30

WDM Optical Networks Planning using Greedy Algorithms

Nina Skorin-Kapov
Faculty of Electrical Engineering and Computing, University of Zagreb
Croatia

1. Introduction

Optical networks have been established as the enabling technology for today's high-speed communication networks. Wavelength Division Multiplexing (WDM) enables the efficient utilization of optical fibers by dividing its tremendous bandwidth into a set of disjoint wavelength bands, referred to as wavelengths. Each wavelength supports one communication channel which corresponds to an end user operating at an arbitrary speed, e.g. peak electronic speed. This helps to overcome the opto-electronic mismatch between the multiple terabit-per-second bandwidth of optical fibers and the gigabit-per-second electronic processing speeds at end users.

In wavelength-routed WDM networks, all-optical directed channels, called *lightpaths*, can be established between pairs of nodes which are not necessarily neighboring in the physical topology. A set of lightpaths creates a so-called virtual topology over the physical interconnection of fibers. Packet-switched traffic is then routed over this virtual topology, independent of the physical topology. Traffic send via a lightpath is transmitted in the optical domain with no opto-electronic conversion at intermediate nodes. Establishing a lightpath requires a transmitter and receiver at the source and destination nodes, respectively, and includes routing it over the physical topology and assigning to it a wavelength.

One of the main challenges in wavelength-routed WDM networks is to successfully solve the Virtual Topology Design (VTD) problem. This problem is usually divided into the following four sub-problems. The first is to determine the set of lightpaths which is to form the virtual topology. This set of lightpaths can be static, scheduled or dynamic. Static lightpaths are established semi-permanently and chosen on the basis of a traffic matrix representing the estimated average traffic floes between node pairs. Scheduled lightpaths, on the other hand, try to exploit the periodic nature of traffic by defining a schedule for establishing and tearing down lightpaths based on periodic traffic trends. Lastly, dynamic lightpaths are established as connection requests arrive with no *a priori* information regarding traffic demands. Unless specified otherwise, the VTD problem usually refers to the static case which we will be discussing in the remainder of this chapter. Thus, we use these terms interchangeably.

The second sub-problem in VTD is to find for each lightpath a corresponding route in the physical topology, while the third is to assign to each a wavelength subject to certain constraints. Lightpaths routed over the same physical links at the same time cannot be

Source: Advances in Greedy Algorithms, Book edited by: Witold Bednorz,
ISBN 978-953-7619-27-5, pp. 586, November 2008, I-Tech, Vienna, Austria

assigned the same wavelength. This is called *the wavelength clash constraint*. If there are no wavelength converters available, which is often the case due to their high prices, the entire lightpath must be established the same wavelength. This is known as the *wavelength continuity constraint*. Sub-problems two and three are commonly referred to as the Routing and Wavelength Assignment (RWA) problem. The RWA problem is often solved separately with the objective to minimize wavelengths and/or lightpath congestion, or maximize the number of established lightpaths subject to a limited number of wavelengths. An example of a 4-node wavelength-routed network, an RWA scheme, and its corresponding virtual topology with five established lightpaths is shown in Fig. 1.

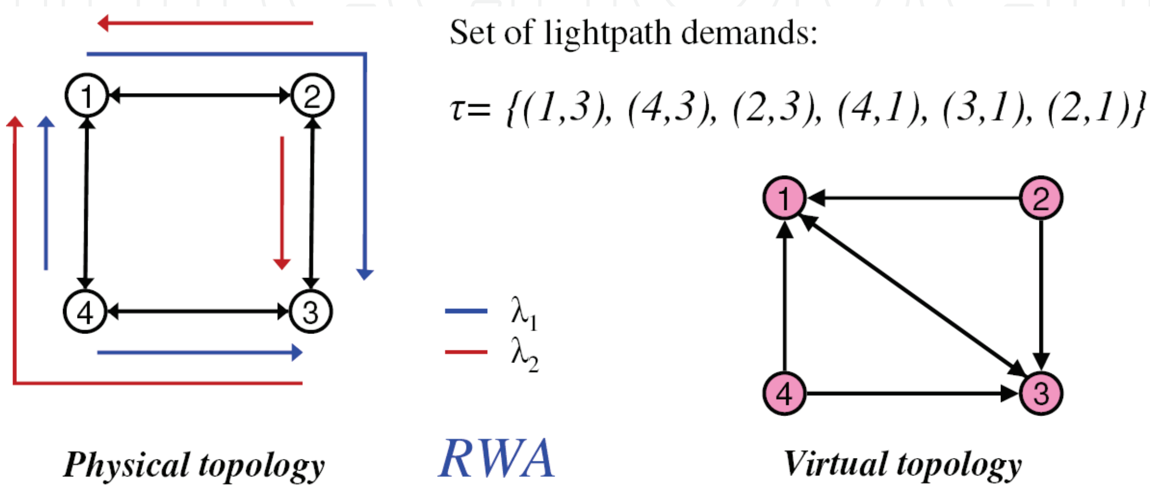


Fig. 1. An example of solving the Routing and Wavelength Assignment problem

Finally, after determining the set of lightpaths and successfully solving the RWA problem, packet-switched traffic must be routed over the virtual topology which is the fourth sub-problem in VTD. Objectives include minimizing the average packet and virtual hop distances, the number of transceivers used, and congestion.

The Virtual Topology Design problem, as well as the RWA problem, is NP-complete. Thus, heuristic algorithms are needed to find sub-optimal solutions for larger problem instances. In this chapter we discuss greedy algorithms based on bin packing for static RWA. Furthermore, we present greedy approaches for solving the first three sub-problems of Virtual Topology Design, which we refer to as the VRWA problem, in conjunction with a linear program for traffic routing (the fourth sub-problem of VTD).

2. The RWA problem

2.1 Problem definition

The Routing and Wavelength Assignment problem is as follows. Given is a graph $G=(V,E)$, where V is the set of nodes and E is the set of bidirectional edges representing a fiber in each direction. Since we are considering the static case, we are given a set of lightpath demands, $\tau = \{(s_1,d_1), \ldots, (s_n,d_n)\}$, where s_i, d_i in V, $i=1,\ldots,n$, are the source and destination nodes, respectively. These lightpaths are to be established semi-permanently. To solve the RWA problem, we need to find a set of directed paths $P=\{P_1,\ldots,P_n\}$ in G, each corresponding to one lightpath demand and assign to each a wavelength subject to the following constraints. Two paths that share a common physical link (in the same direction) cannot be assigned the same wavelength (*the wavelength clash constraint*). Furthermore, we assume that there are no wavelength converters and thus the entire physical path corresponding to a single lightpath must be assigned a unique wavelength (*the wavelength continuity constraint*). Furthermore, we constrain the length in hops of the paths in P by a parameter H.

Our objective is to minimize the number of wavelengths needed to establish the given set of lightpath demands. A secondary objective we consider is minimizing the physical lengths of the lightpaths which is desirable due to transmission impairments and delay.

2.2 Related work

The Routing and Wavelength Assignment problem has been widely studied in the literature. This problem has been proven to be NP-complete (Chlamtac et al., 1992) and several heuristic approaches have been developed to help solve it sub-optimally. Variations have been studied, such as the static, scheduled and dynamic cases, with (un)limited wavelengths, with(out) wavelength converters and/or considering physical impairments in optical fibres ((Choi et al., 2000), (Jia et al., 2002), (Mukherjee, 1997), (Murthy & Gurusamy, 2002)).

In (Ramaswami & Sivarajan, 1995), a mixed integer linear formulation is given for the RWA problem which is highly intractable and, thus, heuristics are needed. Alternative formulations are given in (Ozdaglar & Bertsekas, 2003) which consider a quasi-static view and introduce a cost function which is such that it tends to give integral solutions even when the problem is relaxed.

