

We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900

Open access books available

185,000

International authors and editors

200M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



Lifetime Assessment of Electrical Insulation

Eyad A. Feilat

Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/intechopen.72423>

Abstract

In this chapter, a review of the Weibull probability distribution, probability ranking, and the Weibull graphical estimation technique is presented. A review of single-stress and multiple-stress life models of electrical insulation is also introduced. The chapter also describes the graphical, linear and multiple linear regression techniques used in estimating the parameters of the aging models. The application of maximum likelihood estimation technique for estimating the parameters of combined life models of electrical insulation is illustrated.

Keywords: life models, aging, insulation, Weibull probability, maximum likelihood estimation, least square estimation

1. Introduction

A lifetime analysis of electrical insulation failure is an approach that relies on statistical analysis of data that are attributed to the breakdown of the electrical insulation due to the presence of degrading stresses, such as electrical, thermal and other environmental factors. The lifetime analysis can provide statistical information about the electrical insulation such as lifetime characteristics, probability of failures, lifetime percentiles or any time percentile under normal operating conditions. In this approach, the insulation life is determined by measuring the time-to-breakdown of identical specimens of the solid insulation subjected to life tests [1–5]. Obtaining life data under normal operating conditions is a very time consuming and costly process, rendering it impractical. Besides, it is important to observe modes of failure of the electrical insulation to better understand the prevailing mechanisms of breakdown. Consequently, electrical insulation design engineers and material scientists devised methods to force the insulation to fail in shorter periods of time. These methods seek to accelerate the failures of insulation samples by applying stresses at levels that exceed the levels that the insulation will encounter under normal operating service conditions. Acceleration is accomplished by testing

the insulation (specimen or device) using single or combined high stress levels which could involve electrical, thermal or environmental stresses for either short periods (few seconds or minutes) or long periods (few hours or days) [6]. The accelerated test data are then considered as a base for extrapolation to obtain an estimate of the lifetime of the insulation when the device or material is operated at normal operating conditions for relatively long time periods (decades) of years [7–27].

Times-to-breakdown obtained by accelerated life (aging) tests are analyzed using an underlying lifetime probability distribution. The probability distribution can, correspondingly, be used to make predictions and estimates of lifetime measures of interest at the particular stress level. This is accomplished by projecting or mapping lifetime measures from high stress level to a service level. It can be assumed that there is some model (or function), which can be described mathematically, that maps the lifetime estimate from the high stress level to the service level, and can be as simple as possible (i.e. linear, exponential, etc.) [28–37]. The parameters of the lifetime models can be estimated by combining the proposed life models with the Weibull probability distribution function. Maximum likelihood estimation can be used to estimate the parameters of the combined Weibull-electrical-thermal models using experimental data obtained by measuring time-to-breakdown of the insulation.

2. Weibull probability distribution

The Weibull distribution has been widely recognized as the most common distribution in breakdown testing of solid dielectric insulation and in reliability studies [28–31, 37–40]. Its popularity is attributed to the many shapes it attains for various values of the shape parameter (β). It can model a large variety of data and life characteristics. The distribution also has the important properties of flexibility and a closed form solution for the integral of the Weibull probability density function (*pdf*). This latter property is important for easy determination of the Weibull cumulative probability function (*cdf*) and its corresponding parameters. There are three forms of the Weibull cumulative probability distribution functions, namely, the three-parameter Weibull distribution, the two-parameter Weibull distribution and the mixed Weibull distribution. In life data analysis, the two-dimensional Weibull distribution is often used to describe the time-to-breakdown of solid dielectric insulation with voltage, as it is a convenient way for deriving the V-t characteristic or life models. The two-parameter Weibull *pdf* and *cdf* distributions are defined as:

$$\begin{aligned}
 f(t; \alpha, \beta) &= \frac{\beta}{\alpha} \left(\frac{t}{\alpha} \right)^{\beta-1} \exp \left\{ - \left(\frac{t}{\alpha} \right)^{\beta} \right\} \\
 F(t; \alpha, \beta) &= 1 - \exp \left\{ - \left(\frac{t}{\alpha} \right)^{\beta} \right\} & t \geq 0 \\
 F(t; \alpha, \beta) &= 0 & t < 0
 \end{aligned} \tag{1}$$

where t is the time-to-breakdown; α is the scale parameter, $\alpha > 0$; β is the shape parameter, $\beta > 0$.

$F(t)$ represents the proportion of samples initially tested that will fail by time t . The scale parameter (α) represents the time required for 63.2% of the tested samples to fail. The shape parameter (β), or slope of the Weibull distribution, is a measure of the dispersion of the time-to-breakdown. The unit of α is time, while β is dimensionless.

2.1. Specific characteristics of the Weibull probability distribution

The Weibull *pdf* and/or *cdf* distributions are characterized by the parameters α and β . The scale parameter α is usually a function of the applied voltage when the time is a random variable. A change in the scale parameter α has the same effect on the distribution as a change of the abscissa scale. If α is increased, while β is kept the same, the distribution gets stretched out to the right and its height decreases, while maintaining its shape and location as shown in **Figure 1** [30]. On the other hand, if α is decreased, while β is kept the same, the distribution gets pushed in toward the left (i.e. toward its beginning, or 0) and its height increases. For $\beta < 1$ the distribution has the reversed *J* shape. If $\beta = 1$, the Weibull distribution becomes a two-parameter exponential distribution as shown in **Figure 2**. For $\beta = 2$, it becomes the Rayleigh distribution. For $\beta < 2.6$, the Weibull *pdf* is positively skewed (has a right tail). For $2.6 < \beta < 3.7$, its coefficient of skewness approaches zero (no tail); consequently, it may approximate the normal *pdf*, and for $\beta > 3.7$, it is negatively skewed (left tail) [30].

An interpretation of the shape parameter indicates that if $\beta > 1$, the dielectric fails as a result of wear and the failure rate increases with time. In such a case, the failure rate increases with time. If $\beta < 1$, the failure rate decreases with time. When such behavior is encountered, this indicates that the sample has some units with technical defects.

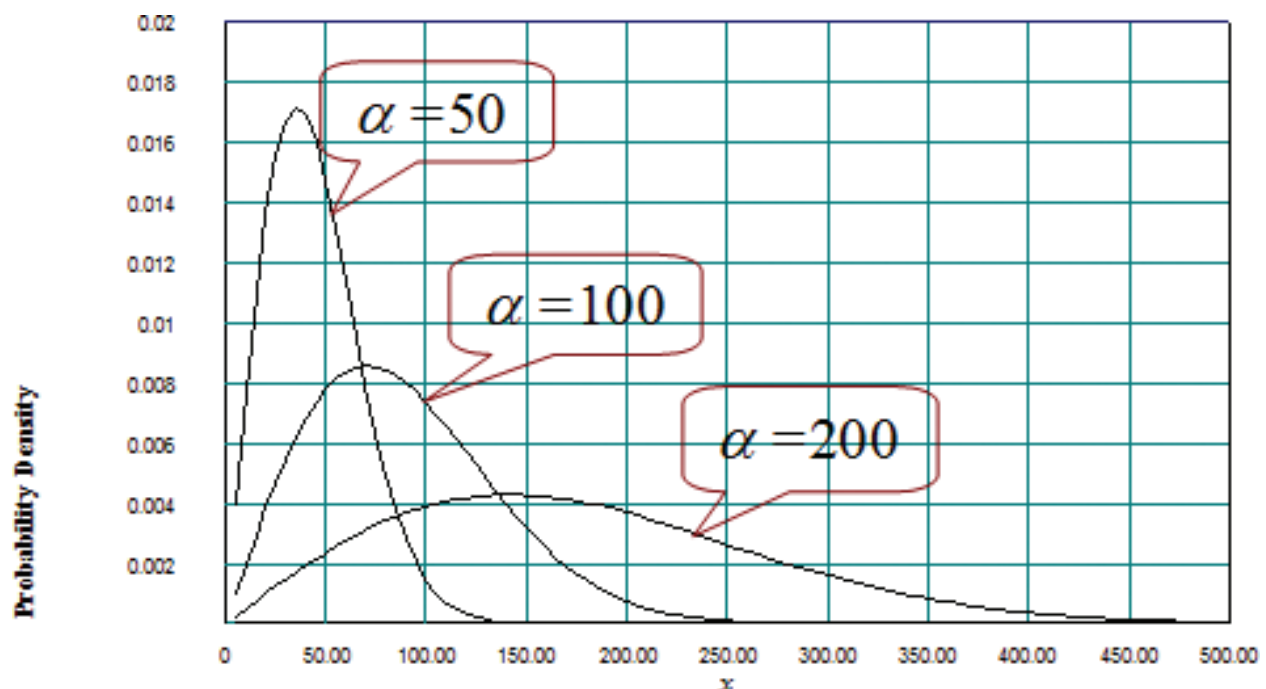


Figure 1. Effect of the scale parameter on Weibull probability distribution.

