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Function Approximation-based Sliding Mode Adaptive Control for Time-varying Uncertain Nonlinear Systems

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1. Introduction

Dead zone characteristics exist in many physical components of control systems. They are nonlinear features particularly in direct current (DC) motor position tracking control systems, mainly caused by the uncertain time-varying nonlinear friction. They can severely limit the control performance owing to their non-smooth nonlinearities. However, dead zone characteristics usually are not easy to be known exactly and may vary with time in practical. In addition to the uncertainties in the linear part of the plant, controllers are often required to accommodate time-varying dead zone uncertainties. In general, there are two usual methods treating the systems with uncertain time-varying dead zone characteristics caused by uncertain nonlinear frictions in DC motor position control systems. The first one is to separate the unknown dead zone from the original DC motor systems and construct an adaptive dead zone inverse, and then compensate the effects of unknown dead zone characteristics (Gang & Kokotovic, 1994; Cho & Bai, 1998; Wang et al., 2004; Zhou et al., 2006). The second method is to deal with both the unknown dead zone characteristics and all the other uncertainties as one uniform uncertainty, thereupon design proper compensator (Wang et al., 2004) or adaptive controller which can counteract the effects of uncertainty (Selmic & Lewis, 2000; Tian-Ping et al., 2005). Furthermore, dead zone uncertainties' bounds remain unknown in many practical DC motor control systems. This problem can't be coped with conventional sliding mode controller (Young et al., 1999; Hung et al., 1993) and general adaptive controller (Gang & Kokotovic, 1994; Cho & Bai, 1998; Wang et al., 2004; Zhou et al., 2006; Wang et al., 2004; Selmic & Lewis, 2000; Tian Ping et al., 2005; Young et al., 1999; Hung et al., 1993). In order to deal with nonlinear systems with unknown bound time-varying uncertainties, adaptive control schemes combined with sliding mode technique have been developed (Chyau-An & Yeu-Shun, 2001; Chyau-An & Yuan-Chih, 2004; Huang & Chen, 2004; Chen & Huang, 2004). These control schemes can transform the unknown bound time-varying uncertainties into finite combinations of Fourier series as long as the uncertainties satisfy Dirichlet condition, so that they can be estimated by updating the Fourier coefficients. Since the coefficients are time-invariant, update laws are easily obtained from the Lyapunov design to guarantee output error convergence.

This chapter is divided into two parts. In the first part, for the position tracking in DC motor with unknown bound time-varying dead zone uncertainties, we'll propose a Function

Approximation-based Sliding Mode Adaptive Controller (short for **FASMAC**). Firstly, we obtain a control law consisting of an unknown bound time-varying uncertain term same as An-Chyau (2001) and another compensative term through sliding mode technique and, afterwards, transform the uncertain term into a combination of a set of orthonormal basis functions with the approach of function approximation technique, where Laguerre function series are employed for their widely application in system model approximation (Wahlberg, 1991; Oliver et al., 1994; Campello et al., 2004) and adaptive controller design (Zervos & Dumont, 1988; Wang, 2004). Then concrete expressions of uncertain term and compensative term can thus be derived based on the Lyapunov design to guarantee output error convergence. This control scheme can not only approximate the unknown bound time-varying uncertainties online but also compensate the error of approximation synchronously. Actual experiments on DC motor position tracking demonstrate the performance of the control scheme. In the second part, we'll extend the sliding mode adaptive controller for SISO system in the first part of the chapter to an adaptive controller for SIMO system with unknown bound time-varying uncertainty. The control strategy only requires that the uncertainty is the piecewise continuous or square integrable in finite time interval, and doesn't demand for the information of the uncertainty's bound and some conditions of the uncertainty, thus it is more suitable for actual SIMO uncertain nonlinear systems. The SIMOAC strategy gives a sliding function according to the sliding mode control basic principle firstly, and then transforms the time-varying uncertainty into the multiplying of a known time-varying basis function vector and an unknown time-invariant coefficient vector, and further obtains the updating law of coefficient vector and an adaptive on-line compensation of approximation error, then adaptive control law are obtained at last.

2. Function Approximation-based Sliding Mode Adaptive Control for SISO system

2.1 Problem statement

The system to be controlled is a DC motor position control system. The simplified plant model is shown in Fig.1, where U_f stands for the equivalent voltage caused by unknown time-varying nonlinear friction, x_1 and x_2 represent the system position state and velocity state, respectively, and T_m is the time constant value of DC motor, K_e is the ratio of speed feedback.

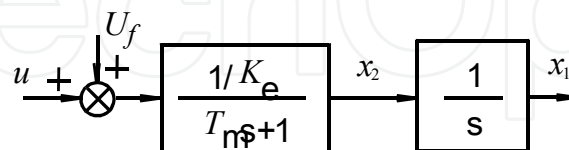


Figure 1. Simplified open-loop model of DC motor position control system

Let U be the known static friction moment of U_f in Fig.1, we obtain system state space equation as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{1}{T_m} x_2 + \frac{1}{T_m K_e} u(t) + \frac{1}{T_m K_e} U + \frac{1}{T_m K_e} (U_f(t) - U) \end{cases} \quad (1)$$

For the purpose of convenience of controller design procedure, let $X(t)=[x_1(t) \ x_2(t)]^T$. Then (1) can be rewritten as

$$\dot{X}(t) = \bar{A}X(t) + \bar{B}u(t) + \bar{U} + D(t) \quad (2)$$

where \bar{A} , \bar{B} and \bar{U} are known constant vectors, while $D(t)$ is unknown time-varying uncertainty, and

$$\bar{A} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T_m} \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ \frac{1}{T_m K_e} \end{bmatrix}, \bar{U} = \begin{bmatrix} 0 \\ \frac{1}{T_m K_e} U \end{bmatrix}, D(t) = \begin{bmatrix} 0 \\ \frac{1}{T_m K_e} (U_f(t) - U) \end{bmatrix} \quad (3)$$

Eq. (2) is a perturbation model of DC motor position system, where $D(t)$ denotes the unknown bound time-varying dead zone uncertainty which mainly origins from uncertain time-varying nonlinear friction. Since conventional control strategy based on precise mathematic model usually can't reach performance requirement, thus, new control scheme needs be developed to improve system performance.

Assumption 1. $\exists C=[c_1 \ c_2] \in \Re^{1 \times 2}$, c_1 and c_2 are both positive or negative, for perturbation model of DC motor position system (2), guarantees that $C\bar{B} \neq 0$ and time-varying uncertain function $CD(t)$ is square integral for any finite time T , $T \in \Re$, that is $CD(t) \in L^2[R^+]$.

Under the Assumption 1, the unknown bound time-varying uncertainty $CD(t)$ can be transformed into a finite combination of Laguerre functions, and then coefficients of Laguerre functions are obtained using Lyapunov direct method.

2.2 Function approximation-based sliding mode adaptive controller design

In this section, we give the details of the FSMAC design. Firstly, a standard linear switch function $s(t)$ is chosen; then, the unknown bound time-varying uncertainty is transformed into a combination of series of orthonormal basis function employing Laguerre functions; thirdly, a control law including the approximation of uncertainty and it's approximation error compensation is proposed; finally, the concrete expression of the control law is obtained through Lyapunov direct method.

Given referenced position as $x_{d1}(t)$, assume that it's not special limit for the velocity of DC motor. Let

$$x_{d2}(t) = \dot{x}_{d1}(t), \quad X_d(t) = [x_{d1}(t) \ x_{d2}(t)]^T \quad (4)$$

Define error function as

$$E(t) = X(t) - X_d(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} = \begin{bmatrix} e_1(t) \\ \dot{e}_1(t) \end{bmatrix} \quad (5)$$

where $e_1(t)$ denotes position error, and $e_2(t)$ denotes velocity error.

We choose a switch function $s(t)$ as

$$s(t) = C(X(t) - X_d(t)) = C \cdot E(t) \quad (6)$$

where $C=[c_1 \ c_2]$ is the constant vector in the Assumption 1.