Most heuristic approaches divide the problem into two sub-problems solved subsequently: the first is to route the set of lightpaths and the second is to assign wavelengths. Given a routing scheme, wavelength assignment is equivalent to the graph coloring problem so existing heuristics for graph coloring are often used. In (Banerjee & Mukherjee, 1996), the authors suggest a multi-commodity flow formulation for routing which is relaxed and then rounded using a randomized approach. Wavelength assignment is solved using graph coloring heuristics. Local random search is used to solve the routing sub-problem in (Hyytia & Virtamo, 1998) while a greedy graph coloring algorithm assigns wavelengths for the obtained routing solution. In (Noronha & Ribeiro, 2006), a tabu search algorithm suggested for color-partitioning is used to perform wavelength assignment on a set of previously calculated alternative routes. Two-step algorithms, such as those mentioned above, can give good results but may have longer execution times than one-step algorithms.

A one-step approach is suggested in (Lee et al., 2002) which gives an integer formulation solved using column generation. This, however, is not practical for larger problems. A simple yet highly efficient greedy algorithm, called *Greedy_EDP_RWA* is suggested in (Manohar et al., 2002). This approach is based on edge disjoint paths and runs as follows. The algorithm creates a partition of the set of lightpaths where each element of the partition contains a subset of the given lightpaths routed on mutually edge disjoint paths which can, thus, be assigned the same wavelength. Hence, the number of wavelengths required is equal to the number of elements in the partition. This algorithm has been shown to give better results than (Banerjee & Mukherjee, 1996) and yet is much faster. We suggested improved greedy algorithms based on bin packing in (Skorin-Kapov, 2006.a) which will be described in more detail in the next subsection. Efficient implementations of these greedy bin packing algorithms were suggested in (Noronha et al., 2008).

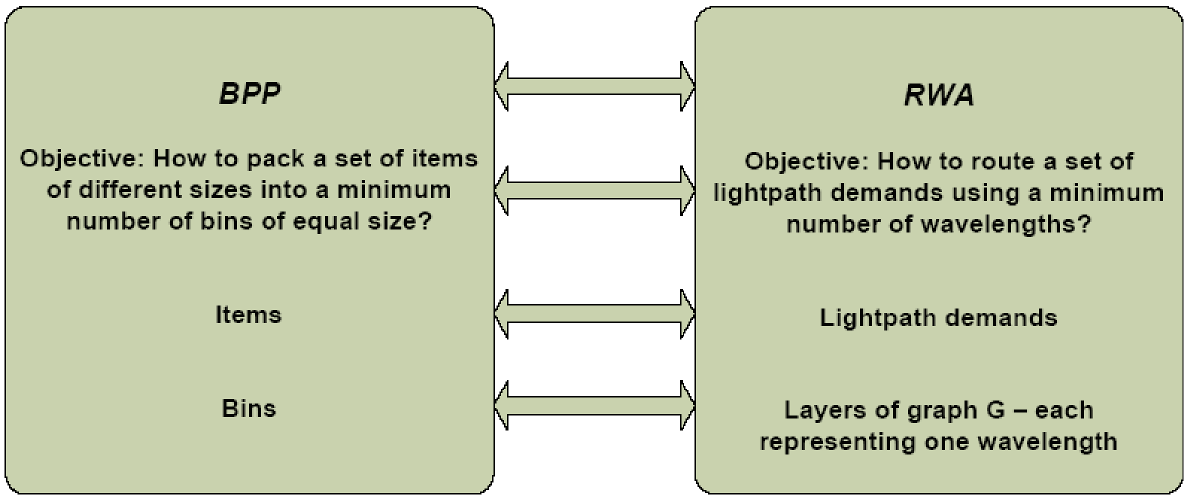


Fig. 2. Analogies between the Bin Packing Problem and Routing and Wavelength Assignment

2.3 Greedy algorithms based on bin packing

In order to efficiently solve RWA using fast greedy algorithms, we adapt classical bin packing heuristics to meet the specific demands of our problem. Bin packing is a well-known NP-hard optimization problem which attempts to pack a given set of items of various sizes into the minimum number of bins of equal size. Various heuristic algorithms have been proposed for bin packing and surveys can be found in (Coffman et el., 1996) and (Coffman et al., 2002). Widely-used greedy heuristics for this problem are the First Fit (FF), Best Fit (BF), First Fit Decreasing (FFD), and Best Fit Decreasing (BFD) algorithms. The First Fit algorithm packs items into the first bin into which it fits, while the Best Fit algorithm pack items into the bin which leaves the least room left over after including the item. Both algorithms pack items in random order, and as such can be used as *online* algorithms which pack items in the order that they appear.

The FFD and BFD algorithms, on the other hand, must have *a priori* knowledge of the entire set of items to be packed. Namely, they sort items in non-increasing order of their size and then pack them according to the FF or BF strategies, respectively. The motivation for this is that first packing the larger items, which are more difficult to pack, and then filling up remaining spaces with smaller items often lead to fewer bins needed. The FFD and BFD algorithms can only be used as *offline* algorithms since they require complete knowledge of the problem (i.e. the set of items), but give much better results than the corresponding online algorithms.

We apply these ideas to help develop efficient greedy algorithms for the static RWA problem. We call these heuristics the FF_RWA, BF_RWA, FFD_RWA, and BFD_RWA algorithms. To apply the Bin Packing Problem (BPP) to RWA, we have to define items and bins in terms of optical networks which we do as follows. Items represent lightpath demands while bins represent layers or copies of the physical topology, i.e., graph *G*, each corresponding to one wavelength. Our objective is to route all the lightpath demands on the minimum number of layers such that lightpaths routed on the same layer are edge disjoint.

2.3.1 The FF_RWA algorithm

The First Fit Routing and Wavelength Assignment (FF_RWA) algorithm runs as follows. Lightpath demands (i.e., *items*) are selected at random and routed on the lowest-indexed layer[1] of graph G (i.e., *bin*) that has a feasible path available and assigns to it the wavelength corresponding to that layer. If there is no feasible path available on any existing layer, i.e. a path shorter than the hop bound H, a new layer is added. Once a path is found, its corresponding edges are deleted, i.e., are marked as used for that wavelength. Note that, using this approach, a lightpath may be routed on a longer path on a lower-indexed layer than might be available on a higher layer. Lightpaths in RWA, as opposed to items in BPP, are not of fixed size but depend on the available links in each layer. This algorithm is basically equivalent to the *Greedy_EDP_RWA* algorithm from (Manohar et al., 2002), differing only in the order in which some steps are executed, but yielding the same results.

2.3.2 The BF_RWA algorithm

The Best Fit Routing and Wavelength Assignment (BF_RWA) algorithm also starts with a single layer and routes lightpath demands in random order. However, instead of routing lightpaths on the first layer on which there is an available path, lightpaths are routed on the layer on which it '*fits best*'. By best fit, we do not mean the layer with the least room left over as in BPP, but rather the one on which the lightpath can be routed on the shortest path. If there are multiple layers which can offer routes of the same path length, the lowest–indexed one is chosen. If there is no feasible path available on any layer, a new one is added. The main motivation for this approach is to use fewer resources for individual lightpaths leaving more room for future demands and ultimately minimizing the number of wavelengths used. Additionally, this approach helps to minimize the physical lengths of the lightpaths.

2.3.3 The FFD_RWA and BFD_RWA algorithms

The First Fit and Best Fit Decreasing Routing and Wavelength Assignment (FFD_RWA and BFD_RWA) algorithms sort the lightpath demands in non-increasing order of the lengths of their shortest paths in G and then proceed according to the FF and BF strategies, respectively. We use a lightpath's shortest path in G as a measure of its size, even though the lightpath will not necessarily be routed on this path. The motivation for this method of sorting is that if *'longer'* lightpaths (i.e. those that are harder to route) are routed first, when most resources are still available, they can be routed on their shortest paths using up less space. *'Shorter'* lightpaths are then more easily routed over the remaining links which can ultimately lead to fewer wavelengths used.

