Chapter 12

Numerical Analysis Methods of Structural Fatigue and Fracture Problems

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Additional information is available at the end of the chapter

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Abstract

Fatigue and fracture problems, which lead to 95% of structural failure, have attracted much attention of engineers and researchers all over the world. Compared with experimental method, numerical simulation method based on empirical models shows its remarkable advantages in structure design because of less cost and higher efficiency. However, the application of numerical simulation method in fatigue lifetime prediction is restricted by low accuracy and poor applicability in some circumstances. Most numerical method is based on empirical models. This chapter first reviews various kinds of empirical models of fatigue and fracture problems, including some modifying methods of basic empirical models, which have been widely applied to fatigue lifetime prediction and indicated their advantages and disadvantages. Then, FEM is introduced as an important method to obtain stress intensity factor or crack growth route. At last, this chapter is finished with existing problems and current trends in fatigue lifetime prediction via numerical method.

Keywords: fatigue lifetime prediction, crack propagation, numerical method, empirical model, Paris law, perturbation approach, extended finite element method, fractal geometry

1. Introduction

With the development of mechanical engineering and manufacturing technology, engineering structures applied in aircrafts and huge machines become much more complex. These structures usually bear constantly changing loads in tour of duty. Although the max stress in structure caused by these dynamic loads is much lower than yield limit and ultimate strength of material, structure is destroyed after a long time. Internal defects in engineering structures appear in producing, processing, and assembling process. Internal defects lead to stress concentration, crack initiation, and propagation and even fatigue failure under dynamic load.

According to statistical data, loss caused by improper structural fatigue lifetime design in America equals 4.4% of gross national product, and 95% of structure failures are related to fatigue break caused by alternating dynamic loads [1]. There are numerous historical examples that result in great loss of human life and economic value. For example, two Comet aircraft crashed in 1954, and the main reason is fatigue of fuselage structure [2]. Mechanical failure caused by fatigue, which concentrates much attention of engineers and researches, has been studied for more than 150 years [3]. However, it is still much difficult to prevent fatigue failure because fatigue of materials is far from being completely comprehended [4].

Metallic materials are widely applied in design of structures and parts in present days; therefore, fatigue of metals is a problem deserving efforts. In fact, the fatigue process is constitutive of crack initiation and crack propagation to total failure, as shown in **Figure 1**, and fatigue lifetime should conclude crack initiation life and crack propagation life.

On one hand, it is widely accepted that the crack initiation phase costs a majority of fatigue lifetime in a high-cycle fatigue regime [5]. Furthermore, crack initiation behavior has a great influence on crack growth prediction in a unified approach for fatigue lifetime prediction [6]. Therefore, knowledge and technology of crack initiation life prediction are significant for evaluation of fatigue lifetime of structures and deserve our efforts to study deeply. On the other hand, there are frequently small cracks and defects in engineering structures due to manufacturing and environment factors; therefore, fatigue crack propagation prediction plays an important role in estimating the structural safety under dynamic loads.

Therefore, people divide structural life prediction problem into two problems: fatigue problem and fracture problem. People pay attention to crack initiation life in fatigue problem and make efforts to construct the relationship between structure life and stress or strain in structure. It is assumed in fatigue problem that there is a small crack existing in structure, and crack propagation behavior is studied in order to predict the remaining life of structure. These two problems have aroused widespread concern nowadays.

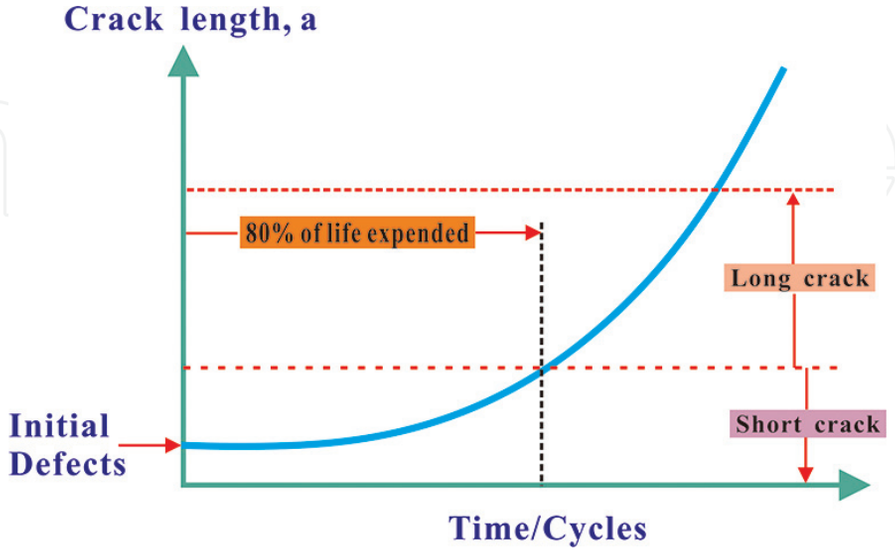

Figure 1. Schematic illustration of crack length versus time/cycles.

