Chapter 7

Dynamics of the Early Stage of Formation of the Earth's-Moon System

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Additional information is available at the end of the chapter

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Abstract

In previous studies it was shown that the energy release during the decay of short-living radioactive elements in small bodies is sufficient for the temperature inside such a protoplanetary core to become larger than the melting temperature of iron. This ensures the realization of the process of differentiation of matter and the development of convection in the inner envelopes. At all stages of proto-Earth's formation, convective heat and mass transfer is the most important factor in the dynamics of the planet. However, the release of heat due to friction in the viscous liquid of the outer regions of the core so far has not been taken into account at all or was taken into account only in the formed envelopes of a planet of constant radius. In this chapter, we present the results of a numerical simulation of the thermal evolution of a 3D spherical segment of a protoplanet of an increasing radius, taking into account the accidental falling of bodies and particles. An algorithm for the numerical solution of the problem is given, taking into account the dissipation of tidal energy in the Earth-Moon system at the stage of planetary accumulation.

Keywords: stage of Earth-Moon forming, temperature distribution, energy of Moon tides, numerical solution of the differential system of equations

1. Introduction

It is a paradox, but there are no significant differences regarding the earliest stages of the forming of the universe and its structure. The basis of modern concepts is the law of gravitation of Einstein A [1]. Unlike Newton's law of gravitation, this potential explicitly takes into account the time dependence and the finite velocity of the gravitational interaction distribution. The initial condition for the Einstein equation is a point singularity in which the mass of the entire substance of the universe is concentrated. Friedman A [2] for the first time obtained no stationary solutions of the gravitational equation had been published. After Hubble's experiment on installed "red shift" [3] in the spectra of stars and galaxies, and then of "relict" radiation [4], the idea of the universe expanding was established. At the same time, our universe is understood as a region, about 13.8 billion light-years in radius, filled with matter formed after the "Big Explosion."

However, new questions arise. What is beyond the boundaries of our universe? Is the Universe only one? Are there any measurable physical fields and processes that will allow us to detect real objects external to our universe? A singular point has arisen in the already existing universe, and can we then trace the development of the instability leading to the 'Big Explosion" of this singular object? Are the laws of physics established in our universe in the outer universes preserved? Clearly, this is only a small part of the issues related to the problems of cosmic physics and planetology.

It is believed now that the most justified are ideas about the formation of stars and planets in protoplanetary clouds inside spiral arms of galaxies. **Figures 1** and **2** show an example of such galaxies.

The evolution of our solar protoplanetary cloud has been studied in detail. This is a vivid example of how complex are the processes of formation of the inner shells of the Earth and its natural moon satellite are insufficiently researched. Despite significant achievements, until recently, it has not been possible to obtain a quantitative explanation of the results obtained on the basis of the analysis of W-Hf and Al-Mg isotope systems, which are interpreted as evidence of the very early separation of chemical reservoirs of the core and mantle in time less than 10 million years [5–7]. While on the basis of the results on the uranium-lead system, the formation of these structures lasted about 100 million years, that is, the separation of their chemical reservoirs occurred long before the end of the growth of the structures themselves.

Figure 1. Impacted galaxies.

Figure 2. Star RS Korma ESO/G. Bono and CTIO. http://www.eso.org/public/images/potw1126a/

2. Modeling of early stage of formation of the Earth and Moon

2.1. The main physical results and mathematical formulation of the problem

Initially, a one-dimensional model was used to solve the problem of early stage of the Earth's formation [8, 9]. Adiabatic stresses and three-dimensional effects due to the location of bodies on the surface of a growing planet have not been taken into account. The main source of internal energy of the protoplanet at this stage of its formation was the release of heat due to the natural radioactive decay of long-living uranium and thorium. The contribution due to the release of heat from the impact of small particles and bodies of the protoplanetary cloud on the surface of the growing planet had been known to be very small, since most of their energy, according to Stefan's law, was released into space. As a result, sufficiently low estimates of the temperature of the inner regions of the protoplanet were obtained by the end of the process of their accumulation (**Figure 3**).

As can be seen from these results, the temperature estimates obtained turned out to be lower than the melting point of iron at a given depth. From this it follows that since the values of the iron coefficient in a solid silicate substance are very small, then for the entire age of the Earth, the separation of a predominantly iron core from a predominantly silicate mantle could not occur. In order for such separation to be realized, the molten state of the substance and the convective heat and mass transfer of the components in the melt are required.