According to (5) and (6), we have

$$s(t) = C \begin{bmatrix} e_1(t) \\ \dot{e}_1(t) \end{bmatrix} = c_1 e_1(t) + c_2 \dot{e}_1(t) = c_1 \left(e_1(t) + \frac{c_2}{c_1} \dot{e}_1(t) \right) \quad (7)$$

where $c_1 \neq 0$. If $E(t)$ lies on the surface $s(t) = 0$, it's easy to guarantee position error $e_1(t)$ asymptotically stable by holding c_1 and c_2 both positive or negative, or alternatively, switch function (6) is a sliding surface (Young et al., 1999; Hung et al., 1993). In the following we will employ the FAT and Lyapunov direct method to derivate a sliding mode adaptive control law that guarantees the stability of sliding surface $s(t)$.

Let $u_m(t) = CD(t)$, $\hat{u}_m(t)$ is the on-line approximation of $u_m(t)$. Under the Assumption 1, there exists a sufficient large N , $u_m(t)$ can be transformed into a combination of a set of Laguerre function series as

$$u_m(t) = W^T Z(t) + \varepsilon(t) \quad (8)$$

Using the same Laguerre function series, the on-line approximation of $u_m(t)$ can be expressed as

$$\hat{u}_m(t) = \hat{W}^T Z(t) \quad (9)$$

where $\varepsilon(t)$ is the approximation error of Laguerre function series, \hat{W} is the approximation of W , and

$$W = [w_1 \ w_2 \ w_3 \ \dots \ w_{N-1} \ w_N]^T, \quad \hat{W} = [\hat{w}_1 \ \hat{w}_2 \ \hat{w}_3 \ \dots \ \hat{w}_{N-1} \ \hat{w}_N]^T \quad (10)$$

$$Z(t) = [\phi_1(t) \ \dots \ \phi_N(t)]^T \quad (11)$$

An excellent property of (8) is its linear parameterization of the time-varying uncertainty into a time-varying basis function $Z(t)$ and a time invariant coefficient vector W , where $Z(t)$ is known while W is an unknown time invariant constant vector. With this transformation, the unknown bound time-varying uncertainty is replaced by a set of unknown constants. Therefore, the approximation of $u_m(t)$ turns to find the update law for \hat{W} in (9) by selecting proper Lyapunov function.

Define

$$\tilde{W} = W - \hat{W} \quad (12)$$

Generally, the bound of $\varepsilon(t)$ can be made small enough by choosing a sufficient large N , and there always exists an error between W and \hat{W} when the control system is running, that is to say, \tilde{W} only converges at a bound, but not asymptotically. Therefore, the main approximation error is $\tilde{W}^T Z(t)$ which is the error between $W^T Z(t)$ and $\hat{W}^T Z(t)$ in many

practical applications. So, it is necessary to compensate this approximation error online when \hat{W} is updated.

Taking the time derivative of Eq. (6) along system trajectory, we have

$$\dot{s}(t) = C\dot{X}(t) - C\dot{X}_d(t) = C(\bar{A}X(t) + \bar{B}u(t) + \bar{U} + D(t)) - C\dot{X}_d(t) \quad (13)$$

On the basis of Eq. (6) and (13), An-Chyau (2001) developed a control law including a time-varying uncertain term and a signum function of sliding surface, where the uncertain term is represented by a set of Fourier series, and then the concrete expression of the control law is obtained with direct Lyapunov method. However, the proposed control scheme can't compensate the on-line approximation error. Adopting the same approach, we propose a control law consisting of an unknown bound time-varying uncertain term same as An-Chyau (2001) and add another compensative term for compensating the on-line approximation error between $W^T Z(t)$ and $\hat{W}^T Z(t)$, and then employ the function approximation technique to transform the uncertain term into a combination of a set of Laguerre series. According to above idea, the form of the proposed control law can be expressed as

$$u(t) = -(C\bar{B})^{-1}C(\bar{A}X(t) + \bar{U} - \dot{X}_d(t)) - (C\bar{B})^{-1}\hat{u}_m(t) - (C\bar{B})^{-1}k \operatorname{sgn}(s(t)) + (C\bar{B})^{-1}u_r(t) \quad (14)$$

in which, the first term $-(C\bar{B})^{-1}C(\bar{A}X(t) + \bar{U} - \dot{X}_d(t))$ on the right side is the control term based on nominal system model; the second term $-(C\bar{B})^{-1}\hat{u}_m(t)$ is from the approximation of unknown bound time-varying uncertainty; the last term $(C\bar{B})^{-1}u_r(t)$ is a compensative control term and $u_r(t)$ is the compensation of on-line approximation error between $W^T Z(t)$ and $\hat{W}^T Z(t)$; while $-(C\bar{B})^{-1}k \operatorname{sgn}(s(t))$ which is used to compensate $\varepsilon(t)$ is a control term including the sign function of sliding surface $s(t)$, and k is a positive constant. Substituting (14) into (13), yields

$$\begin{aligned} \dot{s}(t) &= C\bar{A}X(t) + C\bar{U} + u_m(t) - C\dot{X}_d(t) + (C\bar{B})^{-1}u(t) \\ &= C\bar{A}X(t) + C\bar{U} + u_m(t) - C\dot{X}_d(t) - C\bar{A}X(t) - C\bar{U} - \hat{u}_m(t) + C\dot{X}_d(t) - k \operatorname{sgn}(s(t)) + u_r(t) \\ &= u_m(t) - \hat{u}_m(t) - k \operatorname{sgn}(s(t)) + u_r(t) \end{aligned} \quad (15)$$

From (8)-(12), and (15), we have

$$\dot{s}(t) = \tilde{W}^T Z(t) + \varepsilon(t) - k \operatorname{sgn}(s(t)) + u_r(t) \quad (16)$$

In the following, the on-line update law \hat{W} and expression of $u_r(t)$ can be obtained from a Lyapunov function about \tilde{W} and $s(t)$ properly selected, and then concrete expression of $\hat{u}_m(t)$ also can be obtained through (9). Firstly, we propose the FASMAC using the following theorem, and prove the asymptotic stability of the system under control law (14).

Theorem 1. For DC motor position tracking control system with unknown bound time-varying uncertainty described as (2), select (6) as the sliding surface $s(t)$. There exists real

positive constant η_1, η_2, η_3 and k , when the update law of \hat{W} and compensative term $u_r(t)$ satisfy Eq. (17), under the control of control law (14), the sliding surface $s(t)$ converges to zero and, thus, the position tracking error $e_1(t)$ of uncertain system (2) is asymptotically stable. The update law can be expressed as

$$\dot{\hat{W}} = \frac{\eta_1}{\eta_2} s(t) Z(t), \quad u_r(t) = -\eta_3 \eta_1 s(t) \quad (17)$$

Proof. Let $k, \eta_1, \eta_2, \eta_3$ be the positive constants. We choose the Lyapunov function as

$$V(s(t), \tilde{W}) = \frac{1}{2} \eta_1 (s(t))^2 + \frac{1}{2} \eta_2 \tilde{W}^T \tilde{W} \geq 0 \quad (18)$$

Take time derivative of Eq. (18), yields

$$\begin{aligned} \dot{V}(s(t), \tilde{W}) &= \eta_1 s(t) \dot{s}(t) - \eta_2 \tilde{W}^T \dot{\tilde{W}} \\ &= \eta_1 s(t) (\tilde{W}^T Z(t) + \varepsilon(t) - k \operatorname{sgn}(s(t)) + u_r(t)) - \eta_2 \tilde{W}^T \dot{\tilde{W}} \\ &= \eta_1 s(t) \tilde{W}^T Z(t) - \eta_2 \tilde{W}^T \dot{\tilde{W}} + \eta_1 s(t) \varepsilon(t) - k \eta_1 |s(t)| + \eta_1 s(t) u_r(t) \\ &= \tilde{W}^T \left(\eta_1 s(t) Z(t) - \eta_2 \dot{\tilde{W}} \right) + \eta_1 s(t) \varepsilon(t) - k \eta_1 |s(t)| + \eta_1 s(t) u_r(t) \end{aligned} \quad (19)$$

Substituting (17) into (19), we obtain

$$\begin{aligned} \dot{V}(s(t), \tilde{W}) &= -\eta_3 (\eta_1 s(t))^2 - k \eta_1 |s(t)| + \eta_1 s(t) \varepsilon(t) \\ &\leq -\eta_3 (\eta_1 s(t))^2 - (k - |\varepsilon(t)|) \eta_1 |s(t)| \end{aligned} \quad (20)$$

If the variation bound of $\varepsilon(t)$ can be estimated, that is, there exists a positive constant $\delta > 0$ such that $|\varepsilon(t)| \leq \delta$, with the selection of a sufficient large positive constant k , such that $k \geq \delta$. Then $\dot{V}(s(t), \tilde{W}(t))$ can be derived to be

$$\dot{V}(s(t), \tilde{W}) \leq -\eta_3 (\eta_1 s(t))^2 - (k - \delta) \eta_1 |s(t)| \leq 0 \quad (21)$$

Therefore, it can be easily shown by the Barbalat's lemma (Slotine and Li, 1991) that the sliding surface $s(t)$ converges to zero, and the velocity of convergence can be adjusted by choosing the different values of $k, \eta_1, \eta_2, \eta_3$. Moreover, position error $e_1(t)$ of the uncertain system (2) is asymptotically stable.

