a non-adiabatic cryo-expander is presented on an e-h diagram (see **Figure 6**). Similar to the case of adiabatic throttling (**Figure 3b**), the transiting exergy in the gas flow is defined according to Eq. (9b). As a result, the exergy efficiency is calculated as:

$$\eta_{tr} = \frac{\dot{m} \cdot (\Delta e_T)_{P_{out}} + \dot{Q} \cdot \Theta + \dot{W}_T}{\dot{m} \cdot (\nabla e_P)_{T_{in}}} \tag{19}$$

In the case of an adiabatic cryo-expander η_{tr} is calculated according to Eq. (19), but with the term $\dot{Q}.\Theta_{in}$ equals to zero.

Example 3

An adiabatic turbine (η_T = 0.80) is supplied with air at the rate $\dot{m}$ = 1 kg/s at P_1 = 6 MPa, T_1 = 320 K. The ambient temperature T_0 = 283 K. Calculate the variation of $\dot{E}_{tr}$, $\Delta \dot{E}$ and $\nabla \dot{E}$ and η_{tr} as a function of the outlet pressure P_2 in the range 0.1–3 MPa.

Solution

Given that the expansion of air takes place across ambient temperature, Eqs. (16)–(18) are used to evaluate $\dot{E}_{tr}$, $\Delta \dot{E}$ and $\nabla \dot{E}$. η_{tr} is calculated according to Eq. (19), but with the term $\dot{Q}.\Theta_{in}$ equals to zero. The results are shown in **Table 3**. It is illustrated that $\dot{E}_{tr}$ decreases with P_2 reduction, and as a result $\Delta \dot{E}$ and $\nabla \dot{E}$ and $\dot{W}$ rise, but the increase in $\Delta \dot{E}$ and $\dot{W}$ is offset by the greater increase in $\nabla \dot{E}$ causing η_{tr} to decrease. The negative value of $\dot{E}_{tr}$ in the second last row to the right in **Table 3**, is because the stream at state 2 is under vacuum conditions. It should be
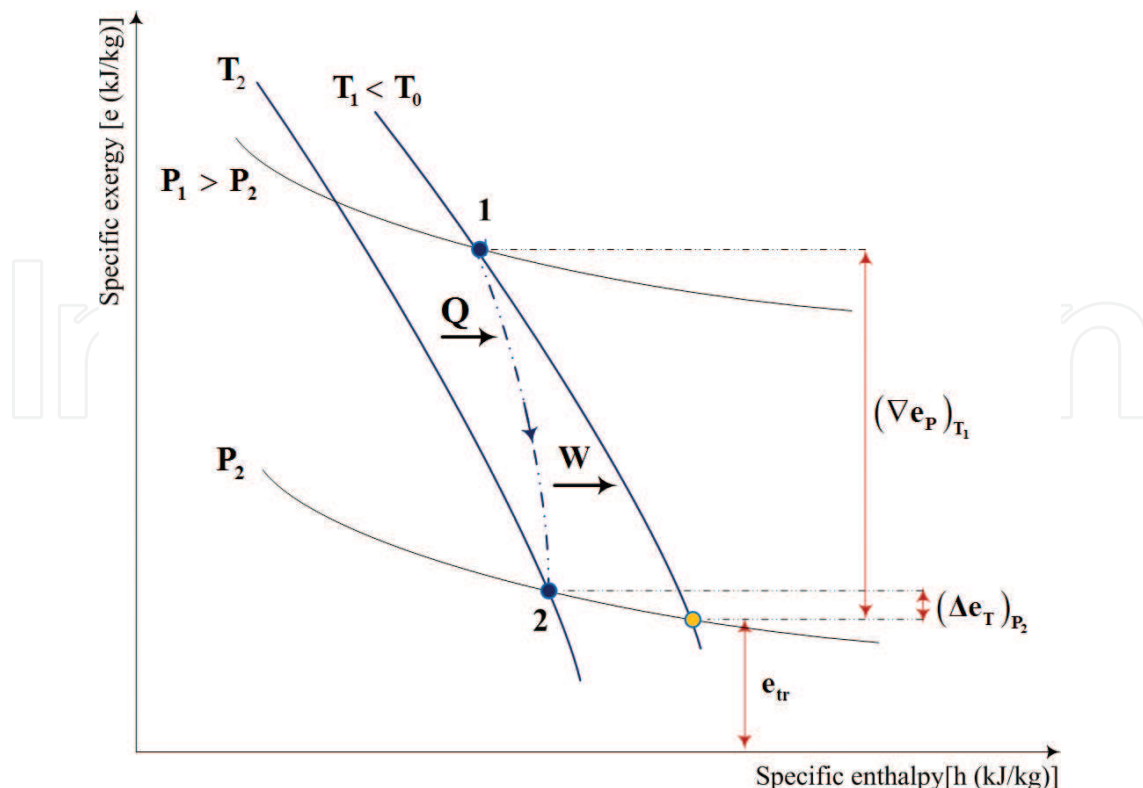