A new model for the accumulation of terrestrial planets was proposed in the work of [11, 12], which uses modern results of isotope geochemical analyses and allowed obtaining reliable estimates of the concentration of short-living naturally radioactive isotopes and, above all, 26Al in the matter of the protoplanetary cloud. On the basis of these data, new estimates of

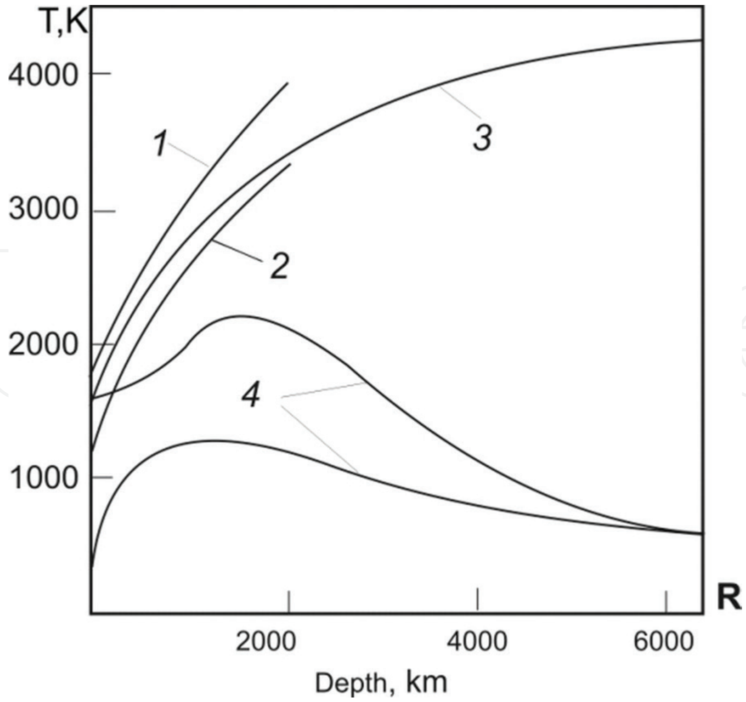

Figure 3. The Earth's temperature distribution after the end of accumulation. 1—The temperature of full mantle material melting, 2—the temperature of the solid mantle, 3—the melting iron temperature, 4—the temperature distribution into the Earth by Safronov [8] (lower curve) and the temperature distribution into the Earth by Lubimova (upper curve) [10].

the temperature distribution in the growing pre-planetary bodies of the planet in the "feeding" zone of the Earth were obtained. It was shown in the work of [11] that the energy release during the decay of short-living radioactive elements in small bodies is sufficient for the temperature inside such a protoplanetary core to become greater than the melting temperature of iron. This ensures the realization of **Table 1** which illustrates the processes of matter density differentiation and the development of convection in the inner envelopes. Originally, a one-dimensional model was used to numerically solve that problem, based on new data on

Radius of the pre-planetary body, km	**Concentration of Al_2O_3 mass. %**			
	1.0	**3.0**	**4.6**	**9.0**
	Temperature into the pre-planetary body center, K			
5	1240	1701	1734	1825
100	1676	1752	1812	1978
150	1690	1793	1876	2104
200	1701	1828	1928	2206
250	1711	1856	1972	2290
300	1718	1878	2006	2359
400	1730	1912	2059	2461

Table 1. The dependence of the temperature in the center of the pre-planetary body from the content of Al_2O_3 [12].

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the content of 26Al in the protoplanetary matter of the Earth. The results are presented in **Figures 4** and **5**. To describe the growth of the mass of the protoplanet and the nearest satellite, the Safronov equation was used [8, 10]:

$$\frac{\partial m}{\partial t} = 2(1+2\theta)\, r^2\, \omega\left(1-\frac{m}{M}\right)\sigma \tag{1}$$

where m is the current mass of the growing body, M is the final mass of the planet, r is the radius of the growing protoplanet, θ is the statistical parameter that takes into account the distribution of bodies and particles by mass and velocity in the protoplanetary supply zone, σ is the surface density of the protoplanetary cloud's matter in the orbit of the growing body, and ω is the angular velocity of the orbital rotation. The change of the temperature in the inner parts of the growing planets was described by a system of equations:

$$\rho\left(\frac{\partial\vec{V}}{\partial t}+\vec{V}\left(\nabla\vec{V}\right)\right) = -\nabla P+\mu\Delta\vec{V}+2\rho\vec{V}\times\vec{\Omega}-\rho\nabla W \tag{2}$$