2.4 Lower bounds

To asses the value of the obtained solutions we compare with simple lower bounds which can be easily calculated even for larger problems. A lower bound on the number of wavelengths is:

$$LB_w = \max\left\{ \max_{i\in V}\left\lceil \frac{\Delta_l}{\Delta_p} \right\rceil, \left\lceil \frac{\sum_{j=1}^{n} l(SP_j)}{2|E|} \right\rceil \right\} \quad (1)$$

---

[1] Initially, only one layer of G is considered.

The first element represents the maximum ratio of logical in (or out) degree Δ_l to physical in (or out) degree Δ_p rounded to the highest integer. The second element represent the sum of the lengths in hops of the shortest paths $l(SP_j)$ for all lightpath demands, divided by the total number of edges $|E|$, multiplied by 2 (since they are bidirectional).
A simple lower on the average physical lengths is simply the sum of all the shortest paths $l(SP_j)$ divide by the number of lightpaths n:

$$LB_H = \frac{\sum_{j=1}^{n} l(SP_j)}{n} \quad (2)$$

2.5 Computational results

The *Greedy_EDP_RWA* algorithm from (Manohar et al., 2002) and the BF_RWA, FFD_RWA, and BFD_RWA were implemented in C++ and run on a PC powered by a P4 2.8GHz processor.[2] Series of 5 random 100-node networks were created with average degrees of 3, 4, and 5. Sets of random lightpath requests were generated where the probability P_l of there being a lightpath between two nodes ranged from 0.2 to 1, in 0.2 increments. The upper bound on the physical hop length H was set to $max(diam(G), \sqrt{|E|})$ as in (Manohar et al., 2002). All algorithms were run with 10 different seeds for each test case.

Test Netw.	P_l	Lightpath requests	LB_W	*Greedy_EDP_RWA (FF_RWA)*	*BF_RWA*	*FFD_RWA*	*BFD_RWA*
1	0.2	2116	21	24.3 (*23,25*)	21.7 (*21,24*)	**21*** (*21,21*)	**21*** (*21,21*)
2		2081	25	28.1 (*26,30*)	25.4 (*25,27*)	**25*** (*25,25*)	**25*** (*25,25*)
3		2067	24	25.8 (*24,27*)	24.4 (*24,26*)	**24*** (*24,24*)	**24*** (*24,24*)
4		2054	29	29.9 (*29,31*)	29.4 (*29,31*)	**29*** (*29,29*)	**29*** (*29,29*)
5		2125	32	33.8 (*32,36*)	**32*** (*32,32*)	**32*** (*32,32*)	**32*** (*32,32*)
1	0.4	4063	39	44.3 (*43,46*)	39.9 (*39,43*)	**39*** (*39,39*)	**39*** (*39,39*)
2		4047	46	50.9 (*49,53*)	47.2 (*47,48*)	**47** (*47,47*)	**47** (*47,47*)
3		4064	50	53.4 (*52,55*)	50.1 (*50,51*)	**50*** (*50,50*)	**50*** (*50,50*)
4		4063	47	51.4 (*50,52*)	48.3 (*47,50*)	**47*** (*47,47*)	**47*** (*47,47*)
5		4099	50	55.8 (*53,59*)	50.3 (*50,51*)	**50*** (*50,50*)	**50*** (*50,50*)
1	0.6	6017	61	66.9 (*63,69*)	61.1 (*61,62*)	**61*** (*61,61*)	**61*** (*61,61*)
2		5995	69	74.6 (*72,78*)	69.1 (*69,70*)	**69*** (*69,69*)	**69*** (*69,69*)
3		6054	67	75.4 (*73,78*)	67.8 (*67,71*)	**67*** (*67,67*)	**67*** (*67,67*)
4		6054	71	77.5 (*75,81*)	71.2 (*71,72*)	**71*** (*71,71*)	**71*** (*71,71*)
5		6113	66	77.9 (*76,83*)	67.6 (*66,70*)	**66*** (*66,66*)	**66*** (*66,66*)
1	0.8	7960	80	86.7 (*84,89*)	80.1 (*80,81*)	**80*** (*80,80*)	**80*** (*80,80*)
2		7988	81	89.8 (*88,93*)	81.8 (*81,83*)	**81*** (*81,81*)	**81*** (*81,81*)
3		8052	88	99.4 (*96,103*)	89.8 (*88,93*)	**88*** (*88,88*)	**88*** (*88,88*)
4		8014	86	94.4 (*91,99*)	86.6 (*86,89*)	**86*** (*86,86*)	**86*** (*86,86*)
5		8017	88	101.4 (*97,106*)	88.9 (*88,90*)	**88*** (*88,88*)	**88*** (*88,88*)
1	1.0	9900	99	108.3 (*106,110*)	99.9 (*99,102*)	**99*** (*99,99*)	**99*** (*99,99*)
2		9900	99	110.7 (*108,113*)	100.4 (*99,102*)	**99*** (*99,99*)	**99*** (*99,99*)
3		9900	99	120 (*118,123*)	105 (*103,109*)	**99*** (*99,99*)	**99*** (*99,99*)
4		9900	99	110.8 (*108,112*)	99.1 (*99,100*)	**99*** (*99,99*)	**99*** (*99,99*)
5		9900	99	122.7 (*119,125*)	106.7 (*105,109*)	**103.6** (*103,104*)	104.5 (*104,105*)

Table 1. The number of wavelengths obtained by the greedy RWA algorithms and the lower bound for 100-node networks with an average degree of 4.

[2] The FF_RWA algorithm was not implemented due to its basic equivalency with *Greedy_EDP_RWA.*

Test Network	P_l	Lightpath requests	LB_{PH}	*Greedy_EDP_RWA (FF_RWA)*	*BF_RWA*	*FFD_RWA*	*BFD_RWA*
1	0.2	2116	2.92	3.86	2.97	3.87	**2.93**
2		2081	2.95	3.88	2.98	3.87	**2.96**
3		2067	2.96	3.89	2.99	3.90	**2.96***
4		2054	3.05	4.03	3.11	4.03	**3.05***
5		2125	3.23	4.25	3.28	4.26	**3.24**
1	0.4	4063	2.91	3.84	2.93	3.85	**2.91***
2		4047	2.97	3.87	2.98	3.88	**2.97***
3		4064	2.97	3.87	2.98	3.88	**2.97***
4		4063	3.05	4.02	3.10	4.00	**3.05***
5		4099	3.24	4.23	3.29	4.25	**3.26**
1	0.6	6017	2.92	3.82	2.93	3.83	**2.92***
2		5995	2.96	3.85	2.97	3.87	**2.96***
3		6054	2.98	3.87	**2.98***	3.88	**2.98***
4		6054	3.05	4.00	3.07	4.00	**3.05***
5		6113	3.24	4.23	3.29	4.23	**3.27**
1	0.8	7960	2.93	3.83	2.94	3.84	**2.93***
2		7988	2.97	3.84	2.98	3.85	**2.97***
3		8052	2.98	3.86	2.99	3.87	**2.98***
4		8014	3.05	4.00	3.08	4.00	**3.05***
5		8017	3.24	4.22	3.27	4.23	**3.26**
1	1	9900	2.94	3.83	**2.94***	3.83	**2.94***
2		9900	2.97	3.85	**2.98**	3.85	**2.98**
3		9900	2.98	3.86	2.99	3.86	**2.98***
4		9900	3.05	4.00	3.07	4.00	**3.05***
5		9900	3.24	4.20	3.28	4.22	**3.27**

Table 2. The average lightpath length (in hops) of the solutions obtained by the greedy RWA algorithms and the lower bound for 100-node networks with an average degree of 4.