Remark 1. Since the approximation error $\varepsilon(t) = \sum_{i=N+1}^{\infty} w_i z_i(t)$, if a sufficient number of basis functions are used, then $\varepsilon(t) \approx 0$ in many practical occasions. Because $\tilde{W}(t)$ is only

bounded, the main approximation error using FAT is the error between $W^T Z(t)$ and $\hat{W}(t)^T Z(t)$. The function of $u_r(t)$ in control law (14) is used to compensate for this approximation error online, and it is seen in (21) that the additional term $-\eta_3(\eta_1 s(t))^2$ is from $u_r(t)$.

2.3 Actual experiments and results analysis

From the procedure of FASMAC design in Section 2.2, the nominal model of the DC motor position system should be identified before doing actual experiments. The DC motor position model can be seen as a combination of a speed model and an integral. The speed model is identified firstly, and then the whole position model can be easily obtained through the integral. The positive and negative speed model should be identified separately owing to their different parameters in actual DC motor system in the paper.

The nominal form of DC motor speed model can be obtained from Section 2.1 as

$$\Omega(s) = \frac{1/K_e}{T_m s + 1} (U(s) + U) = \frac{K}{T_m s + 1} (U(s) + U) \quad (22)$$

There are three parameters needed to identify in (22), T_m , K , U , where $K=1/K_e$. By testing and measuring step response of the DC motor speed system, we obtain the input and output data and further identify the three unknown parameters employing curve fitting and optimizing techniques with MATLAB. Finally, nominal model of DC motor position system with positive and negative speed are obtained, respectively, as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{1}{3.0787} x_2 + \frac{5.3658}{3.0787} u(t) - \frac{5.3658}{3.0757} \times 464.6780 \end{cases} \quad (23)$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{1}{2.7746} x_2 + \frac{5.3169}{2.7746} u(t) + \frac{5.3169}{2.7746} \times 204.7935 \end{cases} \quad (24)$$

Based on (9), (14) and (17), the on-line control law of FASMAC are

$$u(k) = -(\bar{C}\bar{B})^{-1} C(\bar{A}X(k) + \bar{U} - \dot{X}_d(k)) - (\bar{C}\bar{B})^{-1} \hat{u}_m(k) - (\bar{C}\bar{B})^{-1} k \operatorname{sgn}(s(k)) + (\bar{C}\bar{B})^{-1} u_r(k) \quad (25)$$

$$\hat{u}_m(k) = (\hat{W}(k))^T Z(k), \quad u_r(k) = -\eta_3 \eta_1 s(k) \quad (26)$$

$$\hat{W}(k) = \hat{W}(k-1) + \frac{\eta_1}{\eta_2} s(k) Z(k) \quad (27)$$

With the discussion above, the proposed on-line control strategy can be realized as

- 1) Choose proper series number N of Laguerre function.

- 2) Choose proper such parameters of controller as $C, \eta_1, \eta_2, \eta_3, k$.
- 3) Initialize coefficients \hat{W} of Laguerre function series, here we let $\hat{w}_i(0) = 0 (i = 1, 2, \dots, N)$.
- 4) In every sample step k when system is running, do
 - (1) Read system states $X(k)$, calculate reference states $X_d(k)$ and $\dot{X}_d(k)$.
 - (2) Calculate the value of sliding function $s(t)$ according to (6).
 - (3) Calculate $\hat{W}(k)$ according to (27).
 - (4) Calculate $\hat{u}_m(k)$ and $u_r(k)$ according to (26).
 - (5) Calculate $u(k)$ according to (25).
 - (6) $k = k + 1$.

Return to step (1) in 4) and repeat the on-line operating.

Fig.2 shows the actual DC motor device, and the control system consists of a pulse width modulation (PWM) driver, a microcomputer and a build-in card with A/D and D/A channel. The range of digital control signal in this experimental device is in $[-2048 \ 2048]$, and the corresponding voltage after the D/A channel ranges from -20 to 20 voltage. The digital value of position whose range is in $[-180 \ 180]$ degree can be read from the A/D channel. Moreover, the digital value of velocity can also be read from A/D channel directly.



Figure 2. Actual DC motor experiment device

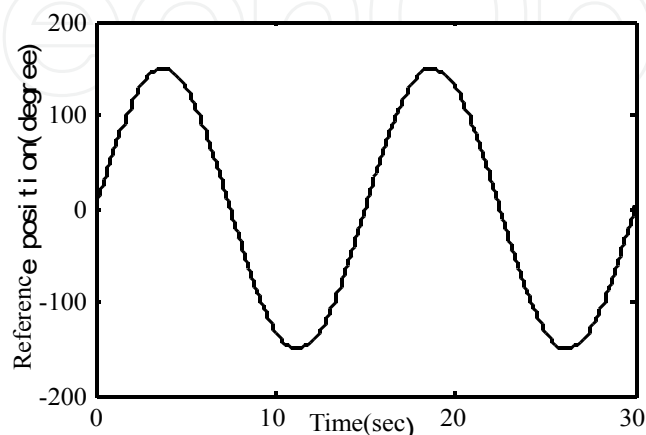


Figure 3. Reference signal

The proposed controller is implemented in a time-interrupt service routine at 10 ms sampling period under MS-DOS environment. The reference trajectory is designed to be

$$y(t) = 150 \sin(2\pi \times 0.0667t) \quad (28)$$

which is showed in Fig. 3. The number of terms of Laguerre function series is 8, and after several adjustments, the actual parameters of FASMAC are chosen as $C = 10 * [10 \ 0.5]$, $\eta_1 = 1$, $\eta_2 = 1$, $\eta_3 = 6$ and $k = 20$. Besides, to do the comparison with the results of FASMAC, experiment of control strategy proposed by An-Chyau (2001) is also implemented on the same DC motor system.

The results of the experiments are shown in Figs. 4-9. Fig.4 shows a comparison of the system output tracking errors between the proposed FASMAC and the controller of An-Chyau. It can be seen that the tracking error under the control of FASMAC mainly lies in $[-2, 2]$, and only reach -7 or 7 when the direction of DC motor speed is changing, whereas, tracking error of An-Chyau's controller lies in $[-10, 10]$ and arrive at peak value of -20 sometimes. Therefore, the tracking performance of the proposed FASMAC is better than the An-Chyau's controller. Fig.5 displays the sliding behaviour of the sliding surface $s(t)$ in our control scheme. Fig. 6, Fig.7 and Fig.8 show the total control value $u(t)$, the uncertain control term $-(C\bar{B})^{-1}\hat{u}_m(t)$ and compensative control term $(C\bar{B})^{-1}u_r(t)$, respectively. Fig.9 depicts the approximation of nonlinear friction $U_f(t)$, which can be calculated as $U + (C\bar{B})^{-1}\hat{u}_m(t)$. Fig.7 reveals that the uncertain term undulates at -100 or 100 in all the control period except the time of the direction of speed is changing. The main reason is that the identified linear nominal model of DC motor system is proper to the actual DC motor when the system is running at high speed, while it's not accurate at low speed because of the complicated dead zone characteristics caused by uncertain time-varying nonlinear friction, especially when the direction of DC motor speed is changing. Therefore, the model error between the identified nominal model and the actual system at low speed in peak value of uncertain control term $-(C\bar{B})^{-1}\hat{u}_m(t)$ when speed direction is changing. Since there is a compensative control term $(C\bar{B})^{-1}u_r(t)$ in control law $u(t)$ in FASMAC, which can compensate for the on-line approximation error of nonlinear friction rapidly, the proposed controller can still guarantee good performance even when the DC motor is running at low speed or its direction is changing, thus, the tracking performance of FASMAC is much better than that of An-Chyau. It should be noted that the compensative term $u_r(t)$ is almost the same as approximation error of nonlinear friction in simulation experiments.

