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Nonlinear Adaptive Tracking-Control Synthesis for General Linearly Parametrized Systems

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1. Introduction

A common problem of engineering practice is to cope with mathematical models of objects with only partly known structure. The model may e.g. involve some unknown (linear or nonlinear) functions that depend on the kind of object (of a given class to which the model refers) and/or of its operation conditions. As an example we take an affine model of SISO system

$$\dot{\boldsymbol{x}} = \boldsymbol{\alpha}(\boldsymbol{x}) + \boldsymbol{\beta}(\boldsymbol{x}) \cdot u \tag{1a}$$

$$y = h(\boldsymbol{x}) \tag{1b}$$

where y, $\boldsymbol{x}$, u denote output, state and control variables respectively, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are smooth vector fields on R^n and $h: R^n \to R$ a smooth function. It is assumed here also that the functions $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are unknown or may be estimated with a considerable inaccuracy.
Considering the system (1) it is possible (under certain conditions (Fabri & Kadrikamanathan, 2001; Sastry & Isidori, 1989)) to obtain a direct input-output relation between u and y, by successive differentiation y with respect of time having

$$y^{(r)} = f(\boldsymbol{x}) + g(\boldsymbol{x})u \tag{2}$$

where r denotes a system relative degree. The whole approach could be well systematized and explained using the concept of Lie derivatives (Isidori, 1989) .
In this chapter the system (1) is uncertain in the sense it is linearly parametrized, or in other words, the unknown functions $\boldsymbol{\alpha}_i$ and $\boldsymbol{\beta}_i$ are assumed to be linear combinations of some known model related functions which represents our elementary knowledge on the model. It is easy to prove (see appendix) that if the functions $\boldsymbol{\alpha}_i$ and $\boldsymbol{\beta}_i$ of system (1a) are of the form of linear combinations of some known functions $\boldsymbol{\alpha}_i$ and $\boldsymbol{\beta}_i$ i.e.

$$\boldsymbol{\alpha}(\boldsymbol{x}) = \sum_{i=1}^{m_1} a_i \boldsymbol{\alpha}_i(\boldsymbol{x}); \qquad \boldsymbol{\beta}(\boldsymbol{x}) = \sum_{i=1}^{m_2} b_i \boldsymbol{\beta}_i(\boldsymbol{x}) \tag{3}$$

where a_i, b_i are real unknown parameters then the scalar functions f, g of system (2) may be represented in similar form:

$$f(\boldsymbol{x})=\sum_{i=1}^{n_1}\theta_i^1 f_i(\boldsymbol{x})+f_0(\boldsymbol{x}) \quad ; \quad g(\boldsymbol{x})=\sum_{i=1}^{n_2}\theta_i^2 g_i(\boldsymbol{x})+g_0(\boldsymbol{x}) \tag{4}$$

with θ_i^1, θ_i^2 unknown parameters and f_i, g_i (called here *model basis functions*) again known trough the $\boldsymbol{\alpha}_i$ and $\boldsymbol{\beta}_i$ (see appendix).
There are a huge amount of nonlinear systems that might be modeled in general form (1),(3). Using described above model transformation one can obtain a parametric model of the form (2),(4) in relative easy way (see section 3.2). The model in this form, referred below as a transformed model, was considered in many papers. One of the known method of tracking control synthesis in the case when we have a rough estimate of the model (2) functions, is a sliding mode control law (Slotine & Li, 1991). The alternative is to use adaptation (for model in the form (2),(4)) which offers more subtle policy but requires more advanced theory.
In our approach the unknown functions *f* and *g* of the transformed model are, as it turned out, linear combinations of some known model related *basis functions* i.e. some elementary knowledge of the model is assumed. The assumption above may, however, be substantially relaxed via applying, as basis functions, some sort of known approximators (Fabri & Kadrikamanathan, 2001; Tzirkel-Hancock & Fallside, 1992). As an example one may adopt a neuro-approximator with Gaussian radial basis functions (Sanner & Slotine, 1992). Systems of this sort are referred to as *functional adaptive* (Fabri & Kadrikamanathan, 2001) and represent a new branch of intelligent control systems. In the real-world applications, however, it seems purposeful to assume that we have at our disposal some (often very limited) knowledge, on the considered plant or process, that should be exploited in reasonable way. In this paper the accent is put-on the later issue.
This chapter is concerned with the problem of adaptive tracking system control synthesis for the described above class (1),(3) of uncertain systems. It has been proven that proportional state feedback plus parameters adaptation via the model basis function concept are able to assure system asymptotic stability. This form of controller permits on-line compensation of unknown model nonlinearities which leads to satisfactory tracking performance. The presented theory is illustrated by the example of ship path-following control system (Zwierzewicz, 2007ab).
It is worth to observe that affine model description (1) is taken here without loss of generality. The general nonlinear system

$$\dot{\boldsymbol{x}}=\boldsymbol{F}(\boldsymbol{x},u) \tag{5a}$$

$$y=h(\boldsymbol{x}) \tag{5b}$$

may be easy expressed in this form by augmenting it with input integrator $\dot{u}=v$ which leads to new state $\boldsymbol{x}_a=\left[\boldsymbol{x}^T \quad u\right]^T$. Now considering *v* as a new input the above system is in the form (1).
The chapter is organized as follows. In section 2., an appropriate portion of the theory is shortly presented, which utility (in the next section) is then verified via an example of ship path-following control system. The next sections contain results of the relevant system simulations, remarks and conclusion.

