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On Risk and Reliability Studies of Climate-Related Building Performance

Krystyna Pietrzyk and Ireneusz Czmoch

Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/intechopen.71684>

Abstract

A design strategy based on integration of the building form and structure with its external environment in order to take advantage of natural forces (wind and buoyancy effects) has been evaluated in terms of risk and reliability measures. Tools for the probabilistic analysis (First-Order Reliability Method (FORM), Monte Carlo) have been presented and applied in the probabilistic modelling and sensitivity analysis of the response function of the studied building physics problem. Sensitivity analysis of the influence of basic random variables on the probability distribution of a response function is straightforward in FORM methodology. The case-based studies of probabilistic modelling of uncertainties coupled to wind speed and temperature difference through the specified building/environment system have been presented (i.e., the distribution models of the air change rate *ACH* and the dynamic *U* value characterising thermal performance of dynamic insulation). Sensitivities of the probability model of *ACH* to the parameters of wind speed and temperature distributions have been estimated for the consecutive values of the air change rate using FORM methodology. Reliability of *ACH* turned out to be most sensitive to the shape parameter of the wind speed distribution (in two-parameter Weibull model). The probabilistic risk analysis along with the effective tools for sensitivity analysis can be used to support design decisions and also to develop better models for evaluation of building performance.

Keywords: building performance, environment, risk, reliability, probabilistic approximation, FORM, sensitivity, climate, climate mitigation, wind, air infiltration, *ACH*, dynamic *U* value

1. Introduction

Developing tools to support decision-making to ensure comfort and safety in built environment while taking into account climate change challenges becomes important. 'Energy is the dominant contributor to climate change, accounting for around 60% of total global greenhouse

gas emissions' [1]. Reducing the carbon intensity of energy is a key objective in long-term climate goals. Hence, choosing a strategy based on integrating the building form and structure with its external environment in order to take advantage of natural forces (for natural ventilation, solar heating, etc.) is an example of design decisions leading towards mitigation of climate change.

Decisions concerning the choice of the design solutions for the particular project have to be taken under uncertainties related to the unknown and variable conditions, i.e. random climatic conditions, uncertain material performance, uncertain user behaviour, etc. Confronted with significant uncertainty, deterministic modelling supporting design process has been proved to be insufficient for decision-making. However, as it is said in [2] 'Existing engineering-based models are unable to propagate uncertainties through the model, and are therefore limited in their ability to display the impact of uncertainties to decision makers'. Facing that challenge, the chapter includes the discussion of the models and the tools applied by the authors for the probabilistic transformation of uncertainties of climatic parameters through a building/environment system for the predictive modelling of building performance.

The method for the quantification of building performance in terms of probability of poor performance (failure) and satisfactory performance (safe behaviour, in general meaning) is presented. Next, the tools for the probabilistic analysis are described (FORM, Monte Carlo) in relation to probabilistic modelling and possible applications of sensitivity analysis. One of the important results of analysis is the probability distribution functions of different performances as the responses of building/environment systems to the environmental loads. Such analysis requires estimation of some climatic parameters in terms of frequency of occurrence and appropriate statistics.

The chapter includes the case-based studies of probabilistic transformation of uncertainties coupled to wind and temperature through the specified building/environment system to show the effect on the distribution model of the air change rate and further on the distribution model of the dynamic thermal transmittance (dynamic U value) of the building envelope. Furthermore, the estimated distribution models could be included in risk/reliability calculations, carried out with FORM tools. The analysis of the sensitivity of the distribution of ACH with respect to the randomness of wind speed and outdoor temperature exemplifies the potential of the FORM tools, which can be effectively used to find out the probabilistic characteristics typical for the combination of the important variables influencing climate-structure interaction.

2. Risk perspective on design for sustainable development

Design for sustainable development can be approached using risk analysis tools. To minimise the risk of undesired consequences while increasing the chance to enhance the quality of life becomes the basic design objective. The design goal can be expressed in other terms—how to secure reliability of design under risk constraints [3]. To be clear about the terminology used further, some definitions are given below.

- **Risk**—a state of **uncertainty** where some possible outcomes have an undesired effect or significant loss [4]. It can be expressed in terms of adverse consequences scaled by the probabilities of undesired outcomes.

- **Qualitative risk**—Relative measure of risk based on ranking or separation into descriptive categories such as low, medium and high or on a scale, e.g., from 1 to 10. The example is the risk matrix of the failure mode [5] where failure severity is described by categories: minor, major, critical and catastrophic, whereas failure frequency is described in terms of very unlikely, remote, occasional, probably and frequent. A combination of both gives a qualitative assessment of risk as low, moderate or high.
- **Quantitative risk**—The most common quantification of risk is the product of likelihood of occurrence and the effect of the hazardous event [6]. Risk is treated as convolution of hazard and vulnerability, and it refers to mutual conditioning of two phenomena [7].
- **Risk assessment** is the determination of quantitative or qualitative estimate of risk related to the well-defined situation and the recognised hazard.
- **Probabilistic risk assessment** denotes the methods dealing with computation both the likelihood of undesired event and severity of possible negative consequences due to occurrence of such event.
- **Risk management process**—‘the systematic process of identification, assessment, monitoring and control of risk’ [8].
- **Reliability**—Ability of a system to satisfactorily perform under the specified conditions of use over an intended period of time. It is quantified by the probability of satisfactory (or safe) performance [9]. **Unreliability** is measured by the **probability of failure** (undesired performance).

2.1. Risk perspective on climate change challenges

Climate change threatens life on our planet. In view of high uncertainty, qualitative or semi-quantitative risk analysis based on the different scenarios is often applied. Following the quantitative definition of risk, one can write

$$Risk = P[hazard] * Consequences \quad (1)$$

$P[hazard]$ is the probability of occurrence of undesired events leading to possible *Consequences* like loss, injury, or discomfort.

Risk reduction could be accomplished by decreasing the probability of undesired event as well as diminishing the scale adverse consequences. Risk reduction of climate change and its consequences can be accomplished by climate change mitigation (decrease of the probability of occurrence of adverse events) or climate change adaptation (decrease of the adverse consequences) described as follows:

Climate change mitigation—‘it consists of actions to limit the magnitude and/or rate of long-term climate change’ [10]. ‘It generally involves reductions in human (anthropogenic) emissions of greenhouse gases’ [11].

Climate change adaptation—‘anticipating the adverse effects of climate change and taking appropriate action to prevent or minimize the damage they can cause, or taking advantage of opportunities that may arise’ [12].

2.2. Risk assessment as a tool supporting design of buildings

Designing for the integration of the building form and structure with its external environment in order to use natural forces to secure comfort (passive strategies) is an example of activities towards mitigation of climate change. If it is supported by probabilistic prediction of a local climate changes, it can be viewed from the climate adaptation perspective too.

The needs for risk reduction related to the hazards induced by climate change become an important boundary condition in the modelling of building/environment system to support building design. Following the definition of risk (Eq. (1)), adverse consequences are indicated by the set {yes = 1, no = 0}, and as a result, the probability $P[\textit{hazard}]$ becomes the discriminating factor for comparison of different design solutions. It means that certain design could be chosen on the basis of comparison of probability of unsatisfactory performance or reliability, evaluated by a set of alternative design proposals. In that way risk assessment becomes a tool supporting design of buildings.

3. Transformation of uncertainties: probabilistic approach

Probability is a measure of uncertainty about future events. Probability of a performance of a building/environment system depends on the theoretical model used and the randomness of the influencing parameters. The epistemic uncertainty about the theoretical models applied together with the aleatory uncertainty coupled to the randomness of important phenomena contributes to the final uncertain outcome, as the result of transformation of uncertainties throughout the model. Methods and tools for probabilistic reliability analysis can be used to estimate the probabilistic response of the structure to the random climatic load. They could be an important part of risk-based design process built upon the framework of risk management methodology as proposed in [13].