From the proof of Theorem 1, we can see that the derivative of Lyapunov function (21) consists of $-\eta_3(\eta_1 s(t))^2$ and $-k\eta_1|s(t)|$, which are contributed by compensative control term $(C\bar{B})^{-1}u_r(t)$ and constant control term $-(C\bar{B})^{-1}k \operatorname{sgn}(s(t))$, respectively. Thus, the sliding surface converges rapidly mainly owing to the existent of compensative control term when its error is large, and when the error falls to a certain extent, the sliding surface error still converges to zero because of the constant control term.

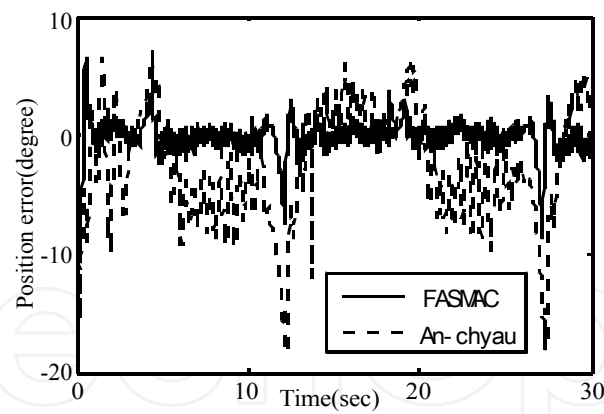


Figure 4. System output tracking error

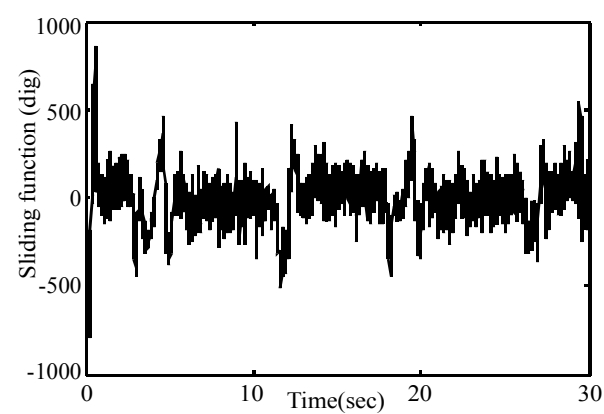


Figure 5. Behaviour of sliding surface $s(t)$

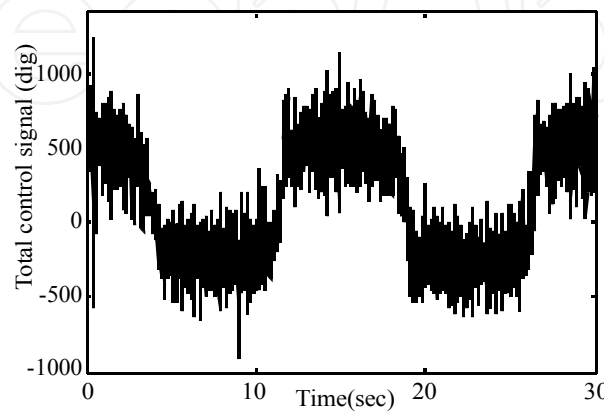


Figure 6. Total control law $u(t)$

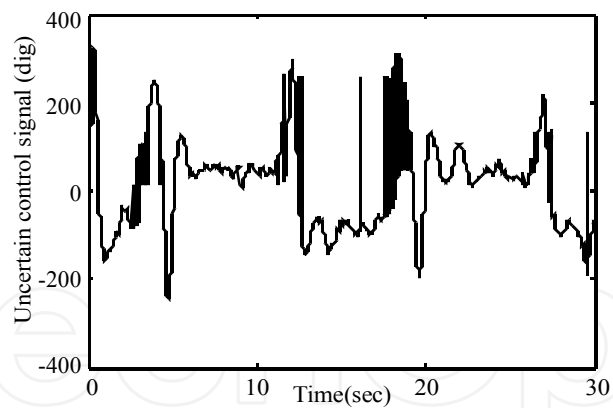


Figure 7. Uncertain term $-(C\bar{B})^{-1}\hat{u}_m(t)$

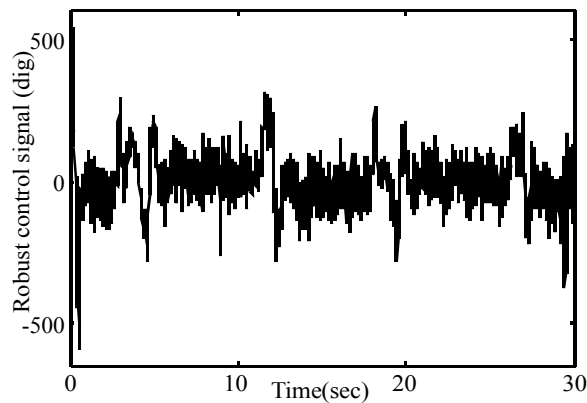


Figure 8. Compensative term $(C\bar{B})^{-1}u_r(t)$

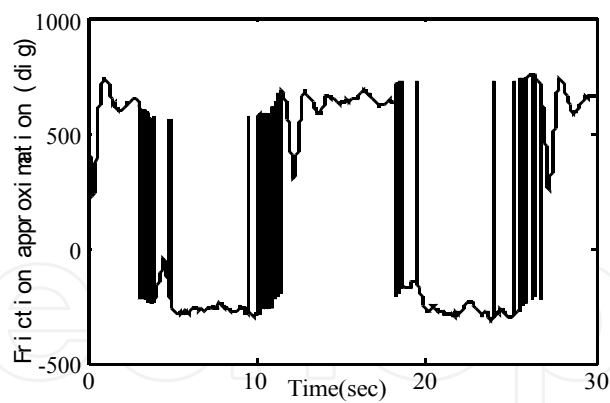


Figure 9. Nonlinear friction $U_f(t)$

3. FAT-based Adaptive Sliding Mode Control of SIMO Nonlinear System with Time-varying Uncertainty

In recent years, the usual methods for uncertain systems with specific structure have used in adaptive controller design include robust control (Zhou & Ren, 2001; Zhou, 2004; Hu & Liu, 2004; Wu et al., 2006), back-stepping(Do & Jiang, 2004; Manosa et al., 2005; Wu et al., 2007), sliding mode control (Huang & Cheng, 2004a; 2004b; Huang & Kuo, 2001; Chu & Tung,

2005; Fang et al., 2006; Chiang et al., 2007), neural network technique (Yang & Calise, 2007; Fu & Chai, 2007; Zhou et al., 2007; Tang et al., 2007) and fuzzy method (Hsu & Lin, 2005; Huang & Chen, 2006; Liu & Wang, 2007). For instance, combining a linear nominal controller with an adaptive compensator, Ruan (2007) and Hovakimyan (2006) realized the high performance stabilizing of inverted pendulum with un-modeling nonlinear dynamics. Since sliding mode control is robust to uncertainties of system structure and parameters, external disturbances and other unexpected factors, when the system lies on the sliding surface, it is obtained more and more attention in the control realm (Hung et al., 1993; Young et al., 1999).

At present, many adaptive control methods for nonlinear uncertain system whose uncertainty satisfies some conditions (Barmish & Leitmann, 1982; Chen & Huang, 1987) or the bound of uncertainty satisfies strict conditions have been developed (L. G. Wu et al., 2006; Z. J. Wu et al., 2007; Fang et al., 2006; Chiang et al., 2007). These research problems are hotspot in the control realm, and some results have been obtained through years of hard work of researchers (Huang & Chen, 2004; Chen & Huang, 2005; Huang & Liao, 2006; Liang et al., 2008). These research works adopt a common technique named function approximation technique (FAT) despite of their different design methods. Utilizing the FAT, the nonlinear time-varying uncertainty can be transformed into a finite combination of basis functions, and Lyapunov direct method can thus be used to find adaptive laws for updating time-invariant coefficients in the approximating series. Using Fourier series, Huang proposed an adaptive sliding control strategy for a class of nonlinear system with unknown bound time-varying uncertainty satisfying the Dirichlet condition, and further obtained the updating law of coefficients in Fourier series by Lyapunov direct method (Chen & Huang, 2005; Huang & Liao, 2006). The Section 2 proposed a FAT-based adaptive sliding mode control method. But the above mentioned control strategies are only suitable for single input single output (SISO) nonlinear systems with certain specific structure, not for SIMO uncertain system. In the second part of the chapter we'll propose a FAT-based adaptive sliding mode control method for SIMO nonlinear control system.