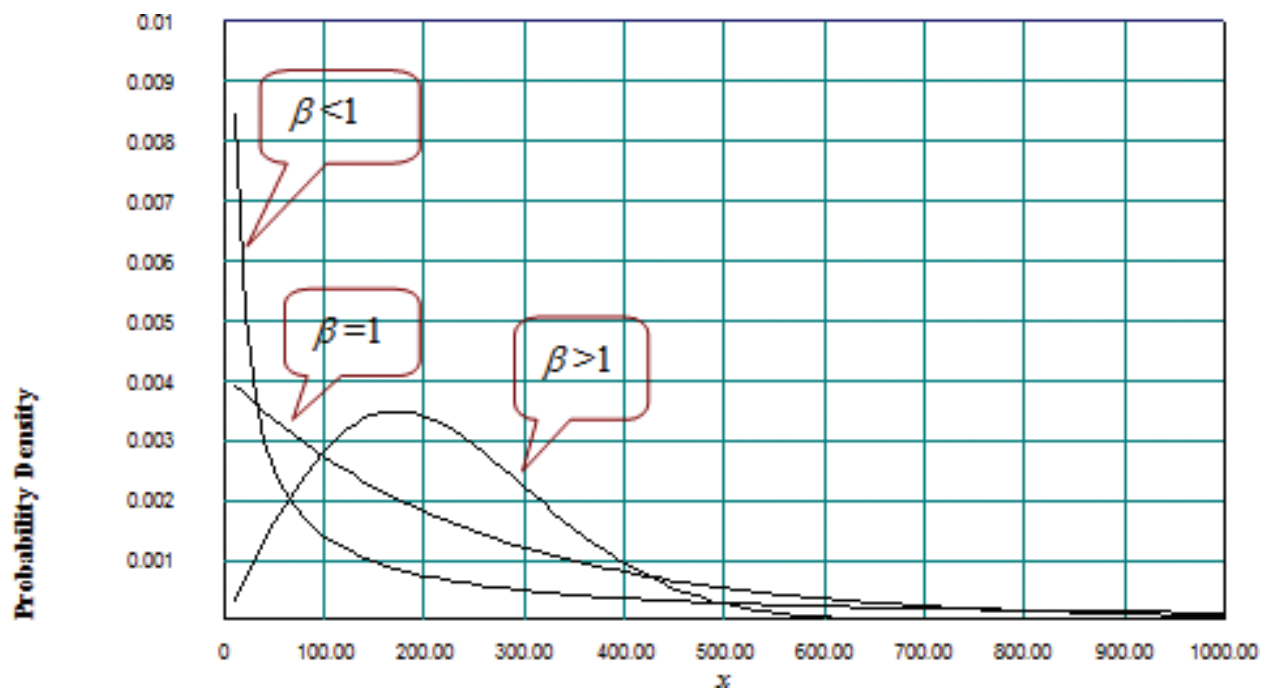


Figure 2. Effect of the shape parameter on the Weibull probability distribution.

2.2. Estimating the cumulative probability of failure data

The cumulative probability of failure $F(t_i)$ is estimated by a value P_i , based on the order statistic rank i and the sample size n . A variety of equations for approximating $F(t_i)$ has been suggested for the plotting position P_i [28, 29]:

1. Mean rank approximation

$$P_i = E\{F(t_i)\} = \frac{i}{n+1} \quad (2)$$

2. Median rank approximation

$$P_i = \text{median of } F(t_i) = \frac{i - 0.3}{n + 0.4} \quad (3)$$

3. Mode rank approximation

$$P_i = \text{mode}\{F(t_i)\} = \frac{i - 1}{n - 1} \quad (4)$$

4. Sample cumulative distribution function (cdf) approximation

$$P_i = \text{sample cdf} = \frac{i - 0.5}{n} \quad (5)$$

The choice of the rank approximation is generally immaterial except when the sample size is small. However, both the mean and median rank approximations are most widely used. In this

work, the median rank approximation is used in estimating the probability of failure. Note that the median rank approximation is also known as Benard's approximation.

2.3. Parameter estimation of Weibull distribution

The Weibull distribution parameters α and β are the variables that govern the characteristics of the Weibull *pdf*. Once the Weibull distribution has been selected to represent the failure data, the associated parameters can be determined from the experimental data. Weibull parameters can be estimated graphically on a probability plotting paper [28–31] or analytically using either Least Squares (LS) or Maximum Likelihood (ML) estimation techniques. An overview of these techniques [1–4] will be presented next.

2.4. Graphical technique

A graphical technique is the simplest method for estimating the Weibull parameters using probability plotting [29, 38]. The breakdown results at each individual stress level are plotted on a specially constructed probability plotting paper (Weibull Probability Paper) as shown in **Figure 3**. A probability plotting looks at the *cdf* of the Weibull distribution and attempts to linearize it by employing a Weibull probability paper. The ordinate axis has a nonlinear scale corresponding to the cumulative probability of failure. The abscissa has a log scale of the time-to-breakdown. To plot the data, the data are ordered from smallest to largest and then a cumulative probability of breakdown to each point is assigned. If the plotted data fit a straight line, the slope of the line can be obtained and thus the shape parameter β can be obtained [14]. The scale parameter α can be determined by finding the time-to-breakdown corresponding to

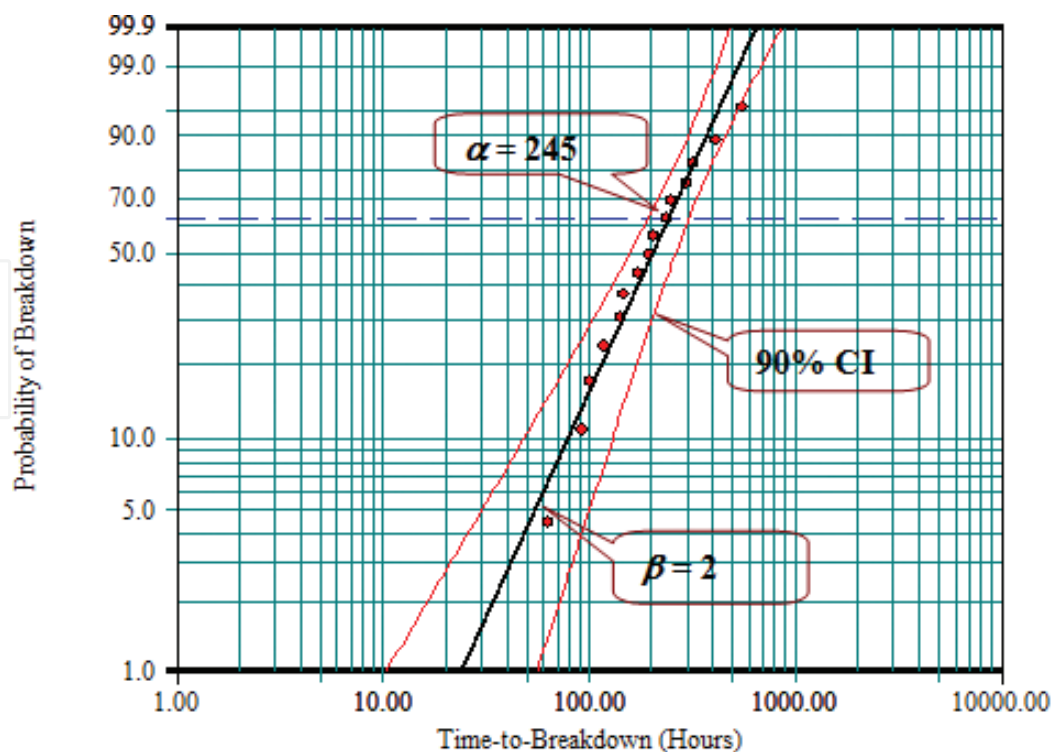


Figure 3. Cumulative probability distribution with confidence interval plotted on Weibull paper.

a cumulative probability of 63.2%. Using this simple, but time consuming approach, the parameters of the Weibull distribution can be determined. This procedure is repeated for each stress level of the accelerated aging tests.