$$\frac{\partial T}{\partial t}+\vec{V}(\nabla T) = \chi\Delta T+Q_1 \tag{3}$$

$$div\vec{V} = 0 \tag{4}$$

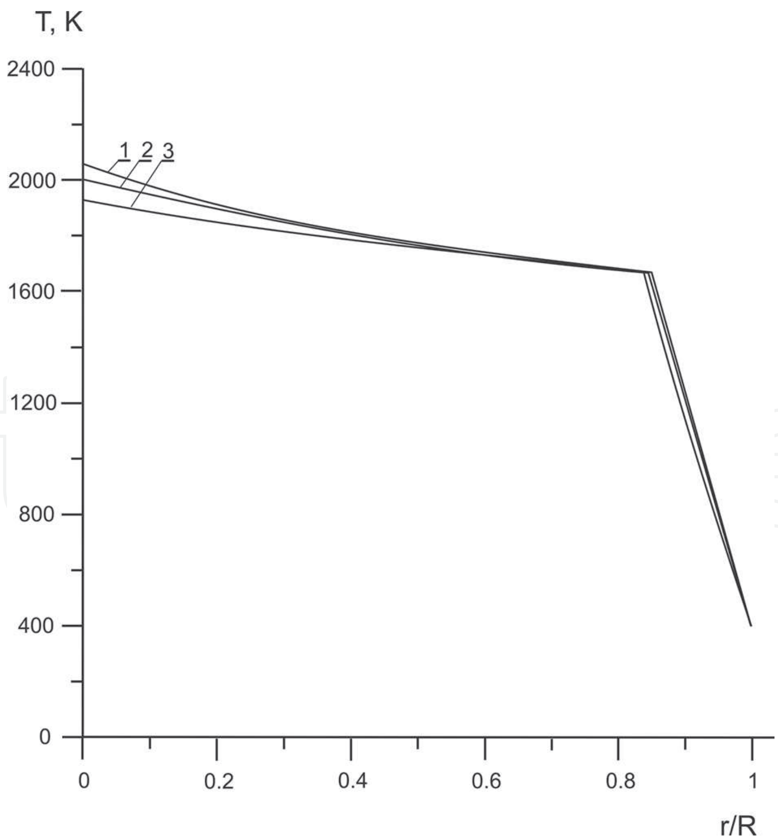

Figure 4. The distribution of the temperature in the growing pre-planetary body. Its radius is 1—400 km, 2—300 km, and 3—250 km.

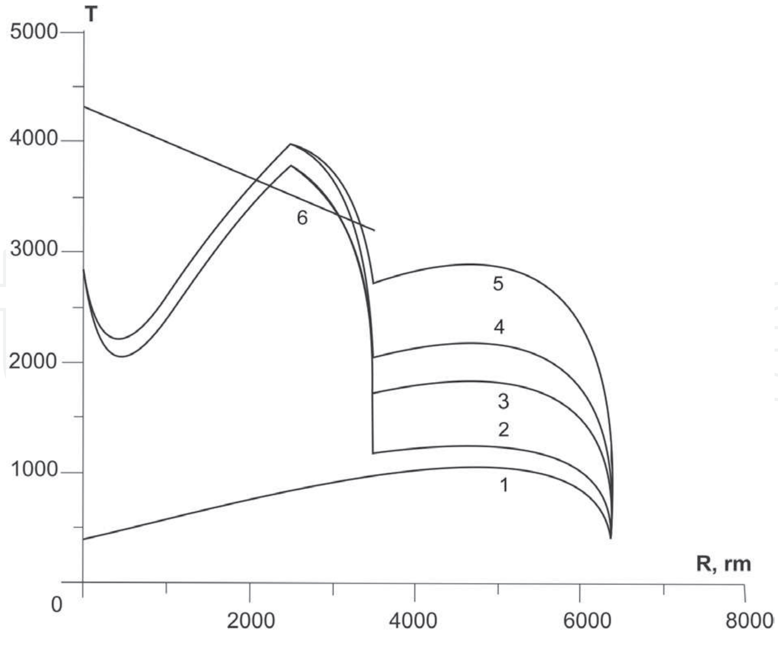

Figure 5. Possible variants of the temperature distributions to the moment, when the Earth has the size 6300 km [12]. 1—The accumulation from little particles. The generation of heat by short-living radioactive elements is not taken into account: ε = 0.001 for growing core and ε = 0.001 for mantle. 2 and 3—The generation of heat by short-living radioactive elements is taken into account: 2—ε = 0.3 for growing core and ε = 0.02 for mantle and 3—ε = 0.4 for growing core and ε = 0.01 for mantle. 4—ε = 04 for growing core and ε = 0.02 for mantle and 5—ε = 0.5 for growing core and ε = 0.05 for mantle. 6—The dependence of the core's melting temperature from pressure.