Figure 6. Gas expansion in a non-adiabatic cryo-expander on an exergy-enthalpy diagram.

P_2 (MPa)	T_2 (K)	$\nabla \dot{E}$ (kW)	$\Delta \dot{E}$ (kW)	$\dot{W}_T$ (kW)	$\dot{D}$ (kW)	$\dot{E}_{tr}$ (kW)	η_{tr} (kW)
3.00	271.8	57.9	0.2	45.6	12.1	274.2	79.1
2.55	263.8	68.3	0.7	53.0	14.6	263.9	78.7
2.50	255.1	80.1	1.5	61.2	17.4	252.0	78.2
1.85	245.4	94.1	2.9	70.3	21.0	238.0	77.7
1.50	234.3	111.1	4.9	80.7	25.5	221.1	77.1
1.15	221.2	132.6	8.2	93.1	31.3	199.6	76.4
0.50	204.9	162.0	13.6	108.4	40.0	170.1	75.3
0.45	182.4	208.9	24.1	129.7	55.1	123.2	73.6
0.10	138.9	333.2	57.7	171.6	103.9	−1.05	68.8

Table 3. Variation in exergy metrics with the outlet pressure for a turbine expansion process.

emphasized that the increase in the $\Delta \dot{E}$ metric reflects the deeper refrigeration of air with increasing pressure drop in the turbine.

3.3. Expansion in a vortex tube

Figure 7a illustrates a counter flow vortex tube [12]. High pressure gas enters the tube through a tangential nozzle (point 1). Colder low-pressure gas leaves via an orifice near the centerline adjacent to the plane of the nozzle (point 2), and warmer low-pressure gas leaves near the periphery at the end of the tube opposite to the nozzle (point 3). The vortex tube requires no work or heat interaction with the surroundings to operate. The cold mass fraction is μ; the hot gas mass fraction is $(1 - \mu)$. The exergy balance around the vortex tube is:

$$e_1 = \mu \cdot e_2 + (1 - \mu) \cdot e_3 + d \tag{20}$$

The expansion processes taking place within a vortex tube are presented on an e-h diagram (**Figure 7b**). The cold stream expands across T_0, the hot expands at $T > T_0$. By applying Eqs. (9a) and (9c) the transiting exergies may be determined for each mass stream, cold (1–2) and hot (1–3).

$$(e_{tr})_{cold} = e(P_2, T_0) \tag{21}$$

$$(e_{tr})_{hot} = e(P_3, T_1) \tag{22}$$

As a result, the exergy produced and consumed within the cold and hot streams are:

$$\Delta \dot{E}_C = \dot{m} \cdot [(\mu) \cdot (e(P_2, T_2) - e(P_2, T_0))] = \dot{m} \cdot (\mu) \cdot (\Delta e_T)_{P_2}$$
$$\Delta \dot{E}_H = \dot{m} \cdot [(1 - \mu) \cdot (e(P_3, T_3) - e(P_3, T_1))] = \dot{m} \cdot (1 - \mu) \cdot (\Delta e_T)_{P_3} \tag{23}$$