Estimating the parameters of the Weibull distribution by a graphical technique using a probability plotting method has some shortfalls. A manual probability plotting is not always consistent in the results. Plotting a straight line through a set of points is a subjective procedure; it differs from person to person. In addition, the probability plot must be constructed for each stress level. This, as a result, takes tremendous time and effort to plot the data. Furthermore, sufficient failures must be observed at each stress level, which is not always possible.

2.5. Least squares technique (LS)

The least squares technique is a linear regression estimation technique that fits a straight line to a set of data points, in an attempt to estimate the parameters associated with the straight lines. The parameters are estimated such that the sum of the squares of the vertical deviations from the points to the line is minimized according to [1–4]

$$J = \sum_{i=1}^N (\tilde{a} + \tilde{b}x_i - y_i)^2 = \min(\tilde{a}, \tilde{b}) \sum_{i=1}^N (\tilde{a} + \tilde{b}x_i - y_i)^2 \quad (6)$$

where \tilde{a} and \tilde{b} are the LS estimates of a and b , and N is the number of data points. To obtain \tilde{a} and \tilde{b} , the performance index J is differentiated with respect to a and b as shown below.

$$\frac{\partial J}{\partial \tilde{a}} = 2 \sum_{i=1}^N (\tilde{a} + \tilde{b}x_i - y_i) \quad (7)$$

$$\frac{\partial J}{\partial \tilde{b}} = 2 \sum_{i=1}^N (\tilde{a} + \tilde{b}x_i - y_i)x_i \quad (8)$$

Setting Eqs. (7) and (8) equal to zero yields

$$\sum_{i=1}^N (\tilde{a} + \tilde{b}x_i - y_i) = \sum_{i=1}^N (\tilde{y}_i - y_i) = - \sum_{i=1}^N (y_i - \tilde{y}_i) = 0 \quad (9)$$

$$\sum_{i=1}^N (\tilde{a} + \tilde{b}x_i - y_i)x_i = \sum_{i=1}^N (\tilde{y}_i - y_i)x_i = - \sum_{i=1}^N (y_i - \tilde{y}_i)x_i = 0 \quad (10)$$

Solving Eqs. (9) and (10) simultaneously yields

$$\tilde{a} = \frac{\sum_{i=1}^N y_i}{N} - \tilde{b} \frac{\sum_{i=1}^N x_i}{N} \quad (11)$$

and

$$\tilde{b} = \frac{\sum_{i=1}^N x_i y_i - \frac{\sum_{i=1}^N x_i \sum_{i=1}^N y_i}{N}}{\sum_{i=1}^N x_i^2 - \frac{\left(\sum_{i=1}^N x_i\right)^2}{N}} \quad (12)$$

The least squares estimation technique is good for functions that can be linearized. Its calculations are easy and straightforward, and the correlation coefficient provides a good measure of the goodness-of-fit of the chosen distribution. However, for some complex distributions, it is difficult and sometimes impossible to implement.

2.6. Maximum likelihood estimation (MLE)

The maximum likelihood parameter estimation seeks to determine the parameters that maximize the probability (likelihood) of the failure data. Statistically, the MLE is considered to be more robust and yields estimators with good statistical properties. The MLE has the following statistical properties:

1. The ML estimators are consistent and asymptotically efficient.
2. The probability distribution of the estimators is asymptotically normal.
3. For small sample sizes, the ML estimators are considered to be more precise than those obtained by LS method. Moreover, the ML estimators can converge into a solution even with only one failure.
4. The MLE technique applies to most models and to different types of data.
5. The ML estimates are unique, and as the size of the sample increases, the estimates statistically approach the true values of the population.

The theory of the MLE method is described as follows: Let t be a continuous random variable representing the time-to-breakdown and characterized by the two-parameter Weibull distribution with *pdf*:

$$f(t; \alpha, \beta) = \left(\frac{\beta}{\alpha}\right) \left(\frac{t}{\alpha}\right)^{\beta-1} \exp \left[-\left(\frac{t}{\alpha}\right)^{\beta} \right] \quad (13)$$

where α and β are unknown constant parameters which need to be estimated. For an experiment with N independent observations, t_1, t_2, \dots, t_N in a given sample, then the likelihood function associated with this sample is the joint density of the N random variables, and thus is a function of the unknown Weibull parameters (α, β) . The likelihood function is defined by [1–4]:

$$L(t_1, t_2, \dots, t_N | \tilde{\alpha}, \tilde{\beta}) = L = \prod_{i=1}^N f(t_i; \tilde{\alpha}, \tilde{\beta}) = \prod_{i=1}^N \frac{\tilde{\beta}}{\tilde{\alpha}} \left(\frac{t_i}{\tilde{\alpha}} \right)^{\tilde{\beta}-1} \exp \left[- \left(\frac{t_i}{\tilde{\alpha}} \right)^{\tilde{\beta}} \right] \quad (14)$$

The logarithmic likelihood function is given by

$$\Lambda = \ln L = \sum_{i=1}^N \ln f(t_i; \tilde{\alpha}, \tilde{\beta}) = \sum_{i=1}^N \ln \left[\frac{\tilde{\beta}}{\tilde{\alpha}} \left(\frac{t_i}{\tilde{\alpha}} \right)^{\tilde{\beta}-1} \exp \left(- \left(\frac{t_i}{\tilde{\alpha}} \right)^{\tilde{\beta}} \right) \right] \quad (15)$$

The parameter estimates $(\tilde{\alpha}, \tilde{\beta})$ are obtained by maximizing L or Λ which is much easier to work with than L . The ML estimators are determined by taking the partial derivatives of Λ with respect to $\tilde{\alpha}$ and $\tilde{\beta}$ set them to zero, that is, $\frac{\partial \Lambda}{\partial \tilde{\alpha}} = 0$, $\frac{\partial \Lambda}{\partial \tilde{\beta}} = 0$, where,

$$\frac{\partial \Lambda}{\partial \tilde{\beta}} = \sum_{i=1}^N \frac{1}{\tilde{\beta}} + \sum_{i=1}^N \ln \left(\frac{t_i}{\tilde{\alpha}} \right) - \sum_{i=1}^N \left(\frac{t_i}{\tilde{\alpha}} \right)^{\tilde{\beta}} \ln \left(\left(\frac{t_i}{\tilde{\alpha}} \right) \right) \quad (16)$$

$$\frac{\partial \Lambda}{\partial \tilde{\alpha}} = - \sum_{i=1}^N \frac{\tilde{\beta}}{\tilde{\alpha}} + \frac{\tilde{\beta}}{\tilde{\alpha}} \sum_{i=1}^N \ln \left(\frac{t_i}{\tilde{\alpha}} \right)^{\tilde{\beta}} \quad (17)$$

The resulting equations give the best estimates $\tilde{\beta}$ and $\tilde{\alpha}$.

$$\frac{1}{\tilde{\beta}} = - \frac{1}{N} \sum_{i=1}^N \ln t_i + \sum_{i=1}^N \frac{(t_i)^{\tilde{\beta}} \ln t_i}{\sum_{i=1}^N (t_i)^{\tilde{\beta}}} \quad (18)$$

$$\tilde{\alpha} = \left[\frac{1}{N} \left(\sum_{i=1}^N (t_i)^{\tilde{\beta}} \right) \right]^{(1/\tilde{\beta})} \quad (19)$$

Eq. (18) is written in terms of $\tilde{\beta}$ only, and can only be solved by an iterative technique, such as the Newton-Raphson iterative technique. Once $\tilde{\beta}$ is obtained, $\tilde{\alpha}$ can be determined using Eq. (19).

Although the methodology for the MLE is simple, the implementation is mathematically intense. The present high-speed computers, however, have made the obstacles of the mathematical complexity of the MLE an easy process. A specialized statistical commercial package Weibull++ is used throughout this work to find the ML estimates of the Weibull distribution parameters [39].

2.7. Failure time percentiles

Once the Weibull distribution parameters are obtained, the failure time percentiles, t_p , can be derived from Eq. (2) as follows (by substituting $F(t; \alpha, \beta) = p$)

$$t_p = \tilde{\alpha}[-\ln(1-p)]^{1/\tilde{\beta}} \quad (20)$$

where t_p is the time-to-breakdown for which a sample will fail with a probability of failure p , and $\alpha = L(V, T, f)$ is function of the applied stresses, (e.g. voltage, temperature, frequency, etc.). If $p = 0.632$, then $t_p = \alpha$, the scale parameter, or the life by which 63.2% of the samples will fail. Likewise, if $p = 0.50$, then $t_p = \tilde{t}$, the median life, or the life by which half of the samples will fail.