Here ρ is the density of the medium, $\vec{V}$ the volume velocity of the flow, P the pressure, μ the kinematic viscosity of the medium, $\vec{\Omega}$ the angular velocity of diurnal rotation, W the potential of the rotating body, T the medium temperature at a given point and time, Q_1 the normalized cumulative power of the release of the specific internal energy, and χ the thermal diffusivity.

Using the Boussinesq approximation for "incompressible fluid" greatly simplifies the consideration of Eqs. ((2)–(4)) [15, 16]. In the energy balance equation, adiabatic heating and energy release of radioactive sources Q_1 are taken into account here, but the contribution of energy dissipation of viscous friction was not taken into account:

$$k\rho\frac{\gamma M}{r}\frac{dr}{dt} = \varepsilon\sigma\left[T^4 - T_1^4\right] + \rho c_P\left[T - T_1\right]\frac{dr}{dt} \tag{5}$$

The system of Eqs. ((1)–(5)) in the spherical sector is solved numerically, and the conditions of periodicity are given on the lateral boundaries, while the conditions (Eq. (5)) are calculated on the outer surface of the growing radius [12].

where ρ is the density of the medium; γ is the gravitational constant; M is the mass of the growing planet; r is its radius; T and T_1 are, respectively, the surface temperature of the body and the external medium; ε is the permeability coefficient and c_P the specific heat; and k is the fraction of the conversion of the potential energy into thermal energy.

The results of the work by [12] show various options for estimating the temperature of the interior regions of the planet by the time of completing its accumulation for numerical models that take into account the difference in transmission ε from the boundary conditions (Eq. (5))

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of thermal radiation, depending on its composition and density. As can be seen from the results presented in **Figure 5**, the temperature of the growing surface sharply decreases as the transparency of the atmosphere increases, due to the increase of the gravitational radius of the protoplanet and the possibility of retaining the silicate content of the cloud.

In the work of [12–14], it was shown that successively changing with the growth of the emerging Earth and the Moon, the main contribution of heat generation in the inner regions of growing bodies is provided first by radioactive sources and, after that, heated from above by the arrival of kinetic energy during a collision with the growing Earth and the Moon. At all stages of the Earth's formation, convective heat and mass transfer is the most important factor in the dynamics of the planet. For the numerical solution of the boundary value problem (Eqs. (1)–(5)), the through-count method proposed in [15, 16] was used. However the one-dimensional model of the thermal structure of the interior regions does not allow one to understand the occurrence of the largest surface structures of continents and oceans.

2.2. New results of 3D modeling of the early stage formation of the Earth-Moon system

In addition, the release of heat due to friction in the viscous melt of the inner envelopes has either not been taken into account at all or has been taken into account only in the already formed envelopes of a planet of constant radius. In this chapter, the first results of numerical modeling of the thermal evolution of the 3D spherical segment of a protoplanet of an increasing radius are considered, taking into account the accidental falling of bodies and particles. In **Figure 6**, the nonuniform temperature distribution on the surface of the growing proto-Earth at a radius of 100 km is shown.