2. Adaptive tracking control synthesis

The control objective is to force the plant (1) output vector $\boldsymbol{y} = [y, \dot{y}, \cdots, y^{(r-1)}]^T$ to follow a specified desired trajectory $\boldsymbol{y}_d = [y_d, \dot{y}_d, \cdots, y_d^{(r-1)}]^T$ with state vector $\boldsymbol{x}$ remaining bounded. It is moreover assumed that reference input y_d and its *r* derivatives are bounded and known as well as that the system zero dynamics is globally exponentially stable (minimum phase condition) .

As the model (1),(3) can be transformed to the form (2),(4) thus, in what follows, our considerations will be referred to the later form.

2.1 The case of exact model

It is assumed in this section that the nonlinear functions *f* and *g* of model (2) are known and $g(\boldsymbol{x}) \neq 0, \ \ \forall \boldsymbol{x} \in R^n$. A substitution of control law

$$u = \frac{-f(\boldsymbol{x}) + v}{g(\boldsymbol{x})} \tag{6}$$

in the system (2) results in exact cancellation of both nonlinearities ($f(\boldsymbol{x})$ and $g(\boldsymbol{x})$) which yields

$$y^{(r)} = v \tag{7}$$

To find control $v(t)$ stabilizing this linear system, a standard poles location technique can be used. If v is chosen as

$$v = y_d^{(r)} - \mu_r e^{(r-1)} - \cdots - \mu_1 e \tag{8}$$

where y_d denotes the reference input which y is required to track, $e := y - y_d$ denotes the output tracking error and coefficients μ_i are chosen such that $\Gamma(s) =: s^r + \mu_r s^{r-1} + \cdots + \mu_1 s = 0$ is Hurwitz polynomial in the Laplace variable *s*, then the tracking error and its derivatives converge to zero asymptotically, because the closed-loop dynamics reduce to the equation

$$e^{(r)} + \mu_r e^{(r-1)} + \cdots + \mu_1 e = 0 \tag{9}$$

which, by virtue of the choice of coefficients μ_i is asymptotically stable (Fabri & Kadrikamanathan, 2001; Sastry & Isidori, 1989; Tzirkel-Hancock & Fallside 1992).

2.2 The case with functional uncertainty

Let us consider now the case when functions *f* and *g* are unknown but have the form (4) with $\theta_i^1, \ \ i = 1, \cdots n_1$, $\theta_i^2, \ \ i = 1, \cdots n_2$ unknown 'true' parameters and the $f_i(\boldsymbol{x}), g_i(\boldsymbol{x})$ known model basis functions. At time *t* our estimates of the functions *f* and *g* are respectively

$$\hat{f}(\boldsymbol{x})=\sum_{i=1}^{n_1}\hat{\theta}_i^1(t)f_i(\boldsymbol{x})+f_0(\boldsymbol{x})\ ;\quad \hat{g}(\boldsymbol{x})=\sum_{i=1}^{n_2}\hat{\theta}_i^2(t)g_i(\boldsymbol{x})+g_0(\boldsymbol{x}) \tag{11}$$

with $\hat{\theta}_i^1,\hat{\theta}_i^2$ standing for the estimates of the parameters θ_i^1,θ_i^2 respectively at time *t*.
Since substitution in the system (2) the control law

$$u=\frac{-\hat{f}(\boldsymbol{x})+v}{\hat{g}(\boldsymbol{x})} \tag{12}$$

no longer guarantees exact cancellation and whereby a resulting system linearity (like in the former case (6)), we will proof below a useful here theorem. Prior to its formulation let us define a sliding surface (Slotine & Li, 1991) which represents some (aggregate) measure of the tracking error

$$\varepsilon(t)=\eta_1 e+\cdots+\eta_{r-1}e^{(r-2)}+e^{(r-1)}:=\Psi(e) \tag{13}$$

as well as introduce some notations

$$f-\hat{f}=\sum_{i=1}^{n_1}(\theta_i^1-\hat{\theta}_i^1)f_i(\boldsymbol{x})=\boldsymbol{\theta}^{1T}\boldsymbol{w}_1;\quad (g-\hat{g})u=\sum_{i=1}^{n_1}(\theta_i^2-\hat{\theta}_i^2)g_i(\boldsymbol{x})u=\boldsymbol{\theta}^{2T}\boldsymbol{w}_2 \tag{14}$$

where

$$\boldsymbol{w}_1=\begin{bmatrix}f_1 & f_2 & \cdots & f_{n_1}\end{bmatrix}^T;\qquad \boldsymbol{w}_2=\begin{bmatrix}g_1 & g_2 & \cdots & g_{n_2}\end{bmatrix}^T u \tag{15}$$

are model *basis functions* and

$$\boldsymbol{\theta}^1=[(\theta_1^1-\hat{\theta}_1^1)\cdots(\theta_{n_1}^1-\hat{\theta}_{n_1}^1)]^T;\ \ \boldsymbol{\theta}^2=[(\theta_1^2-\hat{\theta}_1^2)\cdots(\theta_{n_2}^2-\hat{\theta}_{n_2}^2)]^T \tag{16}$$

are vectors of parameters.
Moreover $\boldsymbol{\theta}=\begin{bmatrix}\boldsymbol{\theta}^{1T} & \boldsymbol{\theta}^{2T}\end{bmatrix}^T$; $\boldsymbol{w}=[\boldsymbol{w}_1^T\ \ \boldsymbol{w}_2^T]^T$.