3. Life models

There are two approaches for studying the electrical breakdown and estimating the insulation lifetimes (under normal operating conditions) of polymeric insulating materials. One approach is based on phenomenological studies which require a complete understanding of the breakdown mechanism. This approach requires physical and/or chemical tests to be performed on the insulating material that may yield to the development of mathematical models functional with the lifetime. An example of this is relating the lifetime of the insulation to the length of trees formed in the insulation bulk as a result of treeing mechanism [40]. The other widely known approach relies on a statistical analysis of failures that are attributed to the breakdown of the electrical insulation due to the presence of degrading stresses, such as electrical, thermal, and other environmental factors [6–36]. In this approach, the insulation life is determined by measuring the time-to-breakdown of identical specimens of the solid insulation subject to life tests. Life tests, however, show that the times-to-breakdown are widely variable. This variation is best modeled by the Weibull probability distribution.

Conducting life tests at realistic working stresses is not possible due to the time constraint, given that most electrical insulation is expected to serve for several decades. Instead, breakdown data are obtained, without paying much attention to the details of the breakdown mechanism, by conducting accelerated life (aging) tests in laboratory experiments so that the insulation life is severely reduced [41, 42]. The main goal of life tests is to establish mathematical models for the aging process and the stresses causing it [32–36]. The constants of these models need to be estimated from life tests where the lifetimes at a variety of stress levels are measured. Once the constants are estimated, the life at any particular stress including normal operating conditions can, in principle, be estimated.

3.1. Single-stress life models

Life models of single stress include the inverse power law and exponential law models for an electrical stress, and the Arrhenius model for a thermal stress [32–34].

3.1.1. Life models for electrical stresses

An electrical stress is considered as one of the main factors causing deterioration of electrical insulation. There are two empirical models that relate the test of an electrical stress to the

time-to-breakdown. One is the inverse power law model, and the second is the exponential law model. The parameters of both models are obtained from experimental data taken at several different high voltage levels with other conditions unchanged. The electrical life models mathematically describe the aging in a solid dielectric insulation that experiences an electric stress [43]. The life models do not characterize the exact type of aging mechanism that takes place. The life models are totally empirical and have no physical meaning other than defining the degradation rate as power or exponential. However, the models have proven to fit reasonably well with experimental data.

3.1.1.1. Inverse power law

The inverse power law (IPL) model is one of the most frequently used in the aging studies under an electrical stress. The inverse power law model is given by:

$$L(V) = k V^{-n} \quad (21)$$

where L , the time-to-breakdown, is usually a Weibull scale parameter α at 63.2% probability, or any other percentile, V is the applied voltage, and k, n , are constants to be determined for the specific tested material or device. The inverse power law is considered valid if the data being plotted on a log–log graph fits a straight line [44].

3.1.1.2. Exponential law

The exponential law is also commonly used for lifetime calculations. The exponential model is given by:

$$L(V) = c \exp(-bV) \quad (22)$$

where L is the time-to-breakdown, V is the applied voltage, and c and b are constants to be determined from the experimental data. The exponential model is verified by plotting the data points on a semi-log graph. The model is considered valid if a straight line is obtained [44].

3.1.2. Life model for a thermal stress

The life of electrical insulation is seriously affected by a thermal stress. This effect can only be recognized by a thermal life test. The life of electrical insulation under a thermal stress is empirically expressed by the well-known Arrhenius equation. This equation describes the thermal aging of materials and shows the dependency of the chemical reaction rate as a function of the temperature. The Arrhenius equation is given by [43]:

$$L(T) = A \exp\left(\frac{B}{T}\right) \quad (23)$$

where L is the time-to-breakdown, T is the absolute temperature, and A, B are constants to be determined experimentally.

3.2. Multi-stress life models

Multi-stress Life models were developed to predict the life of the insulation under combined electrical and thermal stresses. In general these models are limited to the common electrical and thermal aging stresses acting simultaneously [43]. Mainly, some models have empirical nature such Simoni's, Ramu's, and Fallou's [11, 12, 32–36]. These models account for the interactions of electrical and thermal stresses by using a multiplicative law, in which the life under a combined stress is related to the product of the single-stress lives. One possible formula for this interaction can be manifested as the multiplication of the IPL model and Arrhenius relationship, which is given by:

$$L(V, T) = KV^{-n} \exp\left(\frac{B}{T}\right) \quad (24)$$

which is considered to be the basis for both Simoni's and Ramu's electrical-thermal life models. Alternatively, the electrical exponential model is associated with the Arrhenius relationship. This can be expressed as:

$$L(V, T) = C \exp\left(AV + \frac{B}{T}\right) \quad (25)$$

which constitutes the Fallou's electrical-thermal life model.

Another probabilistic life model based on IPL was presented by Montanari et al. [34]. A brief overview of the above mentioned electrical-thermal life models of Simoni, Ramu, Fallou, and the probabilistic model by Montanari will be presented. The above models were developed for a relatively simple dielectric system involving polymer films or slabs. A variety of multi-stress life tests have been also developed for more complex insulation systems, for example, cables and rotating machines stator windings [16, 17]. However, lifetime models as a function of two or more stresses are rarely derived due to excessive cost and time needed for collecting the failure data.

Regarding the frequency as an aging factor little research has been published in developing combined electrical-thermal-frequency life models. In some works, high frequency sinusoidal voltage was applied [41, 42]. The results of these works show that the frequency as an aging factor that causes insulation deterioration. The effect of the frequency is modeled by relating the variation of the parameters of the life model to the frequency [41].

3.2.1. Simoni's model

According to the Simoni's model, the insulation life, in relative terms with respect to a reference life determined by the absence of an electrical stress and at low temperature, is given by:

$$L(V, T) = t_o \left[\frac{V}{V_o} \right]^{-n} \exp\left(-B\Delta\left(\frac{1}{T}\right)\right) \quad (26)$$

where t_o is the time-to-breakdown at room temperature and $V = V_o$, $\Delta\left(\frac{1}{T}\right) = \frac{1}{T} - \frac{1}{T_o}$, and B and n are constants which are determined experimentally.

3.2.2. Ramu's model

The Ramu's model is obtained from a multiplication of classical single-stress laws, and is given by:

$$L(V, T) = K(T)[V]^{-n(T)} \exp \left(-B\Delta \left(\frac{1}{T} \right) \right) \quad (27)$$

where $K(T) = \exp \left(K_1 - K_2\Delta \left(\frac{1}{T} \right) \right)$, $n(T) = \exp \left(n_1 - n_2\Delta \left(\frac{1}{T} \right) \right)$, K_1 , K_2 , n_1 , and n_2 are constants. $\Delta \left(\frac{1}{T} \right)$ is the same as that defined for the Simoni's model.

3.2.3. Fallou's model

Fallou proposed a semi-empirical life model based on the exponential model for electrical aging:

$$L(V, T) = C \exp \left(AV + \left(\frac{B}{T} \right) \right) \quad (28)$$

where C , A , and B are electrical stress constants and must be determined experimentally from time-to-breakdown curves at constant temperatures.

3.2.4. Montanari's probabilistic model

The probabilistic life model of combined electrical and thermal stresses by Montanari *et al.* relates the failure probability p to insulation life L_p . It is based on substituting the scale parameter in the Weibull distribution with the life using the inverse power law. For a given time-to-breakdown probability p , the probabilistic model is given by:

$$L_p(V, T) = L_s(V/V_s)^n [-\ln(1 - p)]^{(1/\beta(T))} \quad (29)$$

where L_p is a lifetime at probability p , L_s is a time-to-breakdown at reference voltage V_s , and β is the shape parameter.

3.3. Estimating life model constants

Eqs. (21) to (23) describe several mathematical models which relate insulation life to a single aging stress, either voltage or temperature. Likewise, Eqs. (24) to (29) describe the electrical-thermal life models. In each model of the above life models, there are several parameters that are needed to be estimated from life testing data. The parameters are estimated from life tests where the lifetimes at a variety of stress levels are measured. Once the parameters are estimated, the life at any particular stress can, in principle, be estimated. This enables a method of estimating the life at normal stress based on failure data collected from accelerated life tests. Traditionally, the parameters of life models are calculated either graphically or analytically using graphical or regression analysis type methods.