In Table 1, the average number of wavelengths of the solutions obtained by the implemented algorithms and the lower bounds for networks with an average degree of 4 are shown. Furthermore, the lowest and highest values for each test case are shown in parenthesis while the best obtained solutions among the tested algorithms are marked in bold. Those solutions which are equal to the lower bound, i.e. that are known to be optimal, are marked as '*'. We can see that the FFD_RWA and BFD_RWA algorithms significantly outperform *Greedy_EDP_RWA* and BF_RWA and give optimal solution for all but two test cases.

In order to further asses the quality of the obtained solutions, we recorded the average path lengths of the lightpaths established for each test case. Table 2 shows the results for networks with an average degree of 4. We can see that here the 'Best Fit' strategy helps obtain significantly shorter lightpaths than the 'First Fit' strategy, while the BFD_RWA algorithm gives the best results in all test cases. The results for networks with average degree of 3 and 5 are omitted for lack of space but can be found in (Skorin-Kapov, 2006.a).

Although all four algorithms are very fast and tractable, running under half a second for the cases tested, the *Greedy_EDP_RWA* and BF_RWA are slightly faster than the FFD_RWA and BFD_RWA algorithms due to the time spent sorting the lightpaths in the latter. However, as a result of sorting lightpaths, FFD_RWA and BFD_RWA usually give the same results for any order of lightpaths (unless all lightpaths are of the same length) and thus only need to be run once, while *Greedy_EDP_RWA* and BF_RWA should be run as multi-start algorithms in order to obtain good solutions.

3. The VTD problem

3.1 Problem definition

The Virtual Topology Design problem includes determining the set of lightpaths to be established on the basis of a traffic matrix, performing RWA, and lastly routing packet-switched traffic over the established virtual topology. Given is the a graph $G=(V, E)$ representing the physical topology and a long-term traffic matrix Λ representing the estimated average traffic flows between pairs of nodes. Furthermore, we have given a limited number of transmitters and receivers, commonly referred to as transceivers T, a maximum number of wavelengths W, as well as an upper bound on the number of hops H in the physical paths of lightpaths.

Various objectives can be considered. The most common optimization criteria used for Virtual Topology Design are the minimization of congestion and average packet hop distance. Congestion is defined as the maximum traffic load on any lightpath. The average packet hop distance is the average number of lightpaths a packet or unit of traffic traverses on its way from source to destination. Traversing multiple lightpaths incurs additional delay due to opto-electronic and electro-optic conversion encountered when going from one lightpath to the next. Both congestion and average packet hop distance are functions of the virtual topology and the traffic matrix, while they are independent of the physical topology and RWA scheme.

An objective criterion which has been gaining more and more attention lately is the minimization of transmitters and receivers since they make up for most of the network cost. An additional objective was proposed in (Skorin-Kapov, 2007), called the virtual hop distance, which minimizes the average hop distance between any two nodes in the virtual topology. Minimizing this criterion ensures that the virtual topology is well connected for all node-pairs, which can postpone costly reconfiguration in case of changing traffic trends. Minimizing the physical lengths of lightpaths is also desirable due to delay and, more importantly, physical impairments which can cause signal degradation. Considering all these objectives and their trade-offs is important to successfully solving the VTD problem.

3.2 Related work

Several approaches have been proposed to solve VTD or a combination of its sub-problems using mixed-integer linear formulations (MILPs) with various constraints. A formulation for complete VTD with the objective to minimize the average packet hop distance with full wavelength conversion is given in (Banerjee & Mukherjee, 2000). Heuristics for the same problem are given in (Mukherjee et al., 1996). The problem with no wavelength conversion is formulated in (Ramaswami & Sivarajan, 1996) with the objective to minimize congestion, but with no a constraint on the number of wavelengths available. Since the formulation is intractable for larger problems, the authors suggest various heuristic algorithms. One of them is the LP Logical Design Algorithm (LPLDA) which solves a relaxation of the proposed MILP and rounds the virtual topology variables; RWA is not considered. Alternative rounding schemes to obtain better solutions from LP-relaxations were proposed in (Skorin-Kapov, 2007).

Another heuristic suggested in (Ramaswami & Sivarajan, 1996), which is best-known, is the Heuristic Topology Design Algorithm (HLDA). HLDA is a greedy algorithm for the VRWA problem with a limited number of wavelengths and no wavelength conversion. Recall that Virtual topology and Routing and Wavelength Assignment (VRWA) problem consists of the first three sub-problems in Virtual Topology Design. The fourth sub-problem, Traffic Routing (TR), is solved subsequently using an LP formulation with the objective to minimize congestion. HLDA attempts to establish lightpaths between nodes in decreasing order of their estimated traffic, where each lightpath is routed on its shortest path and assigned the lowest-indexed wavelength available. After establishing a lightpaths, the value of its corresponding traffic is decreased by the value of the next highest traffic demand (or set to zero if the next highest traffic demand is higher) and then the traffic demands are re-sorted. This enables multiple lightpaths to be established between pairs of nodes with high traffic. Once the procedure ends, additional lightpaths are set up at random between nodes with left-over transmitters and receivers. This algorithm is simple, and yet performs very well with respect to congestion for which it was tested.

In (Krishnaswamy & Sivarajan, 2001), a MILP formulation for VTD including a limit on the number of wavelengths and allowing no wavelength conversion is given. Since the formulation is intractable, its relaxation is solved iteratively 25 times using a cutting plane, after which the lightpath selection and lightpath routing variables are rounded. Wavelength assignment is performed subsequently using a heuristic, while traffic routing over lightpaths is solved with an LP composed of only the traffic constraints from their MILP for VTD. This method gives good results but can be computationally prohibitive and does not guarantee a solution with the constrained number of wavelengths due to the subsequent wavelength assignment heuristic.

In (Zang & Acampora, 1995), the VRWA problem is solved by constraining potential lightpath routes to their shortest paths, and then assigning wavelength subsequently to as many lightpaths as possible in descending order or traffic, subject to the wavelength clash and continuity constraints. This approach utilizes resources well, but significantly limits possibilities by using predetermined shortest paths. In (Puech et al., 2002) a method to reduce the complexity of the first and last sub-problems of Virtual Topology design, i.e. lightpath selection and traffic routing, are given. In (Kuri et al., 2002), a tabu-search algorithm for lightpath selection and traffic routing is presented, while the trade-offs concerning cost and congestion are studied.

3.3 Greedy algorithms each aimed to optimize different objective criteria

Due to the many aspects and evaluation criteria important for VTD and its sub-problems, it is challenging to develop heuristics which perform well for all criteria. We propose 4 greedy heuristics for the VRWA problem (Skorin-Kapov, 2008), each aimed to optimize various optimization criteria, and then solve Traffic Routing using an LP formulation from (Krishnaswamy & Sivarajan, 2001) which minimizes congestion.

3.3.1 The TSO_SP algorithm

The first greedy algorithm considers Traffic Sorted Overall and routes it on the Shortest Path available (the TSO_SP algorithm). A layered graph approach is used, as in the bin packing algorithms for RWA, but with a limited number of layers *W*. First the traffic demands are sorted in non-increasing order giving us an ordering of node-pairs which will be considered as potential lightpath demands. For each node pair in the defined order, a lightpath is established on the layer on which there is the shortest path available. If there is no feasible route available on any of the *W* layers, the lightpath between the node-pair in question is simply not established. If a lightpath is set up, the links along its path are deleted from the corresponding layer, i.e. are marked as used. This approach is similar to HLDA except that multiple lightpaths are not established between pairs of nodes and the transmitters and receivers not used initially are not subsequently assigned to random lightpaths since one of our objectives is to minimize transceiver cost.