Theorem

The closed-loop system (2), (12) and (8) after introduction of parameter update law,

$$\dot{\boldsymbol{\theta}}=-\varepsilon\boldsymbol{w} \tag{17}$$

yields bounded $\boldsymbol{y}(t)$ asymptotically converging to $\boldsymbol{y}_d(t)$.
Proof:
Differentiating (13) and multiplying by a scalar k_d we have

$$\begin{aligned}\dot{\varepsilon}(t)+k_d\varepsilon(t)&=k_d\eta_1 e+(k_d\eta_2+\eta_1)\dot{e}+\cdots+(k_d+\eta_{n-1})e^{(r-1)}+e^{(r)}=\\&=\mu_1 e+\mu_2\dot{e}+\cdots\mu_r e^{(r-1)}+y^{(r)}-y_d^{(r)}=y^{(r)}-v\end{aligned} \tag{18}$$

The coefficients η_i as well as k_d should be selected so that μ_i should have the property mentioned earlier, i.e. that they should ensure an asymptotically stable solution to equation (9).

Transforming now (12) and substituting in (2) yields

$$y^{(r)} - v = f + gu - v \tag{19}$$

$$y^{(r)} - v = f + gu - \hat{f} - \hat{g}u = f - \hat{f} + (g - \hat{g})u \tag{20}$$

so we get the following error equation

$$\dot{\varepsilon}(t) = -k_d \varepsilon(t) + f - \hat{f} + (g - \hat{g})u \tag{21}$$

Making use of (14) the error equations (21) will take a form

$$\dot{\varepsilon}(t) + k_d \varepsilon(t) = \boldsymbol{\theta}^{1T} \boldsymbol{w}_1 + \boldsymbol{\theta}^{2T} \boldsymbol{w}_2 = \boldsymbol{\theta}^T \boldsymbol{w} \tag{22}$$

We prove that the error equation (22) along with the update law (17) yields a bounded $y(t)$ asymptotically converging to $y_d(t)$.
Let us take the Lapunov–like (Slotine & Li, 1991) function of the form

$$V(\varepsilon, \boldsymbol{\theta}) = \frac{1}{2}\varepsilon^2 + \frac{1}{2}\boldsymbol{\theta}^T \boldsymbol{\theta} \tag{23}$$

hence

$$\dot{V} = \varepsilon \cdot \dot{\varepsilon} + \boldsymbol{\theta}^T \dot{\boldsymbol{\theta}} = \varepsilon(-k_d \varepsilon + \boldsymbol{\theta}^T \boldsymbol{w}) - \varepsilon \boldsymbol{\theta}^T \boldsymbol{w} = -k_d \varepsilon^2 \leq 0 \tag{24}$$

If we assume that $k_d > 0$ we have proved that Lapunov function is decreasing along trajectories of (22); thereby establishing bounded ε and $\boldsymbol{\theta}$. However, to verify that $\varepsilon \to 0$ as $t \to \infty$ we use Barbalat's lemma (Slotine & Li, 1991) To check the uniform continuity of $\dot{V}$ it is enough to prove that the second derivative of V i.e.

$$\ddot{V} = -2k_d \varepsilon \dot{\varepsilon} = -2k_d \varepsilon(-k_d \varepsilon + \boldsymbol{\theta}^T \boldsymbol{w}) \tag{25}$$

is bounded. This in turn needs $\boldsymbol{w}$, a continuous function of $\boldsymbol{x}$ to be bounded. Note that if ε and $\boldsymbol{y}_d$ are bounded, it is implied that $\boldsymbol{y}$ is bounded. These facts and assumed stable zero dynamics imply that the state $\boldsymbol{x}$ is bounded. Now (if we could guarantee that $\hat{g}(\boldsymbol{x})$ of (12) is bounded away from zero) it follows that $\boldsymbol{w}$ is bounded. □

Remarks:

Note that, although ε converges to zero the system (22), (17) is not asymptotically stable because $\boldsymbol{\theta}$ is only guaranteed to be bounded.

Prior bounds on the parameters θ_i^2 are frequently sufficient to guarantee that $\hat{g}(\boldsymbol{x})$ is bounded away from zero (Sastry & Bodson, 1989).

One can now observe that adaptive reconstruction of functions f and g in the formula (11) may be interpreted as an extra control leading to much more exact cancellation of system (2) nonlinearities, which in turn make the resulting system closer to linear (see Fig. 1)