3.3.1. Graphical method

The graphical method for estimating the parameters of a life model involves generating two types of plots. First, the time-to-breakdown at each individual stress level is plotted on a probability paper appropriate to the assumed life distribution (i.e. Weibull, Lognormal). The parameters at each stress level are then estimated from the plot. Once the parameters of the life distribution have been estimated at each stress level using probability plotting methods, a second plot is created in which a characteristic lifetime is plotted versus stress on a paper that linearizes the assumed lifetime-stress relationship. For example, a log-log paper linearizes the inverse power law, a semi-log paper linearizes the exponential model, and a log-reciprocal linearizes the Arrhenius relationship. The lifetime characteristic can be any percentile, such as 10% lifetime, the scale parameter, the mean lifetime, etc. The parameters of the lifetime-stress relationship are then estimated from the second plot by solving for the slope and the intercept of the line [45].

In spite of the fact that the graphical method is simple and straightforward, the method suffers from some shortfalls such as:

- It is quite time consuming.
- The graphical method may fail in linearizing the lifetime-stress relationship when the data are plotted on the special paper.
- In accelerated life tests with small data, the separation and individual plotting of the data to obtain the parameters increase the underlying error.
- The estimated parameters, that are assumed constant, are likely to vary when the test is repeated. Confidence intervals on the estimated parameters cannot be established using the graphical methods.

3.3.2. Regression analysis

Calculating the parameters of a life model using regression analysis is relatively straightforward. For most single-stress models, simple linear regression (SLR) is used to estimate the parameters. Similarly, multiple linear regression (MLR) is used for multi-stress models and complicated single-stress models. On the other hand, nonlinear regression methods are employed in cases where life models contain thresholds below certain values where aging does not occur. These nonlinear models are much more difficult to analyze [40]. Therefore, non-statisticians do not usually use nonlinear regression. For this reason, most life models assume that the threshold is close to zero, permitting the use of conventional regression analysis. The MLR method first requires the life model to be linearized into a form such as:

$$y = a_0 + a_1x_1 + a_2x_2 + \cdots + a_kx_k \quad (30)$$

where y is a dependent variable (life, or a mathematical transformation of life), x_k are independent stresses (or transformations of stresses or combinations of stresses), and $a_0, a_1, a_2, \dots, a_k$ are the constants to be determined. The method of least squares can be used to estimate the regression constants in a MLR model. The least squares technique involves finding the values

of the constants that minimize the difference between the predicted lifetimes under a set of stress conditions and the actual lifetime measures under the same conditions. By relating a “characteristic” life of the underlying probability distribution to the aging stress, a set of mathematical equations can be used to calculate the parameters [45]. Each observation $(x_{i1}, x_{i2}, \dots, x_{ik}, y_i)$, satisfies the model in (30), or

$$\begin{aligned} y_i &= a_0 + a_1x_{i1} + a_2x_{i2} + \dots + a_kx_{ik} \\ &= a_0 + \sum_{j=1}^k a_jx_{ij} \quad i = 1, 2, \dots, n \end{aligned} \quad (31)$$

The least squares function is:

$$J = \sum_{i=1}^n \left(y_i - a_0 - \sum_{j=1}^k \tilde{a}_j x_{ij} \right)^2 \quad (32)$$

The function J is to be minimized with respect to $a_0, a_1, a_2, \dots, a_k$. The least squares estimates of $a_0, a_1, a_2, \dots, a_k$ must satisfy:

$$\frac{\partial J}{\partial a_0} \Big|_{a_0, a_1, a_2, \dots, a_k} = -2 \sum_{i=1}^n \left(y_i - \tilde{a}_0 - \sum_{j=1}^k \tilde{a}_j x_{ij} \right) = 0 \quad (33)$$

$$\frac{\partial J}{\partial a_j} \Big|_{\tilde{a}_0, \tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_k} = -2 \sum_{i=1}^n \left(y_i - \tilde{a}_0 - \sum_{j=1}^k \tilde{a}_j x_{ij} \right) x_{ij} = 0 \quad j = 1, 2, \dots, k \quad (34)$$

Simplifying Eqs.(33) and (34), the following least squares normal equations are obtained:

$$\begin{aligned} n\tilde{a}_0 + \tilde{a}_1 \sum_{i=1}^n x_{i1} + \tilde{a}_2 \sum_{i=1}^n x_{i2} + \dots + \tilde{a}_k \sum_{i=1}^n x_{ik} &= \sum_{i=1}^n y_i \\ \tilde{a}_0 \sum_{i=1}^n x_{i1} + \tilde{a}_1 \sum_{i=1}^n x_{i1}^2 + \tilde{a}_2 \sum_{i=1}^n x_{i1}x_{i2} + \dots + \tilde{a}_k \sum_{i=1}^n x_{i1}x_{ik} &= \sum_{i=1}^n x_{i1}y_i \\ \vdots & \\ \tilde{a}_0 \sum_{i=1}^n x_{ik} + \tilde{a}_1 \sum_{i=1}^n x_{ik}x_{i1} + \tilde{a}_2 \sum_{i=1}^n x_{ik}x_{i2} + \dots + \tilde{a}_k \sum_{i=1}^n x_{ik}^2 &= \sum_{i=1}^n x_{ik}y_i \end{aligned} \quad (35)$$

The solution to the $k + 1$ normal equations will be the least square estimators of the regression parameters $a_0, a_1, a_2, \dots, a_k$ of the multi-stress life model. The normal equations can be solved by any method appropriate for solving a system of linear equations. Yet, it is much more convenient to solve the multiple linear regression problem using a matrix approach. For k parameters and n observations, the model relating the parameters to the dependent variable y_n can be expressed in matrix notation as [45]:

$$\mathbf{y} = \mathbf{XA} \quad (36)$$

where,

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_k \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix}$$

In general, \mathbf{y} is an $(n \times 1)$ vector of the observations, \mathbf{X} is an $(n \times k)$ matrix of the levels of stresses, and \mathbf{A} is a $(k \times 1)$ vector of the regression constants. The least squares estimators, $\tilde{\mathbf{A}}$, is the solution for \mathbf{A} that minimizes:

$$J = (\mathbf{y} - \mathbf{XA})'(\mathbf{y} - \mathbf{XA}) \quad (37)$$

by setting $\frac{\partial J}{\partial \mathbf{A}} = 0$. Accordingly,

$$\mathbf{X}'\mathbf{X}\tilde{\mathbf{A}} = \mathbf{X}'\mathbf{y} \quad (38)$$

Eq. (38) is the least squares normal equations in matrix form, which is identical to the scalar form of the normal equations given in Eq. (35). The least squares estimators $\tilde{\mathbf{A}}$ is obtained by multiplying both sides of Eq. (38) by the inverse of $\mathbf{X}'\mathbf{X}$ [45].

$$\tilde{\mathbf{A}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \quad (39)$$

In practice, MLR calculations are always performed using a computer. Many statistical analysis computer packages can quickly and accurately perform the necessary calculations. The more sophisticated commercial packages will also provide a plot of the stress versus lifetime [46].

If the times-to-breakdown are presented with a Weibull distribution, then the conventional linear regression is theoretically not applicable [47]. Only recently commercial computer programs have become available which enable linear regression with Weibull-life data [46, 48]. The calculation method is complicated, but depends on the use of information matrix using a large sample size normal approximation [1]. The regression analysis approach has only been standardized to estimate the parameters of the thermal life model. No other life models have been standardized to data.

Alternatively, another analytical approach will be used in this work to estimate the parameters of the proposed multi-stress life models using the life distribution-life-stress combined model. The MLE will be used for estimating the model parameters. An Accelerated Life Testing Analysis (ALTA) statistical computer package is used in this work to estimate the parameters of the proposed life model of the insulation [48]. This approach will be discussed in the next section. However, MLR will be used whenever the MLE is not possible.

4. Combined Weibull-life model

Three methods for estimating the parameters of accelerated life test models were presented in chapters two and three. First, the graphical method was illustrated using a probability plotting method for obtaining the parameters of the life distribution. The parameters of the life model were then estimated graphically by linearizing the model on a separate lifetime versus stress plot. However, not all life models can be linearized. Hence, instead of estimating the parameters of the life distribution and the life model graphically, an analytical technique based on least squares was presented. However, the accuracy of the graphical method and LS estimation is affected by the probability rank approximation. Furthermore, estimating the parameters of each individual distribution leads to accumulation of uncertainties, depending on the number of failures at each stress level. In addition, the slope (shape) parameters of each individual distribution are rarely equal (common). Using the graphical method or the LS technique, one must estimate a common shape parameter (usually the average) and repeat the analysis. By doing so, further uncertainties are introduced on the estimates, and these are uncertainties that cannot be qualified [48].