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Experimental method and numerical method are two significant ways to analyze fatigue lifetime of structures. Experimental method has been widely applied since a long time ago. However, it is much expensive to predict structural life via experimental method. Furthermore, it is difficult to execute experiments for some complicated structures. Therefore, numerical method based on empirical models becomes much more popular in structural life prediction, and in some cases, those do not need high accuracy because of less cost and higher efficiency.

2. Empirical models in fatigue problem

Approaches to predict fatigue initiation life in literature can be classified into several types. These approaches study the fatigue problem from different perspectives, involving the average or local values of stresses and strains, the initiation of crack and defects, and macro- and microanalysis [7]. Nevertheless, people prefer to use phenomenological models, which reflect general material response at macroscopic scale under cyclic loads, rather than complex micro- or mesoscopic model of material fatigue behavior in structure design [8].

2.1. Empirical models of high-cycle fatigue

Wöhler is the pioneer in this field, who established the traditional stress-based approach in the nineteenth century [9]. He carried out a few fatigue experiments on metallic materials and indicated the relationship between fatigue crack initiation life and cyclic stress. He proposed to apply $S-N$ curves in description of fatigue behavior of metals in his paper. Effectiveness of this method in high-cycle fatigue analysis is demonstrated afterward by many researchers. There are several kinds of expression of $S-N$ curve, mainly including exponential function expression and power function expression. Basquin was the first person who suggested using exponential function to construct the expression of $S-N$ curve in the twentieth century. The typical exponential function expression is written as follows:

$$e^{mS_{\max}}N = C \tag{1}$$

where m and C are constants, which can be determined based on experiment data, N stands for the number of loading cycles, and $S_{\max}$ is the maximum value of stress at specific stress ratio. The power function expression with two parameters is usually expressed in the following form:

$$S_a^m N = C \tag{2}$$

where S_a is the stress amplitude at specific ratio. The power function expression with three parameters is expressed as

$$(S_{\max} - C)^m N = D \tag{3}$$

or

$$S_{\max} = C\left(1 + \frac{A}{N^{\alpha}}\right) \tag{4}$$

where D, A, and α are constants. The parameter C in Eqs. (3) and (4) nearly equals fatigue limit.

2.2. Empirical models of low-cycle fatigue

Stress level is usually high in low-cycle fatigue, and the maximum value of stress is nearly close to the ultimate strength of material. The number of loading cycles in low-cycle fatigue, which is not more than 10^3 times, is much less than that in high-cycle fatigue. Plastic deformation plays an important role in low-cycle fatigue, in which the accumulation of plastic deformation results in structural failure. Because low-cycle fatigue lifetime is much sensitive to the change of stress level, $S - N$ curve is unable to reflect the low-cycle fatigue performance of material. Therefore, $\varepsilon - N$ curve is applied to low-cycle fatigue analysis. The most widely accepted low-cycle fatigue lifetime model based on $\varepsilon - N$ curve is proposed by Basquin [10], which is expressed as follows:

$$\varepsilon_e = \frac{\sigma_a}{E} = \frac{\sigma_f'}{E}\left(2N_f\right)^b \tag{5}$$

where ε_e is the amplitude of elastic strain, E is the elasticity modulus of material, σ_f' is the fatigue strength coefficient of material, and b is the fatigue strength exponent. Because the relationship between plastic strain and fatigue lifetime is not taken into consideration in Basquin formula, Coffin [11] and Manson [12] proposed an empirical model when studying the relationship between fatigue lifetime and plastic strain amplitude. The expression of Coffin-Manson model is.

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'(2N)^c \tag{6}$$

in which ε_a stands for the amplitude of total strain and ε_f' and c stand for the fatigue ductility coefficient and fatigue ductility exponent separately. The relationships between plastic strain, elastic strain, total strain, and fatigue lifetime are shown in **Figure 2**.

2.3. Improved models considering mean stress or stress ratio

There are many factors, such as residual stress, temperature, multiaxial stress, and geometrical feature, that influence structural fatigue lifetime, in which mean stress or stress ratio concentrates the most attention.