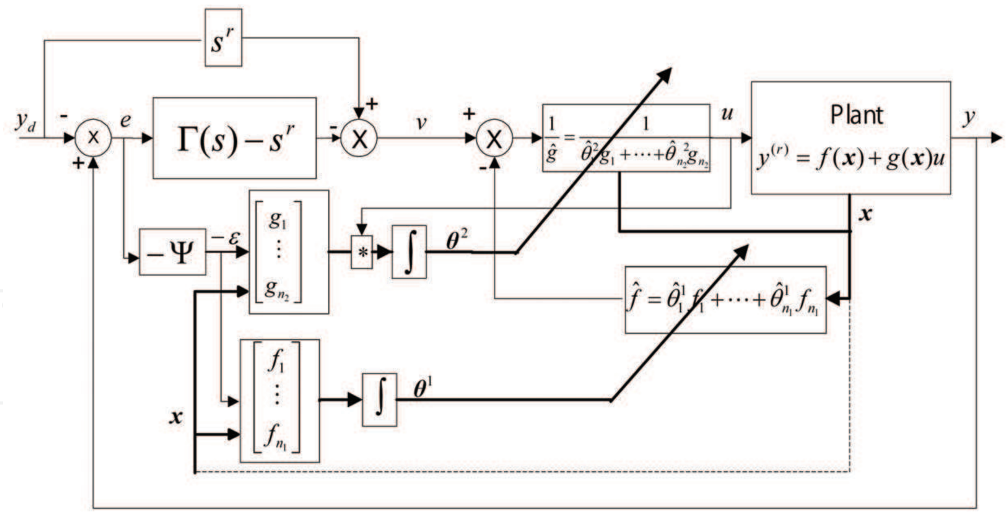

Fig. 1. Model basis functions adaptive control scheme.

3. Adaptive ship path-following control synthesis

Prior to introduction a model that represents further a base for controller synthesis we define some preliminary notions.

3.1 Path-following errors definition

Assume that a path to be followed (preset) is composed of broken line segments defined by a sequence of vertexes (turning points) $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, ... , $P_i(x_i,y_i)$, ..., $P_n(x_n,y_n)$. Let us introduce also the following coordinate systems (Fig.2):
earth-fixed coordinate system (X_g, Y_g) (these coordinates can be measured directly via GPS).
relative (transformed) coordinate system (X_r, Y_r) whose center is located at the point $P_i(x_i,y_i)$ and with the axis OX_r directed along a segment P_iP_{i+1} (i=1,2,...,n)
The relative ship position (x_r,y_r) as well as its relative heading ψ_r can be obtained through the following simple transformation:

$$\begin{bmatrix} x_r \\ y_r \\ \psi_r \end{bmatrix} = \begin{bmatrix} \cos\varphi_{ro} & \sin\varphi_{ro} & 0 \\ -\sin\varphi_{ro} & \cos\varphi_{ro} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_g - x_i \\ y_g - y_i \\ \psi - \varphi_{ro} \end{bmatrix} \tag{26}$$

which express the successive translation and then rotation of the earth-fixed system where $\varphi_r o$ is an angle of its rotation

$$\tan\varphi_{ro} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \tag{27}$$

Now it is reasonable to treat the coordinate y_r and the heading ψ_r as the path-following errors corresponding to the given segment.
For curvilinear reference path the local (relative) coordinate system should be tangent to the path at the point that is closest to the actual ship position. This system has to be then shifted and rotated from time step to time step in such a way, that it remains tangent to the reference path and that the x-coordinate represents the arc length along the path.

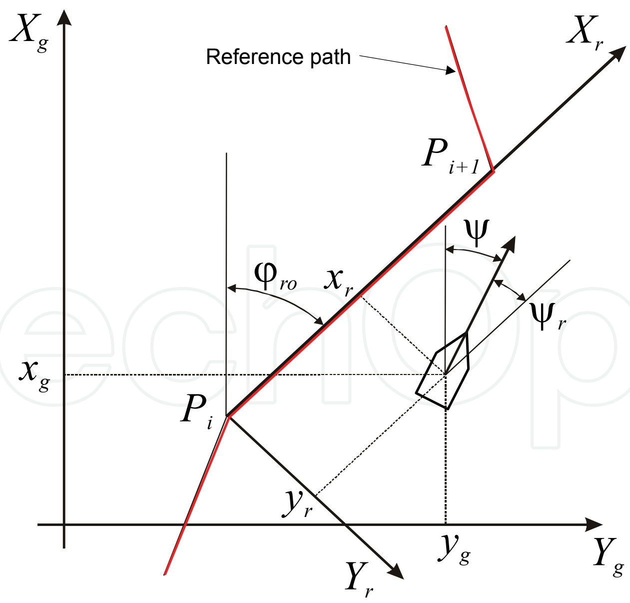

Fig. 2. Earth-fixed and relative coordinate systems

3.2 The case with functional uncertainty

In order to synthesize a path-following controller we apply the adaptive control concepts, presented in the section 2., to the following (partially known), ship motion model presented in the form of so-called error equation

$$\begin{cases} \dot{y}_r = u\sin\psi_r + v\cos\psi_r & (28a) \\ \dot{\psi}_r = r & (28b) \\ \dot{r} = \Phi(r) + c\delta & (28c) \end{cases}$$

with the output

$$y = y_r \tag{28d}$$

where

y_r – relative abscissa of the ship position (cross-track error)