On the other hand, combining the life distribution and the life model relationships in one statistical model that describes both can be accomplished by including the life model into the *pdf* of the life distribution of failure data. Thus, the parameters of that combined model can be estimated using the complete likelihood function (L). Accordingly, a common shape parameter (β) is estimated from the combined model, thus eliminating the uncertainties of averaging the individual shape parameters. All uncertainties are accounted for in the form of confidence intervals that are quantifiable because they are obtained based on the overall model. Besides, the MLE technique is independent of any kind of probability ranks or a plotting method. Therefore, the MLE offers a very powerful method in estimating the parameters of life models [48].

The goal of the ML parameter estimation is to determine the parameters that maximize the probability (likelihood) of the life data. Statistically, the method of the ML is considered to be more robust and yields estimators with good statistical properties (unbiasedness, sufficiency, consistency, and efficiency). Due to its nature, the ML is a powerful tool in estimating the parameters of life models. In addition, the ML provides an efficient method for quantifying uncertainty through confidence intervals.

4.1. Maximum likelihood estimation of combined Weibull-life model

For life data analysis, the two-parameter Weibull distribution *pdf* is commonly used to represent the scatter of the failure data,

$$f(t; \alpha, \beta) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} \exp \left[-\left(\frac{t}{\alpha}\right)^{\beta} \right] \quad (40)$$

where t is the time-to-breakdown, α is the scale parameter (lifetime at 63.2%), and β is the shape parameter or the slope of the Weibull cumulative distribution. The parameters of the life

model can be statistically calculated by combining the life model to the Weibull distribution. For example, the combined Weibull-life model can be derived by setting the scale parameter $\alpha=L(V)$ for the electrical life model, or $\alpha=L(V,T)$ for the electrical-thermal multi-stress life model [13]. The maximum likelihood parameter estimation of combined Weibull-life model will be presented in this chapter for both single-stress and multi-stress life models. The inverse power law (IPL) of the electrical life model and exponential-Arrhenius of the electrical-thermal life model will be used as examples to show the procedure for deriving the parameters of the combined Weibull-life model.

4.2. Weibull-inverse power law electrical life model

The combined Weibull-IPL model can be derived by setting $\alpha = L(V) = kV^{-n}$, yielding the following Weibull *pdf*,

$$f(t, V) = \beta KV^n (KV^n t)^{\beta-1} \exp \left(-(KV^n t)^\beta \right) \quad (41)$$

where $K = 1/k$. This is a three-parameter model (K, β, n) where the parameters can be determined experimentally using life test data.

4.2.1. Parameter estimation of Weibull-IPL model using MLE method

Substituting the IPL electrical life model into the Weibull-Log-Likelihood function yields (Λ) yields:

$$\Lambda = \sum_{j=1}^M \sum_{i=1}^N \ln \left[\tilde{\beta} \tilde{K} \tilde{V}_j^{\tilde{n}} \left(\tilde{K} \tilde{V}_j^{\tilde{n}} t_i \right)^{\tilde{\beta}-1} \exp \left(- \left(\tilde{K} \tilde{V}_j^{\tilde{n}} t_i \right)^{\tilde{\beta}} \right) \right] \quad (42)$$

where M is the number of electrical life test groups; N is the number of times-to-breakdown in j^{th} life test; V_j is the j^{th} life voltage; t_i is the i^{th} time-to-breakdown in the j^{th} group; $\tilde{\beta}$ is an estimate of the Weibull shape parameter; $\tilde{K} = 1/k$, k is the IPL Parameter; \tilde{n} is the second parameter of IPL.

The ML estimates of the parameters can be found by solving for $\tilde{\beta}, \tilde{K}, \tilde{n}$ such that [45].

$$\frac{\partial \Lambda}{\partial \tilde{\beta}} = 0, \quad \frac{\partial \Lambda}{\partial \tilde{K}} = 0, \quad \frac{\partial \Lambda}{\partial \tilde{n}} = 0.$$

where,

$$\frac{\partial \Lambda}{\partial \tilde{\beta}} = \sum_{j=1}^M \sum_{i=1}^N \frac{1}{\tilde{\beta}} + \sum_{j=1}^M \sum_{i=1}^N \ln \left(\tilde{K} \tilde{V}_j^{\tilde{n}} t_i \right) - \sum_{j=1}^M \sum_{i=1}^N \left(\tilde{K} \tilde{V}_j^{\tilde{n}} t_i \right)^{\tilde{\beta}} \ln \left(\tilde{K} \tilde{V}_j^{\tilde{n}} t_i \right) \quad (43)$$

$$\frac{\partial \Lambda}{\partial \tilde{K}} = \sum_{j=1}^M \sum_{i=1}^N \frac{\tilde{\beta}}{\tilde{K}} - \frac{\tilde{\beta}}{\tilde{K}} \sum_{j=1}^M \sum_{i=1}^N \left(\tilde{K} \tilde{V}_j^{\tilde{n}} t_i \right)^{\tilde{\beta}} \quad (44)$$

$$\frac{\partial \Lambda}{\partial \tilde{n}} = \tilde{\beta} \sum_{j=1}^M \sum_{i=1}^N \ln(V_j) - \tilde{\beta} \sum_{j=1}^M \sum_{i=1}^N \ln(V_j) \left(\tilde{K} V_j^{\tilde{n}} t_i \right)^{\tilde{\beta}} \quad (45)$$

4.3. Electrical-thermal life model

In this dissertation, a new electrical-thermal relationship has been proposed for predicting the lifetime of magnet wire insulation at service conditions when the voltage and temperature are the accelerated stresses in a test. This new combined model is given by:

$$L(V, T) = C \exp \left(\frac{A}{V} + \frac{B}{T} \right) \quad (46)$$

where L is the lifetime at 63.2% probability of breakdown; V is the voltage; T is the temperature; C , A , and B : are constants to be estimated by analyzing the joint voltage-temperature life data.

The proposed lifetime relationship can be linearized by finding the natural logarithm of both sides of Eq. (46). A family of linear curves can be obtained by plotting the lifetime versus either of the stresses, V or T , and keeping the other one constant. In this case, the constant B represents the slope of the linearized Arrhenius equation when the voltage is constant, and A represents the slope of the exponential electrical function model when the temperature is constant [45].

Considering that the lifetime is a random variable, the above model can be converted to a probabilistic model by setting the scale parameter α of the Weibull distribution equals to $L(V, T)$ of Eq. (46). Therefore, assuming the time-to-breakdown of the electrical insulation, under combined electrical and thermal stresses, is statistically distributed according to a Weibull distribution, then the Weibull *pdf* can be written as [45]

$$f(t, V, T) = \frac{\beta}{C} \exp - \left(\frac{A}{V} + \frac{B}{T} \right) \left(\frac{t}{C} \exp - \left(\frac{A}{V} + \frac{B}{T} \right) \right)^{\beta-1} \exp - \left(\frac{t}{C} \exp - \left(\frac{A}{V} + \frac{B}{T} \right) \right)^{\beta} \quad (47)$$

This Weibull *pdf* will be used to estimate the parameters of the electrical-thermal life model.

4.3.1. Parameter estimation of Weibull-electrical-thermal life model using MLE

The combined Weibull-electrical-thermal model has four parameters to be estimated using the joint voltage-temperature life data. Using the MLE method, the log-likelihood function of the combined Weibull-electrical-thermal *pdf* is given by [45]:

$$\Lambda = \ln(L) = \sum_{j=1}^M \sum_{i=1}^P \sum_{l=1}^N \ln \left[\frac{\tilde{\beta}}{\tilde{C}} \exp - \left(\frac{\tilde{A}}{\tilde{V}_i} + \frac{\tilde{B}}{\tilde{T}_j} \right) \left(\frac{t_l}{\tilde{C}} \exp - \left(\frac{\tilde{A}}{\tilde{V}_i} + \frac{\tilde{B}}{\tilde{T}_j} \right) \right)^{\tilde{\beta}-1} \exp - \left(\frac{t_l}{\tilde{C}} \exp - \left(\frac{\tilde{A}}{\tilde{V}_i} + \frac{\tilde{B}}{\tilde{T}_j} \right) \right)^{\tilde{\beta}} \right] \quad (48)$$

where M is the number of thermal life test groups at voltage V_i ; P is the number of electrical life test groups at temperature T_j ; N is the number of times-to-breakdown in the j^{th} electrical-thermal life test; V_i is the i^{th} life voltage at the j^{th} life temperature; T_j is the j^{th} life temperature at the i^{th} life voltage; t_l is the l^{th} time-to-breakdown in the j^{th} group; $\tilde{\beta}$ is the estimate of the

Weibull shape parameter; \tilde{C} is an estimate of a parameter of the combined E-T life model; \tilde{A} is an estimate of a parameter of the combined E-T life model; \tilde{B} is an estimate of a parameter of the combined E-T life model.