$$\nabla \dot{E}_C = \dot{m} \cdot [(\mu) \cdot (e(P_1, T_1) - e(P_2, T_0))] = \dot{m} \cdot (\mu) \cdot (\nabla e_{P,T})$$
$$\nabla \dot{E}_H = \dot{m} \cdot [(1 - \mu) \cdot (e(P_1, T_1) - e(P_3, T_1))] = \dot{m} \cdot (1 - \mu) \cdot (\nabla e_P)_{T_1} \tag{24}$$
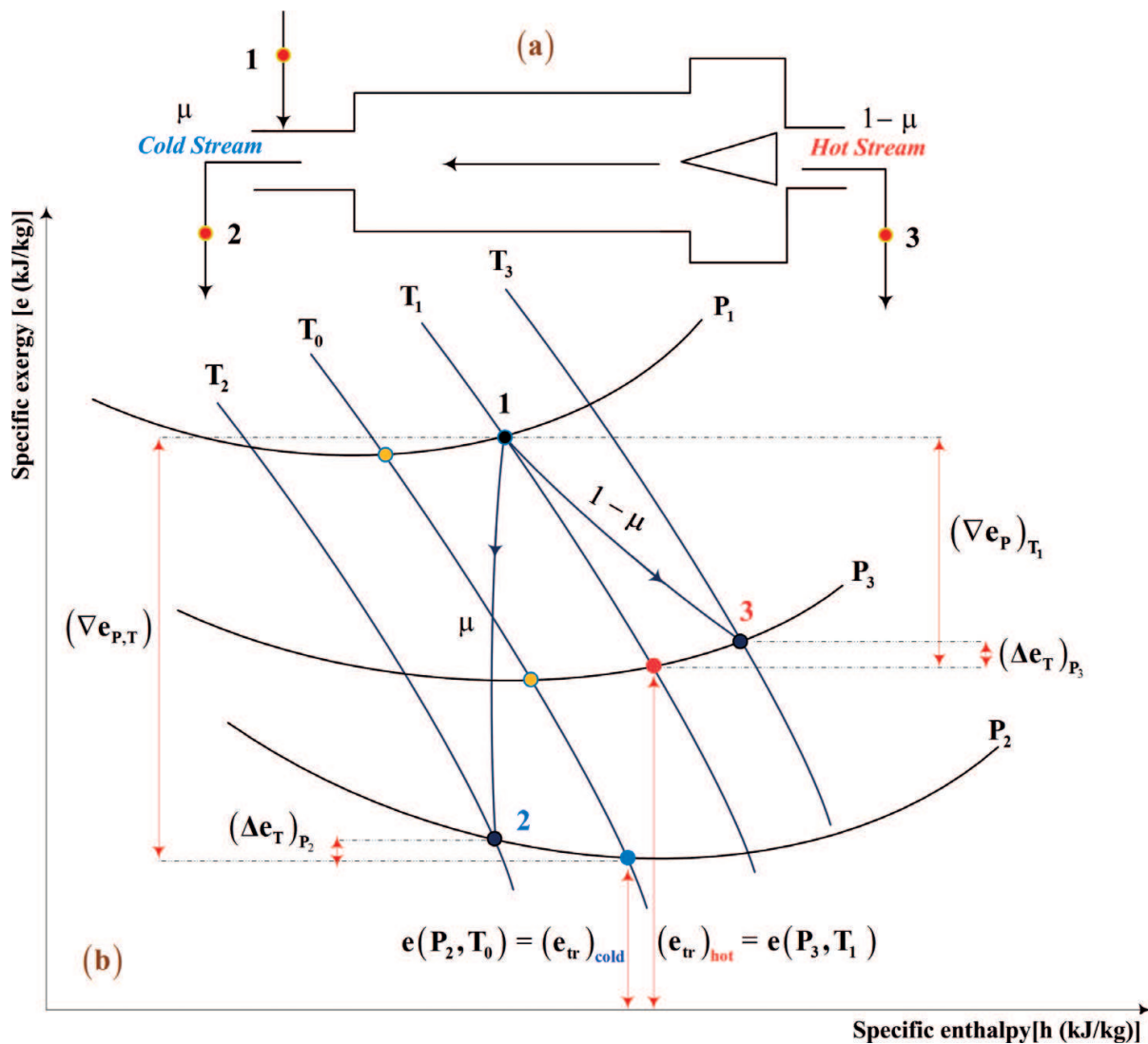
Figure 7. A vortex tube (a) and the presentation of the expansion process on an exergy-enthalpy diagram (b).

