Chapter 5

Recent Advances in the Serviceability Assessment of Footbridges Under Pedestrian-Induced Vibrations

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Additional information is available at the end of the chapter

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Abstract

Current international guidelines determine the effect of pedestrians on footbridges via an equivalent harmonic load. However, the dynamic response of footbridges obtained according to these standards differs from the values recorded experimentally. In order to overcome this issue, a new modelling framework has been recently proposed by several researchers. This novel approach allows considering more accurately three key aspects: (i) the inter- and intra-subject variability, (ii) the pedestrian-structure interaction and (iii) the crowd dynamics. For this purpose, different crowd-structure interaction models have been developed. Despite the large number of proposals, all of them share the same scheme: the crowd-structure interaction is simulated by linking two sub-models, namely (i) a pedestrian-structure interaction sub-model and (ii) a crowd sub-model. Furthermore, the variability of the pedestrian's behaviour may be taken into account via the assumption that the model parameters are random variables. In this chapter, a summary of the state-of-art of this new modelling framework is presented, with special emphasis in a case study where the crowd-structure interaction model developed by the authors is used to simulate the lateral lock-in phenomenon on a real footbridge.

Keywords: footbridges, pedestrian-structure interaction, crowd dynamics, dynamic stability, vibration serviceability assessment

1. Introduction

The increase in the strength of the new structural materials together with the higher aesthetics requirements imposed by current modern society has led to the design of footbridges with greater slenderness, which may be prone to vibrate under pedestrian-induced excitations. There are three factors that characterize this engineering problem: (i) the vibration source, i.e. the pedestrian; (ii) the path, i.e. the structure; and (iii) the receiver, i.e. the pedestrian [1, 2]. In

the last 20 years, after some vibratory events happened in several large-span footbridges [3–5], an intensive research activity has been conducted by the scientific community in order to better characterize the pedestrian-induced vibrations on footbridges. Concretely, these research efforts were mainly focused on two objectives: (i) the accurate definition of the vibration source [6] and (ii) the analysis of a remarkable event, the lateral lock-in phenomenon [7]. On the one hand, the determination of the load induced by pedestrians on footbridges was tackled progressively. Initially, the estimation of the force originated by a single walking or running pedestrian was studied [8, 9]. Subsequently, these results were further extrapolated to the case of a crowd moving on a footbridge [10]. On the other hand, the lateral lock-in instability phenomenon originated by the synchronization of a pedestrian flow walking on a footbridge has been widely studied as well. Based on the outcomes of these researches, different proposals to estimate the number of pedestrians that originates the lateral lock-in phenomenon, as well as limiting values of the modal properties of the structure to avoid the problem, have been provided [7].

As result of all these studies, several standards [11] and design guidelines [12] were published to facilitate designers the assessment of the vibration serviceability limit state of footbridges under pedestrian action. Although these design codes shed light on this issue, they still present some shortages, so that, the dynamic response of the structure obtained numerically based on these recommendations still differs from the values recorded experimentally [13].

In order to overcome these limitations, a new generation of models have been developed and proposed during the last 5 years, giving rise to a new modelling framework. Three key aspects have been additionally taken into account in order to improve the modelling of pedestrian flows and their effect on footbridges [14]: (i) the inter- and intra-subject variability of the pedestrian action, (ii) the pedestrian-structure interaction and (iii) the crowd behaviour. Furthermore, the variability of the pedestrian action is normally simulated via a probabilistic approach, considering that the parameters that characterize the crowd-structure interaction model may be defined as random variables [15]. All these proposed models share a common scheme, and the crowd-structure interaction is simulated via the linking of two sub-models [16, 17]: (i) a pedestrian-structure interaction sub-model and (ii) a crowd sub-model. For the pedestrian-structure sub-model, although different models have been proposed [18], the use of a single-degree-of-freedom (SDOF) system has gained wider popularity in the scientific community. For the crowd sub-model, two approaches have been proposed: either macroscopic or microscopic models [15]. In the first approach, the crowd behaviour is modelled based on fluid mechanics [10], whilse in the second, the position and velocity of each pedestrian follows a multi-agent law [19]. The second approach, which can account explicitly for the inter-subject variability of each pedestrian [20], has been internationally accepted as the best method to simulate numerically the behaviour of pedestrian flows [15]. The linking between the two sub-models is achieved by the implementation of several behavioural conditions [20]. In this way, if certain comfort limits are exceed by the pedestrian-structure interaction sub-model, the velocity and step frequency of each pedestrian in the crowd are modified [20, 21]. The new modelling framework, based on these crowd-structure interaction models, has been applied successfully to determine numerically the response of a footbridge under pedestrian action

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[15–17], to study the change of the modal parameters of real footbridges under the effect of a group of pedestrians [22] and even to analyse the lateral lock-in phenomenon on real footbridges [23].

Nevertheless, despite all these advances, there is not currently any international design guideline which covers comprehensively all aspects of the problem, so it is a challenge for the next years to include all these research results in the design standards of such structures.

The chapter is organized as follows. First, some general recommendations on how to assess the vibration serviceability limit state of footbridges under pedestrian action, according to the more recent design guidelines, are presented in Section 2. Second, the main aspects of the new modelling framework to simulate the crowd-structure interaction are presented in Section 3. As this modelling framework divides the issue in two sub-models, in Section 4, the first sub-model, the pedestrian-structure sub-model is presented and in Section 5, the second sub-model, the crowd sub-model, is described. Later, the interaction between the two sub-models is explained and implemented in Section 6. Subsequently, a case study, the comparison of the analysis of the lateral lock-in phenomenon on the Pedro e Inês footbridge using three different approaches (the experimental values recorded during the field test, the numerical estimation according to the Synpex guidelines [12] and the new modelling framework) is presented in Section 7. The study shows the potential of this new modelling framework to assess more accurately the vibration serviceability limit state of footbridges under pedestrian action. Finally, Sections 8 and 9 present the main conclusions obtained from the chapter and future research lines to be explored, respectively.

2. Brief review of design standards

The international standards for the assessment of the vibration serviceability limit state of footbridges under pedestrian action share two general rules to tackle the pedestrian-induced vibration problem [6]: (i) the establishment of the range of frequencies that characterizes the pedestrian-structure interaction (**Table 1**) and (ii) the treatment of the problem separately in terms of the direction in which the pedestrian action (longitudinal, lateral or vertical) is applied. However, most of these standards only establish the need to assess the dynamic behaviour of the structure, if some of its natural frequencies are within the interaction range (**Table 1**), but do not define a methodology to check the required comfort level.