3.3.2 The TSO_FS algorithm
The TSO_FS algorithm also Sorts Traffic Overall, but routes lightpaths on the First Satisfactory path available. Basically traffic demands are sorted in non-increasing order and corresponding potential lightpaths are routed on the lowest-indexed layer on which there is a satisfactory path available. We consider a path satisfactory of its length is less than the upper bound on the hop length *H*. If there is no satisfactory path available, the lightpath is dropped. The motivation for 'filling up' lower-indexed layers is to leave higher layers empty and potentially minimize the total number of layers used, i.e. the total number of wavelengths used.

3.3.3 The TSBS_SP algorithm
In the TSBS_SP algorithms, Traffic is Sorted By Source and routed on the Shortest Path available. Instead of sorting traffic between all node pairs, or potential lightpaths, in non-increasing order we do the following. For each node separately, we sort the traffic demands originating from that node to all other nodes (i.e. the row in the traffic matrix corresponding to the node in question) in non-increasing order. Then we make a single ordering of traffic demands, i.e. node pairs, by taking the highest traffic demand from each node, starting with the highest one overall and continuing in decreasing order. Then we take the next highest traffic demand, and the third, and so on until all traffic demands are included in the list. An example of such a method of sorting is shown in Fig. 3. Once the traffic demands are sorted, the algorithm tries to establish lightpaths in the specified order by routing them on the layer with the shortest path available. The motivation for this approach, with respect to sorting the lightpaths, is to create a virtual topology which is spread out more evenly and not only concentrated around a few nodes with very high traffic. This could lower the average virtual hop distance as well as prevent unconnected virtual topologies when resources are very scarce which can cause traffic to be blocked between certain nodes, i.e. giving infeasible solutions to the VTD problem.

3.3.4 The TSBS_FS algorithm
The TSBS_FS algorithm also considers Traffic Sorted By Source but routes lightpaths on the First Satisfactory path available. Basically, after sorting the node pairs according to the TSBS strategy, lightpaths are established on the lowest-indexed layer that has a satisfactory path available.

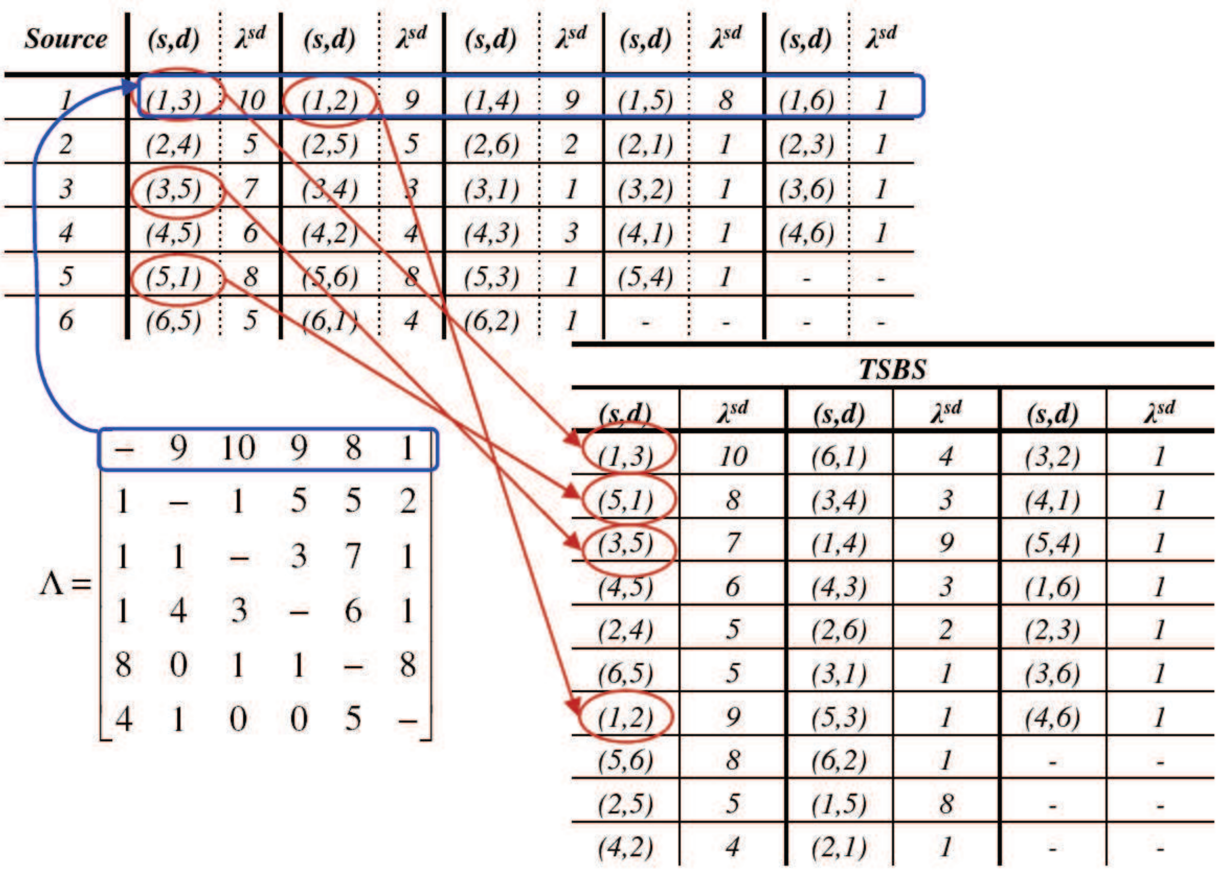


Fig. 3. An example of sorting a traffic matrix using the TSBS method.

3.4 Lower bounds

Lower bounds on the average packet hop distance and congestion were developed in (Ramaswami & Sivarajan, 1996) and are as follows. Assuming $P = (p_{sd})$ is the average traffic distribution matrix, where p_{sd} is the probability that there is a packet from s to d, Π_i for $1 \le i \le N$ is a permutation of $(1,2,\ldots,N)$ such that $p_{i\,\Pi i\,(j)} \ge p_{i\,\Pi i\,(j')}$ if $j \le j'$. If Δ_l is the maximum degree of the virtual topology, the lower bound on the average packet hop distance was shown to be

$$\overline{H_p^{LB}} = \sum_{i=1}^{N} \sum_{k=1}^{m} \sum_{j=n_{k-1}+1}^{N-1} p_{i\pi_i(j)} \tag{3}$$

where m is the largest integer such that

$$N > 1 + \Delta_l + \ldots + \Delta_l^{m-1} = \frac{\Delta_l^m - 1}{\Delta_l - 1} \tag{4}$$

and

$$n_k = \sum_{i=1}^{k} \Delta_l^i, \text{ for } i \le k \le m-1, n_m = N-1, n_0 = 0\,. \tag{5}$$