3.1. Probabilistic analysis with FORM

Development of reliability methods resulted in variety of powerful algorithms to estimate probability of failure for complicated physical and mathematical models of building systems incorporating random variables (i.e. properties or actions). FORM denotes 'First-Order Reliability Method', which has been developed by many researchers in about 40 years ago. Short description of the FORM basics as well as sensitivity tools is presented below. For details, check [14], and for application in building physics, look in [15]. First-order reliability method (FORM) is the most popular approach applied in practice.

Once the response of a system characterised by a set of basic random variables and a mathematical model describing the relationship among them has been established, the probability density function of the response can be estimated with help of FORM tools.

In general, the performance of a system is analysed in the space $\Omega_X = \{X \in R^n\}$ of basic random variables X . For a given failure mode or serviceability requirement, represented by the limit state surface $g(X) = 0$, the space Ω_X is divided into the safe subset, i.e. satisfactory

performance, $\Omega^S = \{X \in R^n; g(X) > 0\}$, and the failure subset, $\Omega^F = \{X \in R^n; g(X) \leq 0\}$, i.e., unsatisfactory performance. If all random variables are continuous with the multivariate joint probability density function (PDF) $f_X(x)$, the failure probability is given by the integral

$$P_f = \int_{\Omega^F} f_x(x) dx \quad (2)$$

The integral (Eq. (2)) can be evaluated exactly for a few cases with the most important one: the linear limit state surface and multidimensional normal (Gaussian) distribution function of variables X .

FORM algorithm starts with the non-linear transformation. Non-normal random vector X is transformed into a standard normal (Gaussian) vector Y with zero mean and unit covariance matrix $C_{YY} = I$. The limit state surface $g(x) = 0$ is mapped into a limit state surface $G(y) = 0$. Next, the design point y^* , i.e. the point on the limit state surface with the minimum distance to the origin of the Y space, is determined by solving the non-linear optimisation problem with a non-linear constrain $G(y) = 0$:

$$\beta = \min \sqrt{y^T y} \quad \text{for } y \text{ on } G(y) = 0 \quad (3)$$

The hyperplane tangential to the limit state surface at the point y^* is given by formula

$$\beta - \alpha^T y = 0 \quad (4)$$

where α is a unit outward normal vector to the hyperplane and β is the distance between the hyperplane and the origin. Since the random vector $Y = Y(X)$ has standard normal distribution, the first-order approximation of the failure probability is given by

$$P_f \cong P[\beta - \alpha^T Y \leq 0] = \Phi(-\beta) \quad (5)$$

where $\Phi(\dots)$ is the Laplace function.

The non-linear constrained optimisation problem (Eq. (3)) can be solved with many standard procedures as well as algorithms developed especially for this purpose, e.g. algorithm for the case of independent, non-normal random variables [16] and algorithm for problems with incomplete probability information [17].

All such solvers are iterative: for the assumed value of design point $x_{(k)}^*$, the values of limit state function $g(x_{(k)}^*)$ and its gradient $\nabla g(x_{(k)}^*)$ are determined. Next, a new position of design point $x_{(k+1)}^*$ is derived, and the process continues until the convergence criteria are fulfilled. If the state of the analysed system is described by the performance function defined by analytical formula, then the gradient can be evaluated easily, and one of algorithms solving the optimisation problem (Eq. (3)) can be applied directly. Otherwise the stochastic finite element method should be applied in order to calculate the value of the limit state function and its gradient vector at following values of design points.

The first-order reliability index β and the failure probability $P_f \cong \Phi(-\beta)$ depend on:

- Parameters $\mathbf{p} = (p_1, \dots, p_d)$ of the probability distributions of basic random variables, e.g. mean value, standard deviation, skewness or location, scale and shape parameters
- Any deterministic parameters $\Theta = (\theta_1, \dots, \theta_g)$ defining the form of the performance function $g(\mathbf{x}, \theta_1, \dots, \theta_g)$

Practical experience shows that the failure probability is usually a strongly non-linear function of the parameter θ , whereas the reliability index β is a rather linear function of the parameter θ . Thus the change in the failure probability due to the change of the parameter θ can be approximated as follows:

$$P_f(\theta + \Delta\theta) \cong \Phi(-\beta - \Delta\beta) \cong \Phi\left(-\beta - \frac{d\beta}{d\theta} \Delta\theta\right) \quad (6)$$

The sensitivity measures of the first-order reliability index do not depend on the curvature of the limit state surface $g(\mathbf{x}) = 0$ at the design point. Therefore, the application of sensitivity measures is limited to small changes of the values of the parameters.

The sensitivity of the first-order approximation of the failure probability $P_f \cong \Phi(-\beta)$ is directly related to the sensitivity of the reliability index β , since

$$\frac{dP_f}{d\theta} = -\varphi(\beta) \frac{d\beta}{d\theta} \quad (7)$$

If θ is a parameter of the limit state function $g(\mathbf{x}, \theta)$, then derivative of the reliability index with respect to the parameter θ is equal to

$$\frac{d\beta}{d\theta} = \frac{1}{|\nabla G(\mathbf{y}^*, \theta)|} \frac{\partial}{\partial \theta} G(\mathbf{y}^*, \theta) \quad (8)$$

where vector \mathbf{Y} contains independent standard normal variables related to the vector of basic random variables by transformation $\mathbf{Y} = \mathbf{Y}(\mathbf{X})$, and the limit state surface $g(\mathbf{x}, \theta) = 0$ defined in the space \mathbf{X} has been mapped into the surface $G(\mathbf{y}, \theta) = 0$. Since the FORM index β is equal to the minimum distance between the origin of the \mathbf{Y} space and the limit state surface $G(\mathbf{y}, \theta) = 0$, thus the design point \mathbf{y}^* is laying on the limit state surface; see **Figure 1**:

$$\beta = -\frac{\nabla G(\mathbf{y}^*, \theta)}{|\nabla G(\mathbf{y}^*, \theta)|} \mathbf{y}^* \quad (9)$$

The limit state surface in the \mathbf{X} space of basic random variables $g(\mathbf{x}, \theta) = 0$ does not depend on any parameter p_{ik} of a random variable X_i with the distribution function $F_i(x_i, p_{ik})$. However, the limit state surface $G(\mathbf{y}, \theta) = 0$ depends on parameter p_{ik} due to the transformation $\mathbf{Y} = \mathbf{Y}(\mathbf{X})$.