According to the authors' opinion, the Synpex guidelines [12] are currently the most comprehensive standard to assess the vibration serviceability limit state of footbridges under pedestrian action. These guidelines [12] divide the checking of the vibration serviceability limit state in seven steps:

i. Evaluate, numerically, the natural frequencies of the footbridge based on a finite element model of the structure.

ii. If some of the natural frequencies of the structure lie inside the interaction ranges (**Table 1**), the comfort class of the footbridge must be further checked.

Standards	Vertical [Hz]	Lateral [Hz]
LRFD American Guide (2009)	<3.00	
Eurocode 1 (2002)	1.60-2.40	0.80–1.20
Eurocode 5 (2003)	<2.50	0.80–1.20
DIN-Fachbericht 102 (2003)	1.60–2.40/3.50–4.50	
SIA 260 (2003)	1.60–4.50	<1.30
BS 5400 (2006)	<5.00	<1.50
Austroads (2012)	1.50–3.00	
Hong Kong Guide (2009)	1.50–2.30	
Ontario Guide (1995)	<3.00	
Setra (2006)	1.00–2.60/2.60–5.00	0.30–1.30/1.30–2.50
Synpex (2007)	1.25–2.30/2.50–4.60	0.50–1.20
EHE-08 (2008)	<5.00	
EAE (2011)	1.60–2.40/3.50–4.50	0.60–1.20
IAP-11 (2011)	1.25–4.60	0.50–1.20

Table 1. Ranges of frequencies of pedestrian-structure interaction according to different international standards [14].

iii. Different design scenarios must be assessed: for each design scenario, the expected traffic class in terms of the pedestrian density, d [P=Person/m^2], (**Table 2**) and its corresponding comfort class in terms of limit acceleration (**Table 3**) must be determined according to the owner's requirements.

Class	Density d [P/m^2]	Characteristics
TC1	<15 P (P = Person)	Very weak traffic
TC2	<0.20 P/m^2	Comfortable and free walking
TC3	<0.50 P/m^2	Unrestricted walking and significantly dense traffic
TC4	<1.00 P/m^2	Uncomfortable situation and obstructed walking
TC5	<1.50 P/m^2	Unpleasant walking and very dense traffic

Table 2. Traffic classes [12].

Class	Degree	Vertical acceleration	Horizontal acceleration
CL1	Maximum	<0.50 m/s^2	<0.10 m/s^2
CL2	Medium	0.50–1.00 m/s^2	0.10–0.30 m/s^2
CL3	Minimum	1.00–2.50 m/s^2	0.30–0.80 m/s^2
CL4	Discomfort	>2.50 m/s^2	>0.80 m/s^2

Table 3. Defined comfort classes with limit acceleration ranges [12].

iv. The damping ratio of the affected vibration mode, ζ_f, is estimated in function of the construction type and the amplitude of the vibrations [12].

v. The maximum acceleration has to be evaluated for each design scenario. For this purpose, it is necessary to define a load model which may be characterized by the following equivalent harmonic loads [12]:

- A pedestrian stream walking is simulated by an equivalent load:

$$p_{wal}(t) = G \cdot \cos\left(2 \cdot \pi \cdot f_s \cdot t\right) \cdot n' \cdot \psi / L_f \text{ [N/m]} \tag{1}$$

- A pedestrian jogging is simulated by a single vertical moving load:

$$P_{jog}\left(t, v_p\right) = 1250 \cdot \cos\left(2 \cdot \pi \cdot f_s \cdot t\right) \cdot \psi \text{ [N]} \tag{2}$$

where $G \cdot \cos\left(2 \cdot \pi \cdot f_s \cdot t\right)$ is the harmonic load due to a single pedestrian, with G being the dynamic load factor (DLF) of the pedestrian step load (280 N for vertical direction, 140 N for longitudinal direction and 35 N for lateral direction); f_s is the step frequency [Hz], which is assumed equal to the considered natural frequency, f_f; ψ is the reduction coefficient that takes into account the probability that the footfall frequency approaches the considered natural frequency and it may be estimated from **Figure 1**, according to the considered natural frequency; v_p is the pedestrian velocity [m/s] which may be assumed around 3 m/s [12] and L_f is the length of the footbridge [m]. In Eq. (1), n' is the equivalent number of pedestrians on the footbridge, which may be determined from:

$$n' = \begin{matrix} 10.8 \cdot \sqrt{\zeta_f \cdot n} \\ 1.85 \cdot \sqrt{n} \end{matrix} \text{ if } \begin{matrix} d < 1.00 \ P/m^2 \\ d \geq 1.00 \ P/m^2 \end{matrix} \tag{3}$$

in terms of the number of pedestrians on the deck, n, and the damping ratio of the considered vibration mode, ζ_f.

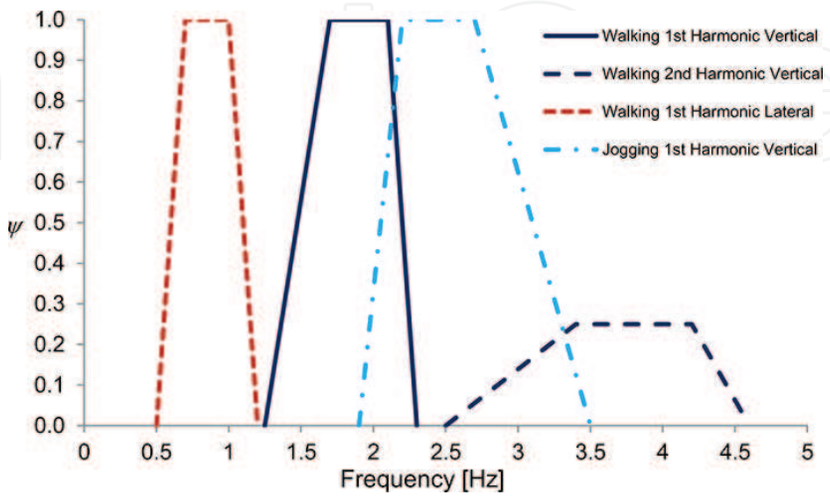

Figure 1. Pedestrian reduction coefficient, ψ, for the equivalent pedestrian load [12].

To estimate the considered natural frequency for each design scenario, the mass of pedestrians has to be taken into account (with a medium pedestrian weight about 70 kg) when its value is greater than 5% of the modal deck mass.

vi. The dynamic response obtained for each considered design scenario must be compared with the trigger acceleration amplitude, 0.10–0.15 m/s^2, which avoids the occurrence of the lateral lock-in phenomenon.

vii. The estimated dynamic acceleration is then compared with the specified comfort class. In case of non-compliance, the designer must adopt measures to improve the dynamic behaviour of the structure, such as for instance: (i) the modification of the mass of the deck, (ii) the modification of the natural frequencies of the structure and/or (iii) the increase of the damping [12].