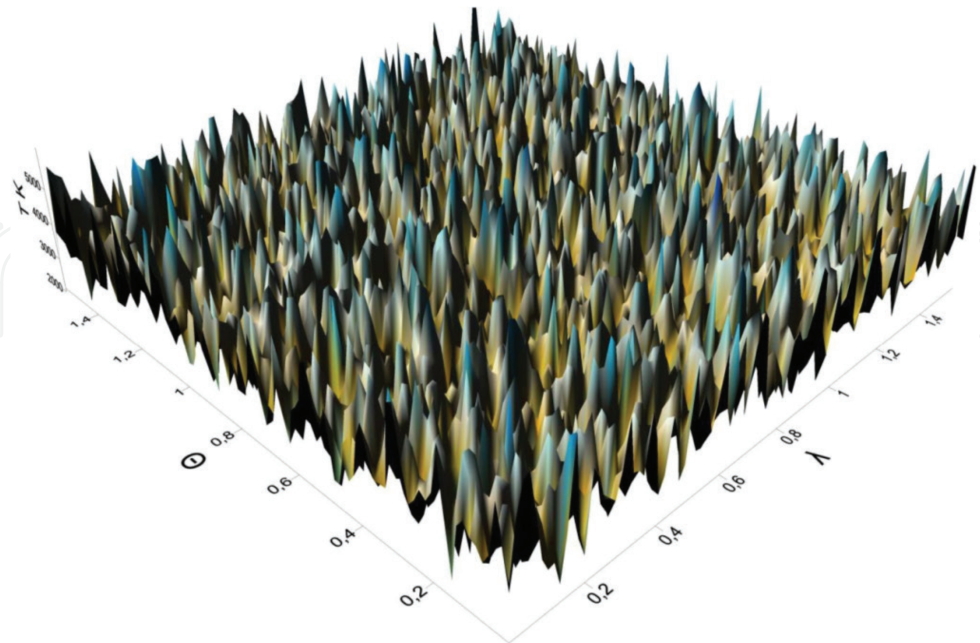

Figure 6. The variant of temperature distribution, stipulated by the random distribution of the accumulated bodies by their values and kinetic energy on the planet's surface at the radius R = 100 km, λ and θ-spherical angles in radians. After 100 km was inserted a comma.

In **Figures 7–9**, the temperature distributions in the inner areas Earth are presented in the form of cross sections of spherical sectors.

In Ref. [13] proposed an algorithm for numerical solution of the problem with allowance for the dissipation of tidal energy in the Earth-Moon system at the stage of planet accumulation. The results obtained showed that the contribution of this source of heat could at times be very significant.

In Ref. [12] showed that the energy release during the decay of short-living radioactive elements in small area, over 50 km in size, is sufficient for the temperature inside such a pre-planetary body to become larger than the melting temperature of iron. This ensures the realization of the process of matter density differentiation and the development of convection in the interior regions of the growing planet. As the Earth grew, the forming area of the outer core remained in the molten state. The use of the 3D model in solving the system of Eqs. ((2)–(4)) with the boundary condition (Eq. (5)) at the stage of formation of the Earth and the Moon made it possible to trace the occurrence and development of thermal heterogeneities in the growing internal areas of the Earth. Similar results were obtained also for the Moon. As can be seen from the results presented in **Figures 7** and **8**, formed due to a random distribution of falling bodies on the forming planet, near-surface thermal heterogeneities do not have time to homogenize and cause a complex structure of distribution of concentration and thermal heterogeneities. However, the Earth and the Moon form a tight double system. Therefore, the thermal evolution of these bodies at the accumulation stage must be analyzed together. The need to take into account the release of heat due to the attenuation of tides during the accumulation of the Earth was noted in the work of [17, 18]. For the mathematical description of the thermal evolution of proto-Earth and proto-Moon growing in the process of accumulation as a close binary system, it is necessary to take into account the evolution of the Moon's orbit due to the tidal interaction and the dissipation of viscous fluid friction energy excited by this tidal interaction.

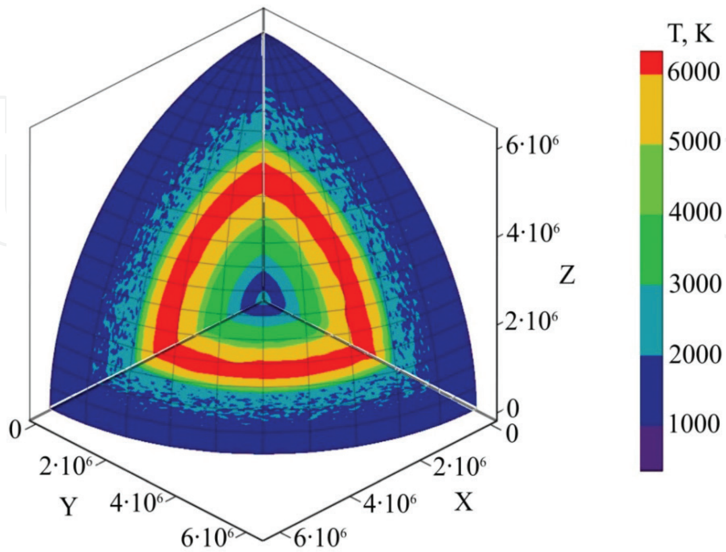

Figure 7. An example of the temperature distribution and initial thermal heterogeneities in the protoplanet up to the end of its accumulation, without taking into account the heat of tidal friction.