Since we consider a limited number of wavelengths W on each link, the virtual degree cannot exceed $W^*\Delta_p$, where Δ_p is the maximum degree of the physical topology, we define the maximum degree of the virtual topology Δ_l to be

$$\Delta_l = min(Tr, W \cdot \Delta_p). \tag{6}$$

Using the lower bound for the average packet hop distance described above, a lower bound on congestion was derived in (Ramaswami & Sivarajan, 1996) as

$$\lambda_{max}^{LB} = \frac{r \cdot \overline{H_p^{LB}}}{E}, \tag{7}$$

where r is the total arrival rate of packets to the network and E is the number of directed links in the virtual topology.
A lower bound on the average virtual hop distance was derived in (Skorin-Kapov, 2007) as follows. Since the average virtual hop distance is independent of the traffic matrix, the lower bound on the average virtual hop distance from any node s in V to all the other nodes in the network is the same for each node s. As noted in (Ramaswami & Sivarajan, 1996), if a network has a maximum logical degree of Δ_l , for some node s in V there can be at most Δ_l nodes one hop away from s, at most Δ_l^2 nodes two hops away, at most Δ_l^3 nodes three hops away, etc. An ideal virtual topology with respect to virtual hop distance from some node s to the remaining nodes in the network would be such a topology in which node s had Δ_l neighbors, each of which had Δ_l neighbors of their own without creating a cycle, and so on, until all the nodes were connected.
Let m be the largest integer such that $N \geq 1 + \Delta_l + \ldots + \Delta_l^{m-1} = (\Delta_l^m - 1)/(\Delta_l - 1)$ holds. In the ideal virtual topology with respect to virtual hop distance from node s, Δ_l nodes would be one hop away from s, Δ_l^2 nodes would be two hops away, etc., up until Δ_l^{m-1} nodes that would be $(m-1)$ hops away. The remaining $(N-1)-(\Delta_l + \ldots + \Delta_l^{m-1})$ nodes would be m hops away. It follows that the lower bound on the average virtual hop distance would be

$$\overline{H_v^{LB}} = \frac{\sum_{k=1}^{m-1} k\Delta_l^k + m\left[(N-1) - \sum_{k=1}^{m-1}\Delta_l^k\right]}{N-1} = \frac{\Delta_l\left[\frac{(m-1)\Delta_l^m - m\Delta_l^{m-1} + 1}{(1-\Delta_l)^2}\right] + m\left(N - \frac{\Delta_l^m - 1}{\Delta_l - 1}\right)}{N-1} \tag{8}$$

Lower bounds on the number of wavelengths, transceivers and average physical hop lengths of the lightpaths are not relevant for our particular problem, i.e., they would be zero since there is no minimum number of lightpaths which must be established.

3.5 Computational results

The greedy algorithms for the VRWA problem described above were implemented in C++ and run on a PC with a P4 2.8 GHz processor. CPLEXv6 solver was used to solve the LP for Traffic Routing. The algorithms were tested on a 14-node reference European core network topology from (Inkret et al., 2003) shown in Fig. 3. The algorithms were tested for two different traffic matrices, p1 and p2, used in (Ramaswami & Sivarajan, 1996) and (Krishnaswamy & Sivarajan, 2001) to test VTD. In traffic matrix p1, most of the traffic is concentrated around 42 pairs, while traffic in p2 is more evenly distributed. The number of transmitters and receivers per node ranged from T= 2 to 13 each, while the number of wavelengths ranged from W=T-1 to W=T+1. The upper bound on the number of physical hops was set to $H = max(diam(G), \sqrt{|E|})$ as for the RWA problem in Section 2.

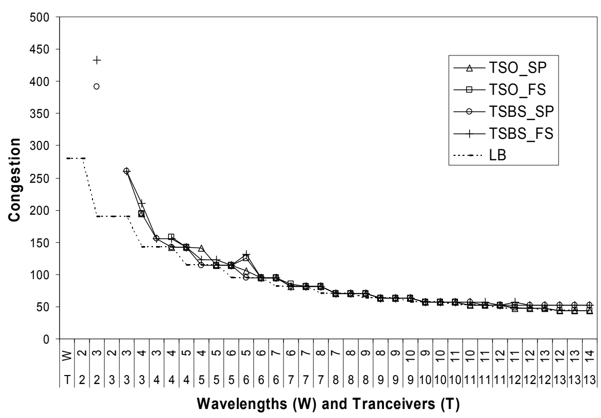
Fig. 3. A reference European core topology from (Inkret et al., 2003).

In Fig. 4, the (a) congestion, (b) average packet hop distance, (c) average virtual hop distance, and (d) number of transceivers used in the solutions obtained for traffic matrix p2 are shown. The results for traffic matrix p1 are similar and are, thus, omitted for the sake of brevity.[3] All four algorithms give similar results for congestion (Fig. 4.(a)), most of which are very close to the lower bound. The exception is for cases with a very small number of transceivers and wavelengths where the algorithms, particularly the TSO algorithms, gave unconnected virtual topologies. For the average packet hop distance, the bound is not very tight so it is difficult to assess their quality (Fig. 4.(b)). Still, we can see that the TSO algorithms tend to perform slightly better than the TSBS algorithms. On the other hand, the TSBS algorithms give better results than the TSO algorithms with respect to the virtual hop distance (Fig. 4.(c)). Here, the bound is fairly tight so we can see that the results are at least near-optimal. However, to establish better connected virtual topologies, the TSBS algorithms use more transceivers than the TSO algorithms (Fig. 4.(d)).

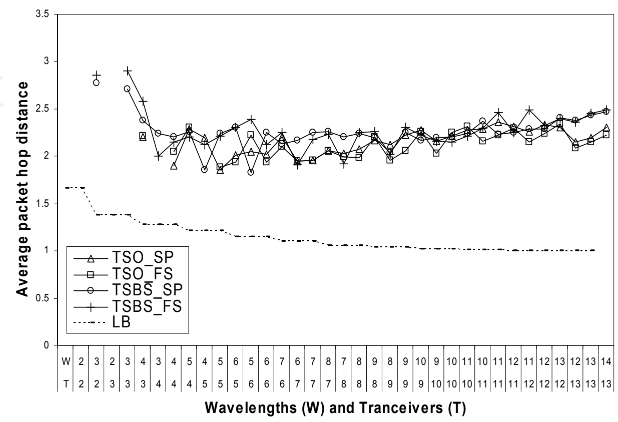
For the European core network, the algorithms usually terminated when all of the available wavelengths were exhausted, and as a result the same number of distinct wavelengths was used by all the algorithms. Furthermore, since the network is fairly small, the physical paths could not differ significantly between solutions. To assess how the algorithms behave with respect to RWA, further testing was done on 5 randomly generated 30-node networks,

[3] Additional numerical results for these algorithms can be found in (Skorin-Kapov, 2008)

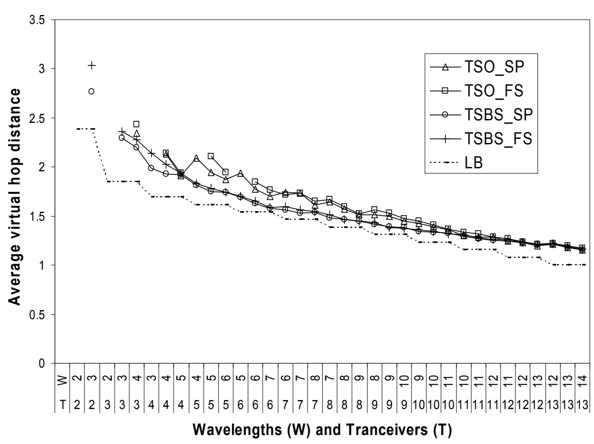
where the probability of there being an edge between two nodes was set to P_l*=0.2* creating fairly dense networks. Traffic matrices were generated using the method suggested in (Banerjee & Mukherjee, 2000) where a fraction *F* of the traffic is uniformly distributed over *[0,C/a],* while the remaining traffic is uniformly distributed over *[0, C*ψ/a].* The values were set to *C=1250*, *a=20, ψ =10* and *F=0.7* as in (Banerjee & Mukherjee, 2000).