Thus $\Delta\dot{E}_C$ and $\Delta\dot{E}_H$ represent the increase in the thermal exergy component due to the cooling of the cold stream and the heating for the hot stream, under conditions of the outlet pressures for each stream. $\nabla\dot{E}_C$ represents the decrease in thermo-mechanical exergy of the cold stream due to the partial thermal exergy destruction because of the temperature drop from T_1 to T_0, and the decrease of mechanical exergy because of pressure drop from P_1 to P_3. $\nabla\dot{E}_H$ is the decrease of mechanical exergy of the hot stream at conditions of constant inlet temperature. The ratio $(\Delta\dot{E}_C + \Delta\dot{E}_H) / (\nabla\dot{E}_C + \nabla\dot{E}_H)$ gives the value of η_{tr}.

Example 4

An adiabatic vortex tube is supplied with air as ideal gas at the rate $\dot{m}$ = 1 kg/s and at P_1 = 0.8 MPa, T_1 = 308 K. The air expands at the cold end to pressure P_2 = 0.1 MPa and at the hot end to the pressure P_3 = 0.15 MPa. The ambient temperature T_0 = 298 K. Calculate the variation of $\dot{E}_{tr}$, $\Delta\dot{E}$ and $\nabla\dot{E}$ and η_{tr} as a function of the cold mass fraction μ in the range 0.2–0.9.

Solution

The results are shown in **Table 4**. It is illustrated that $\dot{E}_{tr,H}$ decreases with the cold mass fraction increase. The $\dot{E}_{tr,C}$ is zero, because it is defined by ambient conditions $P_2 = P_0$ and $T_2 = T_0$. As a result, for the cold stream $\Delta\dot{E}_C$ decreases but $\nabla\dot{E}_C$ increases strongly with μ. An opposite effect is observed for the hot stream, where $\Delta\dot{E}_H$ increases and $\nabla\dot{E}_H$ decreases. As a result, η_{tr} increases, despite the rise in the exergy losses with the increasing cold mass fraction. This can be explained by the fact that the rise in exergy produced in the hot stream surpasses the increase in exergy losses. The exergy efficiency of the vortex tube is relatively low.

3.4. Compression across ambient temperature

In most refrigeration plants and heat pumps compression starts at $T < T_0$ and ends at $T > T_0$. The process is presented on an e-h diagram (see **Figure 8**). According to Eq. (9c), transiting exergy is:

$$e_{tr} = e(P_1, T_0) \tag{25}$$

The produced and consumed exergies are:

$$\Delta\dot{E} = \dot{m} \cdot [e(P_2, T_2) - e(P_1, T_0)] = \dot{m} \cdot (\Delta e_{P,T}) \tag{26}$$

$$\left(\nabla\dot{E} + \dot{W}_C\right) = \dot{m} \cdot [e(P_1, T_1) - e(P_1, T_0)] + \dot{W}_C = \dot{m} \cdot (\nabla e_T)_{P_1} + \dot{W}_C \tag{27}$$

$\Delta\dot{E}$ represents the increase of thermo-mechanical exergy due to the rise in pressure from P to P_2 and the rise in temperature from T_0 to T_2. $\nabla\dot{E}$ represents the destruction of thermal exergy due to the rise in temperature from T_1 to T_0 under conditions of constant pressure P_1, plus the consumed power $\dot{W}_C$. The ratio $\Delta\dot{E}/(\nabla\dot{E} + \dot{W}_C)$ gives the value of exergy efficiency η_{tr}.

Example 5

An adiabatic compressor of a refrigeration plant is supplied with the working fluid R152a at the rate $\dot{m} = 1$ kg/s at $P_1 = 142.9$ kPa and $T_1 = 263$ K (superheated). The fluid is compressed to a pressure of $P_2 = 615.1$ kPa. The ambient temperature $T_0 = 298$ K. Calculate the variation of $\dot{E}_{tr}$, $\Delta\dot{E}$ and $\nabla\dot{E}$ and η_{tr} as a function of the isentropic efficiency η_C in the range 0.75–0.90.

μ (–)	T_2 (K)	T_3 (K)	$\nabla\dot{E}_C$ (kW)	$\Delta\dot{E}_C$ (kW)	$\dot{E}_{tr,C}$ (kW)	$\nabla\dot{E}_H$ (kW)	$\Delta\dot{E}_H$ (kW)	$\dot{E}_{tr,H}$ (kW)	$\dot{D}$ (kW)	η_{tr} (%)
0.35	268.0	329.5	61.9	0.6	0.0	93.1	0.9	21.9	153.5	0.97
0.60	273.0	360.4	106.2	0.7	0.0	57.3	2.3	13.5	160.5	1.83
0.75	278.0	397.6	132.7	0.5	0.0	35.8	3.4	8.4	164.6	2.32

Table 4. Variation in exergy metrics with the cold mass fraction for a vortex tube expansion process.