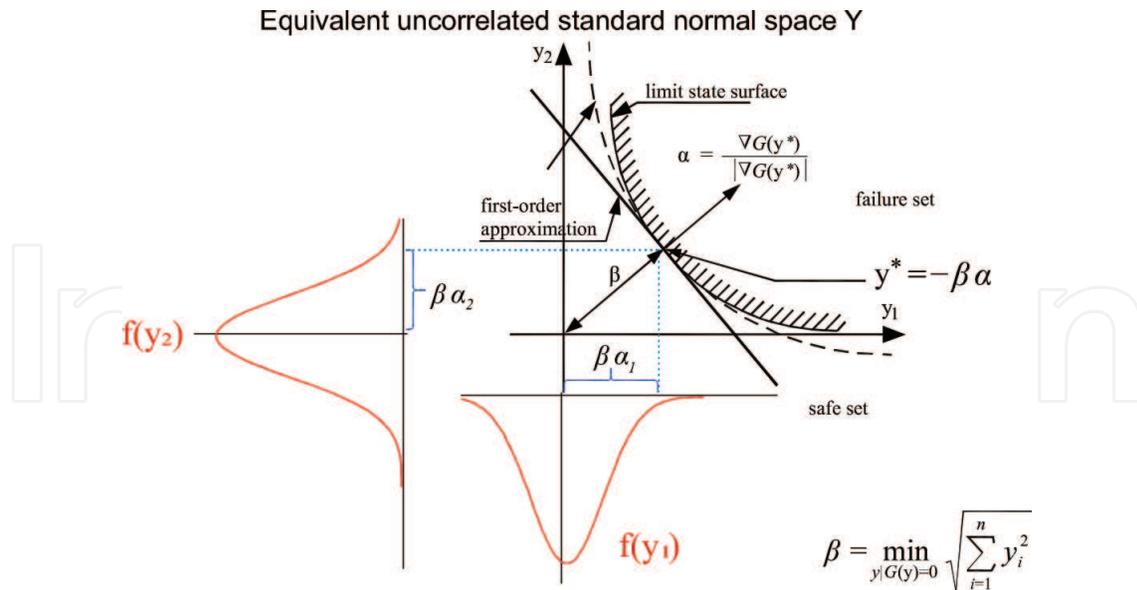


Figure 1. Illustration of sensitivity indices α_i (modified from [14]).

The derivative of the reliability index with respect to the parameter p_{ik} is given by relation

$$\frac{\partial \beta}{\partial p_{ik}} = \frac{1}{\beta} (\mathbf{y}^*)^T \frac{\partial}{\partial p_{ik}} \mathbf{y}^* \quad (10)$$

The derivative of the vector \mathbf{y}^* with respect to parameter p_{ik} have to be evaluated for each specific transformation $\mathbf{Y} = \mathbf{Y}(\mathbf{X})$. For details, see [14].

Sensitivity analysis shows how the uncertainty in the output response function of a system can be allocated to the different uncertainties in the basic variables. Sensitivity analysis is straightforward for FORM methodology. The influence of the basic random variable y_i on the statistics of the response can be quantified by the sensitivity indices α_i [18]:

$$\alpha_i = -\frac{d\beta}{dy_i} \quad \text{for } y_i = y_i^* \quad (11)$$

where \mathbf{y}^* is the design point in the space of normalised reduced random variables.

For uncorrelated random variables, the sensitivity vector α coincides with the direction cosines vector of the random variables [18]. Illustration of sensitivity indices is given in Figure 1.

3.2. FORM versus Monte Carlo simulation

An alternative technique applied for probability estimation of risk or reliability is Monte Carlo simulation (MCS). For the purpose of the zero–one indicator-based MCS, Eq. (2) defining the failure probability is given as follows:

$$P_f = \int_{\Omega^F} f_X(x) dx = \int_{\mathbf{k} \in \mathbb{R}^n} \mathbf{k} h_{\mathbf{K}}(\mathbf{k}) d\mathbf{k} \quad (12)$$

where the random vector \mathbf{K} has the non-negative sampling density function $h_{\mathbf{K}}(\mathbf{k})$ and is defined by the transformation

$$\mathbf{k} = I(g(\mathbf{k}) \leq 0) \frac{f_X(\mathbf{k})}{h_{\mathbf{K}}(\mathbf{k})} \quad (13)$$

and $I(u)$ is an indicator function:

$$I(u) = \begin{cases} 1 & \text{if } u \leq 0 \\ 0 & \text{if } u > 0 \end{cases} \quad (14)$$

In this way the failure probability is equal to the expectation of random vector with the non-negative sampling density function $h_{\mathbf{K}}(\mathbf{k})$:

$$P_f = E[\mathbf{K}] \quad (15)$$

The average of N simulated values of the random vector \mathbf{K} is the estimator of the failure probability, which variance is equal to

$$\text{Var}[\hat{P}_f] = \frac{1}{N(N-1)} \sum_{i=1}^N (k_i - E[\mathbf{K}_i])^2 \quad (16)$$

Monte Carlo simulation technique is a powerful tool to calculate the probability of failure for the system described by non-continuous performance function as well as discrete random variables. However, the basic drawback of the MCS is long CPU time calculation, if the failure probability is of the orders $10^{-2} - 10^{-6}$, since the sample size must be very large in order to obtain estimation of failure probability with low variance and narrow confidence interval. Various variance reduction techniques have been suggested to increase the efficiency of MCS. The basic idea is to assume a sampling density function $h_Y(\mathbf{y})$ that reduces the variance of the estimator \hat{P}_f . In the case of highly complicated systems, when time-consuming method must be applied to evaluate a single value of the limit state function, the MCS with the variance reduction technique is still an approach demanding a lot of computer time. Another drawback of the MCS, especially important, in the context of the chapter, is lack of the sensitivity analysis tools. It is simply impossible to run billions of simulation in order to study sensitivity of the system with respect to specific parameters.

4. Case-based risk/reliability studies of climate-related building performance

4.1. Building/Environment system performance

In the context of the ventilation design, air infiltration constitutes an important complement to air exchange. Furthermore, air infiltration can influence on the properties (thermal and structural)

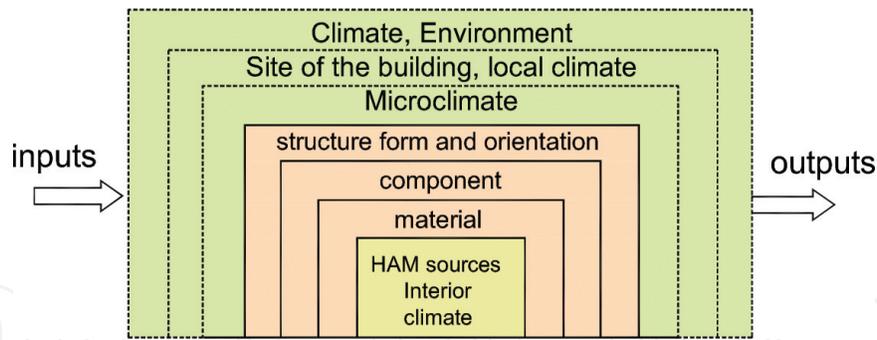


Figure 2. Building/Environment system applied in a traditional building physics analysis [19].

of building components. For some building technologies, e.g. lightweight timber frame with mineral wool filling, and loose mineral wool layers for roof insulation, the dependence of the thermal properties of building components on air infiltration can be observed; thus the interaction between, e.g. thermal transmittance and air infiltration should be taken into account. Therefore, it is important to apply a systemic approach to building/environment system performance taking into account different aspects of building physics. Due to the random natural driving forces governing the rate of air infiltration, the approach based on probabilistic methodology seems to be very well suited to handle these phenomena.

A building can be seen as a system transforming as well as resisting different loads (static and dynamic loads—caused by flow of air, heat and moisture) which is designed to ensure safe and comfortable living conditions inside the enclosure. The structure has to be designed in such a way that the possibilities of adverse consequences of this transformation, for example, loss of stability of the structure, inadequate ventilation or mould growth inside a building, have been minimalised. This systemic approach provides a proper theoretical tool for the analysis of the interrelations between the structure, its environment and its performance. An example of systemic model of a building, applicable in building physics studies, is shown in **Figure 2** [19].

The local environmental conditions interact with building structure to form a microclimate around a building. Sources of heat, air and moisture, including the products of HVAC systems as well as user behaviour, build up the internal load. Physical boundary conditions define the level of integration of the structure with the environment.

The output of the system can be described by the performance of the building (structure and enclosure). The performance can be considered in terms of safety, comfort and energy consumption and described by various parameters depending on physical conditions of the building structure and inside air. Those parameters should fulfil the performance requirements in order to prevent undesired performance (failure state) occurrence.