3.1 Problem statement

Giving the following SIMO uncertain nonlinear system

$$\left. \begin{aligned} \dot{X} &= (A(X) + \Delta A(X, t))X + B(X)u \\ Y &= X \end{aligned} \right\} \quad (29)$$

where, $X, Y \in R^n$, $A(X) \in R^{n \times n}$, $\Delta A(X, t) \in R^{n \times n}$, $B(X) \in R^n$, $u \in R$, and $\Delta A(X, t)$ is a time-varying uncertain function matrix. Let $\Delta A(X, t)X = D(X, t) \in R^n$, then (29) can be rewritten as

$$\left. \begin{aligned} \dot{X} &= A(X)X + B(X)u + D(X, t) \\ Y &= X \end{aligned} \right\} \quad (30)$$

The mathematical model of many actual SIMO electro-mechanical nonlinear systems with un-modeling dynamics in practical engineering can be described by (30), in which $D(X, t)$ is an unmodeling time-varying dynamics. Suppose system (30) satisfies the following assumption.

Assumption 2. $\exists C \in R^{1 \times n}$, for $\forall X \in R^n$, $CB(X) \neq 0$ exists, and time-varying function $CD(t)$ is piecewise continuous or square integrable in finite time interval.

Under the Assumption 2, we give a linear sliding function about system error firstly, and then transforms the approximation problem of unknown bound time-varying function $CD(X, t)$ into the updating of constant coefficient vector by FAT, and design a control law $u(t)$ including approximation of $CD(X, t)$ and compensation of approximation error, and then obtains the concrete expression of updating law of constant coefficient vector and compensation of approximation error through Lyapunov direct method, and thus adaptive sliding control law $u(t)$ can be obtained finally.

3.2 FAT-based sliding mode adaptive controller design

This section gives the details of design procedure of SIMOAC. Firstly, a linear sliding function $S(t)$ is chosen, and then the sliding mode adaptive control law guaranteeing closed-loop system stability can be obtained through Lyapunov direct method.

Let the expected state of system (30) is $X_d(t)$, and defining system state error as

$$E(t) = X(t) - X_d(t) \quad (31)$$

Choosing standard sliding function $S(t)$ as

$$S(t) = CE(t) \quad (32)$$

where, $C \in R^{1 \times n}$ is constant row vector.

Let $u_{m_1}(t) = CD(X, t)$. According to the Assumption 2 and FAT, time-varying scalar function $u_{m_1}(t)$ can be expressed as

$$u_{m_1}(t) = W_1^T Z_1(t) + \varepsilon_1 \quad (33)$$

In (33), W_1 is an unknown n -dimension constant column vector, $Z_1(t)$ is a known n -dimension time-varying basis function column vector, ε_1 is a approximation error. The essential of FAT is that $u_{m_1}(t)$ is approximated by means of the approximation of coefficient column vector W_1 . Let the approximation of $u_{m_1}(t)$ is denoted by $\hat{u}_{m_1}(t)$, then utilizing the same basis function vector, yields

$$\hat{u}_{m_1}(t) = \hat{W}_1^T Z_1(t) + u_c(t) \quad (34)$$

where, $u_c(t)$ is the adaptive compensation of approximation error.

Taking time derivative along system trajectory, and according to (33), yields

$$\dot{S}(t) = C(A(X)X(t) + B(X)u(t) + D(X, t)) - C\dot{X}_d(t)$$

$$\begin{aligned}
&= C(A(X)X(t) - \dot{X}_d(t)) + CD(X, t) + CB(X)u(t) \\
&= C(A(X)X(t) - \dot{X}_d(t)) + CB(X)u(t) + W_1^T Z_1(t) + \varepsilon_1
\end{aligned} \tag{35}$$

According to (Liang et al., 2008), the form of the proposed control law can be chosen as

$$u(t) = -(CB(X))^{-1} [C(A(X)X(t) - \dot{X}_d(t)) + \hat{u}_{m_1}(t)] \tag{36}$$

in which the first term $-(CB(X))^{-1} C(A(X)X(t) - \dot{X}_d(t))$ of right side of Eq. (36) is the control term based on nominal system model, the second one $-(CB(X))^{-1} \hat{u}_{m_1}(t)$ is the approximation of unknown bound time-varying uncertain term $u_m(t) = CD(t)$. Afterwards, a proper Lyapunov function about sliding function $S(t)$, the error square sum performance function and the error of coefficient vector can be constructed, and thus the updating law of coefficient vector $\hat{W}_1(t)$ in uncertainty approximation $\hat{u}_{m_1}(t)$ and concrete expression of adaptive compensation $u_c(t)$ for approximation error can be obtained by Lyapunov direct method.

Let $\tilde{W}_1(t) = W_1(t) - \hat{W}_1(t)$, substituting Eq. (36) into Eq. (35), yields

$$\begin{aligned}
\dot{S}(t) &= u_{m_1}(t) - \hat{u}_{m_1}(t) + u_c(t) \\
&= \tilde{W}_1(t) Z_1(t) - u_c(t) + \varepsilon_1
\end{aligned} \tag{37}$$

Defining system error square sum performance function $f(t)$ as

$$f(t) = E^T(t) Q E(t) \tag{38}$$

in which, $Q \in R^{n \times n}$ is a semi-positive definite diagonal constant matrix.

Taking the time derivative of $f(t)$ along system trajectory, yields

$$\dot{f}(t) = 2E^T(t) Q (A(X)X(t) - \dot{X}_d(t)) + 2(E^T(t) Q B(X)) u(t) + 2E^T(t) Q D(t) \tag{39}$$

Let $u_{m_2}(t) = E^T(t) Q D(t)$, according to the Assumption 2 and FAT, $u_{m_2}(t)$ can also be expressed as

$$u_{m_2}(t) = W_2^T Z_2(t) + \varepsilon_2 \tag{40}$$

where, W_2 is an unknown n-dimension constant column vector, $Z_2(t)$ is a known n-dimension time-varying basis function column vector, ε_2 is the approximation error. Utilizing the same basis function, the approximation of $u_{m_2}(t)$ can also be expressed as

$$\hat{u}_{m_2}(t) = \hat{W}_2(t)^T Z_2(t) \tag{41}$$

Let $\tilde{W}_2(t) = W_2(t) - \hat{W}_2(t)$. According to (36) and (40), (39) can be rewritten as

$$\dot{f}(t) = 2 \left\{ E^T(t) Q \left(I - (CB(X))^{-1} B(X) C \right) \left(A(X) X(t) - \dot{X}_d(t) \right) - (CB(X))^{-1} E^T(t) Q B(X) \hat{u}_{m_1}(t) + u_{m_2}(t) \right\} \quad (42)$$

Let

$$h(t) = E(t)^T Q \left(I - (CB(X))^{-1} B(X) C \right) \left(A(X) X(t) - \dot{X}_d(t) \right) \quad (43)$$

$$g(t) = \eta_1 S(t) + \eta_2 f(t) (CB(X))^{-1} E(t)^T Q B(X) \quad (44)$$

$$p(t) = \eta_2 f(t) \left(h(t) - (CB(X))^{-1} E(t)^T Q B(X) \hat{W}_1(t)^T Z_1(t) + \hat{u}_{m_2}(t) \right) + \eta_1 |S(t)| \delta_1 + \eta_2 |f(t)| \delta_2 \quad (45)$$

Theorem 2. For the SIMO nonlinear system (30) with unknown bound time-varying uncertainty, choosing sliding function $S(t)$ defined as (32) and performance function $f(t)$ defined as (38), then there exist constant scalar value $\eta_i \geq 0, (i = 1, \dots, 7), \delta_1 \geq 0$ and $\delta_2 \geq 0$, when $\dot{\hat{W}}_1(t)$, $\dot{\hat{W}}_2(t)$ and $u_c(t)$ satisfy (46) and (47), sliding surface $S(t) = 0$ and the error square sum performance function $f(t)$ of system (2) are stable under the control of (36).