ψ_r – relative heading (course-error)

r – angular velocity

u – longitudinal velocity

v – transversal velocity

y – system output

δ – rudder deflection as a control variable

c – unknown model parameter

$\Phi(\cdot)$ - unknown function

The equation (28a) is a second equation of the ship kinematical model (compare the first two equations of model (39)) while (28b) and (28c) are in fact the Norrbin ship model (Fossen, 1994; Lisowski, 1981) whose standard form

$$T\ddot{\psi} + F(\dot{\psi}) = k\delta \tag{29}$$

can be transformed into the relevant equations of (28) via definition $\dot{\psi}_r = r$ and substitution of

$$\Phi = -\frac{F(\cdot)}{T} \text{ and } c = k/T \tag{30}$$

The first equation of kinematics, in the model (28), is omitted as x_r represents movement along the path - which is irrelevant here. It is also assumed, for simplicity, that transversal velocity v is of the form $v = -r_1 r$ (compare the last equation of model (39)) where r_1 is unknown.
The double differentiation (which in fact represents a formalism delivered in appendix) of output y with respect to time leads to

$$\ddot{y} = f(\boldsymbol{x}) + g(\boldsymbol{x})\delta \tag{31}$$

where

$$f(\boldsymbol{x}) = ru\cos\psi_r + r_1 r^2 \sin\psi_r - r_1 \cos\psi_r \cdot \Phi(r) \tag{32a}$$

$$g(\boldsymbol{x}) = cr_1 \cos\psi_r \tag{32b}$$

and the state vector $\boldsymbol{x} = [y_r \;\; \psi_r \;\; r]^T$ is assumed to be accessible to measurement.
Simple analysis of this system as well as physical limitations indicate its stable internal (zero) dynamics.
Remark:
In the 'classical' approach to ship control the structure of function F is (according to Norrbin model) often adopted in different ways. Generally it may be assumed in the form

$$F(\dot{\psi}) = a_3\dot{\psi}^3 + a_2\dot{\psi}^2 + a_1\dot{\psi} + a_0 \tag{33}$$

or ignoring the terms of third or second degree we have for example

$$F(\dot{\psi}) = a_3\dot{\psi}^3 + a_1\dot{\psi} + a_0 \tag{34}$$

Now, assuming that a structure of the function F has been predetermined, the coefficients a_i are usually identified via sea trials (Lisowski, 1981).
Owing to that as well as taking into account that Φ has a similar structure as F (below we take the case (33)), it is natural to estimate the (partially) unknown functions (32) of model (31) as follows

$$\begin{aligned} \hat{f}(\boldsymbol{x}) = \sum_{i=1}^{4} \hat{\theta}_i^1 f_i + f_0 = \hat{\theta}_1^1 r^3 \cos\psi_r + \hat{\theta}_2^1 r^2 \cos\psi_r + \\ + \hat{\theta}_3^1 r \cos\psi_r + \hat{\theta}_4^1 \cos\psi_r + \hat{\theta}_5^1 r^2 \sin\psi_r + ru\cos\psi_r \end{aligned} \tag{35a}$$

$$\hat{g}(\boldsymbol{x}) = \sum_{i=1}^{1} \hat{\theta}_i^2 g_i + g_0 = \hat{\theta}_1^2 \cos\psi_r \tag{35b}$$

defining thereby a set of model basis functions f_i, g_i.
It can be seen from (35) that to implement our algorithm besides of the state vector measurements the longitudinal velocity u is also required.
To complete the employing of the theory introduced earlier to our specific case we also need:
the measure of the error

$$\varepsilon(t) = \dot{e} + \eta e = \dot{y}_r + \beta y_r \tag{36}$$

rudder control law

$$\delta = \frac{-\hat{f}(\boldsymbol{x}) + v}{\hat{g}(\boldsymbol{x})} \tag{37}$$

where

$$v(t) = \ddot{y}_d - \mu_2 \dot{e} - \mu_1 e = -\mu_2 \dot{y}_r - \mu_1 y_r \tag{38}$$

and the parameter update law (17).
Note that in our setting above (coordinate transform) $y_d = 0$, so a main task for our controller is to bring output i.e. cross-track error to zero. In fact bringing at the same time ψ_r to zero, in presence of disturbances (e.g. transversal current), is (for the considered here ship (39)) not always possible (Zwierzewicz, 2003) This way the path-following process may be, in our case, accomplished only in the presence of a course error (nonzero drift angle).

4. Ship model and simulations

4.1. Ship motion model

As a simulation model that represents further a real ship dynamics we adopt here the following de Wit-Oppe's (W-O) ship dynamical model (Wit & Oppe, 1979-80).

$$\begin{aligned}
\dot{x} &= u\cos\psi - v\sin\psi \\
\dot{y} &= u\sin\psi + v\cos\psi \\
\dot{\psi} &= r \\
\dot{r} &= -a\,r - br^3 + c\delta \\
\dot{u} &= -f\,u - Wr^2 + S \\
v &= -r_1\,r - r_3 r^3
\end{aligned} \tag{39}$$

where
(x, y) - Cartesian coordinates
ψ - course (heading)
r - angular velocity

u - longitudinal velocity
v - transversal velocity
δ - rudder deflection as a control variable
S - propelling force
Compared to the model (28) one can see that the structure of function Φ adopted there takes the form $\Phi(r) = -br^3 - ar$. Note that this ship characteristic is obviously unknown to the control system designer and has to be adaptively reconstructed.
As the ship model parameters the dynamic maneuvering parameters of the m.s. Compass Island model are adopted. The units of time, length and angle are respectively one minute, one nautical mile and one radian. The parameters were determined as follows a = 1.084 /min, b=0.62min, c = 3.553 rad/min, r_1 = -0.0375 nm/rad, r_2=0, f = 0.86 /min, W= 0.067 nm/rad^2, S=0.215 nm/min^2. The maximum speed of rudder and rudder angle are 3.8 deg/s, and 35 deg, respectively. The ship has got the following characteristics, gross register tonnage 9214 t, deadweight, 13498 t, length, 172 m, draught, 9.14 m, one propeller, and maximum speed, 20 knots. Notice that the adopted parameters make the ship directionally stable (Fossen, 1994; Lisowski, 1981) and that other ship dynamic model (parameters) could be used here as well.