In spite of the fact that the Synpex design guidelines [12] were an important breakthrough, they still present several limitations, which originate that the numerical prediction of the dynamic response of footbridges, obtained using them, under- or over-estimates the values recorded experimentally. As main limitations, the following ones may be enumerated: (i) the change of the dynamic properties of the structure, due to the presence of pedestrians, is estimated in a simplified form, adding directly the pedestrian mass to the structural mass without considering any additional effect on the remaining modal parameters of the structure, (ii) the proposed methods do not fit well to the case where several vibration modes of the footbridge are affected by the pedestrian-induced excitations, (iii) the effect of the non-synchronized pedestrians are not taken into account by these recommendations and (iv) the definition of the pedestrian load is performed under a deterministic approach which does not allow considering the inter- and intra-subject variability of the pedestrian action. In order to overcome these limitations, a new generation of models that configure a new modelling framework has been proposed. A brief description of this new modelling framework is included in the next section.

3. New modelling framework of crowd-structure interaction

The most recent research on this topic proposes and further implements several crowd-structure interaction models to better characterize the dynamic response of footbridges under pedestrian action [14–17]. All these models, which share a common scheme, constitute a new modelling framework for this engineering problem. According to this new approach, the crowd-structure interaction is simulated by linking two individual sub-models (**Figure 2**): (i) a pedestrian-structure interaction sub-model and (ii) a crowd sub-model.

In the first sub-model, although there are several proposals [18, 22, 24, 25] to simulate the pedestrian action (single-degree-of-freedom (SDOF) system, multiple degrees of freedom (MDOF) system and inverted pendulum (IP) system), the most widely adopted alternative is to model the pedestrian either as a SDOF system in vertical direction [18] or as a IP system in lateral direction [20, 24], while the structure is characterized via its modal parameters [22, 26].

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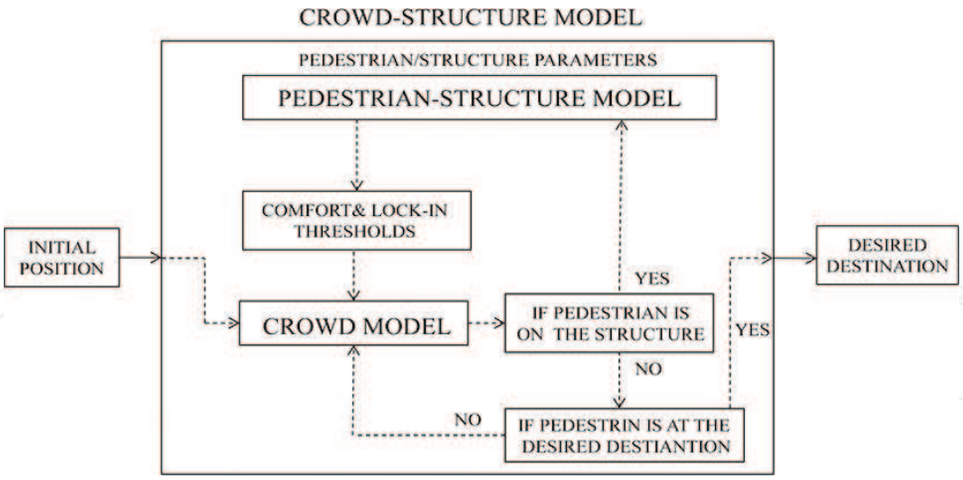

Figure 2. Layout of the new modelling framework.

All the pedestrian-structure interaction models based on the use of a SDOF system share a common formulation to solve the pedestrian-structure interaction [22, 26] but, however, they differ in the values adopted to characterize the modal parameters of the SDOF systems. A wide summary of the pedestrian-structure interaction models proposed by different authors can be found in Ref. [18]. The main output obtained from this sub-model is usually the acceleration experienced by each pedestrian.

In the second sub-model, the crowd is usually simulated via a behavioural model [19] that provides a description of the individual pedestrian position, $\mathbf{x}_p$, pedestrian velocity, $\mathbf{v}_p$, and step pedestrian frequency, f_s. Additionally, in order to take into account the synchronization among pedestrians, an additional parameter must be included. A common manner to simulate this phenomenon is to add a different phase shift, ϕ_p, in the definition of the ground reaction load generated by each pedestrian [14].

The linking between the two sub-models is usually achieved in the different proposals by taking into account the modification of the pedestrian behaviour in terms of the vibration level that he/she experiences [15, 17, 20–24]. Two additional conditions are commonly included for this purpose: (i) a retardation factor, which reduces the pedestrian velocity in terms of the accelerations experienced by each pedestrian; and (ii) a lateral lock-in threshold, which allows simulating the synchronization among the pedestrians and the structure by the modification of both their step frequencies and the phases [20–23]. This new approach has only been implemented, to the best of the authors' knowledge, in vertical and lateral direction, since there are few reported cases of pedestrian-induced vibration problems in longitudinal direction. In order to illustrate briefly this new modelling framework, one of the most recent crowd-structure interaction models, which has been proposed by the authors, is described in the next sections [23]. Subsequently, the potential of the approach to accurately assess the vibration serviceability limit state of footbridges under pedestrian action is illustrated via its implementation for the analysis of a case study. For clarity, the model is described here only for the lateral direction, although it may be easily generalized to the vertical direction [14].

4. Modelling pedestrian-structure interaction

The pedestrian-structure interaction model may follow from the application of dynamic equilibrium equations between a SDOF-system (**Figure 3a**) and the footbridge (**Figure 3b**). The pedestrian mass is divided into sprung, m_a, and unsprung, m_s, components [kg].

As result of this dynamic equilibrium, the following coupled equation system may be obtained [22]:

$$M_i\ddot{y}_i + C_i\dot{y}_i + K_i y_i = \phi_{num\_i}(x_p) \cdot F_{\text{int}} \tag{4}$$

$$m_a\ddot{y}_a + c_p(\dot{y}_a - \dot{y}_s) + k_p(y_a - y_s) = 0 \tag{5}$$

$$m_s\ddot{y}_s + c_p(\dot{y}_s - \dot{y}_a) + k_p(y_s - y_a) = F_p - F_{\text{int}} \tag{6}$$

where y_i is the amplitude of the vibration mode *ith* of the footbridge [m]; y_a is the displacement of the pedestrian sprung mass [m]; y_s is the displacement of the pedestrian unsprung mass [m]; k_p is the pedestrian stiffness [N/m]; c_p is the pedestrian damping [sN/m]; F_p is the ground reaction force [N]; F_{int} is the pedestrian-structure interaction force [N]; M_i is the mass associated with the *ith* vibration mode [kg]; C_i is the damping associated with the *ith* vibration mode [sN/m]; K_i is the stiffness associated with the *ith* vibration mode [N/m]; ϕ_{num_i} is the modal coordinates of the *ith* vibration mode; $x_p = v_{px} \cdot t$ is the pedestrian's longitudinal position on the footbridge [m], being t the time [sec.] and v_{px} the longitudinal component of the pedestrian velocity vector [m/s]; d_p is the distance among pedestrians [m] and $w(x,t)$ is the lateral displacement of the footbridge at position x [m].