Solution

The results are shown in **Table 5**. The transiting exergy does not change with the isentropic efficiency η_C, because is a function of constant parameters T_0 and P_1. The exergy consumed does not change either. The produced exergy decreases. This drop in $\Delta \dot{E}$ is explained by the reduction in "compression reheat". The decrease in $\Delta \dot{E}$ is offset by the greater decrease in $\dot{W}_C$, and as a result η_{tr} increases.

3.5. Compression and expansion in a one phase ejector

A combination of the processes of vapor expansion and compression takes place within a one-phase ejector presented in **Figure 9a**. A primary (pr) stream at high pressure P_1 and temperature
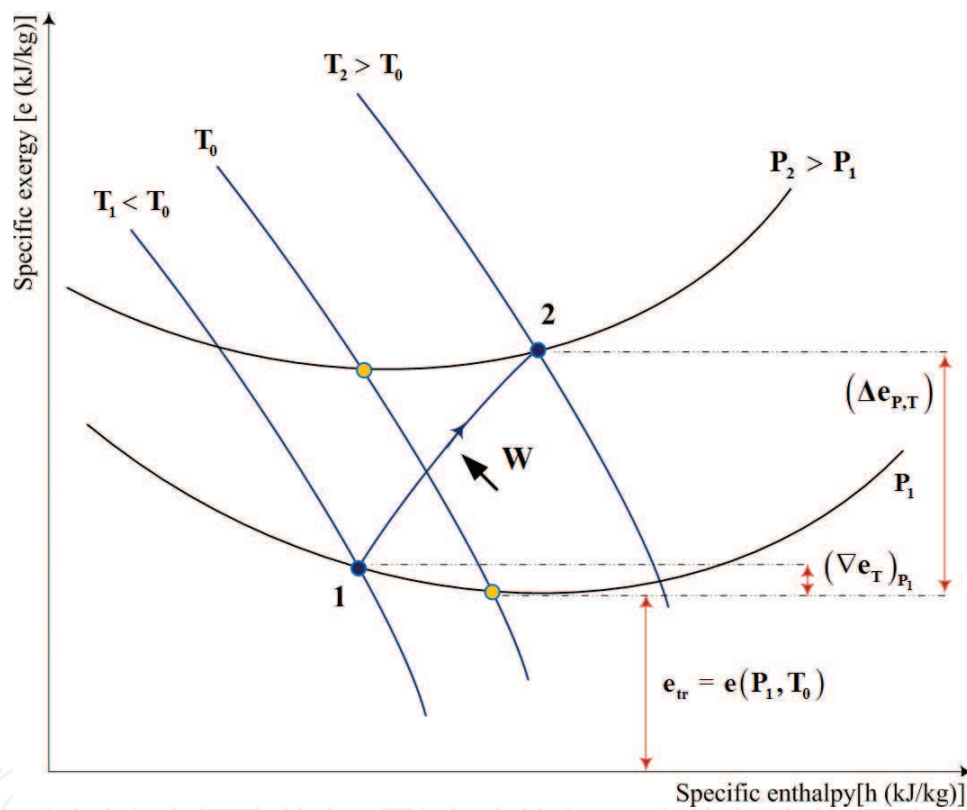

Figure 8. Compression process across ambient temperature (T_0) on an exergy-enthalpy diagram.

η_C (−)	$\dot{W}_C$ (kW)	$\nabla \dot{E}$ (kW)	$\Delta \dot{E}$ (kW)	$\dot{D}$ (kW)	$\dot{E}_{tr}$ (kW)	η_{tr} (kW)
0.75	66.0	2.3	53.3	15.0	12.6	78.0
0.80	61.9	2.3	52.9	11.3	12.6	82.4
0.85	58.3	2.3	52.6	8.0	12.6	86.8
0.90	55.0	2.3	52.3	5.0	12.6	91.3

Table 5. Variation in exergy metrics with the isentropic efficiency for a compression process.