4.2 Simulation results

The *Simulink* simulations are based on the *nonlinear* W-O model of ship dynamics (34) including the controller (37) together with the main feedback linear control component (38), while the adaptation mechanism is realized by *aggregate* tracking error (36), model basis functions (35) as well as parameters update law (17) (Fig. 1).
In Fig. 3 the path to be followed (preset) is a broken line defined by the *way points* (0,0); (0,10); (4,12) and (4, 20). The original ship position, its heading and angular velocity are (0,-0.5), 60° and 0 rad/min respectively. The adopted distance scale is 1 nm while the nominal ship velocity is 0.25 nm/min. In the simulation a transversal current has been, as a load disturbance, introduced (d_y=0.04 nm/min).
To evaluate the accuracy of adaptive process control there is depicted here also a trajectory (blue) driven by controller (37) with fully known dynamics (exact model functions). As we can see the differences are practically negligible.
Fig. 4. describes plots of ship heading versus time. The blue line refers to the case of the fully known ship dynamic model. As one can observe the ship heading, during straight line path segments, is about -10 deg, which in fact indicate a course-error. Such a behavior is, on the other hand, necessary to compensate an effect of currents action. These simulations comply thereby with the relevant comment of section 3.2.
In Fig. 5. it can be seen, that in the case of limited ship model knowledge, the rudder action is substantially more intensive (red line), as compared to the case of full model familiarity.
The last Fig. 6. depicts the plots of cross-track errors versus time. As before the red plot refers to the limited knowledge of the ship dynamics. It proves once more that the differences are relatively small.
An interesting feature of the adaptation process is that the steering process is performed without asymptotic convergence of parameters errors $\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\theta}^{1T} & \boldsymbol{\theta}^{2T} \end{bmatrix}$ to zero (we have proved, at the most, their boundedness). This fact reflects an idea that the main goal of the adaptive system is to drive the error $e := y - y_d$ to zero which does not necessarily imply

that the controller parameters approach their correct values. In fact, the input signal must have certain properties, for the parameters to converge, related to the notion of *persistent excitation* (Astrom & Wittenmark, 1995). This concept, in reference to the closed-loop signals, may be formulated as a requirement of sufficient richness of functions $\boldsymbol{w}$ (15). It is, however, impossible to verify this condition explicitly ahead of time (Sastry & Isidori, 1989; Wang & Hill, 2006).

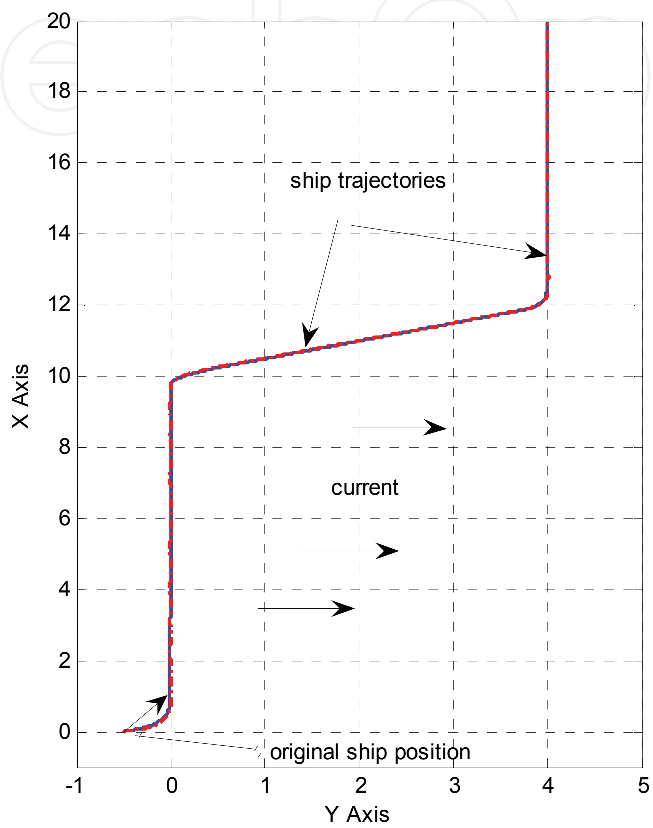

Fig. 3. Ship trajectories, constant current.

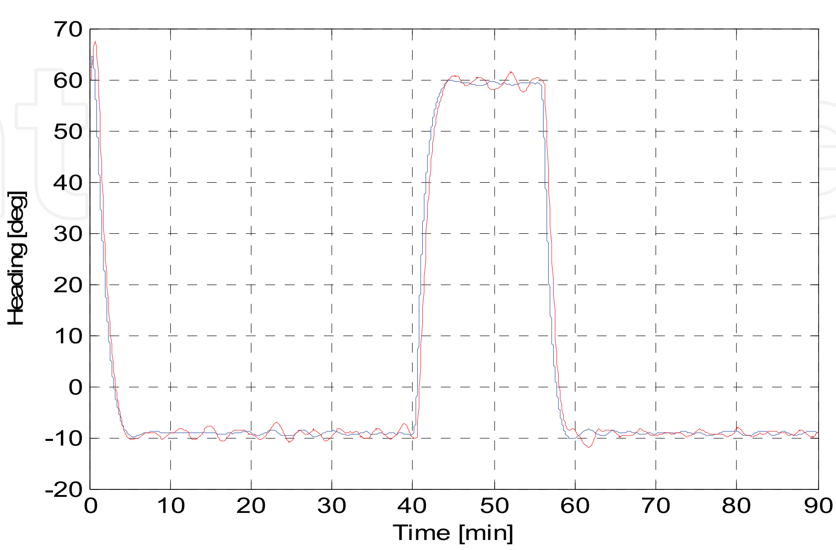

Fig. 4. Ship headings versus time.