2.3.1. Walker formula

Mean stress and stress ratio are of great significance for structural fatigue lifetime. Walker formula considers sensitivity of different materials to mean stress; therefore, it shows well

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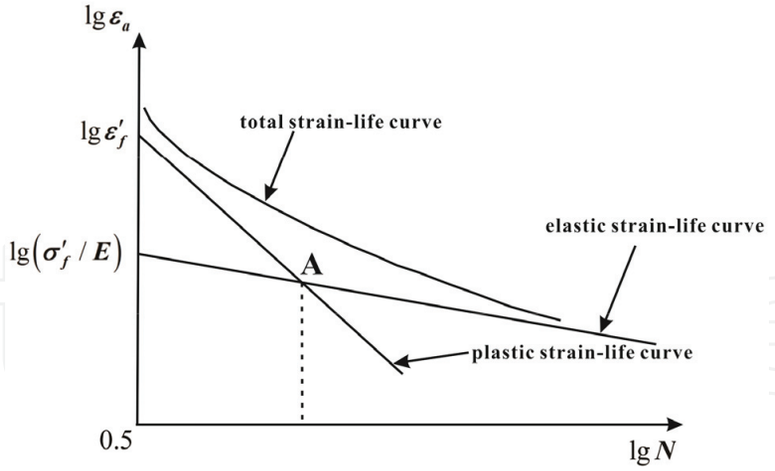

Figure 2. Elastic strain-life curve and plastic strain-life curve.

effect for all materials [13, 14]. An equivalent local strain parameter is defined in Walker formula; its expression is

$$\varepsilon_{eq} = (2\varepsilon_a)^r \left(\frac{\sigma_{\max}}{E}\right)^{1-r} \tag{7}$$

r is the material parameter. In order to construct the relationship between Walker formula and fatigue lifetime, Jaske et al. [15] carried out many experiments on different kinds of materials and proposed following expression based on experimental data:

$$\log N_f = A_0 + A_1 \tanh^{-1}\left[\frac{\log\left(\varepsilon_u \varepsilon_e / \varepsilon_{eq}^2\right)}{\log\left(\varepsilon_u / \varepsilon_e\right)}\right] \tag{8}$$

where A_0 and A_1 are regression coefficients and ε_u and ε_e are the upper and lower limits of this reverse hyperbolic tangent function, respectively. The strain-life curve is shown in **Figure 3**.

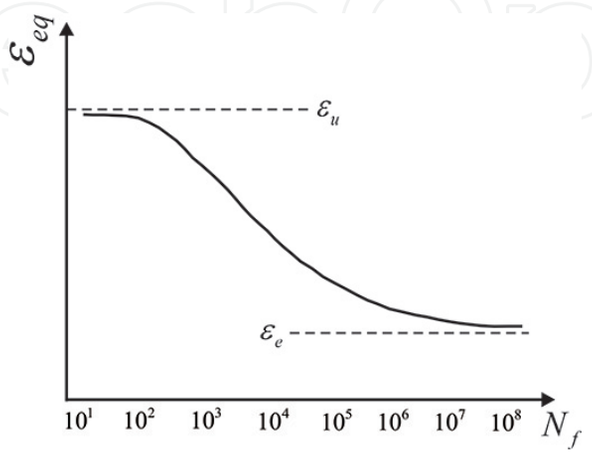

Figure 3. Strain-life curve of Walker formula.

There are too many parameters to be fitted in this method, which need plenty of experimental data. That disadvantage constricts badly the application of Walker formula in engineering.

2.3.2. Morrow's modifying method

Morrow's modifying method and SWT modifying method are two commonly used methods. Morrow mean stress modifying formula is shown as follows [16]:

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)\left(2N_f\right)^b + \varepsilon_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)\left(2N_f\right)^c \tag{9}$$

Considering the greater influence made by mean stress in long life period, further modifying method is given:

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \tag{10}$$

where ε_a is strain amplitude and σ_m is mean stress. Morrow's modifying method aims at elastic strain; therefore, it is only suitable when stress amplitude is constant or mean stress is compression stress.