The numerical vibration modes, $\phi_{num\_i}(x)$, may be obtained by a numerical modal analysis of the structure based on the finite element method:

$$\phi_{num\_i}(x) = \sum_j \phi_i^j \cdot N_j(x) \tag{7}$$

where $N_j(x)$ is the beam shape functions and ϕ_i^j is the nodal values of the vibration modes.

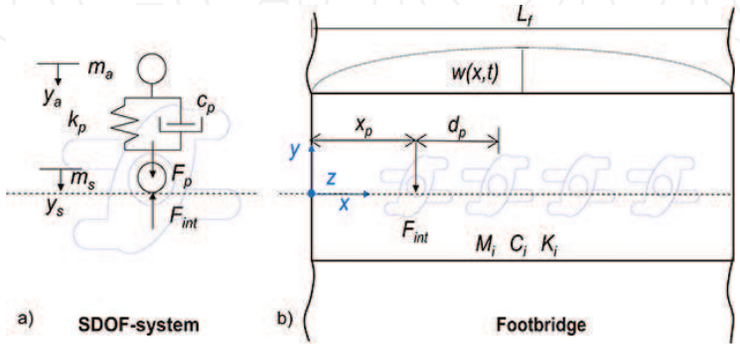

Figure 3. Biomechanical pedestrian-structure interaction model in lateral direction [14]. (a) SDOF-system and (b) Footbridge.

Thus, F_{int} follows from the above Eq. (6) to yield:

$$F_{\text{int}} = F_p - m_s\ddot{y}_s - c_p(\dot{y}_s - \dot{y}_a) - k_p(y_s - y_a) \tag{8}$$

and substituting this equation into the equilibrium equation of the structure:

$$M_i\ddot{y}_i + C_i\dot{y}_i + K_i y_i = \phi_{num\_i}(x_p) \cdot \left(F_p - m_s\ddot{y}_s - c_p(\dot{y}_s - \dot{y}_a) - k_p(y_s - y_a)\right) \tag{9}$$

Applying, at the contact point, the equations of compatibility of displacements, $y_s = w(x_p, t)$, velocities, $\dot{y}_s = \dot{w}(x_p, t)$ and accelerations, $\ddot{y}_s = \ddot{w}(x_p, t)$, between the SDOF system and the structure, the following expressions may be obtained:

$$w(x_p, t) = \sum_{i=1}^{n_m} y_i(t) \cdot \phi_{num\_i}(x_p) \tag{10}$$

$$\dot{w}(x_p, t) = \sum_{i=1}^{n_m} \dot{y}_i(t) \cdot \phi_{num\_i}(x_p) + \sum_{i=1}^{n_m} y_i(t) \cdot v_{px} \cdot \phi'_{num\_i}(x_p) \tag{11}$$

$$\ddot{w}(x_p, t) = \sum_{i=1}^{n_m} \ddot{y}_i(t) \cdot \phi_{num\_i}(x_p) + \sum_{i=1}^{n_m} 2 \cdot \dot{y}_i(t) \cdot v_{p,x} \cdot \phi'_{num\_i}(x_p) + \sum_{i=1}^{n_m} y_i(t) \cdot {v_{p,x}}^2 \cdot \phi''_{num\_i}(x_p)) \tag{12}$$

where $\phi'_{num\_i}(x)$ and $\phi''_{num\_i}(x)$ the first and second spatial derivatives of the *ith* numerical vibration mode and n_m, is the number of considered vibration modes.

It is assumed that the lateral displacement of the footbridge may be decomposed in terms of the amplitude $y_i(t)$ and the modal coordinates of the n_m vibration modes, $\phi_{num\_i}(x)$, and the time variation of the pedestrian velocity is neglected due to its low contribution: Subsequently, the above relations Eqs. (10)–(12) may be substituted in the overall dynamic equilibrium equation of the footbridge, obtaining the following pedestrian-structure interaction model equations to yield in matrix form:

$$\mathbf{M}(t) \cdot \ddot{\mathbf{y}}(t) + \mathbf{C}(t) \cdot \dot{\mathbf{y}}(t) + \mathbf{K}(t) \cdot \mathbf{y}(t) = \mathbf{F}(t) \tag{13}$$

In this manner, the pedestrian-structure interaction model may be represented by a system with $(n_m + 1)$ equations (being n_m the number of the considered vibration modes and *1* the SDOF system that simulates the pedestrian behaviour). In case of a group of *k* pedestrians (**Figure 3b**), the number of equations of system increases to $(n_m + k)$, maintaining the same scheme. A more detailed description of this pedestrian-structure interaction model may be found in Ref. [22].

The lateral ground reaction force, F_p, generated by each pedestrian, may be defined under either a deterministic [8] or a stochastic approach [15]. The second approach allows taking into account the inter- and intra-subject variability of the pedestrian action [15]. Although there are more complex ways [15] to define the lateral ground reaction force, however, it is usually expressed in terms of a Fourier series decomposition [8, 9, 12] as:

$$F_p = m \cdot g \sum_{i=1}^{n_f} \alpha_i \cdot \sin\left(\pi \cdot i \cdot f_s \cdot t - \varphi_i - \phi_p\right) \tag{14}$$

where $m = m_a + m_s$ is the total pedestrian mass, g is the acceleration of gravity, α_i is the Fourier coefficients of the *ith* harmonic of the lateral force, f_s [Hz] is the pedestrian step frequency, φ_i is the phase shifts of the *ith* harmonic of the lateral pedestrian force, ϕ_p is the phase shift among pedestrians and n_f is the total number of contributing harmonics.

According to this formulation, the deterministic or stochastic character of the pedestrian-structure interaction sub-model can be considered depending on the way in which the parameters of the model are defined. If a fixed value is assigned to the parameters, the sub-model will be deterministic; however, if the parameters are defined as random variables, the sub-model will be stochastic.

Finally, **Table 4** shows the values reported in Ref. [14] for the characterization of the pedestrian-structure interaction model. Additionally, a wide summary of the parameters proposed by other researchers can be found in Ref. [18]. These values (**Table 4**) allow defining the pedestrian-structure interaction model in either a deterministic form (considering the average values) or a stochastic form (considering the probabilistic distribution), depending on the purpose of the case under study.