T_1 expands and entrains a secondary stream (s) at low pressure P_2 and temperature $T_2 < T_0$. The ratio $\dot{m}_s/\dot{m}_{pr}$ gives the value of the entrainment ratio (ω) of the ejector. The mixed stream with the parameters $P_2 < P_3 < P_1$ and $T_2 < T_3 < T_1$ leaves the ejector. The exergy balance around the ejector is:

$$\left(\frac{1}{1+\omega}\right) \cdot e_1 + \left(\frac{\omega}{1+\omega}\right) \cdot e_2 = e_3 + d \tag{28}$$

The processes of expansion of the primary stream and compression of the secondary stream are presented on an e-h diagram (**Figure 9b**). The secondary stream is compressed across T_0, meaning that Eq. (9c) is applied to calculate $(e_{tr})_s$. As a result, the transiting exergy for secondary and primary streams are:

$$(e_{tr})_s = e(P_2, T_0) \tag{29}$$

$$(e_{tr})_{pr} = e(P_3, T_3) \tag{30}$$

This means that the exergies produced and consumed may be computed as:

$$\Delta\dot{E}_s = \dot{m}_s \cdot [e(P_3, T_3) - e(P_2, T_0)] = \dot{m}_s \cdot (\Delta e_{P,T})_s \tag{31}$$

$$\begin{aligned}\nabla\dot{E}_s &= \dot{m}_s \cdot [e(P_2, T_2) - e(P_2, T_0)] = \dot{m}_s \cdot \left((\nabla e_T)_{P_2}\right)_s \\ \nabla\dot{E}_{pr} &= \dot{m}_{pr} \cdot [e(P_1, T_1) - e(P_3, T_3)] = \dot{m}_{pr} \cdot (\nabla e_{P,T})_{pr}\end{aligned} \tag{32}$$

$\Delta\dot{E}_s$ is the increase of thermo-mechanical exergy of the secondary stream due to the compression from P_2 to P_3 and the rise in temperature from T_0 to T_3. The exergy consumed $\nabla\dot{E}_s$ within the secondary stream represents the decrease of thermal exergy component because of the temperature rise from T_2 to T_0 (the partial cold destruction). The exergy consumed within the primary stream $\nabla\dot{E}_{pr}$ is the decrease of thermo-mechanical exergy The ratio $\Delta\dot{E}_s/(\nabla\dot{E}_s + \nabla\dot{E}_{pr})$ gives the value of exergy efficiency η_{tr}. The detailed analysis of efficiencies for different parts of an ejector is given in [13].

Example 6

An ejector of a refrigeration plant is supplied with the working fluid R141b. The parameters of the secondary stream are: P_2 = 22.3 kPa, T_2 = 268 K. The pressure of the mixed stream is P_3 = 91 kPa. The ambient temperature T_0 = 289 K. Calculate the variation of $(\dot{E}_{tr})_{pr}$, $(\dot{E}_{tr})_s$, $\Delta\dot{E}$ and $\nabla\dot{E}$ and η_{tr} as a function of the entrainment ratio $\omega = \dot{m}_s/\dot{m}_{pr}$ in the range 0.15–0.25.