2.3.3. SWT modifying method

Expression of Smith-Watson-Topper (SWT) parameter modifying method is [17]

$$\sigma_{\max}\varepsilon_a = \frac{\sigma_f'^2}{E}\left(2N_f\right)^{2b} + \sigma_f'\left(2N_f\right)^{b+c} \tag{11}$$

where

$$\sigma_{\max} = \sigma_m + \sigma_a \tag{12}$$

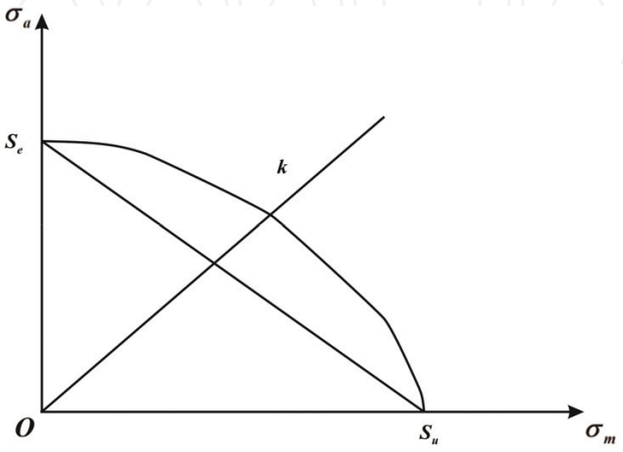

Figure 4. Fatigue limit curve and Goodman simplified straight line.

SWT mean stress modifying method is not valid for compression mean stress, and it will obtain too conservative result when the stretching mean stress is large.

2.3.4. Goodman's modifying method

We can acquire the fatigue limit points of material at different stress ratio $r = \sigma_{\min}/\sigma_{\max}$ under infinite life requirement with the support of large amount of experimental data. Draw these points in rectangular coordinate system whose X-axis is mean stress $\sigma_m = (\sigma_{\min} + \sigma_{\max})/2$ and Y-axis is stress amplitude $\sigma_a = (\sigma_{\max} - \sigma_{\min})/2$; thus, the fatigue limit curve is fitted based on these points. It is unpractical to carry out many experiments on all materials and structures in engineering, so we usually use a simplified straight line to replace the fatigue limit curve. Goodman simplified straight line, which is one of these straight lines, is widely accepted due to its simplicity and conservative estimation [18], as shown in **Figure 4**. Goodman simplified straight line can be expressed in the following relationship:

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u} = 1 \tag{13}$$

where S_e stands for the fatigue strength of material and S_u stands for the ultimate tensile strength of material. However, it has been proved that Goodman modifying method is only appropriate for low-ductility material, such as high-strength steel and cast iron.

3. Empirical models in fracture problem

3.1. Paris law

Paris et al. [19] made great contribution in this field who was pioneer suggesting that crack growth rate, da/dN, was a function of the maximum stress intensity factor $K_{\max}$ in 1961. Then, Liu [20] related the crack growth to the stress intensity factor range ΔK subsequently. Paris and Erdogan [21] proposed the well-known Paris law, which can be presented as follows:

$$\frac{da}{dN} = C(\Delta K)^m \tag{14}$$

where C and m can be obtained from experiment data, and they are usually considered as constants for a particular metal and environment [22]. Since then researchers have made efforts to study on Paris law and its deviation; however, we are still far from a complete comprehension [23].

It is believed that the relationship between crack propagation and ΔK can be divided into three distinct regions, as shown in **Figure 5**. The crack propagation is slow in region A, and concept of a fatigue threshold stress intensity factor range ΔK_{th} is proposed by Mcclintock [24], beneath which cracks are regarded not to grow. In region B, the "mid growth" range, crack propagation is stable, and Paris law is supposed to be held. Region C is associated with fast crack propagation leading to final failure. Therefore, calculation of number of loading cycles in region B, which could be gained from Paris law, is significant for prediction of fatigue crack growth life.

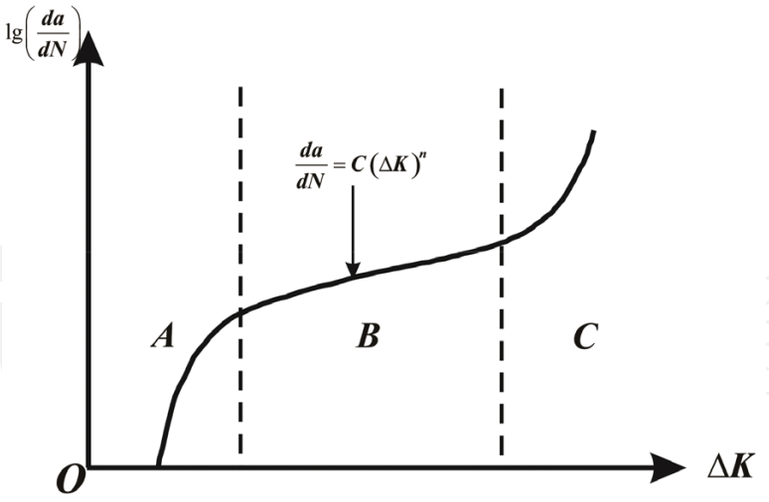

Figure 5. Schematic diagram of the relationship between crack growth and ΔK.