$$\dot{\hat{W}}_1(t) = \eta_1 / \eta_3 S(t) Z_1(t), \quad \dot{\hat{W}}_2(t) = \eta_2 / \eta_4 f(t) Z_2(t) \quad (46)$$

$$u_c(t) = 1 / g(t) \left(p(t) + \eta_5 f(t) + \eta_6 S(t)^2 + \eta_7 |S(t)| \right) \quad (47)$$

Proof: Choosing the Lyapunov function as

$$V(S(t), f(t), \tilde{W}_1(t), \tilde{W}_2(t)) = \frac{1}{2} \eta_1 S(t)^2 + \frac{1}{4} f(t)^2 + \frac{1}{2} \eta_3 \tilde{W}_1^T(t) \tilde{W}_1(t) + \frac{1}{2} \eta_4 \tilde{W}_2^T(t) \tilde{W}_2(t) \geq 0 \quad (48)$$

Taking the time derivative of (48), one yields

$$\begin{aligned} \dot{V}(t) &= \dot{V}(S(t), f(t), \tilde{W}_1(t), \tilde{W}_2(t)) \\ &= \eta_1 S(t) \left(\tilde{W}_1^T(t) Z_1(t) - u_c(t) + \varepsilon_1 \right) \\ &\quad + \eta_2 f(t) \left\{ E^T(t) Q \left(I - (CB(X))^{-1} B(X) C \right) \left(A(X) X(t) - \dot{X}_d(t) \right) \right. \\ &\quad \left. - (CB(X))^{-1} E^T(t) Q B(X) \left(\hat{W}_1(t)^T Z_1(t) - u_c(t) \right) + u_{m_2}(t) \right\} \\ &\quad - \eta_3 \tilde{W}_1^T(t) \dot{\hat{W}}_1(t) - \eta_4 \tilde{W}_2^T(t) \dot{\hat{W}}_2(t) \end{aligned} \quad (49)$$

In time-varying scalar function $u_{m_i}(t)$ ($i = 1, 2$) defined in (33) and (40), the approximation error ε_i satisfies $|\varepsilon_i| \leq \delta_i \geq 0$ only if a sufficient large dimension N is chosen. According to (43), (44) and (45), (49) can be rewritten as

$$\dot{V}(t) \leq \tilde{W}_1(t)^T \left(\eta_1 S(t) Z_1(t) - \eta_3 \dot{\hat{W}}_1(t) \right) + \tilde{W}_2(t)^T \left(\eta_2 f(t) Z_2(t) - \eta_4 \dot{\hat{W}}_2(t) \right) - g(t) u_c(t) + p(t) \quad (50)$$

Substituting (46) and (47) into (50), one yields

$$\dot{V}(t) \leq -\eta_5 f(t) - \eta_6 S(t)^2 - \eta_7 |S(t)| \leq 0 \quad (51)$$

According to Lyapunov stability theorem, sliding surface $S(t) = 0$ and the error square sum performance function $f(t)$ of system (30) are stable. \square

With the discussion above, the proposed on-line control strategy can be realized as

1. According to the characteristic of actual control plant, choosing proper basis function series and the series number N , such as Fourier series and Laguerre series, and then initializing the coefficient vector $W_i = [w_1, w_2, \dots, w_n] (i = 1, 2)$.
 2. Choosing proper weight vector $C \in R^{1 \times n}$, matrix $Q = \text{diag}(q_{11}, q_{22}, \dots, q_{nn}) \in R^{n \times n}$ and learning rate $\eta_j > 0$, ($j = 1, \dots, 7$), where $q_{ii} > 0$, ($i = 1, \dots, n$).
 3. In every sample step k when the system is running, do
 4. Reading system current states $X(k)$, and obtaining error $E(k) = X(k) - X_d(k)$, and then calculating $S(k)$ and $f(k)$ according to (32) and (38).
 5. Calculating coefficient increment $\Delta \hat{W}_i(k)$ ($i = 1, 2$) according to (36), that is $\Delta \hat{W}_1(k) = \eta_1 / \eta_3 S(k) Z_1(k) T_s$, $\Delta \hat{W}_2(k) = \eta_2 / \eta_4 f(k) Z_2(k) T_s$, T_s is sample period.
 6. Calculating $\hat{u}_{m_2}(k)$ according to (41), that is $\hat{u}_{m_2}(k) = (\hat{W}_2(k-1) + \Delta \hat{W}_2(k))^T Z_2(k)$
 7. Calculating $h(k), g(k), p(k)$ according to (43), (44) and (45).
 8. Calculating $u_c(k)$ according to (47).
 9. Calculating time-varying uncertainty term $\hat{u}_{m_1}(k)$ according to (34), that is $\hat{u}_{m_1}(k) = (\hat{W}_1(k-1) + \Delta \hat{W}_1(k))^T Z_1(k) + u_c(k)$.
 10. Calculating sliding mode adaptive control law $u(k)$ according to (36).
 11. $k = k + 1$.
- Return to step 4 and repeat the on-line operating.

3.3 Simulation experiment and result analysis on a double inverted pendulum

This section applies the adaptive controller proposed to the stabilizing control of a double inverted pendulum simulating system, and analyzes the simulation result through the comparison with the result of the linear quadratic regulator (LQR).

Fig. 10 depicts the system diagram of the double inverted pendulum. The system is mainly composed of a car, two rods linked each other, optical-electrical encoder coders measuring displacement information, an alternating current electric motor driving the car which is linked with a belt. In actual system operation the real-time number control signal can be obtained according to current states of the double inverted pendulum, and then this signal can be used to drive the motor to control, and finally the car traverses along the rail.

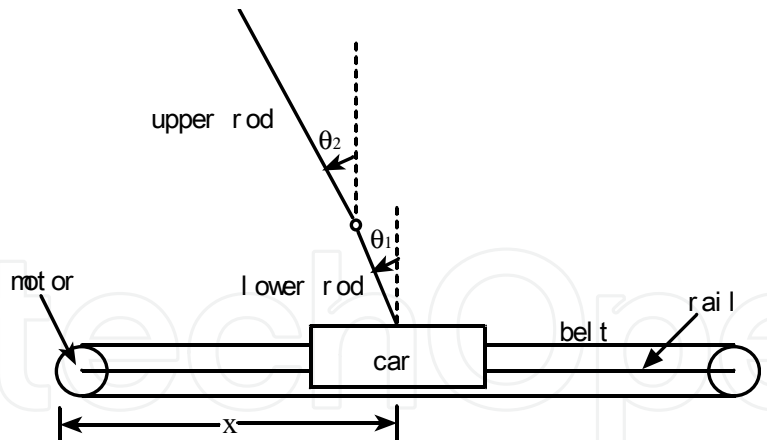


Figure 10. System diagram of double inverted pendulum

According to Fig.10, the mathematical model of the double inverted pendulum can be established by adopting Lagrange method. Choosing system states as $x_1 = x$, $x_2 = \theta_1$, $x_3 = \theta_2$, $x_4 = \dot{x}$, $x_5 = \dot{\theta}_1$, $x_6 = \dot{\theta}_2$, and let $X = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T$, then the linear state equation of the double inverted pendulum nearby equilibrium $X_0 = [0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$ is

$$\begin{aligned}\dot{X} &= AX + Bu \\ Y &= CX\end{aligned}\tag{52}$$

Where

$$\begin{aligned}A &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3(-2gm_1 - 4gm_2)}{2(-4m_1 - 3m_2)l_1} & \frac{9m_2g}{2(-4m_1 - 3m_2)l_1} & 0 & 0 & 0 \\ 0 & \frac{2gm_2(m_1 + 2m_2)l_1^2l_2}{4m_2^2l_1^2l_2^2 - \frac{16}{9}m_2(m_1 + 3m_2)l_1^2l_2^2} & -\frac{4gm_2(m_1 + 3m_2)l_1^2l_2}{3(4m_2^2l_1^2l_2^2 - \frac{16}{9}m_2(m_1 + 3m_2)l_1^2l_2^2)} & 0 & 0 & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 & 0 & 0 & 1 & \frac{3(-2m_1 - m_2)}{2(-4m_1 - 3m_2)l_1} & \frac{2m_2(m_1 + 2m_2)l_1^2l_2 - \frac{4}{3}m_2(m_1 + 3m_2)l_1^2l_2}{4m_2^2l_1^2l_2^2 - \frac{16}{9}m_2(m_1 + 3m_2)l_1^2l_2^2} \end{bmatrix}^T, \\ C &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}\end{aligned}$$

Applying physical parameters in Table 1, the nominal model of the pendulum for simulation can be obtained as

$$\dot{X} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 245.00 & -147.00 & 0 & 0 & 0 \\ 0 & -183.75 & 171.50 & 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.00 \\ 10.00 \\ -1.25 \end{bmatrix} u \tag{53}$$

According to (53), using the LQR algorithm named `lqr.m` in MATLAB and choosing $r = 1.0, Q = diag(180 \ 36 \ 90 \ 7.2 \ 3.6 \ 90)$, the feedback control matrix K can be obtained as $K = [13.4164 \ 183.9220 \ -241.7258 \ 13.8673 \ 2.8233 \ -21.8457]$.