4.2. Case description

4.2.1. Description of the test house

The object of the study is a timber-framed low-rise naturally ventilated building with aspect ratio 2 and slope of the roof of 45° [20]. The building site in the district of Gothenburg has been considered and can be described as a semi-urban area with the surface roughness equal

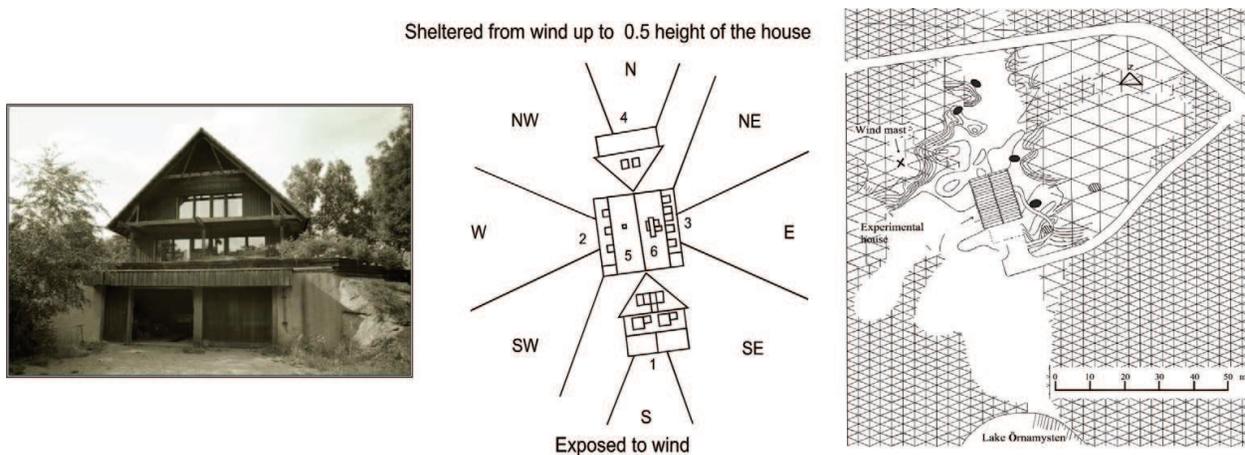


Figure 3. Object of the study – the Building/Environment system [20].

to 0.3 m. Example has been worked out for wind blowing from the west. It is assumed that the building is surrounded by other obstructions (other buildings, topography, vegetation, trees etc.) equivalent to half of its height. The following input data are used: volume of the house $V = 486 \text{ m}^3$, area of the building envelope $A = 336 \text{ m}^2$ and internal temperature $T_{int} = 20^\circ \text{ C}$.

The house was constructed in 1979 with the intention of using it for experimental studies in building physics with focus on ventilation and energy saving. The garage with doors facing south is located in the extended south part of the concrete cellar as shown in **Figure 3**.

4.2.2. Measurement programme

The following parameters have been measured, as is shown in **Figure 4**: (1) leakage characteristics of the house using blower door tests, (2) mean value of pressure difference across the six building components with Validyne pressure transducers, (3) wind speed and wind direction with the anemometer located on a small hill about 25 m from the house, (4) internal and external temperatures and (5) limited number of tracer gas measurements of ACH . The measurement programme has been carried out during 8 months. As a result, hourly mean data have been registered.

The results of the pressure drop measurements have been used to validate the air infiltration through the envelope. An opening under the garage door has been treated separately in the calculation model for air change rate [20, 21].

4.3. Modelling of air change rate

The applied infiltration model takes into account the contribution of wind and stack effect to the total air change rate (ACH) in the following form [22]:

$$ACH = \sqrt{ACH_s^2 + ACH_w^2} \quad (17)$$

where ACH_s is the air change rate caused by stack effect and ACH_w is the air change rate caused by wind.

The model refers to low-rise building with light-weight construction, single ventilation zone, single temperature zone and steady-state conditions of air flow.

The infiltration model developed by Pietrzyk [20] indicates the air change rate ACH as a random function of three basic random variables: **temperature difference**, **wind speed** and **wind direction**. Wind direction is divided into eight sectors and is treated as a uniformly distributed within each sector. Finally, the air change rate conditioned by the wind direction sector is given by the following expression:

$$ACH_d = \sqrt{s_1 \Delta T^2 + s_2 |\Delta T| + s_3 |\Delta T|^{1.5} + w_{d,1} v_d^4 + w_{d,2} v_d^2 + w_{d,3} v_d^3} \quad (18)$$

where d is a wind direction sector; $s_1, s_2, s_3, w_{d,1}, w_{d,2}$ and $w_{d,3}$ are the deterministic coefficients related to the house dimensions, position of neutral pressure layer, level of external and internal pressure coefficients; ΔT is an ext.-int. temperature difference; and v denotes wind speed.

Distributions of air change rate averaged over one hour (1-h) periods at a randomly chosen time in the year have been estimated with the help of the model described by Eq. (18). One-hour mean data ensure steady-state conditions of the airflow through the building envelope. Wind is the most important source of variations in the process of air exchange. However, according to wind energy spectrum presented in [23] for the frequency range 0.00014–0.0033 cycles/hour related to time interval from 5 min to 2 h, the wind speed varies slightly. This range is called spectral gap. Measurements carried out for periods of that duration can be regarded as representing the steady-state conditions [24].

Performance criteria in terms of ACH should take into account the minimum threshold evaluated with respect to unhygienic conditions. Then, probability of unsatisfactory performance is equal to $P[ACH < threshold]$.

Figure 4 presents how the building response such as ACH depends on the uncertain environmental conditions. The wind speed is traced from the meteorological station to the site and eventually to the building envelope which in turn influences the microclimatic conditions near to structure. The zone of wind-structure interaction is included in the model of designed system (see boundary conditions of the system presented by the solid lines). Serviceability performance due to wind action can be evaluated in terms of probability of undesired performance (failure). It is worth noticing that measurement data have been used to model the building performance as well as to validate the results of analysis carried out with the help of the established model.

The probability density function for air change rate as a function of basic random variables 1-h mean wind speed and 1-h mean temperature difference at time points chosen randomly during the year has been estimated with the help of the FORM sensitivity analysis for the performance function

$$g(x_1, x_2; a) = ACH(x_1, x_2) - a \quad (19)$$

where $ACH(x_1, x_2)$ is given by Eq. (18), $x_1 = \Delta T$ and $x_2 = v_d$.