3.2. Improved models

3.2.1. Models considering mean stress or stress ratio

Since Paris law is proposed, much related work is done, and many modifying methods are put forward [22, 25–27]. It is commonly accepted that crack growth rate of material is related to mean stress or stress ratio. Several models, in which Forman formula [28] and Walker formula [29] are most famous, take this factor into consideration. Forman formula also considers the fracture toughness as an important factor; its expression is

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_c - \Delta K} \tag{15}$$

Forman formula is valid for dealing with experimental data of many kinds of materials, especially high-hardness alloy, but it is hard to obtain the fracture toughness K_c for high-ductility material. According to following relationship:

$$R = \frac{K_{\min}}{K_{\max}} \tag{16}$$

$$\Delta K = K_{\max} - K_{\min} \tag{17}$$

Forman formula can be transformed as follows:

$$\frac{da}{dN} = \frac{CK_{\max}(\Delta K)^{m-1}}{K_c - K_{\max}} \tag{18}$$

Forman formula explains the reason why crack growth enlarges sharply when stress intensity factor is close to fracture toughness.

Walker formula is another wide-applied crack propagation model in engineering, which expresses the influence made by stress ratio on crack growth rate. Furthermore, it takes maximum of stress intensity factor into consideration:

$$\frac{da}{dN} = C[(1-R)^m K_{\max}]^n \tag{19}$$

Three parameters C, m, and n can be acquired based on experimental data of crack propagation experiments with different stress ratios. Walker formula is valid when $R > 0$ and $R < 0$. According to the relationship between stress ratio and amplitude of stress intensity factor, another commonly used form of Walker formula is obtained:

$$\frac{da}{dN} = CK_{\max}^m (\Delta K)^n \tag{20}$$

3.2.2. Model based on crack closure theory

In 1971, Elber [30] found that crack opened completely only when the stress was larger than a certain value, and he developed a modified Paris law based on this theory. The stress when crack is completely open is defined as crack opening stress σ_{op}, and the stress when crack begins to close is defined as crack closing stress σ_{cl}. It has been demonstrated that crack opening stress is nearly equal to crack closing stress. The modified formula is written as follows:

$$\frac{da}{dN} = C\left(\Delta K_{eff}\right)^m \tag{21}$$

and

$$\frac{da}{dN} = C(U\Delta K)^m = U^m C(\Delta K)^m \tag{22}$$

U is the crack closure parameter, and its expression is

$$U = \frac{\Delta K_{eff}}{\Delta K} = \frac{\Delta \sigma_{eff}}{\Delta \sigma} = \frac{\left(\sigma_{\max} - \sigma_{op}\right)}{\Delta \sigma} < 1 \tag{23}$$

where efficient stress amplitude $\Delta\sigma_{eff}$ is the difference between maximum stress $\sigma_{\max}$ and crack opening stress σ_{op}.

3.2.3. Model considering crack retardation caused by high load

In Weeler's opinion [31], when structure bears cyclic load with constant amplitude; an occasional overload enlarges the size of plastic zone on crack tip, which would prevent crack from growing to some degree. On the basis of Weeler's research, Willenberg [32] assumed that crack retardation is due to residual compression stress σ_{res}, which is related to plastic deformation

caused by high load. Combining the expression of Forman formula, crack growth rate in retardation period is acquired:

$$\frac{da}{dN} = \frac{C\left(\Delta K_{eff}\right)^m}{\left(1 - R_{eff}\right)K_c - \Delta K_{eff}} \tag{24}$$

The effective stress intensity factor range is

$$\Delta K_{eff} = f\left[\left(\sigma_{\max}\right)_{eff} - \left(\sigma_{\min}\right)_{eff}\right]\sqrt{\pi a} \tag{25}$$

and the effective stress ratio is

$$R_{eff} = \left(\sigma_{\min}\right)_{eff} / \left(\sigma_{\max}\right)_{eff} \tag{26}$$

The maximum and minimum values of effective cyclic stress are

$$\left(\sigma_{\max}\right)_{eff} = \sigma_{\max} - \sigma_{res} \tag{27}$$

$$\left(\sigma_{\min}\right)_{eff} = \sigma_{\min} - \sigma_{res} \tag{28}$$

Then, crack growth rate in retardation period can be estimated as the residual stress σ_{res} is known. However, the residual stress σ_{res} can only be obtained via experimental method.