	Pedestrian modal parameters	
	Lateral	
Definition	**Parameter**	**Value**
Pedestrian total mass	m	$N(75, 15)$kg
Pedestrian sprung mass	m_a	$N(73.216, 2.736)$%
Pedestrian damping ratio	ζ_p	$N(49.116, 5.405)$%
Pedestrian natural frequency	f_p	$N(1.201, 0.178)$Hz
	Walking pedestrian force	
	Lateral	
Definition	**Parameter**	**Value**
First harmonic	α_1	$N(0.086, 0.017)$
Second harmonic	α_2	$N(0.094, 0.009)$
Third harmonic	α_3	$N(0.040, 0.019)$
First phase shift	φ_1	0°
Second phase shift	φ_2	0°
Third phase shift	φ_3	0°

Table 4. Parameters of the pedestrian-structure interaction sub-model reported in Refs. [14, 15], where $N(\mu, \sigma)$ is a Gaussian distribution with mean value, μ, and the standard deviation, σ.

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5. Modelling crowd dynamics

The pedestrian moving inside a crowd may be modelled using either a macroscopic [10] or a microscopic model [15]. The second option is currently the most utilized and it has been successfully implemented by several authors [15, 19–22]. According to this approach, the movement of each pedestrian is governed by the dynamic balance among particles [14]. This model assumes that the different motivations and influences experimented by the pedestrians are described by different social forces [19]. The model is based on Newton dynamics and is able to represent the following rules in relation with the natural pedestrian movement (see Ref. [19] for a more detailed description): (i) the fastest route is usually chosen by pedestrians, (ii) the individual speed of each pedestrian follows a probabilistic distribution function and (iii) the distance among pedestrians in a crowd depends on the pedestrian density, the spatial configuration of the crowd and the pedestrian speed. As an example, the different social forces acting between two pedestrians in a crowd are illustrated in **Figure 4**.

In this manner, the multi-agent model that simulates the behaviour of the crowd consists of the sum of three partial forces: (i) the driving force, $\mathbf{F}_{dri}$, (ii) the repulsive force generated by the interaction among pedestrians, $\mathbf{F}_{ped}$, and (iii) the repulsive force generated by the interaction with the boundaries, $\mathbf{F}_{bou}$. A detailed description of these three forces is carried out in the next sub-sections. The sum of these three forces generates the overall pedestrian-crowd interaction

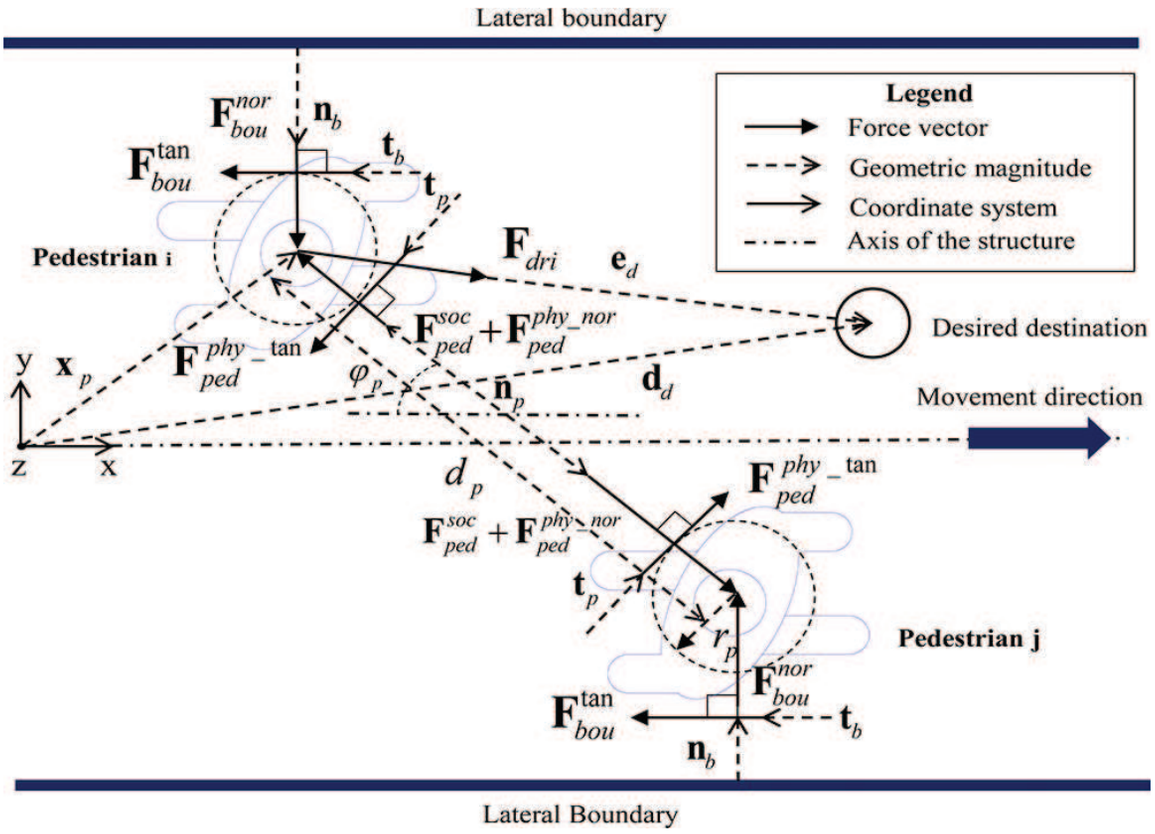

Figure 4. Biomechanical pedestrian-structure interaction model [14].

force, $\mathbf{F}_{pci}$, that describes the movement and direction of each pedestrian in the crowd. This resultant force is defined as follows:

$$\mathbf{F}_{pci} = \mathbf{F}_{dri} + \mathbf{F}_{ped} + \mathbf{F}_{bou} \tag{15}$$

5.1. Driving force

Each pedestrian has a certain motivation to reach his/her desired destination [19], $\mathbf{d}_d$, with his/her desired velocity, v_d, which is represented by the driving force, $\mathbf{F}_{dri}$, as:

$$\mathbf{F}_{dri} = m \cdot \left(\frac{v_d \cdot \mathbf{e}_d}{t_r} - \frac{\mathbf{v}_p}{t_r} \right) \tag{16}$$

where $\mathbf{e}_d$ is the desired direction vector, $\mathbf{v}_p$ is the pedestrian step velocity and t_r is the relaxation time (the time it takes a pedestrian to adapt its motion to its preferences).