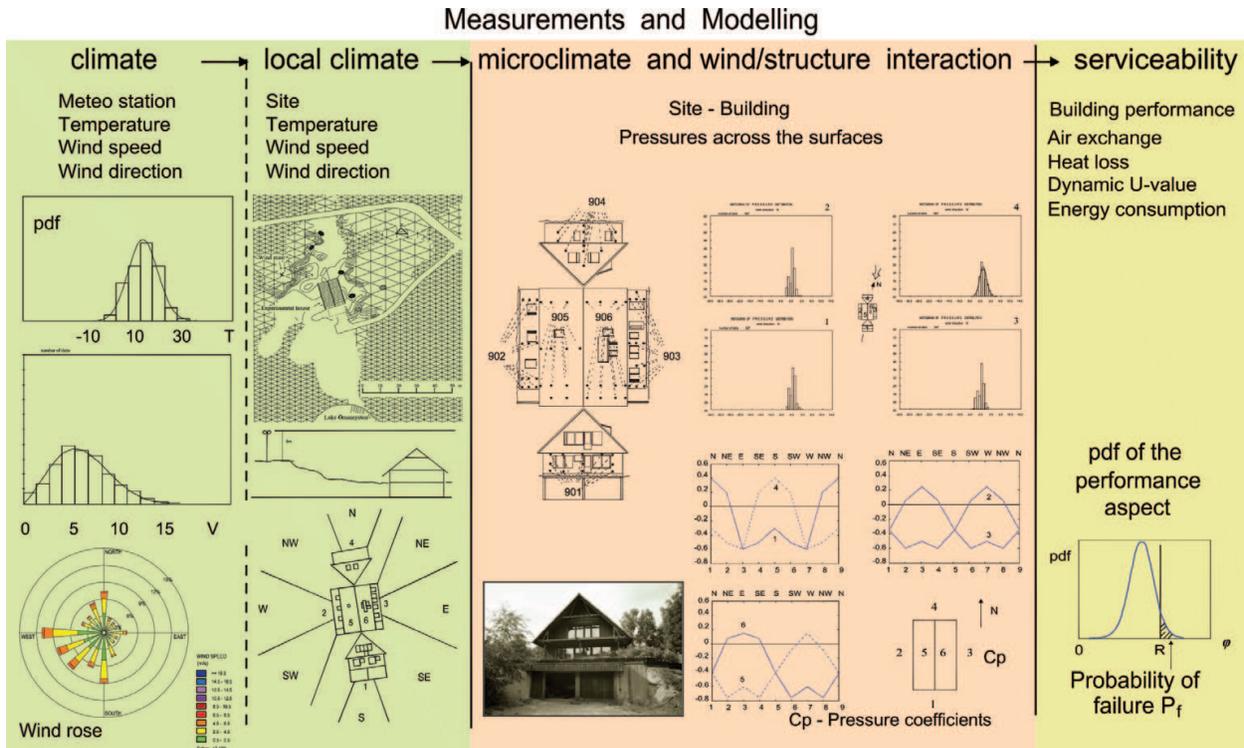


Figure 4. Transformation of uncertainty within the modelling of building performance.

The parametric sensitivity analysis applied to FORM measures (reliability index or failure probability) is used in order to determine the probability density function for the random response $ACH = ACH(x_1, x_2)$. The cumulative distribution function of random function $ACH = ACH(x_1, x_2)$ is actually equivalent to the probability of failure defined for the performance function Eq. (19):

$$F_{ACH}(a) = P[ACH(x_1, x_2) \leq a] = P[g(x_1, x_2; a) \leq 0] \tag{20}$$

Thus, the cumulative probability function can be estimated with the help of the FORM analysis:

$$F_{ACH}(a) \approx \Phi(-\beta_a) \tag{21}$$

where $\Phi(u)$ is the Laplace function and the reliability index β_a has been determined for the limit state surface $g(x_1, x_2) = ACH(x_1, x_2) - a = 0$ defined for a given value of parameter a .

Following the sensitivity measures presented earlier in the chapter, the probability density function of the random response ACH can be estimated with the help of formula:

$$f_{ACH}(a) \approx -\varphi(\beta_a) \frac{d\beta_a}{da} = \frac{\varphi(\beta_a)}{|\nabla G(\mathbf{y}, a)|_{for \mathbf{y}=\mathbf{y}^*}} \tag{22}$$

where $\varphi(u)$ is the probability density function of the standard Gaussian distribution, $G(\mathbf{y}, a)$ is the limit state function in the space $\mathbf{Y} = \mathbf{Y}(\mathbf{X})$ of normalised random variables and \mathbf{y}^* is the design point, i.e. the point on the surface $G(\mathbf{y}, a) = 0$ at the shortest distance to the origin of the

coordinate system. The value of probability density function of random response function $f_{ACH}(ACH)$ can be obtained by means of FORM sensitivity analysis for consecutive values of parameter a ; for details, see [14].

4.3.1. Wind transformation – climate/local climate/microclimate

The input basic random variable for the infiltration model is wind speed in the vicinity of the building envelope. Wind speed and direction are usually measured at the meteorological stations. The mean value of 1-h mean wind speed can be evaluated from the mean value of 10-min mean wind speed obtained from meteorological station using the principle that the mean velocity increases by 5% when the averaging period is reduced from 1 h to 10 min. The transformation of the abovementioned data to the site of the building is often needed especially for wind that changes drastically due to the roughness of the ground surface.

The hourly mean wind speed v is assumed to follow the two-parameter Weibull distribution with probability density function as follows:

$$f(v; c, \lambda) = \frac{\lambda}{c} \left(\frac{v}{c}\right)^{\lambda-1} \exp\left\{-\left(\frac{v}{c}\right)^\lambda\right\} \quad (23)$$

where λ is a shape parameter and c is a scale parameter.

In general, parameters λ, c for wind speed averaged over 1-h should be estimated for different wind direction sectors that shall result in quite different Weibull distributions due to two reasons: directional variability in the terrain surrounding the house and the predominance of certain wind directions. In particular, most building sites are subjected to sheltering effects from topography, trees and buildings. The roughness of the ground surface changes the mean wind speed and its turbulent characteristics and is described by the surface roughness height (aerodynamic roughness length) denoted z_0 . Roughness height depends on the mean element height of the roughness field. The results of laboratory measurements show that the value of z_0 is approximately equal to 1/30 of the height of the roughness elements. **Table 1** presents the classification of roughness height for different types of surfaces with reference to the categories of terrain roughness used in Swedish Code [24].

Transformation of the wind speed between terrains of different surface roughness is possible due to the similarity theory [25], based on the equilibrium boundary layer height, which is according to [26] equal to 1200 m. The wind flows with the gradient velocity v_g along the isobars:

$$v_g = \frac{u_*}{\kappa} \left[\ln\left(\frac{u_*}{f z_0}\right) - A_u \right] \quad (24)$$

where z_0 is the surface roughness height (m); κ is Karman's constant, $\kappa = 0,4$; A_u is const.; assumed -1 ; f is Coriolis parameter (1/s), $f = 1,2 \cdot 10^{-4}$ for latitude of order of 50° [25]; and v_g is gradient velocity (m/s).

u_* is friction velocity depending on the surface shear stress τ_0 as given in Eq. (25):

$$u_* = \sqrt{\tau_0/\rho} \quad (25)$$

where τ_0 is surface shear stress (kg/ms^2) and ρ is air density (kg/m^3).

The mean velocity profile $u(z)$ near the ground, where z is the height above the ground ($z < 100 \text{ m}$), can be expressed by the log-law model described by Eq. (24) assuming ideal conditions, i.e. the uniform height of roughness field and the neutrally stable atmosphere when thermal gradient is weak or absent.

$$u(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right) \quad (26)$$

Eq. (26) is used for wind speeds greater than 10 m/s. For low wind speeds, the influence of thermal gradient both for unstable and stable atmosphere should be taken into account. The modified logarithmic formula can be found in [27].

The 10-min mean wind velocity measured at a meteorological station (usually at the level of $z = 10 \text{ m}$ above the ground) for upwind surface roughness z_{0m} can be transformed to any other location described by upwind surface roughness height z_{0s} through similarity of the wind speed at the gradient height for all terrain types [27]. Hence, the gradient velocity takes the same value for both locations and can be expressed by Eq. (27):

$$v_g = \frac{u_{*m}}{\kappa} \left[\ln\left(\frac{u_{*m}}{f z_{0m}}\right) - A_u \right] = \frac{u_{*s}}{\kappa} \left[\ln\left(\frac{u_{*s}}{f z_{0s}}\right) - A_u \right] \quad (27)$$

where u_{*s} is friction velocity at the building site (m/s) and u_{*m} is friction velocity at the meteorological station (m/s).