3.2.4. Model considering crack propagation threshold

In 1972, Donahue [33] took threshold of stress intensity factor range ΔK_{th} into consideration and proposed a generalized Paris law. The modified expression is

$$\frac{da}{dN} = C(\Delta K - \Delta K_{th})^m \tag{29}$$

The following expression was proposed by McEvily and Greoeger [34] in their research about fatigue crack propagation threshold in 1977:

$$\frac{da}{dN} = C(\Delta K - \Delta K_{th})^2 \left(1 + \frac{\Delta K}{K_c - K_{\max}}\right) \tag{30}$$

in which material constant m equals 2.

Furthermore, if considering stress ratio at the same time, Paris law can be modified into the following expression:

$$\frac{da}{dN} = \frac{C[(\Delta K)^m - (\Delta K_{th})^m]}{(1 - R)K_c - \Delta K} \tag{31}$$

It can be figured out that the above equation is further modified on the basis of Forman formula.

In 1999, McEvily found it out that the following modification is suitable for many alloys' fatigue crack propagation:

$$\frac{da}{dN} = C\left(\Delta K_{eff} - \Delta K_{effth}\right)^2 \tag{32}$$

where ΔK_{effth} stands for the effective stress intensity factor range near crack propagation threshold. This modifying method considers the influences created by crack closure and small crack's elastic-plastic behavior, and it is useful to predict the long crack propagation under cyclic positive stress.

3.2.5. Model based on perturbation series expansion method

Perturbation series expansion method, which is a common method to deal with nonlinear problems, has been widely used in fluid mechanics, structure dynamics, and damage identification. In this method, the parameter in ideal model is regarded to have a small perturbation in order to study the properties of system. This parameter can be expanded into series form:

$$a = \sum_{i=0}^{\infty} a_i \varepsilon^i \tag{33}$$

where ε is a positive small constant.

Qiu and Zheng [35] proposed a novel numerical calculation method to investigate the fatigue crack growth evolution in aluminum alloy sheets accounting for the measurement error. The initial crack length is considered as a modified parameter with a small correction term due to the measurement error; the solution to the crack growth equation is expressed in the form of a perturbation series, and a series of modified equations for predicting the crack length history

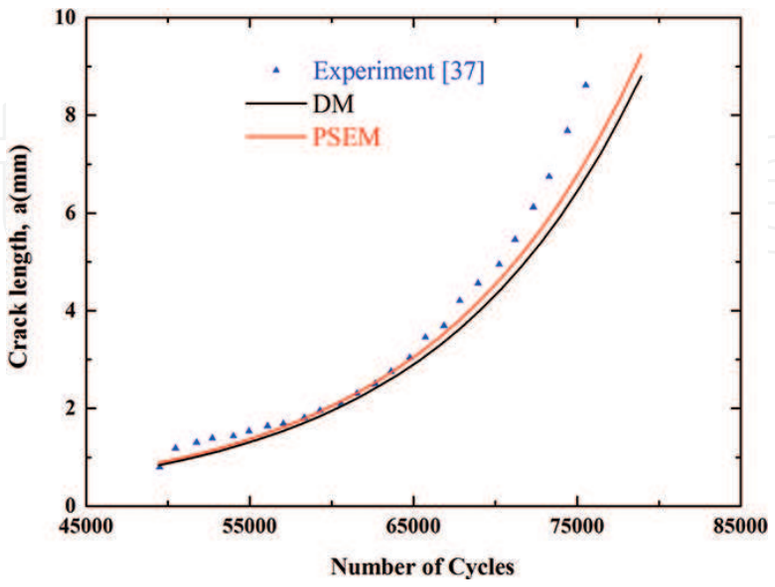

Figure 6. Comparison of the measured and predicted crack length history in Ref. [35].

are derived. The proposed method is verified to be indeed feasible and effective for predicting fatigue crack growth evolution by comparing numerical results with experimental data, as shown in **Figure 6**.

4. Finite element method

There are many kinds of numerical method to obtain stress intensity factor or crack growth route after continuous study of many researchers. Finite difference method (FDM), boundary element method (BEM), mesh-less method, and finite element method (FEM) are four common methods. Many studies have been carried out based on these numerical methods: Christen applied FEM to two-dimensional crack problem and obtained the displacement field and stress field; Nayroles [36] combined the moving least square method (MLSM) with mesh-less method to solve boundary problem. FEM is the most widely used method in above four methods at present [37, 38]. Considering singularity on crack tip, element's density is increased in order to obtain the precious results. Therefore, FEM's rate of convergence is low, and precision is unsatisfactory. People developed precious numerical solution methods based on several kinds of theories, in which semi-analytic numerical solution and new type elements are hot issues.