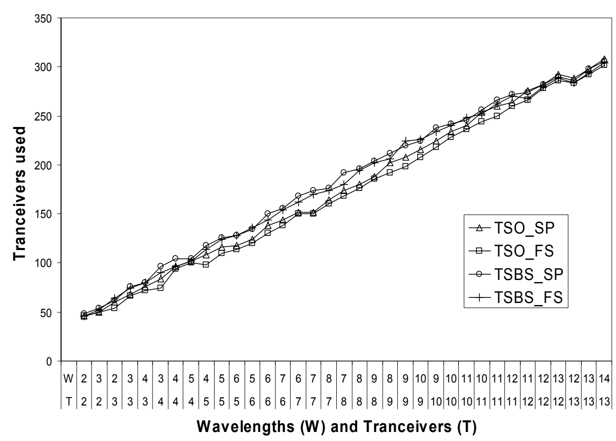
(a)



(b)



(c)

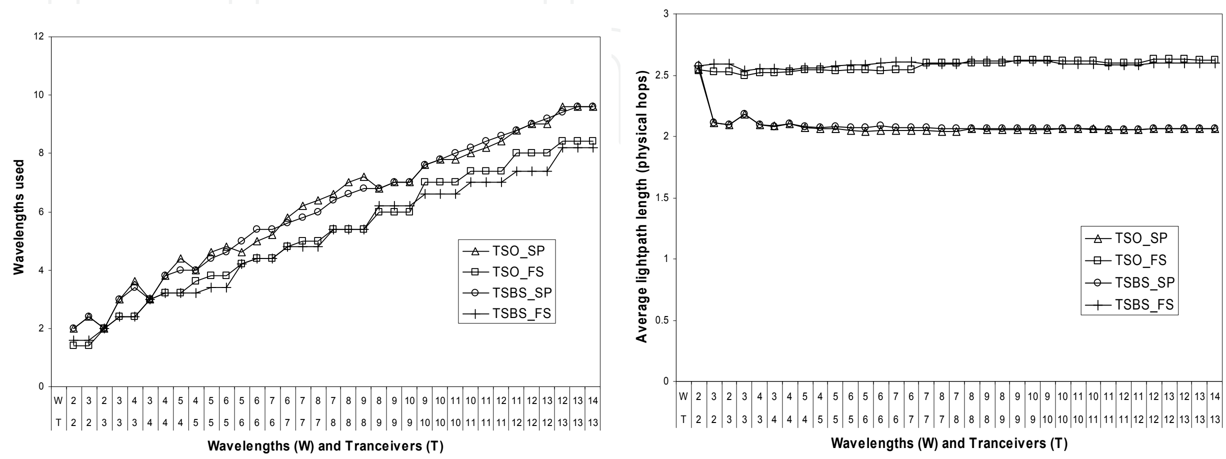


(d)

Fig. 4. The (a) congestion, (b) average packet hop distance, (c) average virtual hop distance, and (d) transceivers used by the proposed algorithms for the reference European core network for traffic matrix p2.

The number of wavelengths used and the lengths of the physical paths are shown in Figs. 5 (a) and (b), respectively. We can see that the FS algorithms use significantly fewer wavelengths than the SP algorithms (Fig. 5 (a)). The results for congestion, average packet and virtual hop distance, and the number of transceivers were almost the same for all algorithms indicating that to establish virtual topologies which perform equally well, the FS algorithms use fewer wavelengths leaving more room for expansion of the virtual topology However since the FS algorithms route paths on the first satisfactory path and not the shortest path, they tend to establish longer lightpaths (Fig. 5 (b)).
From the obtained results, we can see that when sorting traffic demands differently (i.e., TSO vs. TSBS), the TSO algorithms obtain slightly better results for congestion and packet hop distance while TSBS obtains better connected virtual topologies overall. Creating better

connected topologies might be desirable if traffic is prone to change since it can perform well, not only for current traffic trends, but for changing traffic. However, this is a trade-off with cost since establishing well-connected virtual topologies usually requires more transceivers, raising the network cost. Furthermore, if transceivers are very scarce, TSBS could help prevent from establishing unconnected virtual topologies which leave some node-pairs completely disconnected.



(a) (b)

Fig. 5. The (a) number of wavelengths used and the (b) physical lengths of lightpaths for the 30-node networks.

The method of routing and assigning wavelengths (i.e., FS vs. SP) does not significantly affect the objective criteria which are functions of the virtual topology (i.e., congestion, average packet and virtual hop distances). The main advantage of the FS algorithms over the SP algorithms is that they use fewer distinct wavelengths for RWA, particularly in dense networks. However, this is a trade-off with the physical length of lightpaths, which might be critical due to physical impairments.

4. Current and future work

Our current work on Routing and Wavelength Assignment is based on developing routing and wavelength assignment schemes aimed to increase robustness against malicious crosstalk and jamming attacks in transparent optical networks. While faults (i.e., component malfunctions) only affect the connections passing directly through them, attacks can spread and propagate throughout the network, making them more destructive and harder to locate and isolate. Our objective is to arrange lightpath demands in such a way as to minimize the propagation capabilities of such attacks. Furthermore, we aim to minimize the upper bound on the number of wavelengths required for successful wavelength assignment and reduce lightpath congestion. We are currently developing greedy algorithms based on bin packing for attack-aware wavelength assignment. Future work will include extending it to include the routing sub-problem to help obtain improved solutions for the RWA problem.

Furthermore, we are investigating the problem of scheduled Virtual Topology Design. Recall that scheduled VTD involves defining an *a priori* schedule for setting up and tearing down lightpaths based on periodic traffic trends. We proposed efficient greedy algorithms for scheduled RWA in (Skorin-Kapov, 2006.b) which route and assign wavelengths to lightpaths according to a predefined schedule. Currently, in collaboration with P. Pavon-Marino et al. from UPCT, Spain, we are focused on developing algorithms for lightpath selection and scheduling, as well as traffic routing. Preliminary testing of our MILP formulation for the problem was performed using MatPlan WDM (Pavon-Marino et al., 2007). Since this formulation is intractable for larger problem instances, we are working on greedy heuristic approaches to find suboptimal solutions.

5. Conclusions

WDM optical network planning, particularly Virtual Topology Design, is a complex problem and several variations can be considered. Since even the sub-problems of VTD themselves are hard, solving the combined problem for larger instances using exact methods is infeasible. Greedy algorithms have been shown to obtain solutions comparable to those of more complex algorithms in very short time. In the first part of this chapter, we discuss highly-efficient greedy approaches for the static Routing and Wavelength Assignment problem based on bin packing. Suggested are methods of sorting and routing lightpaths which not only reduce the required number of wavelengths, but also reduce the average physical length of established lightpaths. Numerical results indicate that the proposed methods obtain optimal or near-optimal solutions in many cases, and significantly outperform efficient existing algorithms for the same problem. Furthermore, the heuristics are robust and highly tractable and can, thus, be used to solve large problem instances in reasonable time.

In the second part of the chapter, we propose greedy algorithms for the first three sub-problems of Virtual Topology Design, i.e. lightpath selection and RWA, which we call VRWA. Traffic routing is solved subsequently using a linear programming formulation. The greedy algorithms differ with respect to the order in which lightpaths are established, and the method of routing and assigning wavelengths. These variations are intended to improve the performance of the algorithms with respect to different objective criteria such as congestion, average virtual, physical, and packet hop distances, and the number of transceivers and distinct wavelengths used. The fact that they are fast and simple, and can be tailored to meet the needs of the network in question, makes them very attractive for practical use. In general, greedy algorithms have been shown to be very promising candidates for solving complex optical networks planning problems and will play a key role in our future work on scheduled VTD and attack-aware RWA.

6. References

Banerjee, D. & Mukherjee, B. (1996), A Practical Approach for Routing and Wavelength Assignment in Large Wavelength-Routed Optical Newtorks, *IEEE Journal of Selected Areas in Communications*, Vol. 14, (June 1996) pp. 903-908.