5.2. Interactions among pedestrians

The interaction among pedestrians originates a repulsive force [19], $\mathbf{F}_{ped}$, with two components, a socio-psychological force, $\mathbf{F}_{ped}^{soc}$, and a physical interaction force, $\mathbf{F}_{ped}^{phy}$, as:

$$\mathbf{F}_{ped} = \mathbf{F}_{ped}^{soc} + \mathbf{F}_{ped}^{phy} \tag{17}$$

The socio-psychological force reflects the fact that the pedestrians try to maintain a certain distance to other pedestrians in the crowd. This socio-psychological force depends on the distance between pedestrians, reaching its maximum value at the initial distance between two pedestrians, d_p, and tending to zero as such distance increases. The socio-psychological force is defined as:

$$\mathbf{F}_{ped}^{soc} = A_p \cdot \exp\left(\frac{2 \cdot r_p - d_p}{B_p} \right) \cdot \mathbf{n}_p \cdot s_p \tag{18}$$

where A_p is the interaction strength between two pedestrians; B_p is the repulsive interaction range between pedestrians; r_p is the so-called pedestrian radius; $\mathbf{n}_p$ is the normalized vector pointing between pedestrians and s_p is a form factor to consider the anisotropic pedestrian behaviour [19], whose value may be obtained from:

$$s_p = \lambda_p + (1 - \lambda_p) \cdot \frac{1 + \cos\left(\varphi_p\right)}{2} \tag{19}$$

being, λ_p, a coefficient that takes into account the influence of the pedestrians placed in front of the subject on his/her movement, and, φ_p, the angle formed between two pedestrians.

In situations of physical contact among pedestrians ($d_p \leq 2 \cdot r_p$) and high pedestrian density (≥0.80 P=Person/m^2), the physical interaction force, $\mathbf{F}_{ped}^{phy}$, must be considered. This force may be

divided in other two components: (i) the body force, $\mathbf{F}_{ped}^{phy\_nor}$, and (ii) the sliding force, $\mathbf{F}_{ped}^{phy\_\tan}$. The first component simulates the counteracting body action that each pedestrian performs to avoid physical damage in case he/she gets in physical contact with other individuals. The second component considers the pedestrians' tendency to avoid overtaking other subjects quickly at small distances [19]. It is defined as [22]:

$$\mathbf{F}_{ped}^{phy} = \mathbf{F}_{ped}^{phy\_nor} + \mathbf{F}_{ped}^{phy\_\tan} \tag{20}$$

$$\mathbf{F}_{ped}^{phy\_nor} = C_p \cdot H\left(2 \cdot r_p - d_p\right) \cdot \mathbf{n}_p \tag{21}$$

$$\mathbf{F}_{ped}^{phy\_\tan} = D_p \cdot H\left(2 \cdot r_p - d_p\right) \cdot \Delta v_p^t \cdot \mathbf{t}_p \tag{22}$$

being $\mathbf{F}_{ped}^{phy\_nor}$ the normal component of the physical interaction force; $\mathbf{F}_{ped}^{phy\_\tan}$ the tangential component of the physical interaction force; C_p the body force strength due to the contact between pedestrians; D_p the sliding force strength due to the contact between pedestrians; $\mathbf{t}_p$ a normalized tangential vector (which is perpendicular to $\mathbf{n}_p$); $\Delta v_p^t = \left\langle \Delta \mathbf{v}_p \cdot \mathbf{t}_p \right\rangle$ the component of the relative pedestrian velocity in tangential direction; $\Delta \mathbf{v}_p$ the vector of differential velocities between two pedestrians; and $H(\bullet)$ a function which may be defined as [22]:

$$H(\bullet) \cdot = \begin{cases} \bullet & if \quad \bullet > 0 \\ 0 & if \quad \bullet \leq 0 \end{cases} \tag{23}$$

5.3. Interactions with boundaries

The interaction with the boundaries gives rise to forces, $\mathbf{F}_{bou}$. These forces are equivalent to the ones resulting from the interaction with other pedestrian, so they can be formulated in a similar fashion as [22]:

$$\mathbf{F}_{bou} = \mathbf{F}_{bou}^{nor} + \mathbf{F}_{bou}^{\tan} \tag{24}$$

$$\mathbf{F}_{bou}^{nor} = \left\{ A_b \cdot \exp\left(\frac{r_p - d_b}{B_b}\right) + C_b \cdot H\left(r_p - d_b\right) \right\} \cdot \mathbf{n}_b \tag{25}$$

$$\mathbf{F}_{bou}^{\tan} = D_b \cdot H\left(r_p - d_b\right) \cdot \left\langle \mathbf{v}_p, \mathbf{t}_b \right\rangle \cdot \mathbf{t}_b \tag{26}$$

being $\mathbf{F}_{bou}^{nor}$ the component of the boundary interaction force in normal direction; $\mathbf{F}_{bou}^{\tan}$ the component of the boundary interaction force in tangential direction; A_b the pedestrian-boundary interaction strength; B_b the pedestrian-boundary repulsive interaction range; d_b the pedestrian-boundary distance; C_b the body force strength due to the contact with the boundary; D_b the sliding force strength due to the contact with the boundary; $\mathbf{n}_b$ the normalized normal vector between the pedestrian and boundary; $\mathbf{t}_b$ the normalized tangential vector (which is perpendicular to $\mathbf{n}_b$) and $\langle\rangle$ denotes the scalar product [22].

All the parameters for the considered crowd sub-model, based on the social force model, may be obtained from the results provided by different authors [19, 20] as summarized in **Table 5**.

Parameter	Element	Value
Relaxation time	t_r	0.50 sec.
Interaction strength pedestrians	A_p	2000 N
Interaction range pedestrians	B_p	0.30 m
Potential factor	λ_p	0.20
Contact strength pedestrians	C_p	2000 N
Sliding strength pedestrians	D_p	4800 N
Interaction strength boundaries	A_b	5100 N
Interaction range boundaries	B_b	0.50 m
Contact strength boundaries	C_b	2000 N
Sliding strength boundaries	D_b	4800 N
Radius of pedestrian	r_p	0.20 m

Table 5. Parameters of the crowd sub-model reported in Refs. [19, 20].

5.4. Simulation procedure

The simulation of a pedestrian flow requires the determination of four parameters: (i) the pedestrian density, d, (ii) the desired velocity of each pedestrian, v_d, (iii) the phase shift among pedestrians, ϕ_p, and (iv) the distance among pedestrians, d_p.

First, the pedestrian density, d, is established according to the owner's requirements [12]. Second, the values of the desired velocity of each pedestrian can be obtained from the pedestrian step frequencies, f_s, assuming that initially the pedestrian velocity, v_p, is equal to the desired velocity, v_d. For this purpose, the Gaussian distribution of the pedestrian step frequency, $N(1.87, 0.186)$ Hz, reported in Ref. [2], can be adopted as reference. After assigning a step frequency to each pedestrian, its desired velocity is determined from the empirical relation given in Ref. [27]:

$$f_s = 0.35 \cdot |\mathbf{v}_p|^3 - 1.59 \cdot |\mathbf{v}_p|^2 + 2.93 \cdot |\mathbf{v}_p| \tag{27}$$

Subsequently, the initial phase shift among pedestrians, ϕ_p, which allows estimating the number of pedestrian that arrive at the footbridge in phase, is determined considering that it follows a Poisson distribution [14]. Finally, the original distance among pedestrians is calculated considering the width of the footbridge, a predefined geometrical-shaped mesh of pedestrians (triangular or rectangular) and the considered pedestrian density.