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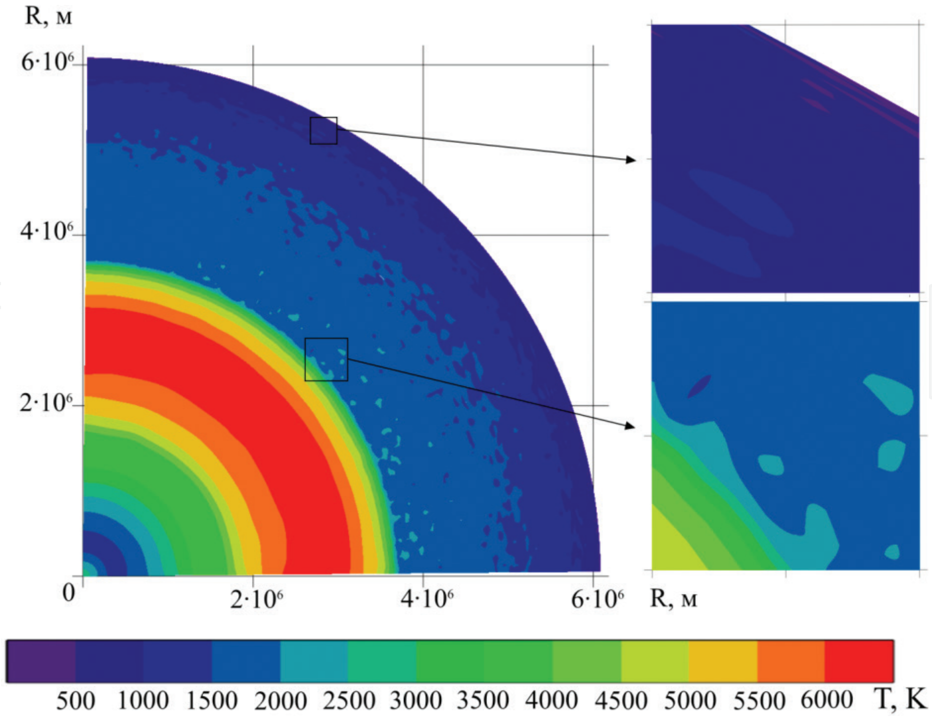

Figure 8. The temperature distribution of the Earth's interior to the end of accumulation along the section of the 3D sector (without taking into account the heat dissipation energy of tidal friction).