4.1. Extended finite element method

Collapsed singular isoparametric elements, which can reflect the singularity on crack tip correctly, were introduced by Barsoum [39]. This method is popular because of its high precision and executing simplicity. In this method, planar eight-node isoparametric element is degenerated into singular isoparametric element, as shown in **Figure 7**. Stress intensity factor is calculated based on the displacements of nodes A and B; the expression is.

$$K_{\mathrm{I}} = \frac{E'}{4}\sqrt{\frac{2\pi}{L}}(4v_A - v_B) \tag{34}$$

In plane stress problem, $E' = E$; in plane strain problem, $E' = \frac{E}{1-\mu^2}$. E, μ, and v are, respectively, elasticity modulus, Poisson ratio, and displacement perpendicular to crack surface. Chen and

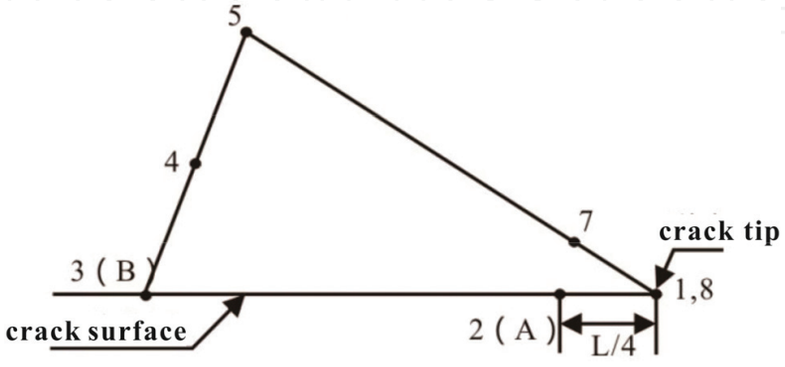

Figure 7. Eight-node singular isoparametric element.

Kuang [40] use interpolation method to acquire the displacements of nodes A and B on the basis of Barsoum's research and obtain the following expression of stress intensity factor:

$$K_{\mathrm{I}} = \frac{E'}{12}\sqrt{\frac{2\pi}{L}}(8v_A - v_B) \tag{35}$$

Lin [41] proposed the 1/4 node displacement method, as shown in **Figure 8**; the corresponding calculation equation of stress intensity factor is

$$K_{\mathrm{I}} = \frac{E'}{2}\sqrt{\frac{2\pi}{L}}v_A \tag{36}$$

Belytschko [42] applied extended finite element method (XFEM) to calculating stress intensity factor and neglected the high-order terms of asymptotic displacement function. The calculation results were not satisfying enough. Karihaloo and Xiao [43] took high-order terms of asymptotic displacement function and outer elements of crack tip into consideration, thus obtaining results of high accuracy. However, calculation efficiency of this method is relatively low. Although researchers have obtained precious results with the help of new type elements, there are still many factors that influence calculation results that need to be studied.

4.2. Fractal finite element method

In the aspect of semi-analytic numerical method, weighted function method and boundary collocation method develop fast. These methods are able to acquire results of high accuracy when dealing with particular models; however, calculation accuracy cannot be guaranteed when dealing with general models.

Fractal finite element method is also a semi-analytic method. Fractal geometry is introduced into ordinary FEM, which not only improves calculation accuracy but also shortens calculation time and saves storage capacity of a computer. In fractal finite element method, an artificial boundary Γ_0 is introduced to divide the structure with crack into two parts: singular field D

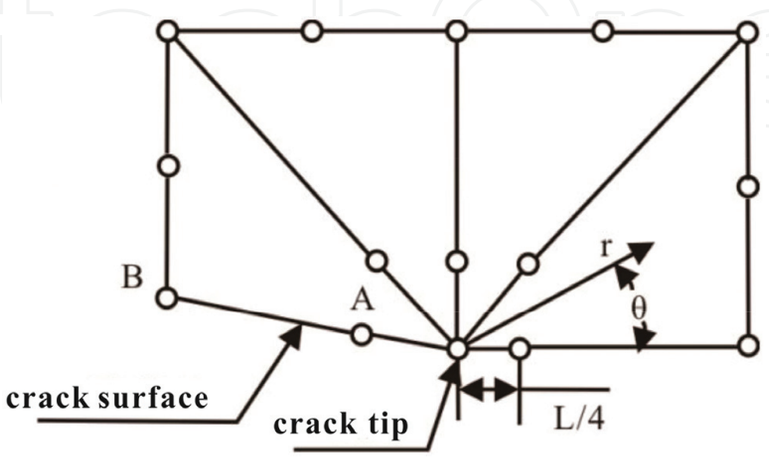

Figure 8. Mesh of 1/4 node element displacement method.

near crack tip and normal field Ω far away from crack tip, as shown in **Figure 9**. Ordinary finite element mesh is constructed in normal field; self-similar mesh needs to be constructed based on fractal theory in singular field.