The parameter estimates $(\tilde{\beta}, \tilde{C}, \tilde{A}, \tilde{B})$ can be found by solving:

$$\frac{\partial \Lambda}{\partial \tilde{\beta}} = 0, \frac{\partial \Lambda}{\partial \tilde{C}} = 0, \frac{\partial \Lambda}{\partial \tilde{A}} = 0, \frac{\partial \Lambda}{\partial \tilde{B}} = 0.$$

where,

$$\begin{aligned} \frac{\partial \Lambda}{\partial \tilde{\beta}} = & \sum_{j=1}^M \sum_{i=1}^P \sum_{l=1}^N \frac{1}{\tilde{\beta}} + \sum_{j=1}^M \sum_{i=1}^P \sum_{l=1}^N \ln \left(\frac{t_l}{\tilde{C}} \exp - \left(\frac{\tilde{A}}{V_i} + \frac{\tilde{B}}{T_j} \right) \right) - \\ & \sum_{j=1}^M \sum_{i=1}^P \sum_{l=1}^N \left(\frac{t_l}{\tilde{C}} \exp - \left(\frac{\tilde{A}}{V_i} + \frac{\tilde{B}}{T_j} \right) \right)^{\tilde{\beta}} \ln \left(\frac{t_l}{\tilde{C}} \exp - \left(\frac{\tilde{A}}{V_i} + \frac{\tilde{B}}{T_j} \right) \right) \end{aligned} \quad (49)$$

$$\frac{\partial \Lambda}{\partial \tilde{C}} = \sum_{j=1}^M \sum_{i=1}^P \sum_{l=1}^N -\frac{\tilde{\beta}}{\tilde{C}} + \frac{\tilde{\beta}}{\tilde{C}} \sum_{j=1}^M \sum_{i=1}^P \sum_{l=1}^N \left(\frac{t_l}{\tilde{C}} \exp - \left(\frac{\tilde{A}}{V_i} + \frac{\tilde{B}}{T_j} \right) \right)^{\tilde{\beta}} \quad (50)$$

$$\frac{\partial \Lambda}{\partial \tilde{A}} = \sum_{j=1}^M \sum_{i=1}^P \sum_{l=1}^N \tilde{\beta} \left(\frac{-1}{V_i} \right) + \sum_{j=1}^M \sum_{i=1}^P \sum_{l=1}^N \left(\frac{\beta}{V_i} \right) \cdot \left(\frac{t_l}{\tilde{C}} \exp - \left(\frac{\tilde{A}}{V_i} + \frac{\tilde{B}}{T_j} \right) \right)^{\tilde{\beta}} \quad (51)$$

$$\frac{\partial \Lambda}{\partial \tilde{B}} = \sum_{j=1}^M \sum_{i=1}^P \sum_{l=1}^N \tilde{\beta} \left(\frac{-1}{T_j} \right) + \sum_{j=1}^M \sum_{i=1}^P \sum_{l=1}^N \left(\frac{\beta}{T_j} \right) \cdot \left(\frac{t_l}{\tilde{C}} \exp - \left(\frac{\tilde{A}}{V_i} + \frac{\tilde{B}}{T_j} \right) \right)^{\tilde{\beta}} \quad (52)$$

4.4. Failure lifetime percentiles

Once the combined Weibull-Lifetime model parameters are estimated, the failure time percentiles, or the time-to-breakdown, t_p , as well as the life lines of the MW insulation at different breakdown probabilities, $p = F_p(t; \alpha, \beta)$ can be derived from Eq. (2) by substituting $\alpha = L(V, T)$. For the Weibull-IPL model, by substituting Eq. (21) into Eq. (20), the lifetime percentile is given by,

$$t_p = kV^{-\tilde{n}} [-\ln(1-p)]^{1/\tilde{\beta}} \quad (53)$$

where \tilde{k} , \tilde{n} , and $\tilde{\beta}$ are the ML estimates of k , n , and β of the combined Weibull-IPL model. Likewise, for the Weibull-Electrical-Thermal model, the lifetime percentile can be obtained by substituting Eq. (46) into Eq. (20). Thus yields,

$$t_p = \tilde{C} \exp \left(\frac{\tilde{A}}{V} + \frac{\tilde{B}}{T} \right) [-\ln(1-p)]^{1/\tilde{\beta}} \quad (54)$$

where \tilde{C} , \tilde{A} , \tilde{B} , and $\tilde{\beta}$ are the ML estimates of C , A , B , and β of the combined Weibull-Electrical-Thermal model.

Nomenclature

<i>ALTA</i>	accelerated life testing analysis
α	Weibull scale parameter
β	Weibull shape parameter
<i>cdf</i>	cumulative distribution function
IPL	inverse power law
LS	least squares
<i>L</i>	lifetime
ML	maximum likelihood
MLE	maximum likelihood estimation
MLR	multiple linear regression
<i>pdf</i>	probability density function
SLR	simple linear regression
<i>t</i>	time-to-breakdown
<i>T</i>	absolute temperature
<i>V</i>	applied voltage
V_s	reference voltage

Author details

Eyad A. Feilat

Address all correspondence to: e.feilat@ju.edu.jo

The University of Jordan, Amman, Jordan

References

- [1] Lawless JF. Statistical Models and Methods for Lifetime Data. 2nd ed. New York: John Wiley & Sons; 2003. DOI: 10.1002/9781118033005
- [2] Nelson WB. Accelerated Testing: Statistical Models, Test Plans, and data Analyses. New York: John Wiley & Sons; 2008. DOI: 10.1002/9780470316795
- [3] Mann RN, Schafer RE, Singpurwalla ND. Methods for Statistical Analysis of Reliability and Life Data. New York: John Wiley & Sons; 1974

- [4] Meeker WQ, Escobar LA. Statistical Methods for Reliability Data. New York: John Wiley & Sons; 1998
- [5] Occhini E. A statistical approach to the discussion of the dielectric strength in electric cables. *IEEE Transactions on Power Apparatus and Systems*. 1971;**90**:2671-2678. DOI: 10.1109/TPAS.1971.292920
- [6] Feilat EA, Grzybowski S, Knight P, Doriott L. Breakdown and aging behavior of composite insulation system under DC and AC high voltages. *WSEAS Transactions on Circuits and Systems*. 2005;**4**:780-787
- [7] Grzybowski S, Dobroszewski R, Grzegorski E: Accelerated endurance tests of polyethylene insulated cables. In: *Proceedings of the 3rd International Conference on Dielectric Materials, Measurements and Applications*; 10-13 September 1979; Birmingham, UK. p. 120-123
- [8] Cacciari M, Montanari GC. Optimum design of life tests for insulating materials, systems and components. *IEEE Transactions on Electrical Insulation*. 1991;**26**:1112-1123. DOI: 10.1109/14.108148
- [9] Mazznti G, Montanari GC, Simoni L. Insulation characterization in multistress conditions by accelerated life tests: an application to XLPE and EPR for high voltage cables. *IEEE Electrical Insulation Magazine*. 1997;**13**:24-34. DOI: 10.1109/57.637151
- [10] Björklund A, Siberg H, Paloniemi P. Accelerated ageing of a partial discharge resistant enameled round wire. In: *Proceedings of the Electrical Electronics Insulation Conference*; 18-21 September 1995; Rosemont, Illinois. pp. 417-419
- [11] Fallou B, Burguiere C, Morel JF. First approach on multiple stress accelerated life testing of electrical insulation. In: *1979 Annual Report of CEIDP*; Pennsylvania, USA. pp. 621-628
- [12] Ramu TS. On the estimation of life of power apparatus insulation under combined electrical and thermal stress. *IEEE Transactions on Electrical Insulation*. 1985;**20**:70-78. DOI: 10.1109/TEI.1985.348759
- [13] Feilat EA, Grzybowski S, Knight P. Multiple stress aging of magnet wire by high frequency voltage pulses and high temperatures. In: *Conference Records of the 2000 IEEE International Symposium on Electrical Insulation (ISEI2000)*; 2-05-04-2000; CA, USA. pp. 157-160
- [14] Grzybowski S, Feilat EA, Knight P, Doriott L. Breakdown voltage behavior of PET thermoplastics at DC and AC Voltages. In: *Proceedings of the IEEE SoutheastCon99*; 25-28 March 1999; Lexington, Kentucky, USA. pp. 284-287
- [15] Grzybowski S, Feilat EA, Knight P, Doriott L. Accelerated aging tests on magnet wires under high frequency pulsating voltage and high temperature. In: *Annual Report of the 1999 Conference on Electrical Insulation and Dielectric Phenomena, (CEIDP99)*; 17-21 October 1999; Austin, Texas, USA. pp. 555-558
- [16] Kim J, Kim W, Park H-S, Kang J-W. Lifetime assessment for oil-paper insulation using thermal and electrical multiple degradation. *Journal of Electrical Engineering and Technology*. 2017;**12**:840-845. DOI: 10.5370/JEET.2017.12.2.840
- [17] Li Y, Tian M, Lei Z, Song J, Zeng J, Lin L, Li Y. Breakdown performance and electrical aging life model of EPR used in coal mining cables. In: *Proceedings of the IEEE*