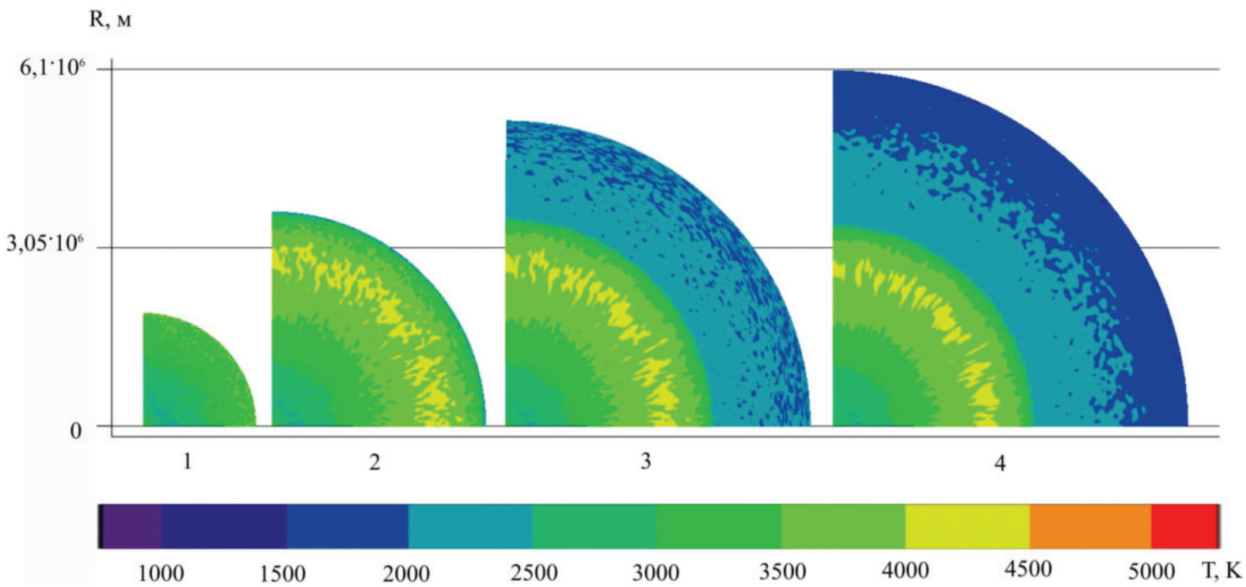

Figure 9. The temperature distribution in the inner areas of the proto-Earth for various values of the achieved radius: 1-R = 1930 km, 2-R = 3670 km, 3-R = 5220 km, and 4-R = 6680 km.

In contrast to the model, for which the Businesque approximation—an incompressible viscous fluid—is still applicable, we can no longer use the incompressibility approximation in Eqs. (2) and (4). In the model under consideration, the tidal interaction between the Sun and the planets will be neglected, which will allow us to use the results of an analysis of the close binary system. For the Earth-Moon system, one can use the approximation for the law of conservation of momentum in the form (Eq. (6)) [17], (**Figure 10**). It does not take into account the interaction with the Sun, which ensures the solar tide:

$$I\,\Omega + \frac{M_1 \cdot m_1}{M_1 + m_1} L^2 \omega \;=\; const \tag{6}$$

where I is the inertial moment of the growing Earth, $\Omega(t)$ is the angular velocity of its rotation about its own axis at time t, $\omega(t)$ is the angular velocity of the orbital motion of the Moon around the Earth for the same moment of time, M_1 is the mass of the proto-Earth for the time moment (t), m_1 is the proto-Moon mass at the same time moment (t), and $L(t)$ is the distance between the centers of mass of the Earth and the Moon.

For the energy of the Earth's own motion and the total energy of the Moon's orbital motion, one can use the expressions (Eq. (7)):

$$E_\Omega \;=\; I\frac{\Omega^2}{2} \quad E_\omega \;=\; -\gamma\frac{M_1(t)\,m_1(t)}{2L} \tag{7}$$

To describe the dynamics of the Earth-Moon system, it is necessary to replace Eq. (3) by Eq. (8):

$$\rho c\left(\frac{\partial T}{\partial t} + \vec{V}(\nabla T)\right) \;=\; \lambda \Delta T + Q_1 + \Phi \tag{8}$$

where Φ is the dissipative function in each of the melt regions, as can be seen from the results of the simpler model, shown in **Figures 7** and **8** for each time point. The function Φ describing the energy dissipation in a viscous fluid in a Cartesian coordinate system has the form [19, 20]:

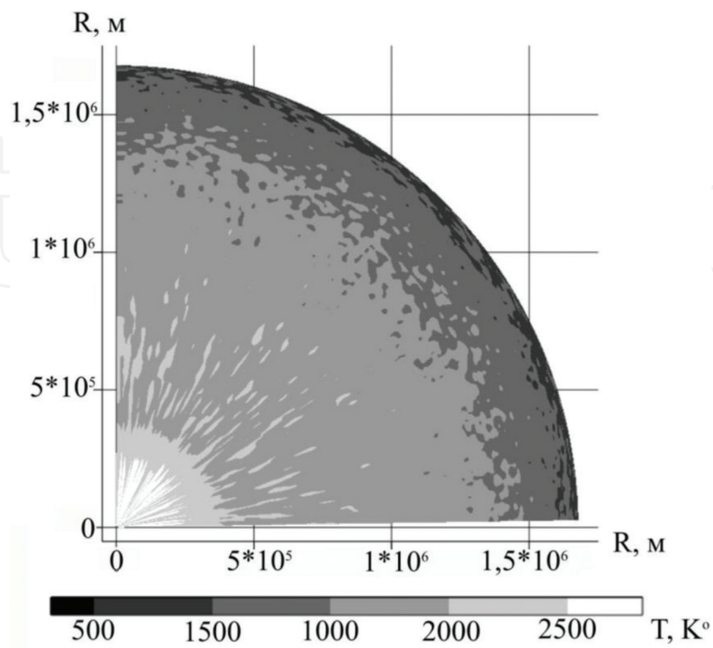

Figure 10. Temperature distribution in the interior areas of the Moon up to the completion of its accumulation.