Self-similar mesh is shown in **Figure 10**. In singular field, infinite curves $\{\Gamma_1, \Gamma_2, \Gamma_3, \cdots\}$ similar to Γ_0 are generated based on the proportionally coefficient $\xi(0 < \xi < 1)$ regarding crack tip as centre. The density of fractal mesh is controlled by ξ. Based on appropriate global interpolation function and fractal transforming technique, plenty of unknown degrees on slave nodes are

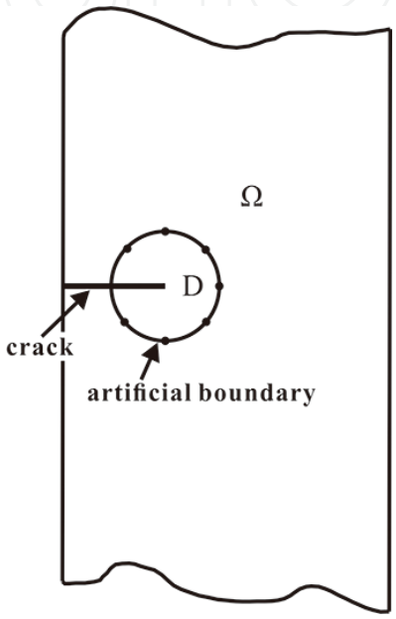

Figure 9. Illustration of division of structure with crack.

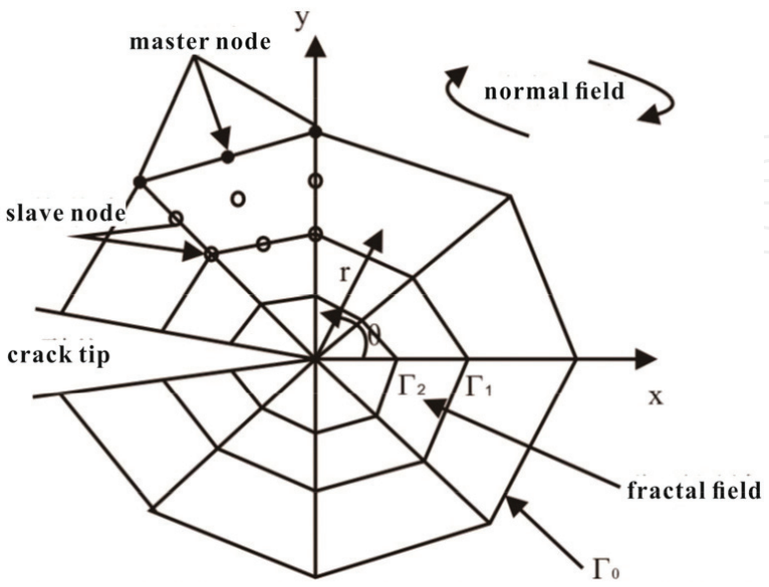

Figure 10. Self-similar mesh in singular field.

transformed into a series of generalized coordinates. Stress intensity factor on crack tip can be calculated via solving generalized coordinates, thus saving calculation time and storage capacity obviously.

5. Conclusion

This chapter reviews the most common empirical models and numerical methods of structural fatigue lifetime prediction. The main advantages and disadvantages of these methods are discussed.

Numerical method based on empirical models, as one of significant ways to analyze structural fatigue life, becomes popular in structural life prediction nowadays because of less cost and higher efficiency.

$S-N$ curve and $\varepsilon-N$ are applied to high-cycle and low-cycle fatigue problems, respectively. And there are many modified models considering mean stress or stress ratio. However, this chapter further shows that part of these models are too complicated to apply to engineering, and other models are only valid in some specific cases.

Paris law is the most significant model of crack propagation problem. But it only considers the stress intensity factor as the factors make influences on crack propagation. Many improved models considering stress ratio, crack closure, crack retardation, and crack propagation threshold have been put forward.

FEM is the most popular numerical method to obtain stress intensity factor or crack growth route. Extended finite element method and fractal finite element method are two mainly developing trends of FEM. However, it is still difficult to achieve high efficiency and accuracy of numerical method at the same time.

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