- International Conference on High Voltage Engineering and Application (ICHVE); 19-22 September 2016; Chengdu, China
- [18] Maussion P, Picot A, Chabert M, Malec D. Lifespan and aging modeling methods for insulation systems in electrical machines: a Survey. In: Proceedings of the 2nd IEEE Workshop on Electrical Machines Design, Control and Diagnostics (WEMDCD'15); 26-27 March 2015; Turin, Italy
 - [19] Akolkar SM, Kushare BE. Remaining life assessment of power transformer. *Journal of Automation and Control*. 2014;**2**:45-48. DOI: 10.12691/automation-2-2-2
 - [20] Kerimli GM. On the mechanism of electrical aging of polymer insulation materials. *Surface Engineering and Applied Electrochemistry*. 2014;**50**:485-490. DOI: 10.3103/S1068375514060052
 - [21] Lahoud N, Faucher J, Malec D, Maussion P. Electrical aging of the insulation of low voltage machines: model definition and test with the design of experiments. *IEEE Transactions on Industrial Electronics*. 2013;**60**:4147-4155. DOI: 10.1109/TIE.2013.2245615
 - [22] Nasrat LS, Kassem N, Shukry N. Aging effect on characteristics of oil impregnated insulation paper for power transformers. *Engineering*. 2013;**5**:1-7. DOI: 10.4236/eng.2013.51001
 - [23] Koltunowicz TL, Cavallini A, Djairam D, Montanari GC, Smit JJ. The influence of square voltage waveforms on transformer insulation break down voltage. In: 2011 Annual Report Conference on Electrical Insulation and Dielectric Phenomena; 16-19-10-2011; Mexico
 - [24] Mazzanti G, Montanari GC, Dissado LA. Electrical aging and life models: the role of space charge. *IEEE Transactions on Dielectrics and Electrical Insulation*. 2005;**12**:876-890. DOI: 10.1109/TDEI.2005.1522183
 - [25] Liao RJ, Liang SW, Sun CX, Yang LJ, Sun HG. A comparative study of thermal aging of transformer insulation paper impregnated in natural ester and in mineral oil. *European Transactions on Electrical Power*. 2010;**20**:518-523. DOI: 10.1002/etep.336
 - [26] Montanari GC, Mazzanti G, Simoni L. Progress in electrothermal life modeling of electrical insulation during the last decades. *IEEE Transactions on Dielectrics and Electrical Insulation*. 2002;**9**:730-745. DOI: 10.1109/TDEI.2002.1038660
 - [27] Kaufhold M, Aninger H, Berth M, Speck J, Eberhardt M. Electrical stress and failure mechanism of the winding insulation in PWM-inverter fed low-voltage induction motors. *IEEE Transactions on Industrial Electronics*. 2000;**47**:396-402. DOI: 10.1109/41.836355
 - [28] Stone GC. The application of Weibull statistics to insulation aging tests. *IEEE Transactions on Electrical Insulation*. 1979;**14**:233-239. DOI: 10.1109/TEI.1979.298226
 - [29] Fothergill JC. Estimating the cumulative probability of failure data points to be plotted on Weibull and other probability papers. *IEEE Transactions on Electrical Insulation*. 1990;**25**:489-499. DOI: 10.1109/14.55721
 - [30] Reliability Engineering Resources[Internet]. Available from: <http://www.weibull.com>
 - [31] Jacquelin J. Inference of sampling on Weibull parameter estimation. *IEEE Transactions on Dielectrics and Electrical Insulation*. 1996;**3**:809-816. DOI: 10.1109/94.556564

- [32] Cygan P, Laghari JR. Models for insulation aging under electrical and thermal multistress. *IEEE Transactions on Electrical Insulation*. 1990;**25**:923-934. DOI: 10.1109/14.59867
- [33] Laghari JR. Complex electrical thermal and radiation aging of dielectric films. *IEEE Transactions on Electrical Insulation*. 1993;**28**:777-788. DOI: 10.1109/14.237741
- [34] Montanari GC, Simoni L. Aging phenomenology and modeling. *IEEE Transactions on Electrical Insulation*. 1993;**28**:755-776. DOI: 10.1109/14.237740
- [35] Gjaerde AC. Multifactor ageing models-origin and similarities. *IEEE Electrical Insulation Magazine*. 1997;**13**:6-13. DOI: 10.1109/CEIDP.1995.483610
- [36] Montanari GC, Cacciari M. A probabilistic insulation life model for combined thermal-electrical stresses. *IEEE Transactions on Electrical Insulation*. 1985;**20**:519-522. DOI: 10.1109/TEI.1985.348776
- [37] Dissado LA, Fothergill JC, Wolfe SV, Hill RM. Weibull statistics in dielectric breakdown; theoretical basis, applications and implications. *IEEE Transactions on Electrical Insulation*. 1984;**19**:227-233. DOI: 10.1109/TEI.1984.298753
- [38] Gibbons DI, Vance LC. A simulation study of estimators for the 2-parameter Weibull distribution. *IEEE Transactions on Reliability*. 1981;**30**:61-66. DOI: 10.1109/TR.1981.5220965
- [39] Weibull++ Life Data Analysis Reference, Version 11. Arizona, USA: ReliaSoft's; 2017
- [40] Eichhorn RM. Treeing in solid extruded electrical insulation. *IEEE Transactions on Electrical Insulation*. 1977;**12**:2-8. DOI: 10.1109/TEI.1977.298001
- [41] Kachen W, Laghari JR. Determination of aging-model constants under high frequency and high electric fields. *IEEE Transactions on Dielectrics and Electrical Insulation*. 1994;**6**:1034-1038. DOI: 10.1109/94.368660
- [42] Cygan P, Krishnakumar B, Laghari JR. High field electrical and thermal aging of polypropylene films. In: *Conference Record of the 1988 IEEE International Symposium on Electrical Insulation*; 5-8 June 1988; Boston, MA, USA. pp. 188-191. DOI: 10.1109/ELINSL.1988.13901
- [43] Grzybowski S, Bandaru S: Effect of multistress on the lifetime characteristics of magnet wires used in flyback transformer. In: *Conference Record of the IEEE International Symposium on Electrical Insulation*; 19-22 September 2004; Indianapolis, IN, USA
- [44] Feilat E A, Grzybowski S, Knight P. Electrical aging models for fine gauge magnet wire enamel of flyback transformer. In: *Proceedings of the IEEE Southeast Con 2000*; 7-9 April 2000; Nashville, TN, USA. pp. 146-149
- [45] Feilat EA. Lifetime characteristics of magnet wires under high frequency pulsating voltage and high temperature [Thesis]. Mississippi State: Mississippi State University; 2000
- [46] SAS/STAT Guide for Personal Computers, Version 6, Chapter 21. The LIFEREG Procedure. SAS Institute, 1987
- [47] Draper NR, Smith H. *Applied Regression Analysis*. John Wiley & Sons; 2014. DOI: 10.1002/9781118625590
- [48] Accelerated Life Testing Analysis, Version 11. Tucson, Arizona, USA. ReliaSoft's; 2017