Solution

The calculation results are shown in **Table 6**. The transiting exergy in the secondary flow is negative because the parameters P_2 and T_0 define the state of the flow under vacuum conditions. The exergy produced and exergy consumed increase with the entrainment factor. The increase in $\Delta\dot{E}_s$ offsets the increase in $(\nabla\dot{E}_s + \nabla\dot{E}_{pr})$, and as a result η_{tr} increases.
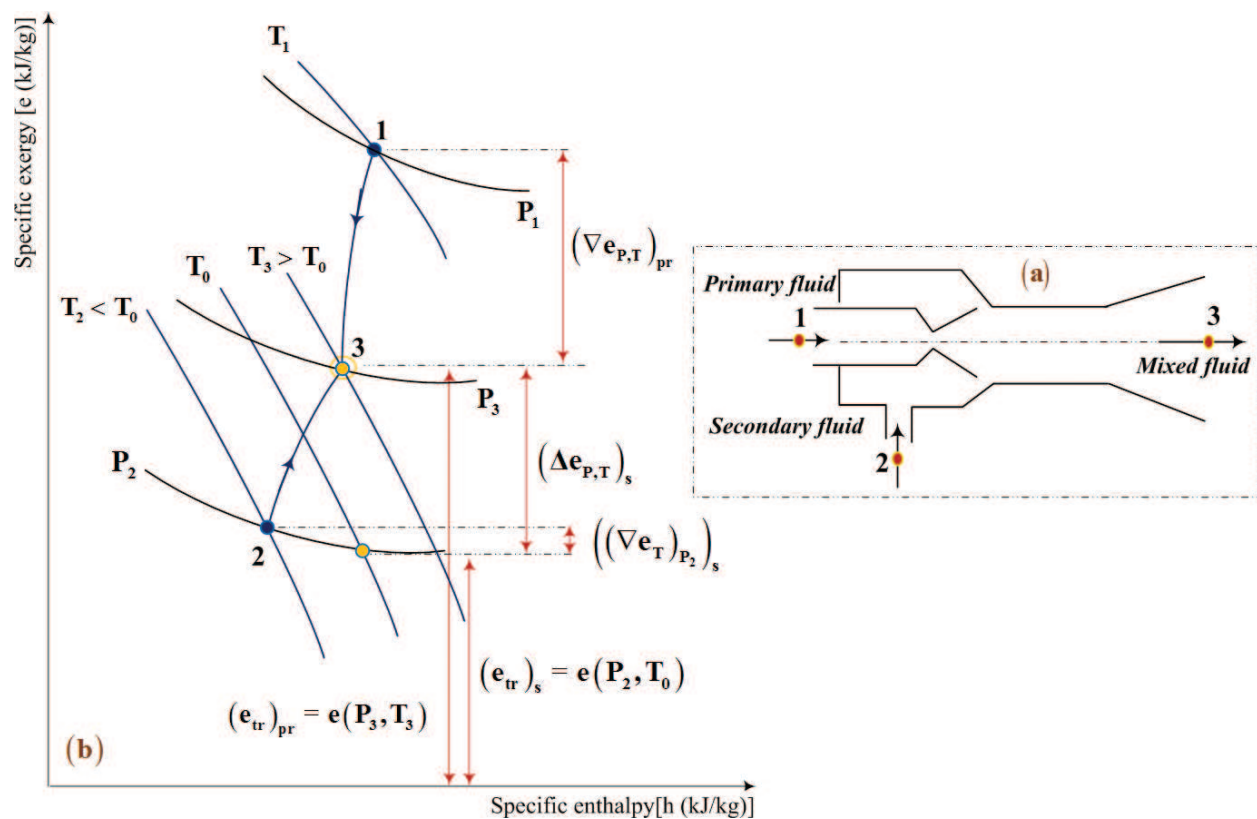
Figure 9. One phase ejector (a) and presentation of expansion and the compression processes (b) on an exergy-enthalpy diagram.

ω (–)	$\nabla\dot{E}_{pr}$ (kW)	$\nabla\dot{E}_s$ (kW)	$\Delta\dot{E}_s$ (kW)	$\dot{D}$ (kW)	$\dot{E}_{tr,pr}$ (kW)	$\dot{E}_{tr,s}$ (kW)	$\dot{E}_{tr}$ (kW)	η_{tr} (%)	P_1 (kPa)	T_1 (K)
0.15	9.99	0.017	1.189	8.818	4.286	−0.546	3.740	11.9	1000	418
0.17	10.07	0.019	1.334	8.755	4.205	−0.619	3.586	13.2	1000	418
0.20	10.19	0.023	1.547	8.666	4.090	−0.729	3.361	15.2	1000	418
0.23	10.29	0.026	1.754	8.562	3.983	−0.838	3.145	17.0	1000	418
0.25	10.36	0.028	1.890	8.498	3.915	−0.911	3.004	18.2	1000	418

Table 6. Variation in exergy metrics with the entrainment factor for compression-expansion processes in an ejector.