Symbol	Description	Parameter	Symbol	Description	Parameter
m_1	Quality of lower rod	0.1033kg	l_1	Length of lower rod	0.06m
m_2	Quality of upper rod	0.2066kg	l_2	Length of upper rod	0.12m
M	Quality of car	0.5kg	g	Acceleration of gravity	9.8000 N/m ²

Table 1. Physical parameters of double inverted pendulum

Furthermore, the nonlinear virtual prototype of the double inverted pendulum can be established utilizing the SimMechanics toolbox in MATLAB, which depicted in Fig. 11. According to the nominal mathematical model of double inverted pendulum expressed as (53), firstly, choosing Laguerre series as basis functions with series number $N=8$, and controller parameters as $C = [13.4164 \ 183.9220 \ -241.7258 \ 13.8673 \ 2.8233 \ -21.8457]$,

$Q = diag([1 \ 2 \ 2 \ 0.1 \ 0.1 \ 0.1]), \eta_1 = 0.5, \eta_2 = 0.5, \eta_3 = 1, \eta_4 = 1, \eta_5 = 0.1, \eta_6 = 5, \eta_7 = 1$, then the proposed SIMOAC can be realized in S-function form in MATLAB. Finally, nonlinear stabilizing control simulating system of the double inverted pendulum depicted in Fig. 12 can be established through the series connection of virtual prototype and S-function controller in simulink environment of MATLAB.

Afterwards, the stabilizing control simulation experiments on double inverted pendulum can be conducd applying LQR algorithm and the proposed SIMOAC strategy in the simulating system in Fig.12, respectively.

The simulation results under the same initial condition $X_0 = [0 \ -0.0873 \ -0.0873 \ 0 \ 0 \ 0]^T$ are depicted in Fig. 13-18, in which Fig. 13 depicts the car displacement the error of the double inverted pendulum system under the control of SIMOAC and LQR. It can be seen clearly that the steady state displacement error is about -0.0250 meter under the control of SIMOAC, while it's about -0.1115 meter under the control of LQR. This shows the predominant performance of the proposed SIMOAC. Besides, Fig. 14 and Fig. 15 depict the angular displacement error of two pendulum rods, respectively, Fig. 16 depicts the adaptive control signal, Fig. 17 depicts on-line approximation of un-modeling nonlinear dynamics of the double inverted pendulum, Fig. 18 depicts the behavior of sliding function $s(t)$.

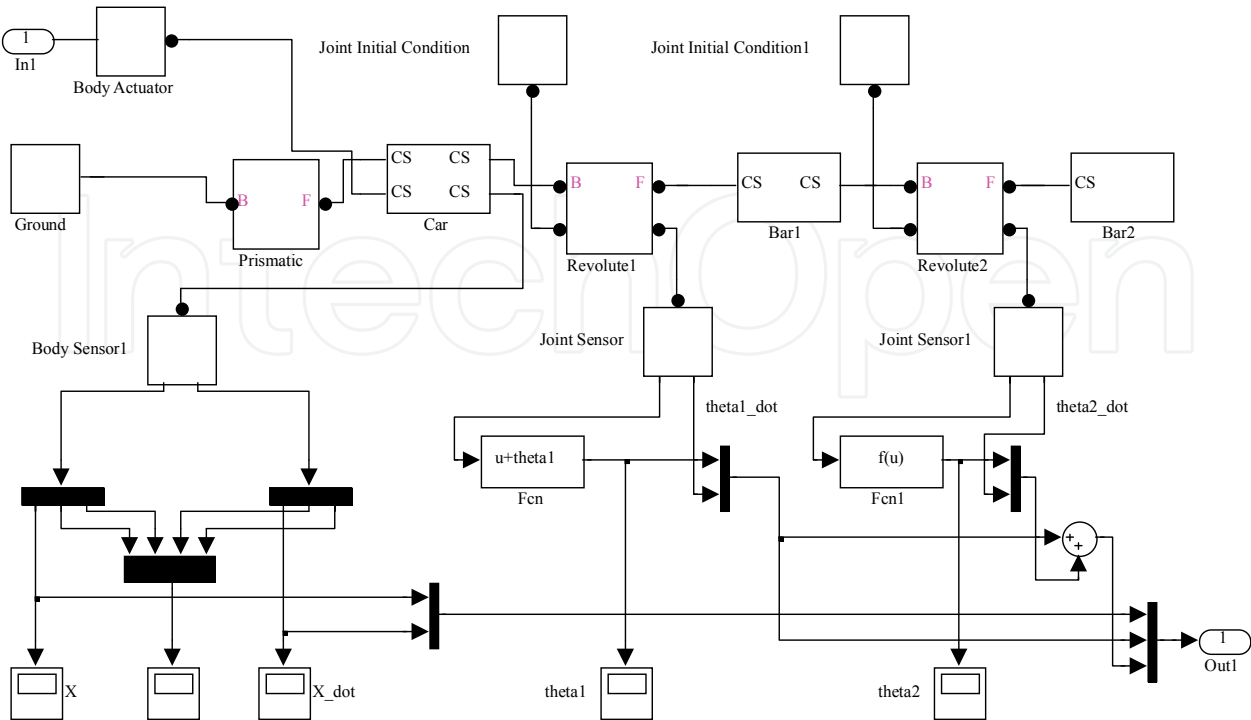


Figure 11. Virtual prototype of double inverted pendulum

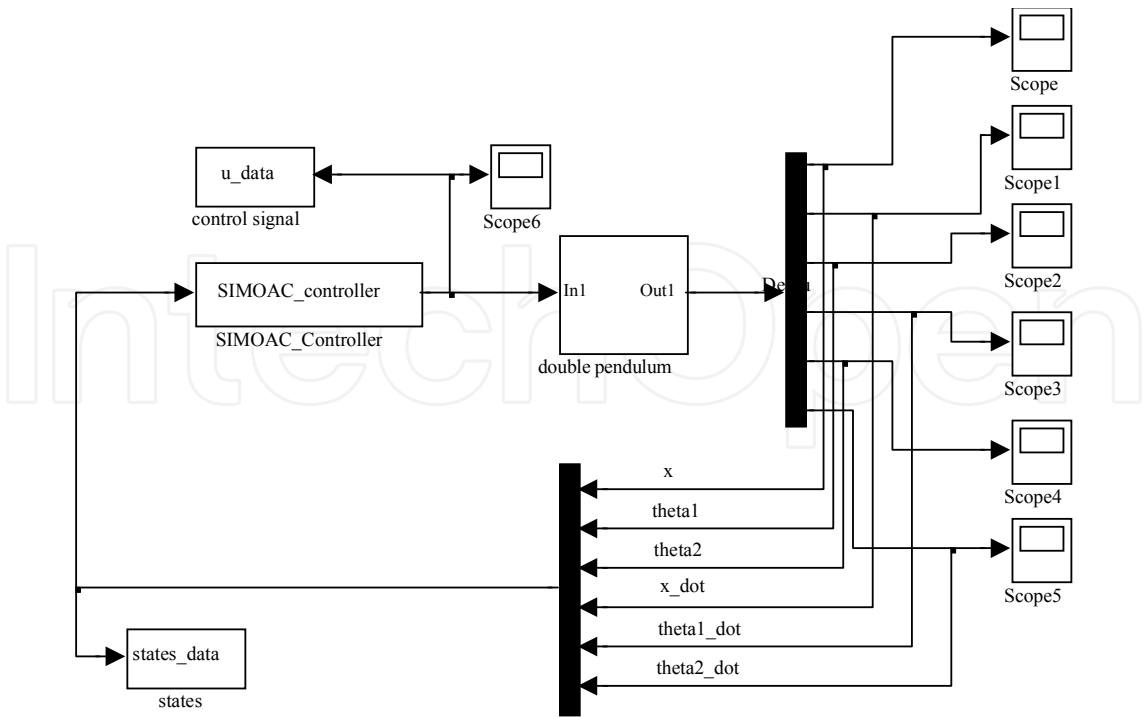


Figure12. Nonlinear stabilizing control simulating system of double inverted pendulum