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$$\Phi = 2\mu\left[\left(\frac{\partial u_x}{\partial x}\right)^2 \left(\frac{\partial u_y}{\partial y}\right)^2 + \left(\frac{\partial u_z}{\partial z}\right)^2 + \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)^2\right] + \lambda\left\langle\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right\rangle^2 \tag{9}$$

where μ and λ are the coefficients of the first and second viscosities: u_x, u_y, and u_z are the projections of the velocity in Cartesian coordinates.

The algorithm for solving this nonlinear system of equations is as follows. For each moment of time, using Eq. (1), new values of the Earth's and Moon's masses and the time step are found at a fixed step along the coordinate r. New boundary conditions for the proto-Earth and like for the proto-Moon are calculated (Eq. (5)). A direct problem is solved in the body of the increased radius, for which the temperature, velocity, and pressure fields are used as the initial conditions from the previous time step (Eqs. (2), (8), (4)), respectively. New distributions of the melting temperature and the function Φ in (Eq. (9)) are computed from the new temperature and pressure distribution. Since the system of equations is nonlinear, the solution is sought using the iteration method, which provides a given relative error. The number of iterations is carried out until the specified accuracy of the solution is achieved. Then the system (Eqs. (6) and (7)) is solved, and new parameters of the satellite orbit are determined. After this, the next step is taken along the spatial grid. But the solution of this nonlinear boundary value problem with the necessary number of iterations at each time step and the necessity of calculating the dissipative function (Eq. (8)) requires very large computational powers. In this case, there are very rough estimates of the viscosity dependences on temperature and pressure, which lead to computational difficulties. Therefore, we confine ourselves to an estimate that can be obtained using the phenomenological approach and using the Q-factor approximation of the interior regions of the Earth and the Moon, based on the results of [17, 18]. We use the quality factor as the fraction of the energy E dissipating into heat during one oscillation period (Eq. (10)):

$$Q^{-1} = \frac{\delta E}{2\pi E} \tag{10}$$

This function is estimated by modern seismic results on the attenuation of the Earth's natural oscillations for the model hydrostatic pressure and available estimates of the current temperature [18].

Tidal interaction redistributes the moments of momentum between the bodies forming a closed system, and its modern value is a well-known value. On the contrary, the total amount of energy in the open system Earth-Moon is not conserved. The value of the rotational energy is not conserved but partially dissipates into heat, and this part of it must be taken into account as a source term in Eq. (8).

3. Conclusions

Newton's law, with sufficient accuracy, described the gravitational interaction of material bodies on the scales of individual stars and their planetary systems. It is not applicable to describing processes in the universe as a whole. In this case, it is already necessary to use the general theory of relativity and the law of gravitational interaction of Einstein.

To simulate the conditions for the evolution of the protoplanetary solar disk, the law of gravitation in the Newton approximation is quite sufficient. The obtained numerical results of the solution of the boundary value problem in the body of the increasing radius of the system of nonlinear equations for modeling the thermal conditions for the formation of the Earth and the Moon in one-dimensional and three-dimensional models were analyzed. It is shown that effects can be revealed in the three-dimensional model of the numerical solution of the problem, which could not be detected in a spherically symmetric model, and they significantly influence on the dynamical evolution of the binary system Earth-Moon.

As the mass increases, the distance between their centers of mass and the speed of the orbital rotation of the Moon around the Earth change. From the law of conservation of angular momentum (Eq. (6)), it follows that the tidal interaction as the mass of the central body increases provides the transfer of the orbital angular momentum of the satellite to the central massive body. At the same time, part of the rotational energy was converted into thermal energy due to viscous friction. Losing the orbital moment, the satellite should approach the central body right up to the orbit of Rosh. The problem of the behavior of the satellite at the Rosh limit requires a careful mathematical study. The power of additional thermal energy and the volume of molten inclusions should increase with the growth of the Earth. Due to the continued transfer of the satellite's orbital moment to the central body, its own angular velocity continued to grow. This should lead to an increase in the orbital angular velocity of the satellite and an increase in the radius of its orbit, which is observed up to the present time.

Thus, the effect of dissipation of viscous friction most significantly manifests itself at the initial stage of the formation of the Earth. The results obtained depend on the parameters, primarily the viscosity versus temperature and pressure, the values of which are known with a high degree of uncertainty. They are to be verified from the results of further research in the areas of physical-mathematical and geological exploration.

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