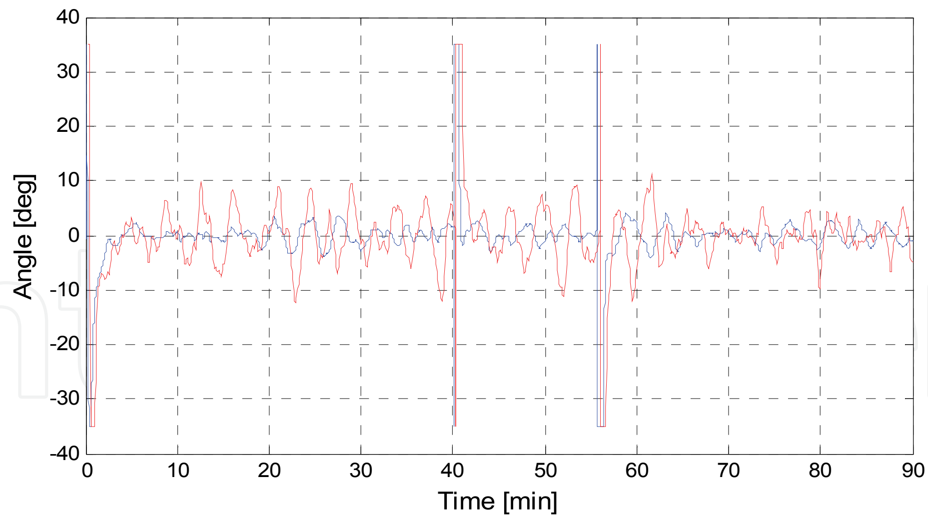

Fig. 5. Rudder deflections versus time.

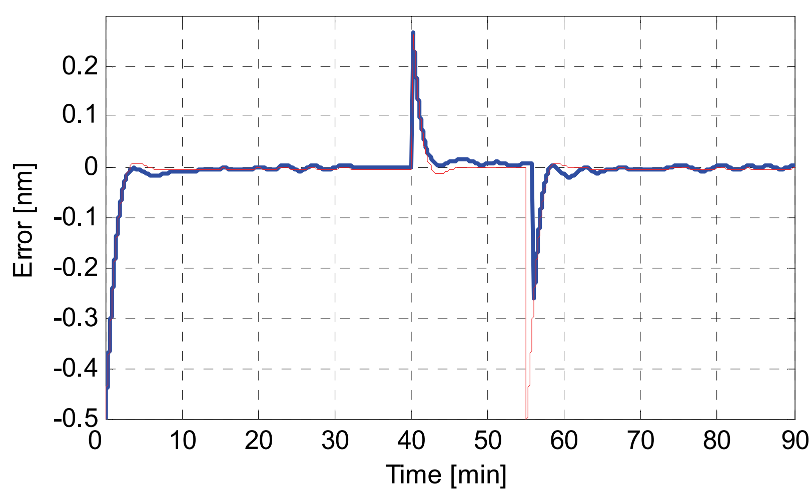

Fig. 6. Cross-track errors versus time.

As a reference input comprises stepwise signals (path) changes, to fulfill the assumptions of its differentialability it has been initially prefiltered. Similarly the wave disturbances were modeled in the form of a white noise driven shaping filter (Fossen, 1994; Zwierzewicz, 2003).

During conducted here simulations, the system performance turned out to be especially sensitive for initial guess of parameter θ_1^2 that had to be picked up in some vicinity of its true value (true value 0.133; picked up 0.5). In this respect, to ensure robustness for the disturbances that arise due, e.g., to the initial guess of parameters and thus inherent approximation errors the system should be additionally augmented with a sliding mode control. This technique is often applied to force the system global stability (Fabri & Kadrikamanathan, 2001; Sanner & Slotine, 1992; Tzirkel-Hancock & Fallside, 1992).

5. Conclusion

In the paper a general class of uncertain, linearly parametrized, nonlinear SISO plants was considered. It has been proven that proportional state feedback plus adaptation via *model* *basis functions* are able to assure their asymptotic stability. As a result of presented theory an adaptive ship path-following system has been proposed. The presented simulations confirm that the system is insensitive for object (ship) model unfamiliarity.

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7. Appendix

We will prove that the system (1),(3) may be easy transformed to the form (2),(4). To this end we recall to the concept of Lie derivative.