Types of surface roughness	Height z_0 (m)	Category
Calm open sea, water area	0.0001	I
Sand surface (smooth)	0.001	I
Snow surface	0.003	I
Bare soil	0.005	I
Airport runway area, mown grass	0.01	I
Farmland with very few buildings, trees, etc.	0.03	I
Farmland with open appearance	0.05	R
Farmland with closed appearance	0.1	II
Many trees and bushes	0.2	II
Shelter belts	0.3	II
Suburbs	0.5	II
City, forest	1.0	III

Table 1. Roughness height for different types and categories of surfaces, acc. to Swedish Code [24].

The friction velocity at the meteorological station u_{*m} is computed from Eq. (28), which has been derived on the basis of Eq. (27), by substituting the friction velocity at the site with friction velocity at the meteorological station.

$$u_{*m} = \frac{u_m(z)\kappa}{\ln\left(\frac{z}{z_{0m}}\right)} \quad (28)$$

where $u_m(z)$ is 10-min mean wind speed measured at the meteorological station at the height z . The mean wind velocity $u_s(z)$ at the site and at the height z characterised by upwind surface roughness z_{0s} can be estimated from Eq. (26):

$$u_s(z) = \frac{u_{*s}}{\kappa} \ln\left(\frac{z}{z_{0s}}\right) \quad (29)$$

The ratio between wind velocity at the site and the wind velocity measured at the meteorological station denoted as η is a function of the surface roughness z_{0m} and z_{0s} :

$$\eta = \frac{u_s(z, z_{0s})}{u_m(z, z_{0m})} \quad (30)$$

It can be shown that non-linear relationship $\eta(u_m)$ can be approximated with errors of order of 7% or less by a constant factor η for specified surface roughness at the building site. As the surface roughness appears in an implicit form in the expression for wind velocity (Eq. (29)), an analytical expression is not available. Instead, values of the factor η have been computed for different combinations of the surface roughness at the site and at the meteorological station (Table 2).

Simple wind transformation between categories of roughness is possible for $z < 20z_0$ [28]. Thus, for $z = 10 \text{ m}$, the transformation is valid for $z_0 < 0,5 \text{ m}$. In the case of non-homogeneous upwind terrain, implementation for multiple roughness changes is required [26].

Values of wind speed measured at the meteorological station can be transformed using Eqs. (27)–(29). The statistical parameters of wind speed evaluated for the site, i.e. the mean value μ_{u_s} and the standard deviation σ_{u_s} , can be easily evaluated on the basis of the mean wind speed measured at the meteorological station μ_{u_m} and the standard deviation of the wind speed measured at the meteorological station σ_{u_m} , since the wind speed at site is related the wind speed measured at the meteorological station by simple Eq. (30). Hence:

	$z_{0m} = 0.01$	$z_{0m} = 0.05$	$z_{0m} = 0.3$
$z_{0s} = 0.01$	1.00	1.16	1.53
$z_{0s} = 0.05$	0.86	1.00	1.31
$z_{0s} = 0.3$	0.66	0.76	1.00

Table 2. Values of the ratio η corresponding to different roughness conditions.

$$\mu_{u_s} = \eta\mu_{u_m} \text{ and } \sigma_{u_s} = \eta\sigma_{u_m} \quad (31)$$

Concluding, the shape parameter λ characterising the distribution of wind speed at the site of the building remains the same as the shape parameter λ for the meteorological station. The scale parameter for the distribution of wind speed at the site is equal to ηc (Eq. (23)).

Hence, probability density function of wind speed transformed to building location is given by 2p-Weibull probability model:

$$f(v; \eta c, \lambda) = \frac{\lambda}{\eta c} \left(\frac{v}{\eta c}\right)^{\lambda-1} \exp\left\{-\left(\frac{v}{\eta c}\right)^\lambda\right\} \quad (32)$$

where c is a scale parameter and λ is a shape parameter of the PDF of wind speed measured at the nearest meteorological station.

Modelling of microclimate around the structure takes into account the influence of structure form, orientation and the quality of the surrounding. Usually, the effect of wind pressure on the façade is estimated with the help of the tabulated values of wind pressure coefficients. In the analysed case pressure differences across the six building components on the structure were measured.

4.3.2. Air flow through the building envelope (influence of wind and temperature)

Some building performance aspects are dependent on the wind-structure interaction. Wind together with temperature difference causes airflow through building envelope.

The probability distribution model of external temperature depends on the specific geographical region. For temperate regions characterised by four seasons evenly distributed over the year, the normal (Gaussian) model with probability density function $\varphi(T; \mu_T, \sigma_T)$, given by Eq. (33), can be used for 1-h mean external temperature at “a random time” [29, 30]:

$$f(T; \mu_T, \sigma_T) = \frac{1}{\sigma_T \sqrt{2\pi}} \exp\left\{-\frac{1}{2} \left(\frac{T - \mu_T}{\sigma_T}\right)^2\right\} \quad (33)$$

Also the full-scale measurements carried out near Gothenburg indicate [20] that the outdoor temperature can be approximated by the normal distribution.

Climatic data consist of 40-year record of observations made on meteorological stations at the airport in Säve, near Gothenburg. External temperature at the building site has been assumed to be equal to the temperature measured at the meteorological station, and its randomness is modelled by the normal distribution with the mean value of 11.1 and the standard deviation of 6.1 as shown in **Figure 5**.

Temperature difference across the building envelope is also described by the normal PDF but shifted towards positive values by the average value of internal temperature.

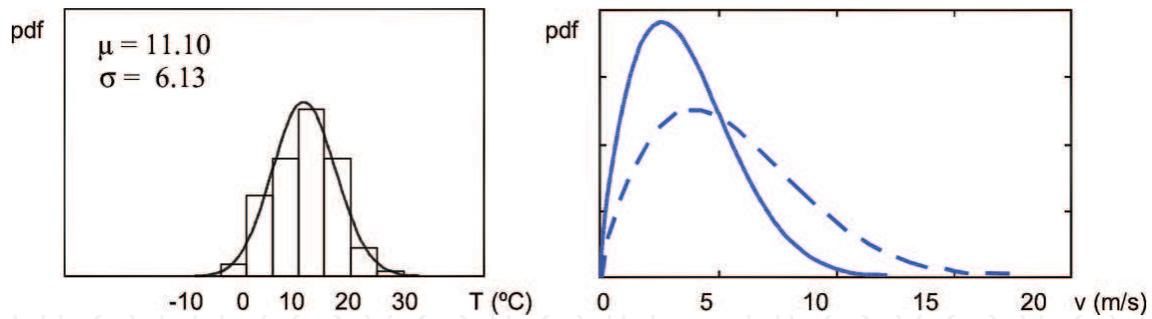


Figure 5. Normal PDF of ext. temperature T ($^{\circ}\text{C}$) (left) and PDF of wind speed (m/s) for data coming from the Säve meteorological station (dashed line) and for local wind (solid line).

The Weibull probability density function for a local wind speed has been evaluated on the basis of 10-min mean values of wind speed measured at meteorological station (see **Figure 5**). The meteorological station is located at the airport with assumed surface roughness 0.01. The ratio between wind velocity at the site and the velocity measured at the meteorological station has been calculated and is equal to 0.66 (**Table 2**). Probabilistic models of local wind speed together with wind speed measured at the meteorological station are given in **Table 3**.