The acceleration vector, $\mathbf{a}_p$, that acts on each pedestrian may be determined as:

$$\mathbf{a}_p = \frac{\mathbf{F}_{pci}}{m} \tag{28}$$

Finally, the evaluation of the remaining variables that govern the crowd model, $\mathbf{v}_p$ and $\mathbf{x}_p$, are then performed using a multi-step method [14].

6. Modelling crowd-structure interaction

The crowd-structure interaction is usually modelled including additional behavioural conditions [20–22]. Concretely, two requirements have been included in this proposal: (i) a comfort and (ii) a lateral lock-in threshold [14, 20].

First, a comfort condition is usually included in the crowd-structure interaction model to take into account the modification of the behaviour of each pedestrian due to the change of his/her comfort level. For this purpose, a retardation factor has been applied to the pedestrian velocity. A minimum comfort threshold 0.20 m/s^2 is selected following the results provided by several researches [20, 28]. In this manner, if the lateral acceleration of each pedestrian, $\ddot{y}_a$, is above this value, the pedestrian velocity is reduced by a retardation factor, r_v, which is a function of the acceleration experienced by the pedestrian. Following the intuitive assumption, reported in Ref. [20], that the pedestrians are likely to react more firmly as the lateral acceleration they feel is higher, a tri-linear function is considered, Eq. (29).

$$r_v(\ddot{y}_a) = \begin{matrix} 1-(0.1/1.05)\cdot\ddot{y}_a \\ 0.9-(0.3/0.65)\cdot(\ddot{y}_a-1.05) \\ 0.6-(0.6/0.4)\cdot(\ddot{y}_a-1.7) \\ 0 \end{matrix} \text{ if } \begin{matrix} \ddot{y}_a \le 1.05\ m/s^2 \\ \ddot{y}_a \le 1.7\ m/s^2 \\ \ddot{y}_a \le 2.1\ m/s^2 \\ \ddot{y}_a > 2.1\ m/s^2 \end{matrix} \tag{29}$$

On the other hand, a maximum lateral limit acceleration, $\ddot{y}_{\lim} = 2.10$ m/s^2, have also been considered [29], so pedestrians stop walking, when the experienced acceleration becomes too high, to keep their balance, and they remain stopped until the footbridge reduces its accelerations. Both to stop walking and to remain stationary before starting to walk again, the same reaction time, $t_{rea} = 2.00$ s, has been adopted. A linear variation has been considered to simulate the variation of the pedestrian velocity during the reaction time. Additionally, a practical lower limit of the pedestrian velocity has been established in order to avoid meaningless small values of this magnitude [20].

$$|v_p| = \begin{cases} 0.1\cdot|v_d| & if \quad \ddot{y}_a < \ddot{y}_{\lim} \cap |v_p| \le 0.1\cdot|v_d| \\ 0 & if \quad \ddot{y}_a \ge \ddot{y}_{\lim} \end{cases} \tag{30}$$

Finally, as lateral lock-in threshold, the criterion suggested by the French standard [11] is usually adopted to simulate the synchronization phenomenon between the movement of the crowd and the footbridge. For this purpose, both the step frequency, f_s, and phase shift, ϕ_p, of each pedestrian are modified to match the natural frequency of the structure, if the lateral acceleration experienced by each subject is above 0.15 m/s^2 and its step frequency is within ±10% of the lateral natural frequency of the structure [30].

7. Application example: Lateral lock-in phenomenon on a real footbridge

In order to illustrate the potential of this new modelling framework, the analysis of the lateral lock-in phenomenon on a real footbridge, the Pedro e Inês footbridge (Coimbra Portugal) has been

performed [23]. The maximum lateral accelerations at the mid-span of the footbridge obtained via three different methods during a lateral lock-in pedestrian test are correlated. The three method used are: (i) the experimental values recorded during a lateral lock-in pedestrian test reported in Ref. [31], (ii) the numerical estimation of the maximum lateral acceleration obtained according to the Synpex guidelines [12] and (iii) the numerical prediction obtained based on the application of the proposed approach [23]. On an updated finite element model of the structure [32].

The footbridge is situated over the Mondego River at Coimbra (Portugal). The structure is configured by five spans (total length of 274.5 m); a central arch of 110 m, two lateral semi-arches of 64 m and two transition spans of 30.5 and 6 m, respectively (**Figure 5**). The deck is configured by a concrete-steel composite box-girder with a variable width between 4 and 8 m. The footbridge presents an anti-symmetrical configuration with respect to the longitudinal axis of the structure. In this way, the intersection of the two parallel decks generates a panoramic square at mid-span of the footbridge (**Figure 5**). As result of the numerical studies performed during the design phase, it was checked that the structure was prone to pedestrian-induced vibrations in lateral direction. Experimental tests were conducted to assess the dynamic response of the footbridge under pedestrian action in lateral direction. The main outcomes of this experimental work were reported in Ref. [31]. These results have been employed in this chapter to illustrate the potential of the new modelling framework. As the pedestrian is forced to walk in a controlled manner during the lateral lock-in pedestrian test, the crowd-structure model previously described has been applied under the deterministic approach.

The natural frequency (around 0.91 Hz) and associated damping ratio (approximately 0.55%) of the first lateral vibration mode of the footbridge were identified experimentally. As the natural frequency of this vibration mode is within the range that characterizes the pedestrian-structure interaction in lateral direction, a lateral lock-in pedestrian test was conducted to determine experimentally the number of pedestrians that originates the lateral instability phenomenon [31]. The analysis focused on characterizing the beginning of the lateral lock-in phenomenon, since during this part of the phenomenon, the modification of the modal properties of the structure induced by the pedestrian-structure interaction is higher [3]. The lateral acceleration, a_{lat}, at mid-span of the structure in terms of the number of pedestrians, which cross along the structure, was recorded in this lateral lock-in pedestrian test. The analysis of the