Chlamtac, I.; Ganz, A. & Karmi, G. (1992). Lightpath communications: An approach to high-bandwidth optical WANs*, IEEE Transactions on Communications,* Vol. 40, (1992) pp. 1171-1182.

Choi, J. S.; Golmie, N.; Lapeyere, F. ; Mouveaux, F. & Su, D. (2000). A Functional Classification Of Routing and Wavelength Assignment Schemes in DWDM Networks: Static Case, *Proceedings of VII Int. Conf. on Optical Communication and Networks*, Nagoya, Japan, Jan. 2000.

Coffman, E. G.; Garey, M. R. & Johnson, D. S. (1996). Packing Approximation Algorithms: A Survey, In: *Approximation Algorithms for NP-Hard Problems*, D. Hochbaum (Ed.), PWS Publishing Co., Boston, MA.

Coffman, E. G.; Csirik, J. & Woeginger, G. (2002). Bin Packing Theory, In: *Handbook of Applied Optimization*, Pardalos, P. & Resende, M. G. C. (Eds.), Oxford University Press, New York.

Hyytia, E. & Virtamo, J. (1998). Wavelength assignment and routing in WDM networks,
Nordic Telegraffic Seminar 14 (1998) pp. 31-40.

Inkret, R.; Kuchar, A. & Mikac, B., *Advanced Infrastructure for Photonic Networks: Extended Final Report of COST Action 266*, Faculty of Electrical Engineering and Computing, University of Zagreb, Zagreb, Croatia, pp.19-21.

Jia, X.; Hu, X.-D. & Du, D.-Z. (2002). *Multiwavelength Optical Networks*, Kluwer Academic Publishers, Norwell, MA.

Krishnaswamy, R. M. & Sivarajan, K. N. (2001). Design of logical topologies: a linear formulation for wavelength-routed optical networks with no wavelength changers, *IEEE/ACM Transactions on Networking*, Vol. 9, No. 2, (April 2001), pp. 186-198.

Kuri, J.; Puech, N. & Gagnaire, M. (2002). A Tabu search algorithm to solve a logical topology design problem in WDM networks considering implementations costs, *Proceedings of SPIE Asian Pacific Optical Conference*, Shangai, China, Oct. 2002.

Lee, K.; Kang, K. C.; Lee, T. & Park, S. (2002). An Optimization Approach to Routing and Wavelength Assignment in WDM All-Optical Mesh Networks without Wavelength Conversion", *ETRI Journal*, Vol 24, No. 2, (2002) pp. 131-141.

Manohar, P.; Manjunath, D. & Shevgaonkar, R. K. (2002). Routing and Wavelength Assignment in Optical Networks From Edge Disjoint Paths Algorithms", *IEEE Communication Letters*, Vol. 6, No. 5, (May 2002) (pp. 211-213).

Mukherjee, B. (1997). *Optical Communication Networks,* McGraw-Hill, New York.

Murthy, C. S. R. & Gurusamy, M. (2002), *WDM Optical Networks: Concepts, Design, and Algorithms*, Prentice Hall, New Jersey.

Mukherjee, B.; Banerjee, D.; Ramamurthy, S. & Mukherjee, A. (1996). Some Principles for Designing a Wide-area WDM Optical Network, *IEEE/ACM Transactions on Networking*, Vol. 4, No. 5, (Oct 1996), pp. 684-696.

Noronha, T. F. & Ribeiro, C. C. (2004). Routing and wavelength assignment by partition coloring, *European Journal of Operational Research*, Vol 171, No. 3 (June 2006, available online Dec. 2004) pp. 797-810.

Noronha, T.F.; Resende, M.G.C.& Ribeiro C.C. (2008). Efficient implementations of heuristics for routing and wavelength assignment, *Proceedings of 7th International Workshop on Experimental Algorithms (WEA 2008),* C.C. McGeoch (Ed.), LNCS, Springer, vol. 5038,(2008) pp. 169-180.

Ozdaglar, A. & Bertsekas, D. (2003). Routing and Wavelength Assignment in Optical Networks", *IEEE/ACM Transactions on Networking,* Vol 11, No. 2, (April 2003) pp. 259-272.

Pavon-Marino, P.; Aparicio-Pardo, R.; Moreno-Munoz, G.; Garcia-Haro, J. & Veiga-Gontan, J. (2007). MatPlan WDM: An Educational Tool for Network Planning in Wavelength-Routing Networks, *Lecture Notes in Computer Science*, 4534, Springer-Verlag, pp. 58-67.

Puech, N.; Kuri, J. & Gagnaire, M. (2002). Topological Design and Lightpath Routing in WDM Mesh Networks: A Combined Approach, *Photonic Network Communications*, Vol. 4, No. 3/4, (July 2002) pp. 443-456.

Ramaswami, R. & Sivarajan, K. N. (1995), Routine and Wavelength Assignment in All-Optical Networks, IEEE/ACM Transactions on Networking, Vol. 3, No. 5, (Oct. 1995), pp. 489-500.

Ramaswami, R. & Sivarajan, K. N. (1996), Design of Logical Topologies for Wavelength-Routed Optical Networks, *IEEE Journal of Selected Areas in Communications*, Vol. 14, No. 5, (June 1996), pp 840-851.

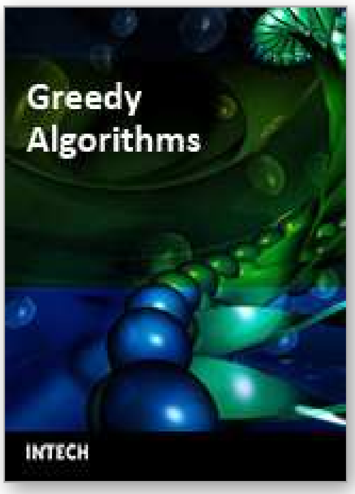
Skorin-Kapov, N. (2006). Heuristic Algorithms for the Routing and Wavelength Assignment of Scheduled Lightpath Demands in Optical Networks, *IEEE Journal of Selected Areas in Communications*, Vol. 24, No. 8, (August 2006) , pp. 2-15.

Skorin-Kapov, N. (2007a). Routing and Wavelength Assignment in Optical Networks Using Bin Packing Based algorithms, *European Journal of Operational Research*, Vol. 177, Issue 2, (March 2007) , pp. 1167-1179.

Skorin-Kapov, N. (2007b), A New Objective Criterion and Rounding Techniques for Determining Virtual Topologies in Optical Networks, *IEEE Communication Letters*, Vol. 11, No. 6, (June 2007), pp.540-542.

Skorin-Kapov, N. (2008). Virtual Topology Design in WDM Optical Networks: Greedy Algorithms and a Study of Various Objectives, (Submitted to: Telecommunications Systems).

Zang, Z. & Acampora, A. S. (1995). A Heuristic Wavelength Assignment Algorithm for Multihop WDM Networks with Wavelength Routing and Wavelength Re-Use, *IEEE/ACM Transactions on Networking*, Vol 3, No. 3, (June 1995) pp. 281-288.



Greedy Algorithms
Edited by Witold Bednorz

ISBN 978-953-7619-27-5
Hard cover, 586 pages
Publisher InTech
Published online 01, November, 2008
Published in print edition November, 2008

Each chapter comprises a separate study on some optimization problem giving both an introductory look into the theory the problem comes from and some new developments invented by author(s). Usually some elementary knowledge is assumed, yet all the required facts are quoted mostly in examples, remarks or theorems.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:

Nina Skorin-Kapov (2008). WDM Optical Networks Planning Using Greedy Algorithms, Greedy Algorithms, Witold Bednorz (Ed.), ISBN: 978-953-7619-27-5, InTech, Available from: http://www.intechopen.com/books/greedy_algorithms/wdm_optical_networks_planning_using_greedy_algorithms

INTECH
open science | open minds

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821