The probability density function of the random function ACH (**Figure 6**) has been evaluated using FORM approach (Eq. (22)). Probabilistic inference leads to the conclusion that the randomness of ACH is best described by the log-normal distribution with the mean value of 0.73 and the standard deviation of 0.38. Mean value and standard deviation are denoted, respectively, by μ and σ . The PDF of the air change rate due to stack effect ACH_s and the PDF of air change rate due to wind ACH_w are also shown in **Figure 6**. Randomness of air change

	Mean value	Standard dev.	Scale parameter	Shape parameter
Meteo	5.65	3.30	6.35	1.77
Local	3.73	2.18	4.19	1.77

Table 3. Stochastic parameters of the wind speed.

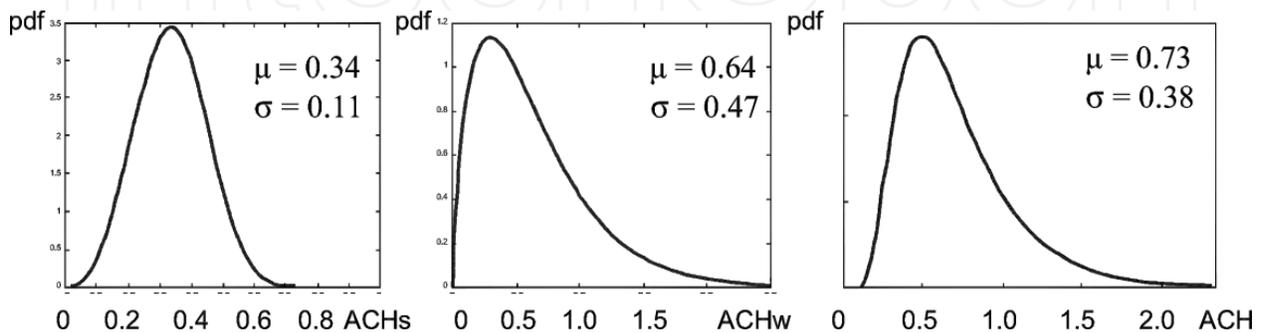


Figure 6. The probability density function for ACH_s (left), ACH_w (middle) and ACH (right) established with the help of FORM analysis.

rate due to stack effect can be described by the normal distribution whereas due to wind by the Weibull distribution skewed to the right.

4.3.3. Sensitivity analysis of the probabilistic variability of air change rate with respect to the variability of wind and temperature

Dependence of ACH on the mean values of input variables follows the trends showed by sensitivity indices for individual variables (see Figures 7–9) [31]. For the values of ACH above 1.0, where $-\alpha_{\Delta T}$ approaches 0 and $-\alpha_v$ is equal to 1, the changes of reliability indices are dependent almost only on the changes of wind speed. Concluding, the wind velocity and temperature difference contribute significantly to the variability of the air change rate with sensitivity indices up to 0.8 for ΔT (for lower ACH) and up to near to 1 for wind speed (for higher ACH (Table 4)).

Sensitivity of ACH distribution with respect to mean values and standard deviations of input variables leads to the following conclusions: (1) strong dependence on wind variation, (2) temperature difference variations affect only low values of ACH (up to 0.4), (3) variations of ΔT affect the lowest values of the ACH distribution, and (4) variations of the wind speed are significant for performance studies of ACH within the whole range of wind speed values.

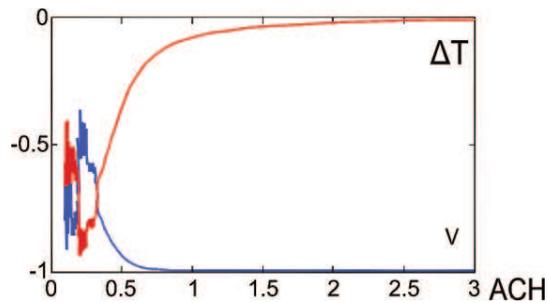


Figure 7. Course of sensitivity index α , for variables ΔT and v .

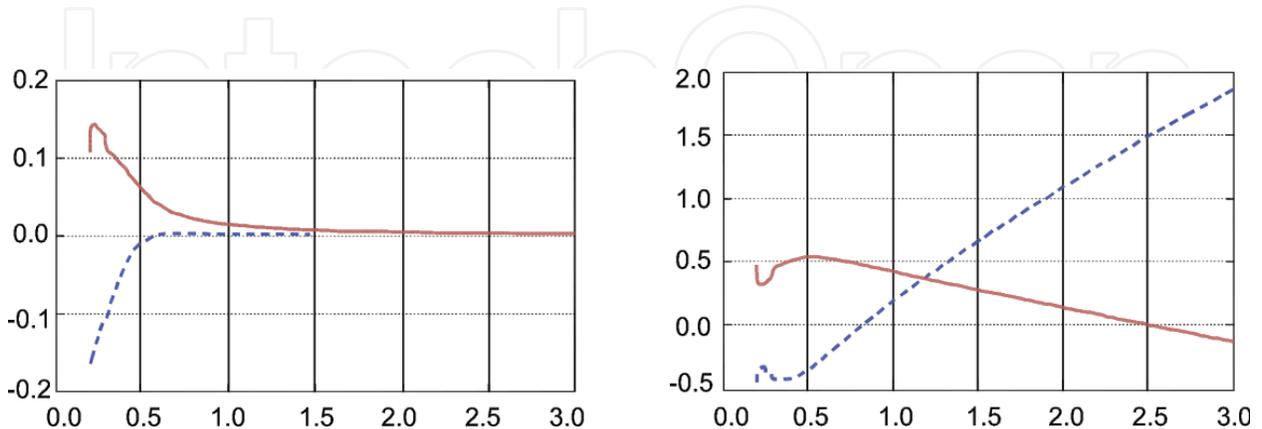


Figure 8. ACH sensitivity to the $\mu_{\Delta T}$ (solid) and the $\sigma_{\Delta T}$ (dashed) (left) and ACH sensitivity to the μ_v (solid) and the σ_v (dashed) (right).

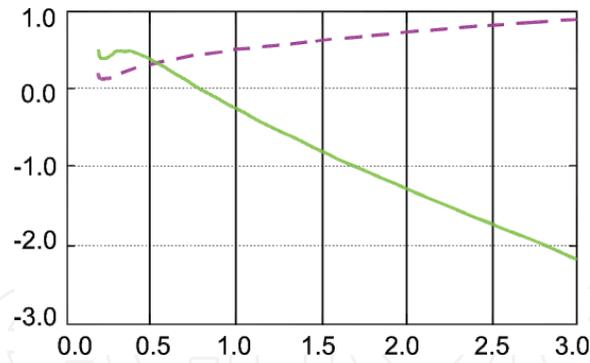


Figure 9. ACH sensitivity to scale (dashed) and shape (solid) parameter of Weibull distribution of wind speed.

	ACH = 0.32	ACH = 0.64	ACH = 3.0
$\alpha_{\Delta T}$	0.8	0.4	0.0
α_v	0.6	0.8	1.0

Table 4. Some results from Figure 7.

Figure 9 shows the measure of sensitivity of the PDF of ACH with respect to scale or shape parameter of the Weibull distribution of wind speed for consecutive values of air change rate. The changes of shape parameter are the most important for the distribution of air change rate, especially for the threshold values close to the tail of distribution.

4.4. Probabilistic modelling of airflow-dependent thermal transmittance

For lightweight timber frame with mineral wool filling, the dependence of the thermal properties of building components on air infiltration is well acknowledged. An example is so-called dynamic wall [32], specially designed to save energy. In such a wall, the ventilation air passes through the insulation exchanging heat with a porous material reducing its conduction heat loss. The air entering the building is preheated by the conduction heat of the insulation (infiltration case), or the air leaving the building heats up the insulating material (exfiltration case) [33]. In the case of dynamic wall, the thermal transmittance becomes the most interesting parameter that can vary with the climatic data. Dynamic wall as a natural heat exchanger is a future of high-performance housing. The interaction between thermal transmittance and airflow through the components should be considered while calculating heat loss through a building envelope. A modelling approach based on probabilistic methods is proposed in [34].