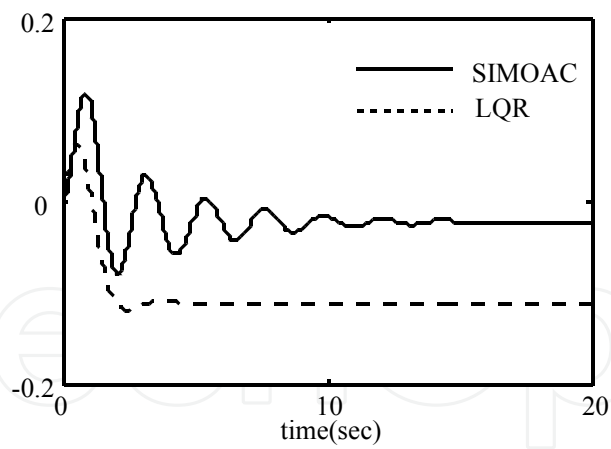


Figure 13. Displacement error of car

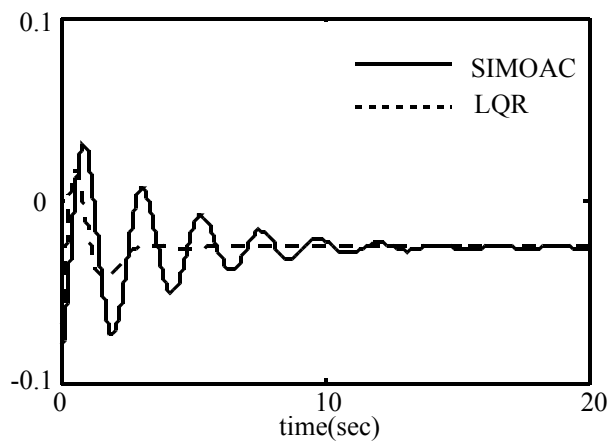


Figure 14. Angular displacement error of the lower rod

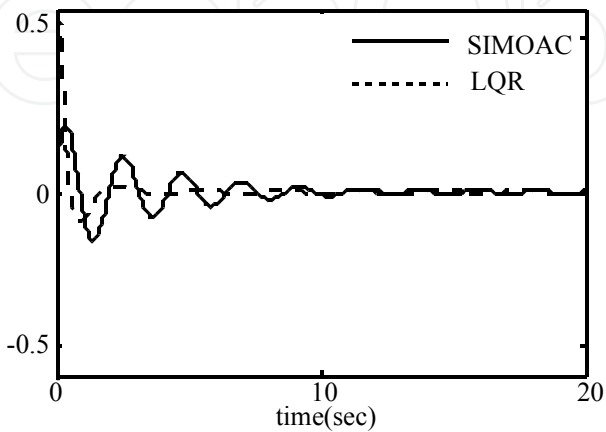


Figure 15. Angular displacement error of the upper rod

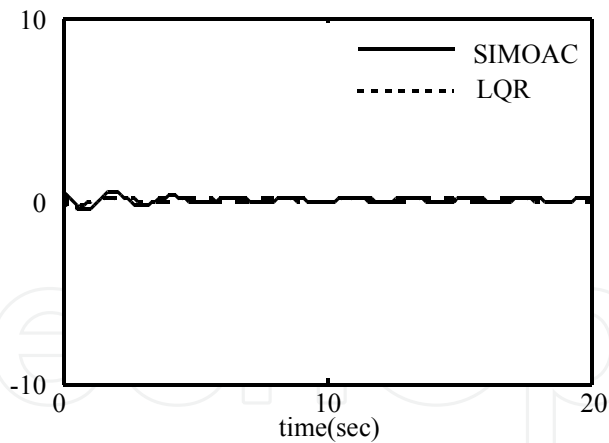


Figure 16. Adaptive control signal

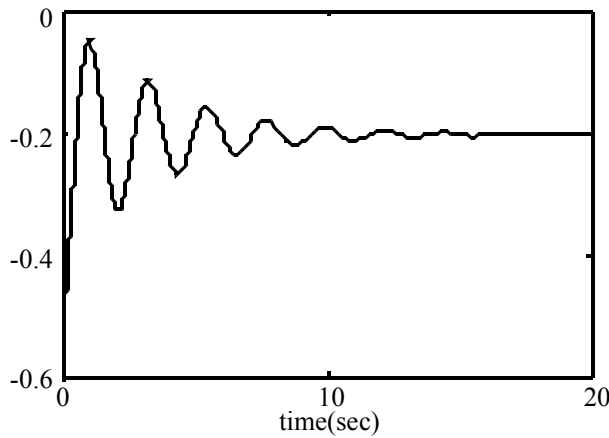


Figure 17. The approximation of uncertainty $-(CB(X))^{-1}\hat{u}_{m_1}(t)$

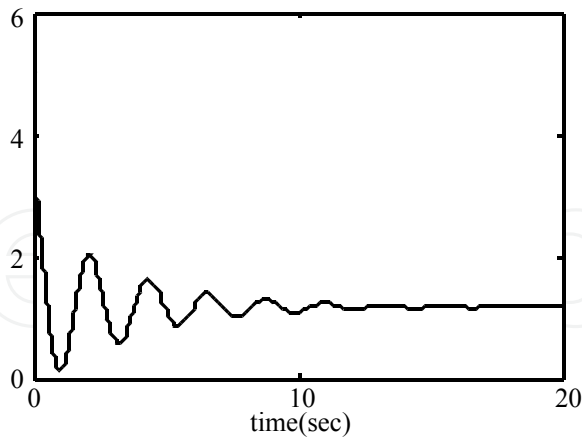


Figure 18. Behavior of sliding function $s(t)$

4. Conclusions

In this chapter, two sliding mode adaptive control strategies have been proposed for SISO and SIMO systems with unknown bound time-varying uncertainty respectively. Firstly, for a typical SISO system of position tracking in DC motor with unknown bound time-varying dead

zone uncertainty, a novel sliding mode adaptive controller is proposed with the techniques of sliding mode and function approximation using Laguerre function series. Actual experiments of the proposed controller are implemented on the DC motor experimental device, and the experiment results demonstrate that the proposed controller can compensate the error of nonlinear friction rapidly. Then, we further proposed a new sliding model adaptive control strategy for the SIMO systems. Only if the uncertainty satisfies piecewise continuous condition or is square integrable in finite time interval, then it can be transformed into a finite combination of orthonormal basis functions. The basis function series can be chosen as Fourier series, Laguerre series or even neural networks. The on-line updating law of coefficient vector in basis functions series and the concrete expression of approximation error compensation are obtained using the basic principle of sliding mode control and the Lyapunov direct method. Finally, the proposed control strategy is applied to the stabilizing control simulating experiment on a double inverted pendulum in simulink environment in MATLAB. The comparison of simulation experimental results of SIMOAC with LQR shows the predominant control performance of the proposed SIMOAC for nonlinear SIMO system with unknown bound time-varying uncertainty.

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The objective of this book is to provide an up-to-date and state-of-the-art coverage of diverse aspects related to adaptive control theory, methodologies and applications. These include various robust techniques, performance enhancement techniques, techniques with less a-priori knowledge, nonlinear adaptive control techniques and intelligent adaptive techniques. There are several themes in this book which instance both the maturity and the novelty of the general adaptive control. Each chapter is introduced by a brief preamble providing the background and objectives of subject matter. The experiment results are presented in considerable detail in order to facilitate the comprehension of the theoretical development, as well as to increase sensitivity of applications in practical problems

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