4. Environmental life cycle analysis and exergy efficiency of cooling systems

Life Cycle Analysis (LCA) is an important tool to analyze environmental problems associated with the production, use, and disposal of products or systems [14]. For every product produced within a system the total inflow and outflow of energy and materials are evaluated. The

environmental burdens are associated by quantifying the energy and materials used, as well as the wastes released into the environment. The impact of these uses and releases on the environment is assessed. The multidimensional approach of LCA causes some problems when different substances need to be compared and general agreement is required. This problem may be avoided if exergy is used as a common quantity as proposed by Life Cycle Exergy Analysis [15]. The crucial idea behind this method is the distinction between renewable and non-renewable resources. In order to illustrate the method, let us consider three defined time periods within the life cycle of an ejector refrigeration system driven by solar energy [16]. At first, exergy is required during the construction stage to build the plant and put it into operation. During this period the spent exergy is stored in materials, such as metals, glass etc. For the second period, maintenance required for the system's operation takes place. Exergy necessary for this maintenance is evaluated. The third period is the clean-up stage, including the plant demolition and the recycling of materials. Exergy used for the clean-up is assessed. The exergy used for the construction, maintenance, and clean-up is assumed to originate from non-renewable resources and is named indirect exergy, $\dot{E}_{ind}$. When the ejector refrigeration system driven by solar energy is put into operation, it starts to deliver a product (cold in this case) with exergy, $\dot{E}_{pr}$. By considering renewable resources (solar in this case) as free, there will be a net exergy output from the plant until the plant is decommissioned. By considering the total life cycle of the plant the net produced exergy becomes $\dot{E}_{net} = \dot{E}_{pr} - \dot{E}_{ind}$. The higher this value is for the three time periods defined above, the more sustainable the system is, because the input of non-renewable resources will be paid back during the system's lifetime. The rise in exergy efficiency of an ejector calculated according to Eqs. (31) and (32) leads to an increase in efficiency of the solar driven refrigeration system [16]. This in turn means that the net produced exergy $\dot{E}_{net}$ increases too. Thus, the evaluation of η_{tr} of an ejector, as presented in Section 3.5, and its subsequent maximization, may lead to the construction and operation of more sustainable solar driven refrigeration plants.

5. Conclusion

The common feature of expansion processes operating below or across ambient temperature is the partial transformation of the mechanical exergy component into the thermal exergy component. Sub-ambient compression processes are characterized by the transformation of work into the mechanical exergy component and the partial destruction of the thermal exergy component below T_0. In order to evaluate the efficiency of these transformations the calculations of the variation in mechanical and thermal exergy components are required. These calculations may be done in many different ways, for example the variation in e_P depends on the chosen temperature conditions, while the variation in e_T depends on the chosen pressure conditions. This multiplicity in the exergy variation evaluation leads to ambiguity in the exergy efficiency definition. The approach based on the exclusion of the "transiting flow" from thermo-mechanical inlet and outlet exergies of an analyzed process overcomes this difficulty. This improvement is possible because the transiting exergy is uniquely defined by a specific combination of the process intensive parameters, namely the inlet and outlet pressures and temperatures, as well as T_0. The transiting exergy approach allows non-ambiguous evaluation

of two thermodynamic metrics: exergy produced and exergy consumed. Their ratio represents the exergy efficiency; the difference between exergy consumed and exergy produced equals the exergy losses within the process. The phenomenological significance of the transiting exergy and the way in which it can be computed for processes below and across T_0 has been illustrated for the cases of an expansion valve, a cryo-expander, a vortex tube, an adiabatic compressor, and a monophasic ejector. The input-output exergy efficiency is not an appropriate criterion for evaluation of these processes.

Acknowledgements

Nomenclature

$\dot{D}$ Destroyed exergy, (kW)

d   Specific exergy losses, (kJ/kg)

e   Specific exergy, (kJ/kg)

$\dot{E}$ Exergy, (kW)

h   Specific enthalpy, (kJ/kg)

$\dot{H}$ Enthalpy, (kW)

$\dot{m}$ (Total) Mass flowrate, (kg/s)

P   Pressure, (MPa, kPa)

$\dot{Q}$ Heat rate, (kW)

s   Specific entropy, (kJ/kg K)

T   Temperature, (K, °C)

$\dot{W}$ Mechanical power, (kW)

Greek symbols

η   Efficiency, (%)

∇   Consumption

Δ   Production

μ   Cold mass fraction, $\mu = \dot{m}_C/\dot{m}$

ω   Entrainment ratio

Subscripts

0	Ambient state
1, 2, 3…	States in a process
C	Cold, Compressor
f	Fuel
H	Hot
in	Inlet
ind	Indirect
int	Internal
max	Maximal
min	Minimal
out	Outlet
pr	Primary, Product
S,s	Secondary
subC	Subcooling
tr	Transiting

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