Probabilistic model of **dynamic U value** takes into account only some of the uncertainties related to the properties of the thermal insulation described by the thermal transmittance U^0 , the climatic load and the internal load coming from the building installations and occupants' behaviour (ventilation strategy). The model described by Eq. (34) can be used to estimate a probability distribution of the dynamic U value of the building envelope consisting of *i*-th elements with total area of A_{tot} :

$$U = \frac{1}{4} \sum_i^n Nu_i U_i^0 A_i \quad (34)$$

The Nusselt number Nu_i is equal to 1 for the element without convection flow. In general, the value of the Nusselt number depends on the velocity of the airflow through the insulation, the direction of the flow and the thickness and the density of the insulation.

The example of approximation of the probability density functions of a dynamic U value has been carried out with the help of FORM techniques. PDF of dynamic U value has been evaluated using FORM sensitivity analysis (see Section 3.1.1). It depends on statistical parameters of the joint distribution of two random variables: thermal transmittance U^0 (varying with the temperature) and wind as well as buoyancy-driven airflow in terms of air change rate ACH (see **Figure 10**). It has been assumed that stochastic information is limited to the parameters of marginal probability density functions of those variables and the correlation coefficient between them.

Probability density functions of thermal transmittance depend on the direction of the airflow through the envelope as well as on the probability model of the air change rate. Respectively, to the contribution of the natural forces (wind, temperature) and mechanical forces, different probability distributions (normal, log-normal, Weibull and gamma) can be fitted to model randomness of the air change rate [35]. In general, the probability density functions of the dynamic U value are skewed to the left—in the case of infiltration—and are skewed to the right, in case of exfiltration. The specific character of the relationship between Nusselt number and air change rate may explain these results. For the case of infiltration, the best fit according to the Kolmogorov-Smirnov test has been obtained for the Weibull distribution, while for the exfiltration case, the three-parameter gamma (or alternatively Gumbel) distribution has been obtained (see **Figure 10**).

The model could be further developed to include uncertainties due to other mechanisms and factors, e.g. influence of wind or radiation on external heat transfer coefficient or the influence of non-homogeneity of the material characteristics.

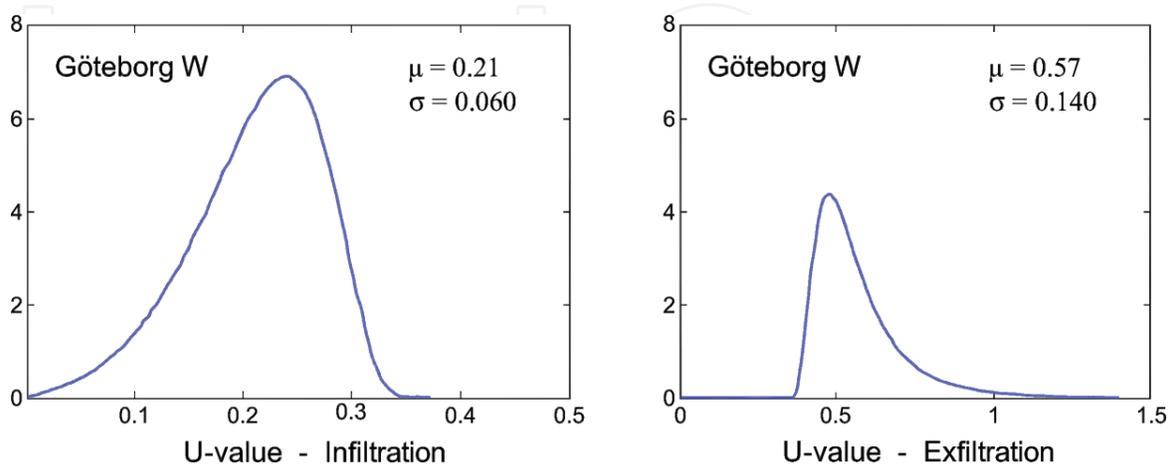


Figure 10. Probability density functions of dynamic U value for the cases of infiltration (left) and exfiltration (right) approximated for the building located near to Gothenburg for western winds.

The probabilistic model for estimation of **heat loss** accounting for interactions between ventilation and transmission heat losses has been presented in [20, 33]. The model predicts the probability density function of the heat loss distribution over a specified period of time (e.g. a heating season) on the basis of the design parameters of the house, temperature characteristics of the site as well as the air change rate due to mechanical or natural ventilation. The probability of heat loss exceeding certain number of kW can be compared for different design options concerning various ventilation strategies (natural or/and mechanical ventilation) and various transmittance properties (tight envelope contra dynamic wall) of the building envelope. Hence, rational engineering decisions promoting low-energy solution contributing to climate change mitigation can be taken into account in the design process.

5. Conclusions

Risk analysis together with appropriate tools can support building design strategies concerning climate change mitigation and adaptation. Lower levels of uncertainty can be handled by means of risk analysis based on system's risk or reliability estimations. In the case of higher order of uncertainties [36], other strategies could be developed based on the concept of resilience.

Risk analysis of building performance enables the selection of the best design based on comparison of probabilities of undesired performance estimated for alternative design solutions. Systemic approach gives opportunity to identify important relationships between variables. For example, air infiltration as a result of climate/structure interaction may be a significant variable in the thermal performance of building envelope. However, in order to handle the whole complexity of the real system, multivariable decision models for different design solutions should be further developed.

The examples of dynamic U values resulting in the different characters of distribution models for the cases of infiltration and exfiltration show that the probabilistic methods and tools can be effectively used to establish the probabilistic characteristics typical for the combination of the important variables influencing climate-structure interaction.

The sensitivity measures are important in the case of risk or reliability-based design. Sensitivity analysis of the distribution of a response variable (random function) with respect to the basic random variables and its parameters is straightforward for FORM methodology, whereas it is not easy in the case of the Monte Carlo simulation. The results of case studies show that the air change rate distribution depends on the temperature difference ΔT and the wind velocity significantly. Dependence of PDF model of ACH on the mean value of input variables is similar in the trends for both studied variables: temperature and wind speed. Sensitivity analysis of ACH probability distribution model to standard deviations of input variables shows the high contribution of wind speed and limited to low values of ACH (up to 0.4) influence of temperature difference.

Approximate transformation of wind speed data from the meteorological station to a specific location, where analysed building is situated, can be carried out by multiplying wind speed by a constant factor η (with 7% error or less), established for a specific ranges of roughness

conditions. The transformation of the probabilistic model of 10-min mean wind speed from meteorological station to the probabilistic model of hourly mean speed for the site of the building results in change of the scale parameter, while the shape parameter remains the same. The form of PDF for *ACH* as well as reliability index is sensitive to the value of the shape parameter of the Weibull distribution of wind speed and much less sensitive to the scale parameter. Hence, the transformation of probabilistic model of wind speed to the local site seems to be robust for the analysed case.

As it was shown, the sensitivity analysis has helped to understand the relationships between model inputs. It can also help to test the model outcome in terms of its robustness in the presence of uncertainty. The probabilistic risk analysis along with the effective tools for sensitivity analysis can be used to support design decisions and also to develop better models for evaluation of building performance.

Author details

Krystyna Pietrzyk^{1*} and Ireneusz Czmochn²

*Address all correspondence to: krystyna.pietrzyk@chalmers.se

1 Department of Architecture and Civil Engineering, Chalmers University of Technology, Gothenburg, Sweden

2 Faculty of Civil Engineering, Warsaw University of Technology, Warsaw, Poland

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