Lie derivative of scalar function $h(\boldsymbol{x})$ with respect to a vector $\boldsymbol{\alpha}(\boldsymbol{x})$, denoted by $L_{\alpha}h(\boldsymbol{x})$ is defined as:

$$L_{\alpha}h(\boldsymbol{x}) = \nabla h(\boldsymbol{x})\boldsymbol{\alpha}(\boldsymbol{x}) \tag{40}$$

where ∇h denotes the gradient of $h(x)$ i.e. $\left[\partial h/\partial x_1 ... \partial h/\partial x_{\mathrm{n}}\right]$. Lie derivative is scalar so the process of taking Lie derivatives could be chained and is denoted as follows

$$L_{\boldsymbol{\alpha}}^{i} h(\boldsymbol{x}) = \nabla(L_{\boldsymbol{\alpha}}^{i-1} h(\boldsymbol{x}))\boldsymbol{\alpha}(\boldsymbol{x}) \tag{41}$$

$$L_{\boldsymbol{\beta}} L_{\boldsymbol{\alpha}}^{i} h(\boldsymbol{x}) = \nabla(L_{\boldsymbol{\alpha}}^{i} h(\boldsymbol{x}))\boldsymbol{\beta}(\boldsymbol{x}) \tag{42}$$

Differentiating y in eqation (1) with respect to time and using Lie derivatives we get e.g.

$$y^{(1)} = \frac{\partial y}{\partial \boldsymbol{x}}\dot{\boldsymbol{x}} = L_{\boldsymbol{\alpha}} h(\boldsymbol{x}) + L_{\boldsymbol{\beta}} h(\boldsymbol{x}) u \tag{43}$$

where $y^{(i)}$ denotes te ith derivative of y with respect to time.
Assume that the system (1) has relative degree equal to r i.e. after r differentiations the following conditions are satisfied

$$L_{\boldsymbol{\beta}} L_{\boldsymbol{\alpha}}^{i-1} h(\boldsymbol{x}) = 0 \qquad \text{for } i = 1,\ldots, (r - 1) \tag{44a}$$

$$L_{\boldsymbol{\beta}} L_{\boldsymbol{\alpha}}^{r-1} h(\boldsymbol{x}) \neq 0 \tag{44b}$$

Calculating now the Lie derivatives of r-th order to the system (1),(3) yields

$$L_{\boldsymbol{\alpha}}^{r} h(\boldsymbol{x}) = \sum_{i_r=1}^{m_1} \cdots \sum_{i_2=1}^{m_1} \sum_{i_1=1}^{m_1} a_{i_1} a_{i_2} \cdots a_{i_r} \nabla(\cdots \nabla(\nabla h \cdot \boldsymbol{\alpha}_{i_1})\boldsymbol{\alpha}_{i_2} \cdots)\boldsymbol{\alpha}_{i_r}(\boldsymbol{x}) = \sum_{i=1}^{n_1} \theta_i^1 f_i(\boldsymbol{x}) \tag{45}$$

and

$$L_{\boldsymbol{\beta}} L_{\boldsymbol{\alpha}}^{r-1} h(\boldsymbol{x}) = \sum_{j=1}^{m_2} \sum_{i_{r-1}=1}^{m_1} \cdots \sum_{i_1=1}^{m_1} a_{i_1} \cdots a_{i_r} b_j \cdot \nabla(\nabla(\cdots(\nabla h \cdot \boldsymbol{\alpha}_{i_1})\cdots)\boldsymbol{\alpha}_{i_{r-1}})\boldsymbol{\beta}_j(\boldsymbol{x}) = \sum_{i=1}^{n_2} \theta_i^2 g_i(\boldsymbol{x}) \tag{46}$$

So the system (1) can be written in the form

$$y^{(r)} = L_{\alpha}^{r} h(\boldsymbol{x}) + L_{\beta} L_{\alpha}^{r-1} h(\boldsymbol{x}) u = \sum_{i=1}^{n_1} \theta_i^1 f_i(\boldsymbol{x}) + \sum_{i=1}^{n_2} \theta_i^2 g_i(\boldsymbol{x}) u \tag{47}$$

which is in fact system (2), (4).
Observe that the free terms $f_0(\boldsymbol{x})$ and $g_0(\boldsymbol{x})$ in formula (4) may be easy obtained by treating one of the coefficients in each sum of (3) as equal to one e.g. $a_1 = 1$ and $b_1 = 1$. This way one of the terms in the formula (45) will take a form $L_{\boldsymbol{\alpha}_1}^{r} h(\boldsymbol{x}) = f_0(\boldsymbol{x})$ or respectively $L_{\boldsymbol{\beta}_1} L_{\boldsymbol{\alpha}_1}^{r-1} h(\boldsymbol{x}) = g_0(\boldsymbol{x})$ - in relation to (46).

Automation and Robotics

Edited by Juan Manuel Ramos Arreguin

ISBN 978-3-902613-41-7
Hard cover, 388 pages
Publisher I-Tech Education and Publishing
Published online 01, May, 2008
Published in print edition May, 2008

In this book, a set of relevant, updated and selected papers in the field of automation and robotics are presented. These papers describe projects where topics of artificial intelligence, modeling and simulation process, target tracking algorithms, kinematic constraints of the closed loops, non-linear control, are used in advanced and recent research.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Zenon Zwierzewicz (2008). Nonlinear Adaptive Tracking-Control Synthesis for General Linearly Parametrized Systems, Automation and Robotics, Juan Manuel Ramos Arreguin (Ed.), ISBN: 978-3-902613-41-7, InTech, Available from: http://www.intechopen.com/books/automation_and_robotics/nonlinear_adaptive_tracking-control_synthesis_for_general_linearly_parametrized_systems

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