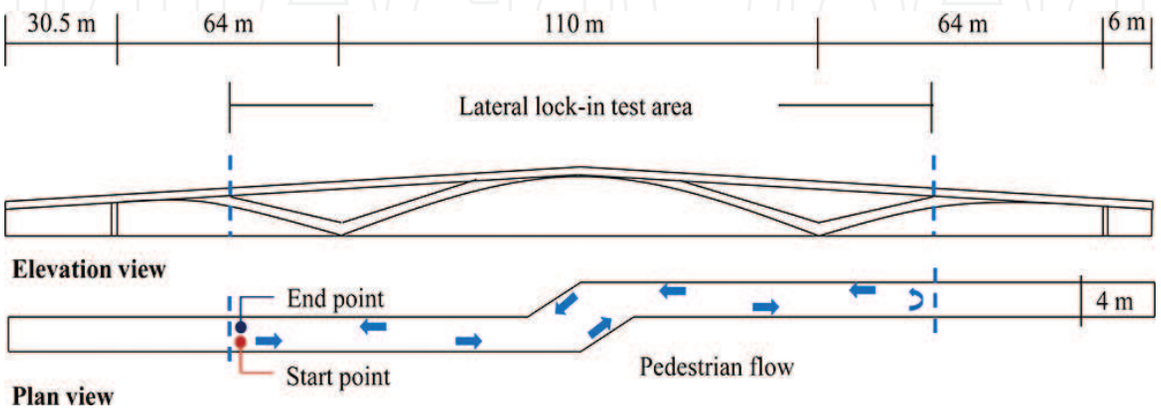

Figure 5. Scheme of the lateral lock-in pedestrian test on Pedro e Inês footbridge [31].

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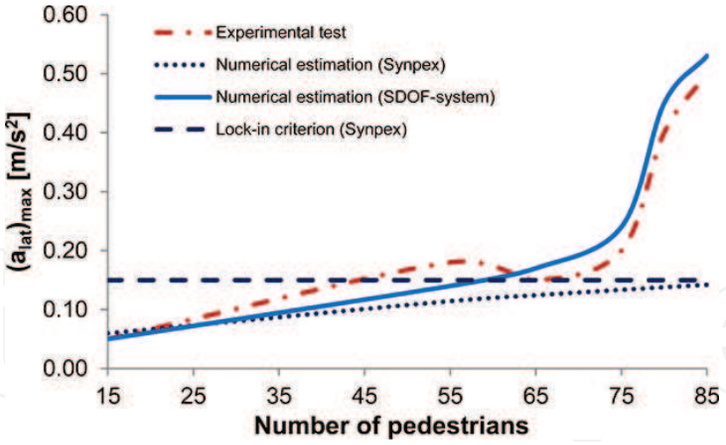

Figure 6. Experimental and numerical variation of the maximum lateral acceleration, $(a_{lat})_{\max}$, during the lateral lock-in pedestrian test [23].

graphical representation of these results (**Figure 6**) allows for identifying the beginning of the instability lateral lock-in phenomenon. As it is illustrated in **Figure 6**, the number of pedestrians that originates the beginning of the lateral lock-in phenomenon is around 75 [31].

Subsequently, a numerical lateral lock-in test based on the proposed approach was performed. Each considered group of pedestrians has been simulated considering as initial spatial distribution, a rectangular-shaped grid with an initial distance among pedestrians $d_p = 0.50$ m in longitudinal direction and an equidistant distribution in lateral direction. During the numerical test, according to the assumptions of the experimental test reported in the literature [31], each considered group of pedestrians walks freely along the footbridge, following the curve path illustrated in **Figure 5**. The number of pedestrians in each group increases gradually between 15 and 85 in increments of 5. The coordinates of the considered lateral vibration modes of the structure follow from the results available in the literature [31].

As result of this numerical analysis, the maximum lateral acceleration at mid-span of the structure in terms of the different groups of pedestrians on the footbridge was obtained. The graphical representation of this relationship is shown in **Figure 6**. A good agreement is achieved between the experimental lateral maximum accelerations and the numerically estimated maximum values, as it is illustrated in **Figure 6**. Additionally, the estimation of the numerical maximum acceleration obtained, applying the methodology proposed by the Synpex guidelines [12], is also shown in **Figure 6**. It is clear from **Figure 6** that the new modelling framework allows obtaining a more accurate numerical analysis of the lateral lock-in phenomenon than these design guidelines. The lateral lock-in criterion established by the Synpex guidelines [12] is also illustrated for reference in **Figure 6**.

8. Conclusions

The assessment of the vibration serviceability limit state of footbridges under pedestrian-induced excitation has usually been performed based on the recommendations of the most

advanced international standards and design guidelines. However, the numerical estimation of the dynamic response of footbridges obtained according to these codes differs from the value recorded experimentally.

In order to overcome this problem, a new generation of crowd-structure interaction models, that constitute a new modelling framework, has been proposed by the scientific community. All these models share, as common characteristic, that they simulate the crowd-structure interaction phenomenon using two sub-models: (i) a pedestrian-structure interaction sub-model and (ii) a crowd sub-model. For the first sub-model, the pedestrian is modelled by a SDOF, MDOF or IP system and the structure via its modal parameters obtained from a finite element model. For the second sub-model, the last tendency is to use a multi-agent method based on the principles of the social force model. The linking between the two sub-models is achieved by the inclusion of several behavioural conditions in the model. Comfort and lateral lock-in threshold are usually considered. Three key aspects are taken into account for this new modelling framework: (i) the inter-and intra-subject variability, (ii) the pedestrian-structure interaction and (iii) the crowd dynamics. The last two aspects are guaranteed by the own formulation of the model, and the first is ensured assuming that the different parameters of the crowd-structure interaction model are random variables.

One of these new crowd-structure interaction models has been described briefly in this chapter, emphasizing the section corresponding to the crowd behaviour.

Finally, the potential of this new modelling framework has been illustrated with a case study, the analysis of the lateral lock-in phenomenon of the Pedro e Inês footbridge (Coimbra, Portugal). As result of this study, a good agreement is achieved between the number of pedestrians which originates the lateral instability phenomenon obtained during the experimental test and the numerical estimation determined via the crowd-structure interaction model.

9. Future trends

Although the use of the crowd-structure interaction model allows improving the estimation of the response of footbridges under pedestrian flows, further studies are being conducted in order to better characterize some aspects of these models. Among others, the following research lines may be cited:

i. The crowd-structure interaction model might be generalized to longitudinal direction via the estimation of the parameters of the SDOF-system in that direction.

ii. In order to better characterize the inter- and intra-subject variability, the statistical distributions that characterize the parameters of the pedestrian-structure interaction model should be improved via the analysis of the behaviour of other groups of pedestrians on different types of footbridges.

iii. The relationship between the parameters of the pedestrian-structure interaction model and the step frequency of the pedestrian should be further analysed.

iv. The parameters of the crowd sub-model, normally based on the results of researches of general purpose, should be estimated concretely for the case of pedestrians moving on footbridges, to improve still more the accuracy of the crowd-structure interaction model.

v. This new modelling framework allows establishing the comfort requirements directly in terms of the maximum accelerations experienced by the pedestrians (instead of the maximum accelerations reached by the deck of the footbridge). A new research line can be opened to establish more accurate thresholds which allow characterizing the vibration serviceability limit state better [14].

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